# WEIGHTED NEUTROSOPHIC SOFT SETS APPROACH IN A MULTICRITERIA DECISION MAKING PROBLEM 

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#### Abstract

The paramount importance of decision making problem in an imprecise environment is becoming very much significant in recent years. In this paper we have studied weighted neutrosophic soft sets which are a hybridization of neutrosophic sets with soft sets corresponding to weighted parameters. We have considered here a multicriteria decision making problem as an application of weighted neutrosophic soft sets.


Keywords - Soft sets, neutrosophic sets, neutrosophic soft sets, weighted neutrosophic soft sets.

## 1 Introduction

In 1999, Molodtsov initiated the novel concept, the concept of 'soft set theory' [ 1 ] which has been proved as a generic mathematical tool to deal with problems involving uncertainties. Due to the inadequacy of parametrization in the theory of fuzzy sets [ 2 ], rough sets [ 3 ], vague sets [4], probability theory etc. we become handicapped to use them successfully. Consequently Molodtsov has shown that soft set theory has a potential to use in different fields [ 1 ]. Recently, the works on soft set theory is growing very rapidly with all its potentiality and is being used in different fields [5-10]. A detailed theoretical study may be found in [ 10 ]. Depending on the characteristics of the parameters involved in soft set different hybridization viz. fuzzy soft sets [ 11 ], soft rough sets [ 12 ], intuitionistic fuzzy soft sets [ 13 ], vague soft sets [ 14 ], neutrosophic soft sets [ 15 ] etc. have been introduced. The soft set theory is now being used in different fields as an application of it. Some of them have been investigated in [ $6-10,16$ ]. Based soft set [ 1 ] and neutrosophic sets [ 17 ] a hybrid structure 'neutrosophic soft sets' has been initiated [15]. The parameters considered here are neutrosophic in nature. Imposing the weights on the parameters ( may be in a particular parameter also) a weighted neutrosophic soft sets has been introduced [ 18 ]. In this paper we

[^0]use this concept to solve a multi-criteria decision making problem. In section 2 of this paper we briefly recall some relevant preliminaries centered around our problem. Some basic definitions on weighted neutrosophic soft sets relevant to this work are available in section 3. A decision making problem has been discussed and solved in section 4. Conclusions are there in the concluding Section 5.

## 2 Preliminaries

Most of the real life problems in the fields of medical sciences, economics, engineering etc. the data involve are imprecise in nature. The classical mathematical tools are not capable to handle such problems. The novel concept 'soft set theory' initiated by Molodtsov [ 1 ] is a new mathematical tool to deal with such problems. For better understanding we now recapitulate some preliminaries relevant to the work.

Definition 2.1 [ 1 ] Let U be an initial universe set and E be a set of parameters. Let $\mathrm{P}(\mathrm{U})$ denotes the power set of $U$. Consider a nonempty set $A, A \subset E$.
A pair ( $F, A$ ) is called a soft set over $U$, where $F$ is a mapping given by $F: A \rightarrow P(U)$.
A soft set over $U$ is a parameterized family of subsets of the universe $U$. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of $\varepsilon$-approximate elements of the $\operatorname{soft} \operatorname{set}(F, A)$.

Definition 2.2 [ 10 ] For two soft sets ( F, A ) and (G, B ) over a common universe U, we say that (F, A ) is a soft subset of (G, B ) if
(i) $\mathrm{A} \subset \mathrm{B}$, and
(ii) $\forall \varepsilon \in \mathrm{A}, \mathrm{F}(\varepsilon)$ and $\mathrm{G}(\varepsilon)$ are identical approximations.

We write ( $\mathrm{F}, \mathrm{A}$ ) $\simeq(\mathrm{G}, \mathrm{B})$.
( $F, A$ ) is said to be a soft super set of ( G, B ), if ( G, B ) is a soft subset of ( $F, A$ ). We denote it by ( $\mathrm{F}, \mathrm{A}$ ) $\tilde{\mathrm{D}}(\mathrm{G}, \mathrm{B})$.

Let A and B be two subsets of E , the set of parameters. Then $A \times B \subset E \times E$. Now we are in the position to define 'AND', 'OR' operations on two soft sets over a common universe.

Definition 2.3 [ 10 ] If ( $\mathrm{F}, \mathrm{A}$ ) and ( $\mathrm{G}, \mathrm{B}$ ) be two soft sets over a common universe U then '( $\mathrm{F}, \mathrm{A}$ ) AND ( $\mathrm{G}, \mathrm{B}$ )’ denoted by ( $\mathrm{F}, \mathrm{A})^{\wedge}(\mathrm{G}, \mathrm{B})$ is defined by

$$
(\mathrm{F}, \mathrm{~A})^{\wedge}(\mathrm{G}, \mathrm{~B})=(\mathrm{H}, \quad A \times B)
$$

where $\mathrm{H}(\alpha, \beta)=\mathrm{F}(\alpha) \cap \mathrm{G}(\beta), \quad \forall(\alpha, \beta) \in A \times B$.
Definition 2.4[10] If (F, A ) and (G, B ) be two soft sets over a common universe $U$ then '( F, A ) OR (G, B )' denoted by (F, A ) $\vee$ ( G, B ) is defined by

$$
(\mathrm{F}, \mathrm{~A}) \vee(\mathrm{G}, \mathrm{~B})=(\mathrm{O}, \mathrm{~A} \times \mathrm{B})
$$

where, $\mathrm{O}(\alpha, \beta)=\mathrm{F}(\alpha) \cup \mathrm{G}(\beta), \quad \forall(\alpha, \beta) \in A \times B$.

The non-standard analysis was introduced by Abraham Robinson in 1960. The non-standard analysis is a formalization of analysis and a branch of mathematical logic that rigorously defines the infinitesimals. Informally, an infinitesimal is an infinitely small number. Formally, x is said to be infinitesimal if and only if for all positive integers n one has $|x|<\frac{1}{n}$. Let $\varepsilon>0$ be a such infinitesimal number. Let's consider the non-standard finite numbers $1^{+}=1+\varepsilon$, where ' 1 ' is its standard part and ' $\varepsilon$ ' its non-standard part, and ${ }^{-} 0=0-\varepsilon$, where ' 0 ' is its standard part and ' $\varepsilon$ ' its non-standard part.

Definition 2.5 [ 17 ] A neutrosophic set $A$ on the universe of discourse $X$ is defined as

$$
\mathrm{A}=\left\{\left\langle\mathrm{x}, \mathrm{~T}_{\mathrm{A}}(\mathrm{x}), \mathrm{I}_{\mathrm{A}}(\mathrm{x}), \mathrm{F}_{\mathrm{A}}(\mathrm{x})\right\rangle \mathrm{x} \in \mathrm{X}\right\}
$$

where $\left.\mathrm{T}_{\mathrm{A}}, \mathrm{I}_{\mathrm{A}}, \mathrm{F}_{\mathrm{A}}: \mathrm{X} \rightarrow\right]^{-} 0,1^{+}\left[\right.$and ${ }^{-} 0 \leq T_{\mathrm{A}}+I_{A}+F_{A} \leq 3^{+}$.
Here $\mathrm{T}_{\mathrm{A}}, \mathrm{I}_{\mathrm{A}}, \mathrm{F}_{\mathrm{A}}$ are respectively the true membership, indeterministic membership and false membership function of an object $x \in X$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-} 0,1^{+}$[. But in real life applications in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^{-} 0,1^{+}$[. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.

Definition 2.6 [ 15 ] Let $U$ be an initial universe set and $E$ be a set of parameters which is of neutrosophic in nature. Consider $A \subset E$. Let $P(U)$ denotes the set of all neutrosophic sets of U.

The collection ( $\mathrm{F}, \mathrm{A}$ ) is termed to be the neutrosophic soft set ( NSS ) over U , where F is a mapping given by $\mathrm{F}: \mathrm{A} \rightarrow \mathrm{P}(\mathrm{U})$.

For an illustration we consider the following example.
Example 2.7 Let $U$ be the set of objects under consideration and $E$ is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider $\mathrm{E}=\{$ beautiful, large, very large, small, average large, costly, cheap, brick build \}. In this case to define a neutrosophic soft set means to point out beautiful objects, large objects, very large objects etc. and so on. Suppose that there are five objects in the universe $U$ given by $U=\left\{o_{1}\right.$, $\left.\mathrm{o}_{2}, \mathrm{o}_{3}, \mathrm{o}_{4}, \mathrm{o}_{5}\right\}$ and the set of parameters $\mathrm{A}=\left\{\mathrm{e}_{1}, \mathrm{e}_{2}, \mathrm{e}_{3}, \mathrm{e}_{4}\right\}$ where $\mathrm{e}_{1}$ stands for the parameter 'large', $\mathrm{e}_{2}$ stands for the parameter 'very large', $\mathrm{e}_{3}$ stands for the parameter 'small' and $\mathrm{e}_{4}$
stands for the parameter 'average'. Suppose that the NSS ( F, A ) describes the length of the objects under consideration for which,

$$
\begin{gathered}
\mathrm{F}(\text { large })=\left\{\begin{array}{c}
\left\langle\mathrm{o}_{1}, 0.6,0.4,0.7\right\rangle,\left\langle\mathrm{o}_{2}, 0.5,0.6,0.8\right\rangle,\left\langle\mathrm{o}_{3}, 0.8,0.7,0.7\right\rangle,\left\langle\mathrm{o}_{4}, 0.6,0.4,0.8\right\rangle, \\
\left.\left\langle\mathrm{o}_{5}, 0.8,0.6,0.7\right\rangle\right\},
\end{array}\right.
\end{gathered}
$$

$\mathrm{F}($ very large $)=\left\{\left\langle\mathrm{o}_{1}, 0.5,0.3,0.6\right\rangle,\left\langle\mathrm{o}_{2}, 0.8,0,5,0.7\right\rangle,\left\langle\mathrm{o}_{3}, 0.9,0.7,0.8\right\rangle,\left\langle\mathrm{o}_{4}, 0.7,0.6\right.\right.$, $\left.0.7\rangle,\left\langle\mathrm{o}_{5}, 0.6,0.7,0.9\right\rangle\right\}$,

$$
\begin{gathered}
\mathrm{F}(\text { small })=\left\{\left\langle\mathrm{o}_{1}, 0.3,0.8,0.9\right\rangle,\left\langle\mathrm{o}_{2}, 0.4,0.6,0.8\right\rangle,\left\langle\mathrm{o}_{3}, 0.6,0.8,0.4\right\rangle,\left\langle\mathrm{o}_{4}, 0.7,0.7,0.6\right\rangle,\right. \\
\left.\left\langle\mathrm{o}_{5}, 0.6,0.7,0.9\right\rangle\right\},
\end{gathered}
$$

$\mathrm{F}($ average $)=\left\{\left\langle\mathrm{o}_{1}, 0.8,0.3,0.4\right\rangle,\left\langle\mathrm{o}_{2}, 0.9,0.6,0.8\right\rangle,\left\langle\mathrm{o}_{3}, 0.8,0.7,0.8\right\rangle,\left\langle\mathrm{o}_{4}, 0.6,0.7,0.\right\rangle\right.$, $\left\langle\mathrm{o}_{5}, 0.7,0.6,0.8\right\rangle$ \}.

So, F (large) means large objects, F (small) means the objects having small length etc. For the purpose of storing a neutrosophic soft set in a computer, we could represent it in the form of a table as shown below ( corresponding to the neutrosophic soft set in the above example ). In this table, the entries $c_{i j}$ correspond to the object $o_{i}$ and the parameter $e_{j}$, where $c_{i j}=($ truemembership value of $o_{i}$, indeterminacy-membership value of $o_{i}$, falsity-membership value of $\left.o_{i}\right)$ in $F\left(e_{j}\right)$. The tabular representation of the neutrosophic soft set ( $\left.\mathrm{F}, \mathrm{A}\right)$ is as follow:

Table 1. The Tabular form of the NSS (F, A ).

| $\mathbf{U}$ | $\mathbf{e}_{\mathbf{1}}=$ large | $\mathbf{e}_{2}=$ very large | $\mathbf{e}_{3}=$ small | $\mathbf{e}_{4}=$ average |
| :---: | :--- | :---: | :--- | :--- |
| $\mathbf{o}_{\mathbf{1}}$ | $(0.6,0.4,0.7)$ | $(0.5,0.3,0.6)$ | $(0.3,0.8,0.9)$ | $(0.8,0.3,0.4)$ |
| $\mathbf{o}_{\mathbf{2}}$ | $(0.5,0.6,0.8)$ | $(0.8,0.5,0.7)$ | $(0.4,0.6,0.8)$ | $(0.9,0.6,0.8)$ |
| $\mathbf{o}_{\mathbf{3}}$ | $(0.8,0.7,0.7)$ | $(0.9,0.7,0.8)$ | $(0.6,0.8,0.4)$ | $(0.8,0.7,0.8)$ |
| $\mathbf{o}_{\mathbf{4}}$ | $(0.6,0.4,0.8)$ | $(0.7,0.6,0.7)$ | $(0.7,0.7,0.6)$ | $(0.6,0.7,0.5)$ |
| $\mathbf{o}_{5}$ | $(0.8,0.6,0.7)$ | $(0.6,0.7,0.9)$ | $(0.6,0.7,0.9)$ | $(0.7,0.6,0.8)$ |

Definition 2.8 [ 15 ] Let (F, A ) and (G, B ) be two neutrosophic soft sets over the common universe U. (F, A ) is said to be neutrosophic soft subset of (G, B ) if A $\subset B$ and $\mathrm{T}_{\mathrm{F}(\mathrm{e})}(\mathrm{x}) \leq \mathrm{T}_{\mathrm{G}(\mathrm{e})}(\mathrm{x}), \mathrm{I}_{\mathrm{F}(\mathrm{e})}(\mathrm{x}) \leq \mathrm{I}_{\mathrm{G}(\mathrm{e})}(\mathrm{x}), \mathrm{F}_{\mathrm{F}(\mathrm{e})}(\mathrm{x}) \geq \mathrm{F}_{\mathrm{G}(\mathrm{e})}(\mathrm{x}), e \in A$.

We denote it by $(\mathrm{F}, \mathrm{A}) \subset(\mathrm{G}, \mathrm{B})$. ( $\mathrm{F}, \mathrm{A}$ ) is said to be neutrosophic soft super set of (G, B) if (G, B )is a neutrosophic soft subset of (F, A ).

Definition 2.9 [ 15 ] AND operation on two neutrosophic soft sets.
Let (H, A ) and (G, B ) be two NSSs over the same universe U. Then the 'AND' operation on them is denoted by ' $(\mathrm{H}, \mathrm{A})^{\wedge}(\mathrm{G}, \mathrm{B})$ ' and is defined by $(\mathrm{H}, \mathrm{A})^{\wedge}(\mathrm{G}, \mathrm{B})=(\mathrm{K}, \boldsymbol{A} \times \boldsymbol{B})$, where the truth-membership value, indeterminacy-membership value and falsity-membership value of ( $\mathrm{K}, \boldsymbol{A} \times \boldsymbol{B}$ ) are as follows:

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{K}(\alpha, \beta)}(\mathrm{m})=\min \left(\mathrm{T}_{\mathrm{H}(\alpha)}(\mathrm{m}), \mathrm{T}_{\mathrm{G}(\beta)}(\mathrm{m})\right), \\
& I_{K(\alpha, \beta)}(\mathrm{m})=\frac{I_{H(\alpha,}(m)+I_{G(\beta, c}(m)}{2}, \text { and } \\
& \mathrm{F}_{\mathrm{K}(\alpha, \beta)}(\mathrm{m})=\max \left(\mathrm{F}_{\mathrm{H}(\alpha)}(\mathrm{m}), \mathrm{F}_{\mathrm{G}(\beta)}(\mathrm{m})\right), \quad \forall \alpha \in \mathrm{A}, \forall \beta \in \mathrm{~B} .
\end{aligned}
$$

The decision maker may not have equal choice for all the parameters. He/she may impose some conditions to choose the parameters for which the decision will be taken. The conditions may be imposed in terms of weights ( positive real numbers $\leq 1$ ). This imposition motivates us to define weighted neutrosophic soft sets.

## 3 Weighted Neutrosophic Soft Sets

Definition 3.1 [ 18 ] A neutrosophic soft set is termed to be a weighted neutrosophic soft sets (WNSS) if the weights ( $\mathrm{w}_{\mathrm{i}}$, a real positive number $\leq 1$ ) be imposed on the parameters of it. The entries of the weighted neutrosophic soft set $\mathrm{d}_{\mathrm{ij}}=w_{i} \times c_{i j}$, where $\mathrm{c}_{\mathrm{ij}}$ is the ij -th entry in the table of neutrosophic soft set.

For an illustration we consider the following example.
Example 3.2 Consider the example 2.7. Suppose that the decision maker has no equal preference for each of the parameters. He may impose the weights of preference for the parameters ' $\mathrm{e}_{1}=$ large' as ' $\mathrm{w}_{1}=0.8$ ', ' $\mathrm{e}_{2}=$ very large' as ' $\mathrm{w}_{2}=0.4$ ', ' $\mathrm{e}_{3}=$ small' as ' $\mathrm{w}_{3}=0.5$ ', ' $\mathrm{e}_{4}=$ average large' as ' $\mathrm{w}_{4}=0.6$ '. Then the weighed neutrosophic soft set obtained from ( $\mathrm{F}, \mathrm{A}$ ) denoted as $\left(\mathrm{H}, \mathrm{A}^{\mathrm{w}}\right)$ and its tabular representation is as below:

Table 2: Tabular form of the weighted $\operatorname{NSS}\left(\mathrm{H}, \mathrm{A}^{\mathrm{w}}\right)$.

| $\mathbf{U}$ | $\mathbf{e}_{\mathbf{1}}, \mathbf{w}_{\mathbf{1}}=\mathbf{0 . 8}$ | $\mathbf{e}_{\mathbf{2}}, \mathbf{w}_{\mathbf{2}}=\mathbf{0} .4$ | $\mathbf{e}_{\mathbf{3}}, \mathbf{w}_{\mathbf{3}}=\mathbf{0} . \mathbf{5}$ | $\mathbf{e}_{\mathbf{4}, \mathbf{w}}^{\mathbf{4}} \mathbf{=} \mathbf{0 . 6}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{o}_{\mathbf{1}}$ | $(0.48,0.32,0.56)$ | $(0.20,0.12,0.24)$ | $(0.15,0.40,0.45)$ | $(0.48,0.18,0.24)$ |
| $\mathbf{o}_{\mathbf{2}}$ | $(0.40,0.48,0.64)$ | $(0.32,0.20,0.28)$ | $(0.20,0.30,0.40)$ | $(0.54,0.36,0.48)$ |
| $\mathbf{o}_{3}$ | $(0.64,0.56,0.56)$ | $(0.36,0.28,0.32)$ | $(0.30,0.40,0.20)$ | $(0.48,0.42,0.48)$ |
| $\mathbf{o}_{4}$ | $(0.48,0.32,0.64)$ | $(0.28,0.24,0.28)$ | $(0.35,0.35,0.30)$ | $(0.36,0.42,0.30)$ |
| $\mathbf{o}_{5}$ | $(0.64,0.48,0.56)$ | $(0.24,0.28,0.36)$ | $(0.30,0.35,0.45)$ | $(0.42,0.36,0.48)$ |

Definition 3.3 [ 18 ] AND operation on two weighted neutrosophic soft sets.
Let ( $\mathrm{H}, \mathrm{A}^{\mathrm{W}_{1}}$ ) and ( $\mathrm{G}, \mathrm{B}^{\mathrm{W}_{2}}$ ) be two WNSSs over the same universe U . Then the 'AND' operation on them is denoted by ' $\left(H, A^{w_{1}}\right)^{\wedge}\left(G, B^{W_{2}}\right)$ ' and is defined by $\left(H, A^{W_{1}}\right) \wedge(G$, $\left.B^{W_{2}}\right)=\left(K, A^{W_{1}} \times B^{W_{2}}\right)$, where the truth-membership value, indeterminacy-membership value and falsity-membership value of $\left(\mathrm{K}, \mathrm{A}^{\mathrm{w}_{1}} \times \mathrm{B}^{\mathrm{W}_{2}}\right)$ are as follows:
$\mathrm{T}_{\mathrm{K}\left(\alpha,{ }^{\mathrm{w}}{ }_{1, \beta}{ }^{\mathrm{w}}{ }_{2)}(\mathrm{m})=\min \left(\mathrm{w}_{1}, \mathrm{w}_{2}\right) \cdot \min \left(\mathrm{T}_{\mathrm{H}(\alpha)}(\mathrm{m}), \mathrm{T}_{\mathrm{G}(\beta)}(\mathrm{m})\right), \forall \alpha \in \mathrm{A}, \forall \beta \in \mathrm{B}, ~\right.}^{\text {, }}$
$I_{K\left(\alpha^{w_{1}}, \beta^{w_{2}}\right)}(m)=\frac{I_{H\left(\alpha^{m_{i}}\right)}(m)+I_{G\left(\beta^{w^{2}}\right)}(m)}{2}, \forall \alpha \in \mathrm{~A}, \forall \beta \in \mathrm{~B}$,
$\mathrm{F}_{\mathrm{K}\left({ }^{\mathrm{w}}{ }^{1}, \beta{ }^{\mathrm{w}}{ }^{\mathrm{w}}\right)(\mathrm{m})=\max \left(\mathrm{w}_{1}, \mathrm{w}_{2}\right) \cdot \max \left(\mathrm{F}_{\mathrm{H}(\alpha)}(\mathrm{m}), \mathrm{F}_{\mathrm{G}(\beta)}(\mathrm{m})\right), \forall \alpha \in \mathrm{A}, \forall \beta \in \mathrm{B} .}$
Definition 3.4 Comparison Matrix. It is a matrix whose rows are labelled by nobject $\mathrm{o}_{1}, \mathrm{o}_{2}$, $\ldots ., \mathrm{o}_{\mathrm{n}}$ and the columns are labelled by m weighted parameters $\mathrm{e}_{1}, \mathrm{e}_{2}, \ldots \ldots, \mathrm{e}_{\mathrm{m}}$. The entries $\mathrm{c}_{\mathrm{ij}}$ of the comparison matrix are evaluated by $\mathrm{c}_{\mathrm{ij}}=\mathrm{a}+\mathrm{b}-\mathrm{c}$, where ' $a$ ' is the positive integer calculated as 'how many times $\mathrm{T}_{\mathrm{oi}}\left(\mathrm{e}_{\mathrm{j}}\right)$ exceeds or equal to $\mathrm{T}_{\mathrm{ok}}\left(\mathrm{e}_{\mathrm{j}}\right)$ ', for $\mathrm{i} \neq \mathrm{k}, \forall \mathrm{i}=1,2, \ldots$, $n$, ' $b$ ' is the positive integer calculated as 'how many times $I_{o i}\left(e_{j}\right)$ exceeds or equal to $I_{o k}$ $\left(e_{j}\right)$ ', for $\mathrm{i} \neq \mathrm{k}$ and $\forall \mathrm{i}=1,2, \ldots ., \mathrm{n}$ and ' c ' is the integer 'how many times $\mathrm{F}_{\mathrm{oi}}\left(\mathrm{e}_{\mathrm{j}}\right)$ exceeds or equal to $\mathrm{F}_{\mathrm{ok}}\left(\mathrm{e}_{\mathrm{j}}\right)^{\prime}$, for $\mathrm{i} \neq \mathrm{k}$ and $\forall \mathrm{i}=1,2, \ldots$, n .

Definition 3.5 Score of an Object. The score of an object $\mathrm{o}_{\mathrm{i}}$ is $\mathrm{S}_{\mathrm{i}}$ and is calculated as

$$
\mathrm{S}_{\mathrm{i}}=\sum_{j} c_{i j}, \forall \mathrm{i}=1,2, \ldots ., \mathrm{n} .
$$

Here we consider a problem to choose an object from a set of given objects with respect to a set of choice parameters P . We follow an algorithm to identify an object based on multiobserver ( considered here three observers with their own choices ) input data characterized by colours ( $F, A^{w}$ ), size ( $G, B^{w}$ ) and surface textures ( $H, C^{w}$ ) features. The algorithm to choose an appropriate object depending upon the choice parameters is given below.

### 3.6 Algorithm

1. input the neutrosophic soft sets ( $\mathrm{H}, \mathrm{A}$ ), ( $\mathrm{G}, \mathrm{B}$ ) and ( $\mathrm{H}, \mathrm{C}$ ) (for three observers )
2. input the weights $\left(\mathrm{w}_{\mathrm{i}}\right)$ for the parameters $\mathrm{A}, \mathrm{B}$ and C
3. compute weighted neutrosophic soft sets $\left(H, A^{w}\right),\left(G, B^{w}\right)$ and $\left(H, C^{w}\right)$ corresponding to
the NSSs (H, A ), ( G, B ) and (H, C ) respectively
4. input the parameter set P as preferred by the decision maker
5. compute the corresponding NSS ( $\mathrm{S}, \mathrm{P})$ from the WNSSs $\left(\mathrm{H}, \mathrm{A}^{\mathrm{w}}\right),\left(\mathrm{G}, \mathrm{B}^{\mathrm{w}}\right)$ and $\left(\mathrm{H}, \mathrm{C}^{\mathrm{w}}\right)$ and place in tabular form
6. compute the comparison matrix of the NSS ( $\mathrm{S}, \mathrm{P}$ )
7. compute the score $\mathrm{S}_{\mathrm{i}}$ of $\mathrm{o}_{\mathrm{i}}, \forall \mathrm{i}=1,2, \ldots, \mathrm{n}$
8. the decision is $\mathrm{o}_{\mathrm{k}}$ if $\mathrm{S}_{\mathrm{k}}=\max _{i} S_{i}$
9. if k has more than one values then any one of $\mathrm{o}_{\mathrm{i}}$ may be chosen.

Based on the above algorithm we consider the following multi-criteria decision making problem.

## 4 Application in a Decision Making Problem

Let $U=\left\{o_{1}, o_{2}, o_{3}, o_{4}, o_{5}\right\}$ be the set of objects characterized by different lengths, colours and surface texture. Consider the parameter set, $\mathrm{E}=\{$ blackish, dark brown, yellowish, reddish, large, small, very small, average, rough, very large, coarse, moderate, fine, smooth, extra fine $\}$. Also consider $\mathrm{A}=\{$ very large, small, average large $\}, \mathrm{B}=\{$ reddish, yellowish, blackish $\}$ and $\mathrm{C}=\{$ smooth, rough, moderate $\}$ be three subsets of the set of parameters E . Let the NSSs ( F, A ), ( G, B ) and ( H, C ) describe the objects 'having different lenghts', 'objects having different colours' and 'surface structure features of the objects' respectively. These NSSs as computed by the three observers Mr. X, Mr. Y and Mr. Z respectively, are given below in their respective tabular forms in table 3,4 and 5 . Now suppose that the decision maker imposes the weights on the parameters $\mathrm{A}, \mathrm{B}$ and C and the repective weighted neutrosophic soft sets are ( $\mathrm{F}, \mathrm{A}^{\mathrm{w}}$ ), ( $\mathrm{G}, \mathrm{B}^{\mathrm{w}}$ ) and ( $\mathrm{H}, \mathrm{C}^{\mathrm{w}}$ ). The WNSS ( $\mathrm{F}, \mathrm{A}^{\mathrm{w}}$ ) describes the 'objects having different lengths', the WNSS ( $\mathrm{G}, \mathrm{B}^{\mathrm{w}}$ ) describes the 'different colours of the objects' and the WNSS ( $\mathrm{H}, \mathrm{C}^{\mathrm{w}}$ ) describes the 'surface structure feature of the objects'. We consider the problem to identify an object from $U$ based on the multiobservers neutrosophic data, specified by different observers ( we consider here three observers ), in terms of WNSSs ( $\mathrm{F}, \mathrm{A}^{\mathrm{w}}$ ), ( $\mathrm{G}, \mathrm{B}^{\mathrm{w}}$ ) and ( $\left.\mathrm{H}, \mathrm{C}^{\mathrm{w}}\right)$ as described above.

Table 3: Tabular form of the WNSS ( $\mathrm{F}, \mathrm{A}^{\mathrm{w}}$ ).

| U | $\mathbf{a}_{1}=$ very large | $\mathbf{a}_{2}=$ small | $\mathrm{a}_{3}=$ average large |
| :---: | :---: | :---: | :---: |
| 01 | ( 0.5, 0.6, 0.8 ) | ( 0.7, 0.3, 0.5 ) | ( 0.6, 0.7, 0.3 ) |
| 02 | ( 0.6, 0.8, 0.7 ) | ( 0.3, 0.6, 0.4 ) | ( 0.8, 0.3, 0.5 ) |
| 03 | ( 0.3, 0.5, 0.8 ) | ( 0.8, 0.3, 0.2 ) | ( 0.3, 0.2, 0.6 ) |
| 04 | ( 0.8, 0.3, 0.5 ) | ( 0.3, 0.5, 0.3 ) | ( 0.6, 0.7, 0.3 ) |
| 05 | ( 0.7, 0.3, 0.6 ) | ( 0.4, 0.6, 0.8 ) | ( 0.8, 0.3, 0.8 ) |
| weight | $\mathrm{w}_{1}=\mathbf{0 . 5}$ | $\mathbf{w}_{\mathbf{2}}=\mathbf{0 . 6}$ | $\mathrm{w}_{\mathbf{3}}=\mathbf{0 . 3}$ |
| 01 | ( $0.25,0.30,0.40$ ) | ( $0.42,0.18,0.30$ ) | ( $0.18,0.21,0.09$ ) |
| 02 | ( 0.30, 0.40, 0.35 ) | ( 0.18, 0.36, 0.24 ) | ( 0.24, 0.09, 0.15 ) |
| 03 | ( 0.15, 0.25, 0.40 ) | ( 0.48, $0.18,0.12$ ) | ( 0.09, 0.06, 0.18 ) |
| 04 | ( $0.40,0.15,0.25$ ) | ( 0.18, 0.30, 0.18 ) | ( 0.18, 0.21, 0.09 ) |
| 05 | ( $0.35,0.15,0.30$ ) | ( 0.24, 0.36, 0.48 ) | ( 0.24, 0.09, 0.24 ) |

Table 4: Tabular form of the WNSS ( G, B ${ }^{\mathrm{w}}$ ).

| $\mathbf{U}$ | $\mathbf{b}_{\mathbf{1}}=$ reddish | $\mathbf{b}_{\mathbf{2}}=$ yellowish | $\mathbf{b}_{\mathbf{3}}=$ blackish |
| :--- | :---: | :---: | :---: |
| $\mathbf{0}_{\mathbf{1}}$ | $(0.5,0.7,0.3)$ | $(0.7,0.8,0.6)$ | $(0.8,0.3,0.4)$ |
| $\mathbf{0}_{\mathbf{2}}$ | $0.6,0.7,0.3)$ | $(0.8,0.5,0.7)$ | $(0.6,0.7,0.3)$ |
| $\mathbf{0}$ | $(0.8,0.5,0.6)$ | $(0.7,0.3,0.6)$ | $(0.8,0.3,0.5)$ |
| $\mathbf{0}$ | $(0.7,0.2,0.6)$ | $(0.8,0.6,0.5)$ | $(0.6,0.7,0.3)$ |
| $\mathbf{0}$ | $(0.8,0.4,0.7)$ | $(0.6,0.5,0.8)$ | $(0.7,0.4,0.2)$ |


| weight | $\mathbf{w}_{\mathbf{1}}=\mathbf{0 . 6}$ | $\mathbf{w}_{\mathbf{2}}=\mathbf{0 . 4}$ | $\mathbf{w}_{\mathbf{3}}=\mathbf{0 . 7}$ |
| :---: | :---: | :---: | :---: |
|  | $(0.30,0.42,0.18)$ | $(0.28,0.32,0.24)$ | $(0.56,0.21,0.28)$ |
| $\mathbf{0}_{\mathbf{1}}$ | $(0.36,0.42,0.18)$ | $(0.32,0.20,0.28)$ | $(0.42,0.49,0.21)$ |
| $\mathbf{0}_{\mathbf{2}}$ | $(0.48,0.30,0.36)(0.28,0.12,0.24)$ | $(0.56,0.21,0.35)$ |  |
| $\mathbf{0}_{3}$ | $(0.42,0.12,0.36)(0.32,0.24,0.20)$ | $(0.42,0.49,0.21)$ |  |
| $\mathbf{0}_{4}$ | $(0.48,0.24,0.42)(0.24,0.20,0.32)$ | $(0.49,0.28,0.14)$ |  |
| $\mathbf{0 5}$ |  |  |  |

Table 5: Tabular form of the WNSS ( $\mathrm{H}, \mathrm{C}^{\mathrm{w}}$ ).

| U | $\mathrm{c}_{1}=$ smooth | $c_{2}=$ rough | $c_{3}=$ moderate |
| :---: | :---: | :---: | :---: |
| 01 | ( 0.8, 0.5, 0.6 ) | ( 0.8, 0.7, 0.3 ) | ( 0.8, 0.6, 0.4 ) |
| 02 | ( 0.7, 0.6, 0.7 ) | ( 0.7, 0.5, 0.6 ) | ( 0.7, 0.5, 0.6 ) |
| 03 | ( 0.8, 0.7, 0.6 ) | ( 0.6, 0.3, 0.7 ) | ( 0.8, 0.2, 0.4 ) |
| 04 | ( 0.7, 0.5, 0.7 ) | $(0.8,0.7,0.4)$ | ( 0.7, 0.8, 0.7 ) |
| 05 | ( 0.8, 0.7, 0.4 ) | ( 0.7, 0.4, 0.8 ) | $(0.8,0.6,0.5)$ |
| weight | $\mathrm{w}_{1}=\mathbf{0 . 6}$ | $\mathrm{w}_{2}=0.8$ | $\mathrm{w}_{3}=0.5$ |
| 01 | ( 0.48, 0.30, 0.36 ) | ( 0.64, 0.56, 0.24 ) | ( 0.40, 0.30, 0.20 ) |
| 02 | ( 0.42, 0.36, 0.42 ) | $(0.56,0.40,0.48)$ | $(0.35,0.25,0.30)$ |
| 03 | ( 0.48, 0.42, 0.36 ) | $(0.48,0.24,0.56)$ | ( $0.40,0.10,0.20)$ |
| 04 | ( 0.42, 0.30, 0.42 ) | $(0.64,0.56,0.32)$ | $(0.35,0.40,0.35)$ |
| 05 | ( 0.48, 0.42, 0.24 ) | $(0.56,0.32,0.64)$ | $(0.40,0.30,0.25)$ |

In the above two WNSSs ( $\mathrm{F}, \mathrm{A}^{\mathrm{w}}$ ) and ( $\mathrm{G}, \mathrm{B}^{\mathrm{w}}$ ) given in their respective tabular form in 3 and 4 , if the evaluator wants to perform the operation ' $\left(\mathrm{F}, \mathrm{A}^{\mathrm{w}}\right)$ AND ( $\mathrm{G}, \mathrm{B}^{\mathrm{w}}$ )' then we will have $3 \times 3=9$ parameters of the form $\mathrm{e}_{\mathrm{ij}}$, where $\mathrm{e}_{\mathrm{ij}}=\mathrm{a}_{\mathrm{i}} \wedge{ }^{\wedge} \mathrm{b}_{\mathrm{j}}$, for $\mathrm{i}=1,2,3$ and $\mathrm{j}=1,2,3$ and $\mathrm{e}_{\mathrm{ij}} \in E \times E$. On the basis of the choice parameters of the evaluator if we consider the WNSS with parameters $\mathrm{R}=\left\{\mathrm{e}_{11}, \mathrm{e}_{21}, \mathrm{e}_{22}, \mathrm{e}_{31}, \mathrm{e}_{32}\right\}$ we have the $\operatorname{WNSS}\left(\mathrm{K}, \mathrm{R}^{\mathrm{w}}\right)$ obtained from the WNSSs $\left(\mathrm{F}, \mathrm{A}^{\mathrm{w}}\right)$ and $\left(\mathrm{G}, \mathrm{B}^{\mathrm{w}}\right)$. So $\mathrm{e}_{11}=($ very large, reddish $), \mathrm{e}_{22}=($ small, yellowish $)$ etc. Computing ' $\left(\mathrm{F}, \mathrm{A}^{\mathrm{w}}\right.$ ) AND ( $\mathrm{G}, \mathrm{B}^{\mathrm{w}}$ )' for the choice parameters R , we have the tabular representation of the WNSS ( $\mathrm{K}, \mathrm{R}^{\mathrm{w}}$ ) as below:

Table 6: Tabular form of the $\operatorname{WNSS}\left(\mathrm{K}, \mathrm{R}^{\mathrm{w}}\right)$.

| $\mathbf{U}$ | $\mathbf{e}_{\mathbf{1 1}}$ | $\mathbf{e}_{\mathbf{2 1}}$ | $\mathbf{e}_{\mathbf{2 2}}$ | $\mathbf{e}_{\mathbf{3 1}}$ | $\mathbf{e}_{\mathbf{3 2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 1}$ | $(0.25,0.36,0.48)$ | $(0.30,0.30,0.30)$ | $(0.28,0.25,0.36)$ | $(0.15,0.615,0.18)$ | $(0.18,0.265,0.24)$ |
| $\mathbf{0 2}$ | $(0.30,0.41,0.56)$ | $(0.18,0.39,0.24)$ | $(0.12,0.28,0.42)$ | $(0.18,0.255,0.30)$ | $(0.24,0.145,0.28)$ |
| $\mathbf{0 3}$ | $(0.15,0.275,0.48)$ | $(0.48,0.24,0.36)$ | $(0.28,0.15,0.36)$ | $(0.09,0.18,0.36)$ | $(0.09,0.09,0.24)$ |
| $\mathbf{0 4}$ | $(0.35,0.135,0.36)$ | $(0.18,0.21,0.36)$ | $(0.12,0.27,0.30)$ | $(0.18,0.165,0.36)$ | $(0.18,0.175,0.20)$ |
| $\mathbf{0 5}$ | $(0.35,0.195,0.42)$ | $(0.24,0.30,0.48)$ | $(0.16,0.28,0.48)$ | $(0.24,0.285,0.48)$ | $(0.18,0.145,0.32)$ |

Computing the WNSS ( $\mathrm{S}, \mathrm{P}$ ) from the WNSSs ( $\mathrm{K}, \mathrm{R}^{\mathrm{W}}$ ) and ( $\mathrm{H}, \mathrm{C}^{\mathrm{W}}$ ) for the specified parameters $\mathrm{P}=\left\{\mathrm{e}_{11} \wedge \mathrm{c}_{1}, \mathrm{e}_{21}{ }^{\wedge} \mathrm{c}_{2}, \mathrm{e}_{21} \wedge \mathrm{c}_{3}, \mathrm{e}_{31} \wedge \mathrm{c}_{1}\right\}$, where the parameter $\mathrm{e}_{11} \wedge \mathrm{c}_{1}$ means ( very large, reddish, smooth ), $\mathrm{e}_{21}{ }^{\wedge} \mathrm{c}_{2}$ means ( small, reddish, rough ) etc. The tabular form of the WNSS ( $\mathrm{S}, \mathrm{P}$ ) is as below:

Table 7: Tabular form of the WNSS ( S, P ).

| $\mathbf{U}$ | $\mathbf{e}_{1 \mathbf{1}}{ }^{\wedge} \mathbf{c}_{\mathbf{1}}$ | $\mathbf{e}_{2 \mathbf{1}}{ }^{\wedge} \mathbf{c}_{\mathbf{2}}$ | $\mathbf{e}_{2 \mathbf{1}}{ }^{\wedge} \mathbf{c}_{\mathbf{3}}$ | $\mathbf{e}_{31}{ }^{\wedge} \mathbf{c}_{\mathbf{1}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{o}_{\mathbf{1}}$ | $(0.25,0.4375,0.48)$ | $(0.30,0.58,0.40)$ | $(0.25,0.425,0.30)$ | $(0.15,0.45,0.36)$ |
| $\mathbf{o}_{\mathbf{2}}$ | $(0.30,0.6675,0.42)$ | $(0.18,0.488,0.48)$ | $(0.15,0.388,0.36)$ | $(0.18,0.455,0.42)$ |
| $\mathbf{o}_{3}$ | $(0.15,0.51,0.48)$ | $(0.36,0.295,0.56)$ | $(0.40,0.20,0.36)$ | $(0.09,0.4975,0.36)$ |
| $\mathbf{o}_{4}$ | $(0.35,0.3375,0.42)$ | $(0.18,0.542,0.48)$ | $(0.15,0.488,0.42)$ | $(0.18,0.388,0.42)$ |
| $\mathbf{o}_{5}$ | $(0.35,0.4725,0.42)$ | $(0.24,0.385,0.64)$ | $(0.20,0.425,0.48)$ | $(0.24,0.4725,0.48)$ |

Then the tabular form of the comparison matrix for the WNSS ( $\mathrm{S}, \mathrm{P}$ ) is as below:

Table 8: Tabular form of the comparison matrix of the WNSS (S, Q ).

| $\mathbf{U}$ | $\mathbf{e}_{11}{ }^{\wedge} \mathbf{c}_{\mathbf{1}}$ | $\mathbf{e}_{\mathbf{2 1}}{ }^{\wedge} \mathbf{c}_{\mathbf{2}}$ | $\mathbf{e}_{\mathbf{2 1}}{ }^{\wedge} \mathbf{c}_{\mathbf{3}}$ | $\mathbf{e}_{\mathbf{3 1}}{ }^{\wedge} \mathbf{c}_{\mathbf{1}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{o}_{\mathbf{1}}$ | $\mathbf{- 2}$ | 7 | 6 | 1 |
| $\mathbf{o}_{\mathbf{2}}$ | 4 | 1 | 0 | 2 |
| $\mathbf{o}_{3}$ | $\mathbf{- 1}$ | 1 | 2 | 3 |
| $\mathbf{o}_{\mathbf{4}}$ | 2 | 2 | 2 | $\mathbf{- 2}$ |
| $\mathbf{0}$ | 4 | $\mathbf{- 1}$ | 1 | 3 |

Computing the score for each of the objects we have the respective scores as below:

| $\mathbf{U}$ | Score |
| :---: | :---: |
| $\mathbf{o}_{\mathbf{1}}$ | $\mathbf{1 2}$ |
| $\mathbf{o}_{\mathbf{2}}$ | 7 |
| $\mathbf{\mathbf { o } _ { 3 }}$ | 5 |
| $\mathbf{\mathbf { 0 } _ { \mathbf { 4 } }}$ | 4 |
| $\mathbf{0 5}$ | 7 |

Clearly, the maximum score is 12 and scored by the object $\mathrm{o}_{1}$. The selection will be in favour of the object $\mathrm{o}_{1}$. The second choice will be in favour of either $\mathrm{o}_{2}$ or $\mathrm{o}_{5}$ as they have the same score 7 . Next the decision maker may choose the objects $o_{3}$ and $o_{4}$ as the score 5 and 4 are scored by them respectively.

## 5 Conclusion

Since its initiation the soft set theory is being used in variety of many fields involving imprecise and uncertain data. In this paper we present an application of weighted neutrosophic soft sets for selection of an object. Here the selection is based on multicriteria input data of neutrosophic in nature. We also introduce an algorithm to select an appropriate object from a set of objects based on some specified parameters.

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