# A hybrid neutrosophic multiple criteria group decision making approach for project selection 

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#### Abstract

The project selection is one of the most important phases of a project life cycle. The project selection is considered as a Multi-Criteria Decision-Making (MCDM) problem. This research aims to study the integration between Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) into Decision-Making Trial and Evaluation Laboratory (DEMATEL) under neutrosophic environment to provide a new technique for making a decision regarding the choice of appropriate project (project selection) as one of the most important phases of the project life cycle. Projects are selected by comparing them against many criteria. Criteria are evaluated based on expert's opinion. Sometimes experts cannot give reliable information due to the non-deterministic environment. The neutrosophic set theory will be used to handle and overcome the ambiguity or lack of confirmation of information. The criteria are weighted by DEMATEL, then the best project alternative is selected using TOPSIS. In the proposed model, each pairwise comparison judgments is symbolized as a trapezoidal neutrosophic number. Experts will focus only on $(\mathrm{n}-1)$ judgments for n alternatives to overcome the difficulties of $[(\mathrm{n} *(\mathrm{n}-1)) / 2]$ consistence judgments in case of increasing number of alternatives. A numerical example is developed to show the validation of the suggested model in the neutrosophic environment.


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## 1. Introduction and related works

A project is a set of related activities that are employed to accomplish some goals. Any project has a life cycle. It has been widely recognized that the selection of a project is a critical phase of project life cycle. A life cycle of a project consists of four stages, as shown in Fig. 1. The fastest and most important stage in the life cycle of a project is the project selection after the identification and evaluation of the project. Project life cycle always starts with the client by choosing the appropriate project from a set of available

[^0]alternatives (projects) for investment or for any other purposes. Once the project is selected, the second stage is the planning of the project by defining and determining the scope of the work, basic schedule, time tradeoffs, and resource consideration in a project. The third stage is project implementation, and finally, the project completion.

In this research, we focus on the fastest and most important stage of the project life cycle, i.e. the project selection phase. Project selection is considered as a multi-criteria decision-making (MCDM) problem, where the choice of the preferred project among several projects depends on the differentiation between projects based on certain criteria. There are many studies (Aragonés-Beltrán, Chaparro-González, Pastor-Ferrando, \& Pla-Rubio,


Fig. 1. Project life cycle.


Fig. 2. Main and general criteria for project selection.

2014; Greco, Figueira, \& Ehrgott, 2005; Lee \& Kim, 2000, 2001; Meredith \& Mantel, 2011; Pohekar \& Ramachandran, 2004; San Cristóbal, 2011; Santhanam \& Kyparisis, 1995; Schwalbe, 2015; Zavadskas, Turskis, Tamošaitiene, \& Marina, 2008) that discussed the most important criteria on which to choose the best project among several projects. There are several important criteria related to the project selection. These criteria are investment, rate of return, risk, likely profit, pay back, similarity to existing businesses, expected life, flexibility, environmental impact, and competition, as shown in Fig. 2. Multicriteria evaluation for project selection is a comparison between several alternatives of the project against some of criteria, as shown in Fig. 3, where rarely would one


Fig. 3. Multi-criteria (MCDM) evaluation.
project emerge as the best on all chosen criteria. If that happens, it is a dominant project and it should be clearly chosen, but if this is not the case, as it happens in most of the real situations, we should compare the different alternatives on different sets of chosen criteria. The criteria (Fig. 4) are divided into tangible criteria and intangible criteria. The tangible criteria are the measurable criteria in units (e.g., payback period criterion measured in years and investments measured in millions of dollars, and so on). The intangible criteria are non-measurable criteria (such as risk measured not in a unit that may be expressed by very high, high, medium, low or very low). In case of intangible criteria, we should develop a scale. In this paper, we use the scale of $(0-1)$ instead of $(1-9)$. There are many techniques used for evaluated the criteria and selecting the best alternative among several ones considering several criteria such as AHP, ANP, Delphi, MOORA, and so on. In this research, we weighted the criteria using the neutrosophic DEMATEL and then select the best project alternative using neutrosophic TOPSIS. Multi-criteria decision-making problem (MCDM) is a formal and systematic way of decision-making on complex problems (Daneshvar Rouyendegh, 2011). Hwang and Yoon (Wang and Yoon, 1981) proposed one of the most used methods for MCDM; this method is TOPSIS (Technique for order preference by similarity to an ideal solution). Then the proposed set theories have provided the different multi-criteria decision-making methods. TOPSIS method is used to weight and compare set of alternatives against a set of criteria and select the best one. The alternatives are compared by the distance between alternatives and the optimal solution, where the best alternative is of the shortest distance from the optimal solution and the worst alternative is of the largest distance from the optimal solution. Many research focus on MCDM methods used fuzzy data (Bayrak, Celebi, \& Taşkin, 2007; Carlsson and Fullér, 1996; Chan, Kumar, Tiwari, Lau, \& Choy, 2008; Chen, 2000; Chu, 2002; Haq \& Kannan, 2006; Izadikhah, 2009; Jahanshahloo, Lotfi, \& Izadikhah, 2006a, 2006b; Önüt, Kara, \& Işik, 2009; Tsaur, Chang, \& Yen, 2002). Fuzzy sets focus only on the membership value and don't aware about non membership functions and indeterminacy value. Fuzzy sets unable to deal with ambiguity and non deterministic conditions. So we used neutrosophic set to deal and overcome the lack of certain information and uncertainty conditions. Boran, Genç, Kurt, and Akay (2009) suggested TOPSIS method under intuitionistic fuzzy


Fig. 4. Types of criteria based on their measurable.
environment. Ye (2010) extended the TOPSIS technique in interval-valued intuitionistic fuzzy sets. Science and Human Affairs Program of the Battelle Memorial Institute of Geneva founded DEMATEL method in the period from 1972 to 1979. Today it's become one of the most widely used tool for evaluating and weighting different criteria related to specific problem Chiu, Chen, Tzeng, \& Shyu, 2006; Liou, Tzeng, \& Chang, 2007; Tzeng, Chiang, \& Li, 2007; Wu and Lee, 2007; Lin and Tzeng, 2009). Yang, Shieh, Leu, and Tzeng (2008) applied DEMATEL to study and analyze the relationship of reasons and effect among weighted criteria or to conclude interrelationship among factors (Broumi, Bakali, Talea, \& Smarandache, 2016). In this research, we combine the TOPSIS into DEMATEL under neutrosophic set to solve the project selection problem.

## 2. Preliminaries

Neutrosophic theory was developed by Florentin Smarandache in 1998. In this section, we present definitions involving neutrosophic sets, single-valued neutrosophic sets, trapezoidal neutrosophic numbers, and operations on trapezoidal neutrosophic numbers.
Definition 1 El-Hefenawy, Metwally, Ahmed, \& ElHenawy, 2016.. Let $X$ be a space of points and $x \in X$. A neutrosophic set $A$ in $X$ is defined by a truth-membership function $T_{A}(x)$, an indeterminacy-membership function $I_{A}(x)$ and a falsity-membership function $F_{A}(x), T_{A}(x)$, $I_{A}(x)$ and $F_{A}(x)$ are real standard or real nonstandard subsets of $]-0,1+\left[\right.$. That is $\left.T_{A}(x): X \rightarrow\right]-0,1+$ $\left[I_{A}(x): X \rightarrow\right]-0,1+\left[\right.$ and $\left.F_{A}(x): X \rightarrow\right]-0,1+[$. There is no restriction on the sum of $T_{A}(x), I_{A}(x)$ and $F_{A}(x)$, so $0-\leq$ $\sup (x)+\sup x \leq 3+$.

Definition 2 (Abdel-Baset, Hezam, \& Smarandache, 2016; El-Hefenawy et al., 2016; Hezam, Abdel-Baset, \& Smarandache, 2015; Saaty, 2006.). Let $X$ be a universe of discourse. A single valued neutrosophic set $A$ over $X$ is an object taking the form $A=\left\{\left\langle x, T_{A}(x), I_{A}(x), F_{A}(x),\right\rangle: x\right.$ $\in X\}$, where $T_{A}(x): X \rightarrow[0,1], I_{A}(x): X \rightarrow[0,1]$ and $F_{A}(x)$ : $X \rightarrow[0,1]$ with $0 \leq T_{A}(x)+I_{A}(x)+F_{A}(x) \leq 3$ for all $x \in$ $X$. The intervals $T_{A}(x), I_{A}(x)$ and $F_{A}(x)$ represent the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of $x$ to $A$, respectively. For convenience, a Single Valued Neutrosophic (SVN) number is represented by $A=(a, b, c)$, where $a, b$, $c \in[0,1]$ and $a+b+c \leq 3$.

Definition 3 Mahdi, Riley, Fereig, \& Alex, 2002.. Suppose $\alpha_{a}, \theta_{a}, \beta_{a} \in[0,1]$ and $a_{1}, a_{2}, a_{3}, a_{4} \epsilon \mathrm{R}$, where $a_{1} \leq a_{2} \leq$ $a_{3} \leq a_{4}$. Then, a single valued trapezoidal neutrosophic number $a=\left\langle\left(a_{1}, a_{2}, a_{3}, a_{4}\right) ; \alpha_{a}, \theta_{a}, \beta_{a}\right\rangle$ is a special neutrosophic set on the real line set $R$, whose truthmembership, indeterminacy-membership and falsitymembership functions are defined as:
$T_{a}(x)=\left\{\begin{array}{cc}\alpha_{a}\left(\frac{x-a_{1}}{a_{2}-a_{1}}\right) & \left(a_{1} \leq x \leq a_{2}\right) \\ \alpha_{a} & \left(a_{2} \leq x \leq a_{3}\right) \\ \alpha_{a}\left(\frac{a_{4}-x}{a_{4}-a_{3}}\right) & \left(a_{3} \leq x \leq a_{4}\right) \\ 0 & \text { otherwise }\end{array}\right.$
$I_{a}(x)=\left\{\begin{array}{cc}\frac{\left(a_{2}-x+\theta_{a}\left(x-a_{1}\right)\right)}{\left(a_{2}-a_{1}\right)} & \left(a_{1} \leq x \leq a_{2}\right) \\ \alpha_{a} & \left(a_{2} \leq x \leq a_{3}\right) \\ \frac{\left(x-a_{3}+\theta_{a}(a 4-x)\right)}{\left(a_{4}-a_{3}\right)} & \left(a_{3} \leq x \leq a_{4}\right) \\ 1 & \text { otherwise }\end{array}\right.$
$F_{a}(x)=\left\{\begin{array}{cc}\frac{\left(a_{2}-x+\beta_{a}\left(x-a_{1}\right)\right)}{\left(a_{2}-a_{1}\right)} & \left(a_{1} \leq x \leq a_{2}\right) \\ \alpha_{a} & \left(a_{2} \leq x \leq a_{3}\right) \\ \frac{\left(x-a_{3}+\beta_{a}(a 4-x)\right)}{\left(a_{4}-a_{3}\right)} & \left(a_{3} \leq x \leq a_{4}\right) \\ 1 & \text { otherwise },\end{array}\right.$
where $\alpha_{a}, \theta_{a}$ and $\beta_{a}$ typify the maximum truth-membership degree, the minimum indeterminacy-membership degree and the minimum falsity-membership degree, respectively. A single valued trapezoidal neutrosophic number $a=$ $\left\langle\left(a_{1}, a_{2}, a_{3}, a_{4}\right) ; \alpha_{a}, \theta_{a}, \beta_{a}\right\rangle$ may express an ill-defined quantity of the range, which is approximately equal to the inter$\operatorname{val}\left[a_{2}, a_{3}\right]$.

Definition 4 (Izadikhah, 2009; Liou et al., 2007.). Let $a=$ $\left\langle\left(a_{1}, a_{2}, a_{3}, a_{4}\right) ; \alpha_{a}, \theta_{a}, \beta_{a}\right\rangle$ and $b=\left\langle\left(b_{1}, b_{2}, b_{3}, b_{4}\right) ; \alpha_{b}, \theta_{b}\right.$, $\left.\beta_{b}\right\rangle$ be two single valued trapezoidal neutrosophic numbers, and $\mathrm{Y} \neq 0$ be any real number. Then:

1. Addition of two trapezoidal neutrosophic numbers:

$$
a+b=\left\langle\left(a_{1}+b_{1}, a_{2}+b_{2}, a_{3}+b_{3}, a_{4}+b_{4}\right) ; \alpha_{a} \text { ád }_{b}, \theta_{a} \text { á' }^{\prime} \theta_{b}, \beta_{a} \text { á }^{\prime} \beta_{b}\right\rangle
$$

2. Subtraction of two trapezoidal neutrosophic numbers:

$$
a-b=\left\langle\left(a_{1}-b_{4}, a_{2}-b_{3}, a_{3}-b_{2}, a_{4}-b_{1}\right) ; \alpha_{a} \text { ád }_{b}, \theta_{a} a^{\prime} \theta_{b}, \beta_{a} \text { áa }^{\prime} \beta_{b}\right\rangle
$$

3. Inverse of trapezoidal neutrosophic numbers:
$\mathrm{a}^{-1}=\left(\left(\frac{1}{a_{4}}, \frac{1}{a_{3}}, \frac{1}{a_{2}}, \frac{1}{a_{1}}\right) ; \alpha_{a}, \theta_{a}, \beta_{a}\right\rangle \quad$ where $(\mathrm{a} \neq 0)$
4. Multiplication of trapezoidal neutrosophic numbers by constant value:
$\mathrm{Ya}= \begin{cases}\left\langle\left(\mathrm{Y} a_{1}, \mathrm{Y} a_{2}, \mathrm{Y} a_{3}, \mathrm{Y} a_{4}\right) ; \alpha_{a}, \theta_{a}, \beta_{a}\right\rangle & \text { if }(\mathrm{Y}>0) \\ \left\langle\left(\mathrm{Y} a_{4}, \mathrm{Y} a_{3}, \mathrm{Y} a_{2}, \mathrm{Y} a_{1}\right) ; \alpha_{a}, \theta_{a}, \beta_{a}\right\rangle & \text { if }(\mathrm{Y}<0)\end{cases}$
5. Division of two trapezoidal neutrosophic numbers:

$$
\tilde{\tilde{a}} \underset{\tilde{b}}{ }= \begin{cases}\left\langle\left(\frac{a_{1}}{b_{4}}, \frac{a_{2}}{b_{2}}, \frac{a_{3}}{b_{2}}, \frac{a_{2}}{b_{1}}\right) ; \alpha_{\tilde{a}} \Lambda \alpha_{\tilde{b}}, \theta_{\tilde{a}} \mathrm{v} \theta_{\tilde{b}}, \beta_{\tilde{a}} V \beta_{\tilde{b}}\right\rangle & \text { if }\left(a_{4}>0, b_{4}>0\right) \\ \left\langle\left(\frac{a_{4}}{b_{4}}, \frac{a_{3}}{b_{3}}, \frac{a_{2}}{b_{2}}, \frac{a_{1}}{b_{1}}\right) ; \alpha_{\tilde{a}} \Lambda \alpha_{\tilde{b}}, \theta_{\tilde{a}} \mathrm{v} \theta_{\tilde{b}}, \beta_{\tilde{a}} V \beta_{\tilde{b}}\right\rangle & \text { if }\left(a_{4}<0, b_{4}>0\right) \\ \left\langle\left(\frac{a_{4}}{b_{1}}, \frac{a_{3}}{b_{2}}, \frac{a_{2}}{b_{3}}, \frac{a_{1}}{b_{4}}\right) ; \alpha_{\tilde{a}} \Lambda \alpha_{\tilde{b}}, \theta_{\tilde{a}} \mathrm{v} \theta_{\tilde{b}}, \beta_{\tilde{a}} V \beta_{\tilde{b}}\right\rangle & \text { if }\left(a_{4}<0, b_{4}<0\right)\end{cases}
$$

6. Multiplication of trapezoidal neutrosophic numbers:

## 3. Methodology

Fuzzy set theory was applied in many studies, but it focuses only on membership value. The intuitionistic fuzzy set theory developed by Atanassov, deals with membership and non-membership value. The neutrosophic set theory is developed by Smarandache, and it treats the uncertainty and ambiguity by adding the indeterminacy besides truthiness and falsity values. In this section, the framework of the proposed model is shown in Fig. 5, we present the proposed TOPSIS - DEMATEL based on the neutrosophic set model as follows:

### 3.1. The neutrosophic DEMATEL technique

Step1: We start with neutrosophic DEMATEL method for evaluating and weighting the important criteria affecting the project selection problem. To weight the criteria, we should do the following:

1. Select those experts who have great experiences in project management.


Fig. 5. The framework of the proposed model.
$\tilde{a} \tilde{b}= \begin{cases}\left\langle\left(a_{1} b_{1}, a_{2} b_{2}, a_{3} b_{3}, a_{4} b_{4}\right) ; \alpha_{\tilde{a}} \Lambda \alpha_{\tilde{b}}, \theta_{\tilde{a}} \mathrm{v} \theta_{\tilde{b}}, \beta_{\tilde{a}} V \beta_{\tilde{b}}\right\rangle & \text { if }\left(a_{4}>0, b_{4}>0\right) \\ \left\langle\left(a_{1} b_{4}, a_{2} b_{3}, a_{3} b_{2}, a_{4} b_{1}\right) ; \alpha_{\tilde{a}} \Lambda \alpha_{\tilde{\tilde{L}}}, \theta_{\tilde{a}} \mathrm{v} \theta_{\tilde{b}}, \beta_{\tilde{a}} V \beta_{\tilde{b}}\right\rangle & \text { if }\left(a_{4}<0, b_{4}>0\right) \\ \left\langle\left(a_{4} b_{4}, a_{3} b_{3}, a_{2} b_{2}, a_{1} b_{1}\right) ; \alpha_{\tilde{a}} \Lambda \alpha_{\tilde{b}}, \theta_{\tilde{a}} \mathrm{v} \theta_{\bar{b}}, \beta_{\tilde{a}} V \beta_{\tilde{b}}\right\rangle & \text { if }\left(a_{4}<0, b_{4}<0\right)\end{cases}$

Table 1
Pairwise comparison among criteria with the degree of $(\alpha, \beta$, and $\theta)$.

| C | Y1 | Y2 | $\ldots$ | Yn |
| :---: | :---: | :---: | :---: | :---: |
| Y1 | $\left(\mathrm{L}_{11}, \mathrm{~m}_{111}, \mathrm{~m}_{11 \mathrm{u}}, \mathrm{u}_{11} ; \alpha, \beta, \theta\right)$ | $\left(\mathrm{L}_{12}, \mathrm{~m}_{121}, \mathrm{~m}_{12 \mathrm{u}}, \mathrm{u}_{12} ; \alpha, \beta, \theta\right)$ | $\ldots$ | $\left(\mathrm{L}_{1 \mathrm{n}}, \mathrm{m}_{1 \mathrm{nl}}, \mathrm{m}_{1 \mathrm{nu}}, \mathrm{u}_{1 \mathrm{n}} ; \alpha, \beta, \theta\right)$ |
| Y2 | $\left(\mathrm{L}_{21}, \mathrm{~m}_{211}, \mathrm{~m}_{21 \mathrm{u}}, \mathrm{u}_{21} ; \alpha, \beta, \theta\right)$ | $\left(\mathrm{L}_{22}, \mathrm{~m}_{221}, \mathrm{~m}_{22 \mathrm{u}}, \mathrm{u}_{22} ; \alpha, \beta, \theta\right)$ | $\ldots$ | $\left(\mathrm{L}_{2 \mathrm{n}}, \mathrm{m}_{2 \mathrm{nl}}, \mathrm{m}_{2 \mathrm{nu}}, \mathrm{u}_{2 \mathrm{n}} ; \alpha, \beta, \theta\right)$ |
| $\ldots$ | ...... | . | $\ldots$ | $\cdots \cdots \cdots$.... |
| Yn | $\left(\mathrm{L}_{\mathrm{n} 1}, \mathrm{~m}_{\mathrm{n} 11}, \mathrm{~m}_{\mathrm{n} 1 \mathrm{u}}, \mathrm{u}_{\mathrm{n} 1} ; \alpha, \beta, \theta\right)$ | $\left(\mathrm{L}_{\mathrm{n} 2}, \mathrm{~m}_{\mathrm{n} 21}, \mathrm{~m}_{\mathrm{n} 2 \mathrm{u}}, \mathrm{u}_{\mathrm{n} 2} ; \alpha, \beta, \theta\right)$ | $\ldots$ | $\left(\mathrm{L}_{\mathrm{nn}}, \mathrm{m}_{\mathrm{nnl}}, \mathrm{m}_{\mathrm{nnu}}, \mathrm{u}_{\mathrm{nn}} ; \alpha, \beta, \theta\right)$ |

Table 2
Crisp value relative to each expert.

| Criteria | Y 1 | Y 2 | $\ldots$ | Yn |
| :--- | :--- | :--- | :--- | :--- |
| Y1 | $\mathrm{CV}_{11}$ | $\mathrm{CV}_{12}$ | $\ldots$ | $\mathrm{CV}_{1 \mathrm{n}}$ |
| Y2 | $\mathrm{CV}_{21}$ | $\mathrm{CV}_{22}$ | $\ldots$ | $\mathrm{CV}_{2 \mathrm{n}}$ |
| $\ldots$ | $\ldots \ldots \ldots$ | $\ldots \ldots \ldots$ | $\ldots$ | $\ldots \ldots .$. |
| Yn | $\mathrm{CV} V_{n 1}$ | $C V_{n 2}$ | $\ldots$ | $\mathrm{CV}_{n n}$ |

2. List and identify the most important criteria affecting the project selection problem.
3. Each expert makes a pairwise comparison among the important related criteria ( $\mathrm{Y} 1, \mathrm{Y} 2, \ldots, \mathrm{Yn}$ ) in a trapezoidal neutrosophic number ( $1_{\mathrm{nm}}, \mathrm{m}_{\mathrm{nml}}, \mathrm{m}_{\mathrm{nmu}}, \mathrm{u}_{\mathrm{nm}}$ ), and also express the maximum truth-membership degree $(\alpha)$, the minimum indeterminacy-membership degree ( $\beta$ ), and the minimum falsity membership degree $(\theta)$ of single valued neutrosophic numbers $\left(1_{n \mathrm{~nm}}, \mathrm{~m}_{\mathrm{nml}}, \mathrm{m}_{\mathrm{nmu}}, \mathrm{u}_{\mathrm{nm} ;} ; \alpha\right.$, $\beta, \theta$ ), using a scale form $(0-1)$ and focusing only on ( $\mathrm{n}-1$ ) consensus judgments (Abdel-Basset, Mohamed, \& Sangaiah, 2017), as shown in Table 1.
4. Calculate the crisp value of each expert's opinion, as shown in Table 2, using the following equations:
$\mathrm{S}\left(\mathrm{a}_{i j}\right)=1 / 16[a 1+b 1+c 1+d 1] \times(2+\alpha \mathrm{a}-\theta \mathrm{a}-\beta \mathrm{a})$
$\mathrm{A}\left(\mathrm{a}_{i j}\right)=1 / 16[a 1+b 1+c 1+d 1] \times(2+\alpha \mathrm{a}-\theta \mathrm{a}+\beta \mathrm{a})$
5. Combine all experts' opinions in one integration matrix and calculate the average of expert's opinions by dividing all experts' opinion for each criterion by the number of experts ( n ) considered in the problem. Calculate average value for each value for each expert by dividing each value by the number of experts (n) as shown in Eq. (6), and then combine all averaged values of the all of expert's opinion in one matrix called the initial directed relation matrix $A$, where $a$ is $n \times n$ matrix of pairwise comparisons by all expert, $\mathrm{S}=\left[\mathrm{S}_{\mathrm{ij}}\right]_{n}{ }_{\mathrm{n}}$, where S is the degree of each criterion $i$ on criterion $j$.

$$
\begin{equation*}
\mathrm{CV}_{11}=\frac{\mathrm{CV} 1 \ln 1+\mathrm{CV} 11 \mathrm{n} 2+\cdots+\mathrm{CV} 11 \mathrm{~nm}}{n} \tag{6}
\end{equation*}
$$

6. Normalizing the initial direct relation matrix (A) using Eqs. (7) and (8).

$$
\begin{align*}
& \mathrm{K}=\frac{1}{\operatorname{Max}(1 \leq \mathrm{i} \leq \mathrm{n}) \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{aij}}  \tag{7}\\
& \mathrm{~S}=\mathrm{K} \times \mathrm{A} \tag{8}
\end{align*}
$$

7. Obtaining the total relation matrix ( T ) by applying Eq. (9), where I is the identity matrix of the same size of S matrix obtained in the previous step.

$$
\begin{equation*}
\mathrm{T}=\mathbf{S} \times(I-S)^{-1} \tag{9}
\end{equation*}
$$

8. Calculate the sum of rows (D) and the sum of columns $(R)$, then calculate $(R+D)$ and $(R-D)$, furthermore make a causal diagram between $(\mathrm{R}+\mathrm{D})$ and $(\mathrm{R}-\mathrm{D})$, and arrange the criteria relative to their importance by weighting them.
Step 2: After weighting the criteria, we apply the neutrosophic TOPSIS method to compare between the set of projects alternatives against set weighted criteria obtained from step 1. To select the best project among several projects using neutrosophic TOPSIS, we should do the following:
9. Obtain the decision matrix between different project alternatives $(\mathrm{Pi})$ and criteria $(\mathrm{Yj})$ based on the opinion

Table 4
Crisp value of pairwise comparison relative to each expert.

|  | Y 1 | Y 2 | $\ldots$ | Yn |
| :--- | :--- | :--- | :--- | :--- |
| P1 | $\mathrm{CV}_{11}$ | $\mathrm{CV}_{12}$ | $\ldots$ | $\mathrm{CV}_{1 \mathrm{n}}$ |
| P2 | $\mathrm{CV}_{21}$ | $\mathrm{CV}_{22}$ | $\ldots$ | $\mathrm{CV}_{2 \mathrm{n}}$ |
| $\ldots$ | $\ldots \ldots \ldots$ | $\ldots \ldots \ldots$ | $\ldots$ | $\ldots \ldots \ldots$ |
| Pm | $\mathrm{CV}_{\mathrm{m} 1}$ | $\mathrm{CV}_{\mathrm{m} 2}$ | $\ldots$ | $\mathrm{CV}_{\mathrm{mn}}$ |

Table 3
Decision matrix of pairwise comparisons based for each expert.

|  | Y1 | Y2 | $\ldots$ | Yn |
| :---: | :---: | :---: | :---: | :---: |
| P1 | $\left(l_{11}, \mathrm{~m}_{111}, \mathrm{~m}_{11 \mathrm{u}}, \mathrm{u}_{11} ; \alpha, \beta, \theta\right)$ | $\left(\mathrm{l}_{12}, \mathrm{~m}_{121}, \mathrm{~m}_{12 \mathrm{u}}, \mathrm{u}_{12} ; \alpha, \beta, \theta\right)$ | $\ldots$ | $\left(\mathrm{l}_{1 \mathrm{n}}, \mathrm{m}_{1 \mathrm{nl}}, \mathrm{m}_{1 \mathrm{nu}}, \mathrm{u}_{1 \mathrm{n}} ; \alpha, \beta, \theta\right)$ |
| P2 | $\left(\mathrm{l}_{21}, \mathrm{~m}_{211}, \mathrm{~m}_{21 \mathrm{u}}, \mathrm{u}_{21} ; \alpha, \beta, \theta\right)$ | $\left(\mathrm{l}_{22}, \mathrm{~m}_{221}, \mathrm{~m}_{22 \mathrm{u}}, \mathrm{u}_{22} ; \alpha, \beta, \theta\right)$ | $\ldots$ | $\left(\mathrm{l}_{2 \mathrm{n}}, \mathrm{m}_{2 \mathrm{nl}}, \mathrm{m}_{2 \mathrm{nu}}, \mathrm{u}_{2 \mathrm{n}} ; \alpha, \beta, \theta\right)$ |
| $\mathrm{Pm}$ | $\ldots \ldots$. $\left(l_{m 1}, \mathrm{~m}_{\mathrm{m} 11}, \mathrm{~m}_{\mathrm{mlu}}, \mathrm{u}_{\mathrm{m} 1} ; \alpha, \beta, \theta\right)$ | $\ldots \ldots \ldots$ $\left(\mathrm{l}_{\mathrm{m} 2}, \mathrm{~m}_{\mathrm{m} 21}, \mathrm{~m}_{\mathrm{m} 2 \mathrm{u}}, \mathrm{u}_{\mathrm{m} 2} ; \alpha, \beta, \theta\right)$ | $\ldots$ | $\ldots \ldots \ldots$ $\left(\mathrm{l}_{\mathrm{mn}}, \mathrm{m}_{\mathrm{mnl}}, \mathrm{m}_{\mathrm{mnu}}, \mathrm{u}_{\mathrm{mn}} ; \alpha, \beta, \theta\right)$ |



Fig. 6. Main criteria for fighter aircraft selection problem example.
of decision makers based on trapezoidal neutrosophic single values with $(\alpha, \beta, \theta)$ and after using the numerical scale ( $0-1$ ) for intangible criteria, as shown in Fig. 3 and expressed in Table 3.
2. Determine the crisp value of the decision matrix obtained in the previous matrix by Eqs. (4) and (5), to obtain the following Table 4.
3. In Table 4, we evaluated each project alternative (Pi) by a set of criteria $(\mathrm{Yj})$ because the criteria have not the same measuring units, and some of them are tangible and some are not tangible, as shown in the introduction. The next step is getting the normalized decision matrix, R , using the equation (10). The elements of normalized decision matrix are fractions between 0 and 1 .
$\mathrm{r}_{\mathrm{ij}}=\mathrm{Y}_{\mathrm{ij}} /$ sqroot $\left(\operatorname{sum}, \mathrm{i}=1, \cdots . ., \mathrm{n}\right.$ of $\left.\mathrm{Y}_{\mathrm{ij}}^{2}\right)$
4. Obtain the weighted decision matrix V taking into consideration the fact that the individual criteria have a certain weight (obtained from step 1 neutrosophic DEMATEL). We get V by multiplying each column of R by the corresponding weights, where $\mathrm{W}_{1{ }^{*} \mathrm{n}}$ is the result of step 1 .
$\mathrm{V}_{\mathrm{m} * \mathrm{n}}=\mathrm{R}_{\mathrm{m} * \mathrm{n}} * \mathrm{~W}_{1 * \mathrm{n}}$
5. Obtain IDEAL ( $\mathrm{A}^{*}$ ) and Negative IDEAL ( $\mathrm{A}^{-}$) solutions from the weighted decision matrix V. where ( $\mathrm{A}^{*}$ ) is the best possibilities for each criterion among all alternatives in V and it's the largest value if profit and the smallest value in case of cost criterion measures. And ( $\mathrm{A}^{-}$) is the worst possibilities for each criterion among all alternatives in V and it's the smallest value if profit criterion measure and largest if cost measurable criterion.
Table 5
Pairwise comparisons among six criteria for the first expert

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y1 | (0.5,0.5,0.5,0.5) | (0.5,0.6,0.9,0.2;0.4,0.2,0.3) | (0.8,0.7,0.9,1.0;0.7,0.2,0.5) | (0.5,0.8,1.0,0.0;0.7,0.2,0.5) | (0.8,0.4,1.0,0.9;0.4,0.2,0.3) | (0.7,0.5,0.4,0.2;0.5, 0.4,0.3) |
| Y2 | (0.4,0.3,0.9,0.8;0.9,0.7,0.4) | (0.5,0.5,0.5,0.5) | (0.0,0.1,0.3, $0.4 ; 0.8,0.2,0.6)$ | (0.5,0.2,0.8,1.0,0.7,0.6,0.5) | (0.4,0.3,0.2,0.5;0.9,0.7,0.3) | (0.3,0.2,0.4,0.9;0.4,0.3,0.7 |
| Y3 | (0.7,0.3,0.5,0.8;0.2,0.4,0.3) | (0.6,0.1,0.7,1.0;0.3,0.1,0.5) | (0.5,0.5,0.5,0.5) | (0.3,0.5,0.9,0.2;0.5,0.1,0.2) | (0.7,0.6,0.5,0.4;0.4,0.3,0.5) | (0.4,0.7,0.3, $0.4 ; 0.5,0.4,0.2)$ |
| Y4 | (0.8,0.9,0.4,0.9;0.9,0.5,0.4) | (0.2,0.4,0.5,0.6;0.7,0.6,0.3) | (0.1,0.5,0.3,0.7;0.3, $0.4,0.6)$ | $(0.5,0.5,0.5,0.5)$ | (0.2,0.5,0.6,0.8;0.7,0.3,0.4) | (0.8,0.9,0.8,0.2;0.8,0.2,0.4) |
| Y5 | (0.3,0.4,0.1,0.2;0.7,0.6,0.5) | (0.7,0.4,0.3, 0.1;0.7,0.4,0.3) | (0.8,0.4,0.2,0.5;0.7,0.3,0.2) | (0.9,0.2,0.6,0.3;0.5,0.2,0.9) | (0.5,0.5,0.5,0.5) | (0.4,0.3, 0.2,0.5;0.9,0.7,0.4) |
| Y6 | (0.3,0.4,0.5,0.8;0.2,0.4,0.3) | $(0.2,0.4,0.5,0.6 ; 0.7,0.6,0.3)$ | (0.4,0.7,0.5,0.1;0.1,0.3,0.4) | (0.2,0.3, $0.5,0.1 ; 0.7,0.1,0.3)$ | (0.7,0.6,0.5,0.6;0.6,0.5,0.4) | (0.5,0.5,0.5,0.5) |

Table 6
The crisp matrix for expert 1.

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y1 | 0.5 | 0.261 | 0.425 | 0.375 | 0.368 | 0.203 |
| Y2 | 0.27 | 0.5 | 0.1 | 0.25 | 0.166 | 0.158 |
| Y3 | 0.216 | 0.255 | 0.5 | 0.261 | 0.22 | 0.214 |
| Y4 | 0.375 | 0.191 | 0.13 | 0.5 | 0.263 | 0.371 |
| Y5 | 0.1 | 0.169 | 0.261 | 0.219 | 0.5 | 0.158 |
| Y6 | 0.188 | 0.191 | 0.191 | 0.244 | 0.255 | 0.5 |

6. Calculate the separation measures from ideal $\left(\mathrm{S}_{\mathrm{i}}^{*}\right)$ and negative ideal ( $\mathrm{S}_{\mathrm{i}}^{-}$) Eqs. (12) and (13) solution for all alternatives $\mathrm{i}=1, \ldots, \mathrm{~m}$. where:

$$
\begin{align*}
\mathrm{S}_{\mathrm{i}}^{*} & =[\text { sqroot }(\text { sum of squares for } \mathrm{j} \\
& \left.\left.=1, \cdots \ldots, \mathrm{n} \text { of }\left(\mathrm{v}_{\mathrm{ij}}-\mathrm{v}_{\mathrm{j}}^{*}\right)\right)\right]  \tag{12}\\
\mathrm{S}_{\mathrm{i}}^{-} & =[\text {sqroot }(\text { sum of squares for } \mathrm{j} \\
& \left.\left.=1, \cdots \ldots, \mathrm{n} \text { of }\left(\mathrm{v}_{\mathrm{ij}}-\mathrm{v}_{\mathrm{j}}^{-}\right)\right)\right] \tag{13}
\end{align*}
$$

7. Determine the relative closeness ideal solution; for each alternative calculate the relative closeness to the ideal solution ( $\left.C_{i}^{*}, i=1, \ldots, m\right)$ by Eq. (14). The closeness rating is a number between 0 and 1 with 0 being the worst possible alternative and 1 being the best possible alternative.
$\mathrm{C}_{\mathrm{i}}^{*}=\frac{\mathrm{S}_{\mathrm{i}}^{-}}{\left(\mathrm{S}_{\mathrm{i}}^{*}+\mathrm{S}_{\mathrm{i}}^{-}\right)}$
8. Make a decision for selecting the preference alternative project and determine the preference order by arranging alternatives in descending order, based on the relative closeness value for each alternative.

## 4. Illustrative example

This example illustrates the process of evaluating several projects and selects the best project using neutrosophic TOPSIS-DEMATEL, which is employed for weighting the different criteria affecting the process of projects evaluation. Then, a comparison is performed between the alternative projects and the weighted criteria. In this example, we consider four projects under the fighter aircraft selection. We consider six important criteria affecting the fighter aircraft selection. The six important criteria and their measurable units are presented in Fig. 6.

First, we apply the neutrosophic DEMATEL technique for weighting the main six criteria (in Fig. 6) for this problem, and then we apply the TOPSIS technique in the neutrosophic environment to select the best project. For more details, we follow the next steps:

Step 1: Start with neutrosophic DEMATEL by implementing the following:

1. Select the experts in project management field; we select three experts in this example.
Table 7
Pairwise comparisons among six criteria for the second expert.

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y1 | (0.5,0.5,0.5,0.5) | (0.3,0.4,0.7,0.5;0.6,0.2,0.1) | (0.9,0.6,0.8,0.9;0.8,0.4,0.3) | (0.4,0.9,1.0,0.0;0.7,0.4,0.3) | (0.3,0.2,0.4,0.9;0.5,0.3,0.1) | (0.8,0.4,1.0,0.9;0.4,0.2,0.3) |
| Y2 | (0.6,0.2,0.8,0.9;0.9,0.7,0.4) | (0.5,0.5,0.5,0.5) | (0.3,0.1,0.6,0.4;0.6,0.2,0.4) | (0.7,0.2,0.6,1.0;0.7,0.6,0.5) | (0.3,0.0,0.3, $0.4 ; 0.8,0.2,0.6)$ | (0.3,0.2,0.4,0.9;0.4,0.3,0.7) |
| Y3 | (0.6,0.3,0.5,0.9;0.2,0.4,0.1) | (0.9,0.1,0.7,1.0;0.5,0.1,0.5) | (0.5,0.5,0.5,0.5) | (0.8,0.5, 0.9,0.2;0.5,0.4,0.2) | (0.3,0.6,0.5, 0.4;0.4,0.3,0.2) | (0.3,0.7,0.3, 0.4;0.4,0.5,0.2) |
| Y4 | (0.6,0.9,0.4,0.6;0.6,0.5,0.4) | (0.7,0.4,0.5,0.6;0.7,0.4,0.3) | (0.1,0.5,0.5,0.7;0.3, $0.4,0.2)$ | (0.5,0.5,0.5,0.5) | (0.6,0.5,0.6,0.8;0.7,0.3,0.1) | (0.9,0.7,0.8,0.2;0.8,0.2,0.4) |
| Y5 | (0.6,0.4,0.1,0.2;0.7,0.6,0.5) | (0.3,0.4,0.3, $0.1 ; 0.7,0.4,0.4)$ | (0.2,0.1,0.3, $0.4 ; 0.8,0.2,0.4)$ | (0.9,0.6,0.9,0.2;0.4,0.2,0.1) | (0.5,0.5,0.5,0.5) | (0.4,0.6,0.2,0.5;0.9,0.6,0.4) |
| Y6 | (0.6,0.4,0.5,0.8;0.2,0.4,0.3) | (0.3,0.4,0.5, 0.6;0.7,0.5,0.3) | (0.3,0.7,0.5,0.1;0.1, $0.5,0.4)$ | (0.4,0.3, $0.5,0.1 ; 0.2,0.4,0.3)$ | (0.8,0.6,0.5, $0.6 ; 0.6,0.5,0.3)$ | (0.5,0.5, $0.5,0.5$ ) |

Table 8
The crisp matrix for the second expert 2.

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y1 | 0.5 | 0.273 | 0.42 | 0.263 | 0.236 | 0.368 |
| Y2 | 0.281 | 0.5 | 0.175 | 0.25 | 0.125 | 0.158 |
| Y3 | 0.244 | 0.321 | 0.5 | 0.285 | 0.214 | 0.181 |
| Y4 | 0.266 | 0.275 | 0.191 | 0.5 | 0.359 | 0.358 |
| Y5 | 0.13 | 0.131 | 0.138 | 0.341 | 0.5 | 0.202 |
| Y6 | 0.216 | 0.214 | 0.12 | 0.122 | 0.281 | 0.5 |

2. Identify the main criteria affecting the fighter aircraft selection problem, as presented in Fig. 6.
3. Make pairwise comparison matrix for each expert based on the trapezoidal neutrosophic number to evaluate each criterion against the others, as shown in Tables 5, 7, and 9.
4. Calculate the crisp value for each pairwise comparison matrix (for each expert opinion) using Eqs. (4), and (5). These crisp values are presented in Tables 6, 8, and 10.
5. Generate the initial directed matrix (s) by integrating the three matrices of expert's opinion using Eq. (6). The initial directed matrix is displayed in Table 11.
6. Generate the generalized direct relation matrix by normalizing the initial directed matrix using Eq. (7) to get the value of K, and then apply Eq. (8) to get the generalized direct relation matrix, as carried forth in Table 12.

| Row 1 | 2.077 |
| :--- | :--- |
| Row 2 | 1.451 |
| Row 3 | 1.704 |
| Row 4 | 1.944 |
| Row 5 | 1.462 |
| Row 6 | 1.494 |

$$
\mathrm{K}=\frac{1}{2.077}
$$

7. Calculate the total relation matrix using Eq. (9), as introduced in Table 13, where (I) is the identity matrix.
8. Calculate the sum of each row and column in the total relation matrix ( T ), then draw causal diagram between the summation of rows and columns as a horizontal line and the differences between rows and column as vertical axes, as pictured in Fig. 7.

Sum of rows and columns

| Col 1 | 4.1226 | Row 1 | 5.4187 |
| :--- | :--- | :--- | :--- |
| Col 2 | 4.2583 | Row 2 | 3.695 |
| Col 3 | 4.049 | Row 3 | 4.3606 |
| Col 4 | 4.5037 | Row 4 | 5.0038 |
| Col 5 | 4.4394 | Row 5 | 3.6215 |
| Col 6 | 4.4172 | Row6 | 3.6906 |

Table 9
Pairwise comparisons among six criteria for the third expert

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y1 | (0.5,0.5,0.5,0.5) | (0.6,0.4,0.7,0.6;0.6,0.2,0.1) | (0.7,0.6,0.8,0.8;0.8,0.4,0.3) | (0.3,0.9,1.0,0.8;0.3,0.4,0.4) | (0.4,0.2,0.4,0.8;0.5,0.3,0.1) | (0.7,0.4,1.0,0.5;0.4,0.2,0.3) |
| Y2 | (0.3,0.2,0.8,0.9;0.9,0.7,0.4) | (0.1,0.5,0.5,0.5) | (0.6,0.1,0.6,0.3;0.6,0.2,0.4) | (0.6,0.2,0.6,1.0;0.7,0.6,0.6) | (0.4,0.0,0.3,0.3;0.8,0.2,0.6) | (0.2,0.2,0.4,0.6;0.4,0.3,0.7) |
| Y3 | (0.3,0.3,0.5,0.9;0.2,0.4,0.1) | (0.5,0.1,0.7,0.9;0.5,0.1,0.5) | (0.5,0.5,0.5,0.5) | (0.7,0.5,0.9,0.2;0.5,0.4,0.1) | (0.4,0.6,0.5,0.5;0.4,0.3,0.2) | (0.5,0.7,0.3, 0.4;0.4,0.5,0.2) |
| Y4 | $(0.5,0.9,0.4,0.6 ; 0.5,0.3,0.4)$ | $(0.5,0.4,0.5,0.7 ; 0.7,0.4,0.3)$ | $(0.6,0.5,0.5,0.6 ; 0.3,0.4,0.2)$ | $(0.5,0.5,0.5,0.5)$ | $(0.5,0.5,0.6,0.9 ; 0.7,0.3,0.1)$ | (0.9,0.7,0.8,0.7;0.8,0.2,0.4) |
| Y5 | (0.6,0.4,0.1,0.2;0.7,0.6,0.5) | (0.6,0.4,0.3,0.3;0.7,0.4,0.4) | (0.7,0.1,0.3, $0.5 ; 0.8,0.2,0.4)$ | (0.4,0.6,0.9,0.2;0.4,0.2,0.2) | (0.5,0.5,0.5,0.5) | (0.4,0.6,0.2,0.8;0.9,0.6,0.4) |
| Y6 | (0.8,0.4,0.5,0.8;0.2,0.4,0.3) | (0.4,0.4,0.5, 0.5;0.7,0.5, 0.3) | (0.5,0.7,0.5, 0.3;0.1, 0.5, 0.4) | (0.2,0.3, $0.5,0.1 ; 0.2,0.4,0.1)$ | (0.7,0.6,0.5,0.4;0.6,0.5,0.3) | (0.5,0.5, 0.5, 0.5) |

Table 10
The crisp matrix for the third expert.

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y1 | 0.5 | 0.331 | 0.381 | 0.282 | 0.236 | 0.309 |
| Y2 | 0.248 | 0.5 | 0.2 | 0.225 | 0.125 | 0.123 |
| Y3 | 0.213 | 0.261 | 0.5 | 0.288 | 0.238 | 0.202 |
| Y4 | 0.27 | 0.263 | 0.234 | 0.5 | 0.359 | 0.426 |
| Y5 | 0.13 | 0.19 | 0.22 | 0.263 | 0.5 | 0.238 |
| Y6 | 0.234 | 0.214 | 0.15 | 0.117 | 0.248 | 0.5 |

Table 11
The initial directed matrix.

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y1 | 0.5 | 0.288 | 0.409 | 0.307 | 0.28 | 0.293 |
| Y2 | 0.266 | 0.5 | 0.158 | 0.242 | 0.139 | 0.146 |
| Y3 | 0.224 | 0.279 | 0.5 | 0.278 | 0.224 | 0.199 |
| Y4 | 0.304 | 0.243 | 0.185 | 0.5 | 0.327 | 0.385 |
| Y5 | 0.12 | 0.163 | 0.206 | 0.274 | 0.5 | 0.199 |
| Y6 | 0.213 | 0.206 | 0.153 | 0.161 | 0.261 | 0.5 |

Table 12
The generalized direct relation matrix X .

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y1 | 0.2405 | 0.138528 | 0.196729 | 0.147667 | 0.13468 | 0.140933 |
| Y2 | 0.127946 | 0.2405 | 0.075998 | 0.116402 | 0.066859 | 0.070226 |
| Y3 | 0.107744 | 0.134199 | 0.2405 | 0.133718 | 0.107744 | 0.095719 |
| Y4 | 0.146224 | 0.116883 | 0.088985 | 0.2405 | 0.157287 | 0.185185 |
| Y5 | 0.05772 | 0.078403 | 0.099086 | 0.131794 | 0.2405 | 0.095719 |
| Y6 | 0.102453 | 0.099086 | 0.073593 | 0.077441 | 0.125541 | 0.2405 |


|  | Col + Row | Col-Row |
| :--- | :--- | :--- |
| 1 | 9.5413 | -1.2961 |
| 2 | 7.9533 | 0.5633 |
| 3 | 8.4096 | -0.3116 |
| 4 | 9.5075 | -0.5001 |
| 5 | 8.0609 | 0.8179 |
| 6 | 8.1078 | 0.7266 |

9. Weight the six criteria based on the causal diagram. The importance of all criteria is evaluated and ranked based on the expert's opinion and introduced in the causal

Table 13
The total relation matrix $T$.

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y1 | 0.9547 | 0.8651 | 0.9025 | 0.9166 | 0.8880 | 0.8918 |
| Y2 | 0.6164 | 0.7566 | 0.5401 | 0.6411 | 0.5680 | 0.5728 |
| Y3 | 0.6711 | 0.7220 | 0.8108 | 0.7538 | 0.7101 | 0.6928 |
| Y4 | 0.7956 | 0.7783 | 0.7128 | 0.9574 | 0.8641 | 0.8956 |
| Y5 | 0.5102 | 0.5502 | 0.5513 | 0.6478 | 0.7664 | 0.5956 |
| Y6 | 0.5746 | 0.5861 | 0.5315 | 0.5870 | 0.6428 | 0.7686 |

diagram as follows: the reliability criterion is the most important criterion for project selection (Y5), and the least important criterion is the ferry range (Y1).
Based on the expert's opinion and neutrosophic DEMATEL method, the weights of considered criteria relative to their importance are $(0.1,0.2,0.1,0.1,0.3$, and 0.2).

Step 2: Apply the Neutrosophic TOPSIS for ranking the four projects and select the best one, by performing the following:

1. Obtain the decision matrix between the four project alternatives (P1-P4) and the six criteria (Y1-Y6) [Tables 14-17]. These values are crisp values, based on the opinion of decision-makers expressed by trapezoidal neutrosophic single values with ( $\alpha, \beta, \theta$ ), using the numerical scale (0-1) for intangible criteria [Fig. 8].


Fig. 7. The causal diagram for the six criteria.

Table 14
The decision matrix of the fighter aircraft selection.

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| P1 | $1500^{\sim}$ | $5.5^{\sim}$ | $2^{\sim}$ | $20,000^{\sim}$ | Avg $^{\sim}$ | V.high $^{\sim}$ |
| P2 | $2700^{\sim}$ | $6.5^{\sim}$ | $2.5^{\sim}$ | $18,000^{\sim}$ | Low $^{\sim}$ | Avg $^{\sim}$ |
| P3 | $200^{\sim}$ | $4.5^{\sim}$ | $1.8^{\sim}$ | $21,000^{\sim}$ | High $^{\sim}$ | High $^{\sim}$ |
| P4 | $1800^{\sim}$ | $5^{\sim}$ | $2.2^{\sim}$ | $20,000^{\sim}$ | Avg $^{\sim}$ | Avg $^{\sim}$ |

Table 15
The decision matrix of the fighter aircraft selection with the numerical scale of intangible criteria.

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| P1 | $1500^{\sim}$ | $5.5^{\sim}$ | $2^{\sim}$ | $20,000^{\sim}$ | $0.5^{\sim}$ | $0.9^{\sim}$ |
| P2 | $2700^{\sim}$ | $6.5^{\sim}$ | $2.5^{\sim}$ | $18,000^{\sim}$ | $0.3^{\sim}$ | $0.5^{\sim}$ |
| P3 | $200^{\sim}$ | $4.5^{\sim}$ | $1.8^{\sim}$ | $21,000^{\sim}$ | $0.7^{\sim}$ | $0.7^{\sim}$ |
| P4 | $1800^{\sim}$ | $5^{\sim}$ | $2.2^{\sim}$ | $20,000^{\sim}$ | $0.5^{\sim}$ | $0.5^{\sim}$ |

Table 16
The decision matrix in trapezoidal values based on trapezoidal neutrosophic single values with $(\alpha, \beta, \theta)$.

|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P1 | (800,1200,1500,1800;0.6,0.5,0.3) | (3.5,4,5.5,5.7;0.5,0.3,0.4) | (1,1.5,2,2.5;0.5,0.4,0.3) | (18000,19000,20000,20500;0.5,0.3,0.4) | (0.3,0.4,0.5,0.6;0.5,0.6,0.2) | 0.7,0.8,0.9,0.95;0.8,0.5,0.3) |
| P2 | (1700,2100,2700,2009;0.9,0.5,0.4) | (5,5.5,6.5,7;0.9,0.5,0.3) | (1.5,2,2.5,3;0.2,0.3,0.1) | 17500,17800,18,000,20000;0.8,0.6,0.4) | (0.2,0.25,0.3, $0.4 ; 0.9,0.5,0.3)$ | (0.4,0.54,0.5,0.6;0.5,0.7,0.3) |
| P3 | (90,150,200,220;0.8,0.6,0.4) | (4,4.2,4.5,5.5;0.8,0.7,0.6) | (1.3,1.5,1.8,1.9;0.5, 0.6,0.2) | (20000,20800,21000,21500;0.9,0.8,0.7) | (0.5,0.6,0.7,0.75;0.5,0.6,0.4) | (0.5,0.6,0.7,0.8;0.7,0.6,0.5) |
| P4 | (1200,1500, 1800, $2000 ; 0.5,0.6,0.2)$ | (4,4.5,5,5.2;0.6,0.5,0.4) | (1.8,2,2.2,2.5;0.7,0.5,0.4) | (18500,19100,20,000,20500;0.6,0.3,0.1) | (0.3,0.4,0.5,0.6;0.6,0.2,0.1) | (0.3,0.4,0.5, $0.6 ; 0.3,0.5,0.4)$ |



Fig. 8. The scale for intangible criteria.
2. Generate the normalized decision matrix ( R ) using Eq. (10), as presented in Table 18; notice that all $\mathrm{r}_{\mathrm{ij}}$ is between 0 and 1 .
$\mathrm{W}=(0.1,0.2,0.1,0.1,0.3,0.2)$
3. Obtain the weighted decision matrix V by multiplying each column of R with the corresponding criterion weight (the output of step 1), using Eq. (11), as presented in Table 19.
4. In the weighted matrix (Table 19), we determine for each criterion the best value (the largest value) and the worst value (the smallest value). This is done for all benefits criteria such as (Y1, Y3, Y4, Y5, and Y6), but in case of cost criteria we select the smallest value as the best value, and the largest value as the worst value, such as criterion Y2 in our example, where Y2 represents the acquisition cost. Obtain the ideal (the best possible solution) and negative (the worst possible solution) ideal solution $\mathrm{A}^{*}$, and $\mathrm{A}^{-}$.
$\mathrm{A}^{*}=(0.078886,0.072335,0.05809,0.063816,0.18124$, 0.152606 )
$\mathrm{A}^{-}=(0.004985,0.137315,0.039623,0.03697,0.105724$, 0.057398 )
5. Calculate the separation measures from ideal and negative ideal solution $\mathrm{Si}^{*}, \mathrm{Si}^{-}$using Eqs. (12) and (13), as shown in Table 20.
6. Compute the relative closeness to the ideal solution for each alternative by using Eq. (14); the relative closeness values are expressed in Table 21:
7. Finally, rank four alternatives based on their relative closeness value. Determine the preference order by arranging the alternatives of the relative closeness values for alternatives in the descending order of $\mathrm{Ci}^{*}, \mathrm{i}=1,2,3,4$. Thus, the rank of alternatives in the fighter aircraft selection problem using neutrosophic TOPSIS-DEMATEL emerges as A1, A4, A3, and A2.

Table 17
The equivalent crisp values of the decision matrix.

|  | Y1 | Y2 | Y3 | Y4 | Y5 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| P1 | 596.25 | 2.10375 | 0.7875 | 8718.75 | 0.19125 | 0.41875 |
| P2 | 1175 | 3.15 | 1.0125 | 6221.25 | 0.182813 |  |
| P3 | 74.25 | 1.659375 | 0.690625 | 7288.75 | 0.239063 |  |
| P4 | 690.625 | 1.986875 | 0.95625 | 10738.75 | 0.25875 |  |

Table 18
The normalized decision matrix.

|  | Y1 | Y2 | Y3 | Y4 | Y5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| P1 | 0.400302 | 0.458533 | 0.451807 | 0.518117 | 0.446533 |
| P2 | 0.788855 | 0.686574 | 0.580895 | 0.369702 | 0.352412 |
| P3 | 0.049849 | 0.361677 | 0.396228 | 0.433139 | 0.558167 |
| P4 | 0.463662 | 0.433059 | 0.548623 | 0.638157 | 0.633114 |

Table 19
The weighted decision matrix V , with best and worst values.

|  | $Y 1$ | Y2 | Y3 | Y4 | Y5 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| P1 | 0.04003 | 0.091707 | 0.045181 | 0.051812 | 0.13396 | $\mathbf{0 . 1 0 5 7 2 4}$ |
| P2 | $\mathbf{0 . 0 7 8 8 8 6}$ | $\mathbf{0 . 1 3 7 3 1 5}$ | $\mathbf{0 . 0 5 8 0 9}$ | $\mathbf{0 . 0 3 6 9 7}$ | 0.16745 |  |
| P3 | 0.046366 | 0.072335 | $\mathbf{0 . 0 3 9 6 2 3}$ | 0.043314 | 0.066623 |  |
| P4 |  | 0.056612 | $\mathbf{0 . 0 6 3 8 1 6}$ | 0.094752 |  |  |

Table 20
The separation measures from ideal and negative ideal solution $\mathrm{Si}^{*}$, and $\mathrm{Si}^{-}$.

| Separation measures from |  |
| :--- | :--- |
| Ideal solution | Negative ideal solution |
| $\mathrm{S}^{*}=0.0666$ | $\mathrm{~S}^{-}=0.1158$ |
| $\mathrm{~S} 2^{*}=0.1343$ | $\mathrm{~S} 2^{-}=0.0767$ |
| $\mathrm{~S} 3^{*}=0.0988$ | $\mathrm{~S}^{-}=0.0973$ |
| $\mathrm{~S} 4^{*}=0.1017$ | $\mathrm{~S}^{-}=0.1046$ |

Table 21
The relative closeness to the ideal solution for each alternative.

| Alternatives | Relative closeness value |
| :--- | :--- |
| 1 | $\mathrm{C}^{*}=0.634868$ |
| 2 | $\mathrm{C}^{*}=0.363507$ |
| 3 | $\mathrm{C}^{*}=0.496175$ |
| 4 | $\mathrm{C} 4^{*}=0.507029$ |

## 5. Conclusion and future work

Neutrosophic set is the most comprehensive set, which includes both fuzzy set and intuitionistic fuzzy set, as it considers the indeterminacy function in addition to truthmembership and falsity membership, being suitable in analyzing real situations. Also, in real life situations, accurate judgments are rarely since ambiguity and uncertainty surround the decision-making process. To solve the problem
of project selection, the important criteria should be identified well, and then the selection process should be performed among several alternative projects. In this research, we considered parameters of TOPSISDEMATEL comparison matrices as trapezoidal neutrosophic numbers. TOPSIS is combined with the DEMATEL for more powerful and accurate weighted criteria, helping the selection of the best project alternative. Neutrosophic TOPSIS-DEMATEL model presented here is used for assisting the decision of project selection phase of project life cycle. We consider only ( $\mathrm{n}-1$ ) consensus judgment for each expert, for n numbers of alternatives. As well, we consider the $(0-1)$ scale for intangible criteria. The project selection is a very important phase of any project life cycle after identification and appraisal of projects. In the future, we enhance the proposed model to solve the different phases of a project's life cycle. Moreover, we plan to solve the selection project problem with more complex techniques dealing with Multi-Criteria Decision-Making problems.

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