

## **A Methodology to Combine Interval Neutrosophic Focal Elements and Their Basic Probability Assignment in Evidence Theory**

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### **ABSTRACT**

In this paper, a method is proposed to combine interval neutrosophic focal elements and their corresponding Basic Probability Assignments (BPA) of three variables using Dempster Shafer Theory (DST) of evidence under ordinary arithmetic operations and Modified Arithmetic operations on interval numbers. The validity of the proposed method has been verified with the help of a numerical example.

**Keywords:** Dempster-Shafer Theory (DST), Basic Probability Assignments, Neutrosophic set, Interval Neutrosophic Number (INN), Modified arithmetic operations

**Mathematical Subject Classification (2010):** 60A86, 97K50, 11K65

### **1. Introduction**

Probability theory is proposed only for randomness uncertainty and it is inappropriate to represent epistemic uncertainty. To overcome the constraint of probabilistic method, Dempster put forward a mathematical theory of evidence in 1976 and now it is known as Evidence Theory or Dempster-Shafer Theory (DST). In a finite discrete space, Dempster-Shafer Theory can be interpreted as a generalization of Probability theory where probabilities assigned to sets as opposed to mutually exclusive singletons. In traditional probability theory, evidence is associated with only one possible event. In Dempster- Shafer Theory, evidence can be associated with multiple possible events. Further, Evidence Theory is based on two dual non additive measures, namely Belief measure and Plausibility measure. Belief and Plausibility measures can conveniently be characterized by a function  $m: \rho(X) \rightarrow [0,1]$  such that  $m(\emptyset) = 0$  and  $\sum_{A \in \rho(X)} m(A) = 1$ . This function is known as Basic Probability Assignment (BPA). Every set  $A \in \rho(X)$  for which  $m(A) > 0$  is usually called a focal element of  $m$ . The Dempster Rule of combination is critical to the original conception of Dempster- Shafer theory. The measure of Belief and plausibility are derived from the combined basic assignments. Dempster's rule combines multiple belief functions through their basic probability

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assignment (m). These belief functions are defined on the same frame of discernment, but are based on independent assignment or bodies of evidence. The Dempster rule of combination is purely a conjunctive operation. The combination rule results in a belief function based on conjunctive pooled evidence.

The standard way of combining evidence is expressed by the formula

$$m_{1,2}(A) = \frac{\sum_{B \cap C = A} m_1(B).m_2(C)}{1 - \sum_{B \cap C = \emptyset} m_1(B).m_2(C)}, \text{ for all } A \neq \emptyset \text{ and } m_{1,2}(\emptyset) = 0.$$

Neutrosophic set theory was proposed by Smarandache [2] in 1999. The neutrosophic set introduced from a philosophical point of view is difficult to be applied in practical problems since its truth membership function (T), the indeterminacy membership function (I) and the false membership function (F) lie in the non standard interval ]0-, 1+[. As a simplified form of the neutrosophic set, Wang et al [4] defined an Interval Neutrosophic Set when its three functions are restricted in the real standard interval [0,1].

The paper is organized as follows: Section 1 introduces Dempster- Shafer Theory and Neutrosophic set theory. Section 2 deals with the basic definitions about Neutrosophic set and Interval Neutrosophic Set and their arithmetic operations. Algebraic Combination of Interval neutrosophic Focal elements are given in Section 3 and the effectiveness of the proposed method is illustrated by means of an example. Finally, some concluding remarks are given in section 4.

## 2. Preliminaries

**Definition 2.1.** Let  $X$  be a space of points (objects), with a generic element in  $X$  denoted by  $x$ . A neutrosophic set  $A$  in  $X$  is characterized by a truth membership function  $T_A$ , indeterminacy  $I_A$  and a falsity membership function  $F_A$ .  $T_A(x)$ ,  $I_A(x)$  and  $F_A(x)$  are real standard or non standard subsets of  $]0, 1+[$ . That is

$$\begin{aligned} T_A : X &\rightarrow ]0, 1+[ , \\ I_A : X &\rightarrow ]0, 1+[ , \\ F_A : X &\rightarrow ]0, 1+[ . \end{aligned}$$

There is no restriction on the sum of  $T_A(x)$ ,  $I_A(x)$  and  $F_A(x)$ , so  $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$

**Definition 2.2.** Let  $X$  be a space of points (objects), with a generic element in  $X$  denoted by  $x$ . An interval neutrosophic set (INS)  $A$  in  $X$  is characterized by a truth membership function  $T_A$ , indeterminacy  $I_A$  and a falsity membership function  $F_A$ . For each point  $x$  in  $X$ ,  $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$ .

**Definition 2.3.** Let  $X$  be a space of points and  $A$  be an interval neutrosophic set. Then the interval neutrosophic number (INN) is denoted by  $\langle [T^L, T^U], [I^L, I^U], [F^L, F^U] \rangle$ .

## 2.4. Arithmetic operations on interval neutrosophic set

### Addition:

The addition of two interval neutrosophic sets  $A$  and  $B$  is an interval neutrosophic set  $C$ , written as  $C = A+B$ , whose truth membership, indeterminacy membership and falsity

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membership functions are related to those of A and B by

$$\begin{aligned} \inf T_C(x) &= \min(\inf T_A(x) + \inf T_B(x), 1), \\ \sup T_C(x) &= \min(\sup T_A(x) + \sup T_B(x), 1), \\ \inf I_C(x) &= \min(\inf I_A(x) + \inf I_B(x), 1), \\ \sup I_C(x) &= \min(\sup I_A(x) + \sup I_B(x), 1), \\ \inf F_C(x) &= \min(\inf F_A(x) + \inf F_B(x), 1), \\ \sup F_C(x) &= \min(\sup F_A(x) + \sup F_B(x), 1), \end{aligned}$$

for all  $x$  in  $X$ .

**Subtraction:**

The subtraction of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as  $C = A-B$ , whose truth membership, indeterminacy membership and falsity membership functions are related to those of A and B by

$$\begin{aligned} \inf T_C(x) &= \min(\inf T_A(x), \inf F_B(x)), \\ \sup T_C(x) &= \min(\sup T_A(x), \sup F_B(x)), \\ \inf I_C(x) &= \min(\inf I_A(x), 1 - \sup I_B(x)), \\ \sup I_C(x) &= \min(\sup I_A(x), 1 - \inf I_B(x)), \\ \inf F_C(x) &= \min(\inf F_A(x), \inf T_B(x)), \\ \sup F_C(x) &= \min(\sup F_A(x) + \sup T_B(x)), \end{aligned}$$

for all  $x$  in  $X$ .

**Multiplication:**

The multiplication of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as  $C = A \times B$ , whose truth membership, indeterminacy membership and falsity membership functions are related to those of A and B by

$$\begin{aligned} \inf T_C(x, y) &= \inf T_A(x) + \inf T_B(y) - \inf T_A(x). \inf T_B(y), \\ \sup T_C(x, y) &= \sup T_A(x) + \sup T_B(y) - \sup T_A(x). \sup T_B(y), \\ \inf I_C(x) &= \inf I_A(x). \sup I_B(y), \\ \sup I_C(x) &= \sup I_A(x). \inf I_B(y), \\ \inf F_C(x) &= \inf F_A(x). \sup F_B(y), \\ \sup F_C(x) &= \sup F_A(x). \inf F_B(y), \end{aligned}$$

for all  $x$  in  $X$  and  $y$  in  $Y$ .

**Scalar Multiplication:**

The scalar multiplication of interval neutrosophic set A is an interval neutrosophic set B, written as  $B = a.A$ , whose truth membership, indeterminacy membership and falsity membership functions are related to those of A by

$$\begin{aligned} \inf T_B(x) &= \min(\inf T_A(x). a, 1) \\ \sup T_B(x) &= \min(\sup T_A(x). a, 1) \\ \inf I_B(x) &= \min(\inf I_A(x). a, 1) \\ \sup I_B(x) &= \min(\sup I_A(x). a, 1) \\ \inf F_B(x) &= \min(\inf F_A(x). a, 1) \\ \sup F_B(x) &= \min(\sup F_A(x). a, 1) \end{aligned}$$

**Division:**

The division of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as  $C = A/B$ , whose truth membership, indeterminacy membership and falsity membership functions are related to those of A and B by

$$\begin{aligned} \text{If } T_A^L < T_B^L \quad \text{and } T_A^U < T_B^U, \text{ then } \inf T_C(x) &= \frac{\inf T_A(x)}{\inf T_B(x)}, \\ \sup T_C(x) &= \frac{\sup T_A(x)}{\sup T_B(x)}. \\ \text{If } T_A^L > T_B^L \quad \text{and } T_A^U > T_B^U, \text{ then } \inf T_C(x) &= \frac{\inf T_A(x) - \inf T_B(x)}{1 - \inf T_B(x)}, \\ \sup T_C(x) &= \frac{\sup T_A(x) - \sup T_B(x)}{\sup T_B(x)}. \\ \text{If } I_A^L < I_B^L \quad \text{and } I_A^U < I_B^U, \text{ then } \inf I_C(x) &= \frac{\inf I_A(x)}{\inf I_B(x)}, \\ \sup I_C(x) &= \frac{\sup I_A(x)}{\sup I_B(x)}. \\ \text{If } I_A^L > I_B^L \quad \text{and } I_A^U > I_B^U, \text{ then } \inf I_C(x) &= \frac{\inf I_A(x) - \inf I_B(x)}{1 - \inf I_B(x)}, \\ \sup I_C(x) &= \frac{\sup I_A(x) - \sup I_B(x)}{1 - \sup I_B(x)}. \\ \text{If } F_A^L < F_B^L \quad \text{and } F_A^U < F_B^U, \text{ then } \inf F_C(x) &= \frac{\inf F_A(x)}{\inf F_B(x)}, \\ \sup F_C(x) &= \frac{\sup F_A(x)}{\sup F_B(x)}. \\ \text{If } F_A^L > F_B^L \quad \text{and } F_A^U > F_B^U, \text{ then } \inf F_C(x) &= \frac{\inf F_A(x) - \inf F_B(x)}{1 - \inf F_B(x)}, \\ \sup F_C(x) &= \frac{\sup F_A(x) - \sup F_B(x)}{1 - \sup F_B(x)}. \end{aligned}$$

### 2.5. Modified arithmetic operations on interval neutrosophic set

The midpoint of the interval neutrosophic number  $A = \langle [T^L, T^U], [I^L, I^U], [F^L, F^U] \rangle$  is

$$m(A) = \left\langle \frac{T^U + T^L}{2}, \frac{I^U + I^L}{2}, \frac{F^U + F^L}{2} \right\rangle = \langle m_T(A), m_I(A), m_F(A) \rangle$$

Suppose  $A = \langle [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U] \rangle$  and  $B = \langle [T_B^L, T_B^U], [I_B^L, I_B^U], [F_B^L, F_B^U] \rangle$  are two INN's. The modified arithmetic operations on INN are given below.

#### Addition:

$$A + B = \langle \min(m(A) + m(B) - k, 1), \min(m(A) + m(B) + k, 1) \rangle$$

where  $k = \frac{(T_A^U + T_B^U) - (T_A^L + T_B^L)}{2}$  for truth membership function.

$$k = \frac{(I_A^U + I_B^U) - (I_A^L + I_B^L)}{2} \text{ for indeterminacy membership function.}$$

$$k = \frac{(F_A^U + F_B^U) - (F_A^L + F_B^L)}{2} \text{ for falsity membership function.}$$

#### Subtraction:

$$A - B =$$

$\langle \max(0, m(A) - m(B) - k), \max(0, m(A) - m(B) + k) \rangle$  for  $m(A) \geq m(B)$

$\langle \max(0, m(B) - m(A) - k), \max(0, m(B) - m(A) + k) \rangle$  for  $m(A) < m(B)$

where  $k = \frac{(T_A^U + T_B^U) - (T_A^L + T_B^L)}{2}$  for truth membership function.

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$$k = \frac{(I_A^U + I_B^U) - (I_A^L + I_B^L)}{2} \text{ for indeterminacy membership function.}$$

$$k = \frac{(F_A^U + F_B^U) - (F_A^L + F_B^L)}{2} \text{ for falsity membership function.}$$

**Multiplication:**

$$A \times B = \langle m(A)m(B) - k, m(A)m(B) + k \rangle$$

where  $k = \min(m(A)m(B) - \alpha, \beta - m(A)m(B))$   
 $\alpha = \min(T_A^L \cdot T_B^L, T_A^L T_B^U, T_A^U T_B^L, T_A^U \cdot T_B^U)$  and  $\beta = \max(T_A^L \cdot T_B^L, T_A^L T_B^U, T_A^U T_B^L, T_A^U \cdot T_B^U)$ .  
 Similarly, we can find  $\alpha$  and  $\beta$  for indeterminacy and falsity functions.

**Scalar multiplication:**

Let A be an interval neutrosophic set and  $\lambda$  be any scalar, then

$$A = \begin{cases} \langle [\lambda T_A^L, \lambda T_A^U], [\lambda I_A^L, \lambda I_A^U], [\lambda F_A^L, \lambda F_B^U] \rangle \text{ for } \lambda \geq 0 \\ \langle [\lambda T_A^U, \lambda T_A^L], [\lambda I_A^U, \lambda I_A^L], [\lambda F_A^U, \lambda F_B^L] \rangle \text{ for } \lambda < 0 \end{cases}$$

**Division:**

$$A/B = \begin{cases} \langle m(A)/m(B) - k, m(A)/m(B) + k \rangle \text{ for } m(A) \leq m(B) \\ \langle m(B)/m(A) - k, m(B)/m(A) + k \rangle \text{ for } m(A) > m(B) \end{cases}$$

where  $k = \min(m(A)/m(B) - \alpha, \beta - m(A)/m(B))$  for  $m(A) \leq m(B)$   
 $k = \min(m(B)/m(A) - \alpha, \beta - m(B)/m(A))$  for  $m(A) > m(B)$   
 $\alpha = \min(T_A^L \cdot T_B^L, T_A^L T_B^U, T_A^U T_B^L, T_A^U \cdot T_B^U)$  and  $\beta = \max(T_A^L \cdot T_B^L, T_A^L T_B^U, T_A^U T_B^L, T_A^U \cdot T_B^U)$ .  
 Similarly, we can find  $\alpha$  and  $\beta$  for indeterminacy and falsity functions.

**3. Algebraic combination of interval neutrosophic focal elements**

Let  $X_1$  and  $X_2$  be two variables whose values are represented by Dempster- Shafer structure with interval neutrosophic focal elements  $A_1, A_2, A_3, \dots, A_n$  and  $B_1, B_2, B_3, \dots, B_n$  which are considered as intervals and their corresponding Basic Probability Assignments (BPA) are as follows:

$$\mu(A_i) = a_i \text{ and } (B_j) = b_j, \quad i = 1, 2, 3, \dots, n \text{ \& } j = 1, 2, 3, \dots, m$$

where  $\sum_{i=1}^n a_i = 1$  and  $\sum_{j=1}^m b_j = 1$ .

First we combine all the interval neutrosophic focal elements using arithmetic operations which will produce 'nm' number of interval neutrosophic focal elements and there after the corresponding basic probability assignment of resulting interval neutrosophic focal elements will be calculated as follows.

**Addition of interval neutrosophic focal elements:**

$$\mu(A_i + B_j) = \frac{\mu(A_i) + \mu(B_j)}{\sum_i \sum_j \mu(A_i) + \mu(B_j)}$$

**Subtraction of interval neutrosophic focal elements:**

$$\mu(A_i - B_j) = \frac{\mu(A_i)(1 - \mu(B_j))}{\sum_i \sum_j \mu(A_i)(1 - \mu(B_j))}$$

**Multiplication of interval neutrosophic focal elements:**

$$\mu(A_i B_j) = \frac{\mu(A_i)\mu(B_j)}{\sum_i \sum_j \mu(A_i)\mu(B_j)}$$

**Division of interval neutrosophic focal elements:**

$$\mu(A_i/B_j) = \frac{\mu(A_i)/\mu(B_j)}{\sum_i \sum_j \mu(A_i)/\mu(B_j)}$$

**3.1. Basic probability assignment of interval neutrosophic focal elements**

**Numerical example**

Suppose Basic Probability Assignments of two parameters is assigned by an expert and which are given in the following tables:

**BPA of the first parameter**

Interval Neutrosophic Focal Elements	BPA
$\langle [0.2,0.4], [0.3,0.5], [0.3,0.5] \rangle$	<b>0.25</b>
$\langle [0.5,0.7], [0.0,0.2], [0.2,0.3] \rangle$	<b>0.40</b>
$\langle [0.6,0.8], [0.2,0.3], [0.2,0.3] \rangle$	<b>0.35</b>

**BPA of the second parameter:**

Interval Neutrosophic Focal Elements	BPA
$\langle [0.5,0.7], [0.1,0.3], [0.1,0.3] \rangle$	<b>0.17</b>
$\langle [0.2,0.3], [0.2,0.4], [0.5,0.8] \rangle$	<b>0.43</b>
$\langle [0.4,0.6], [0.0,0.1], [0.3,0.4] \rangle$	<b>0.40</b>

**Addition of Interval Neutrosophic Focal Elements and their BPA:**

Adding the Interval neutrosophic focal elements using ordinary and modified arithmetic operations, the resulting interval neutrosophic focal elements and their corresponding BPA are calculated and are given in the following table.

Focal Elements (Ordinary)	Focal Elements (Modified)	BPA
$\langle [0.7,1.0], [0.4,0.8], [0.4,0.8] \rangle$	$\langle [0.7,1.0], [0.4,0.8], [0.4,0.8] \rangle$	0.070
$\langle [0.4,0.7], [0.5,0.9], [0.8,1.0] \rangle$	$\langle [0.4,0.7], [0.5,0.9], [0.8,1.0] \rangle$	0.113
$\langle [0.6,0.1], [0.3,0.6], [0.6,0.9] \rangle$	$\langle [0.6,0.1], [0.3,0.6], [0.6,0.9] \rangle$	0.108
$\langle [0.0,1.0], [0.1,0.5], [0.3,0.6] \rangle$	$\langle [0.0,1.0], [0.1,0.5], [0.3,0.6] \rangle$	0.095
$\langle [0.7,1.0], [0.2,0.6], [0.7,1.0] \rangle$	$\langle [0.7,1.0], [0.2,0.6], [0.7,1.0] \rangle$	0.138
$\langle [0.9,1.0], [0.0,0.3], [0.5,0.7] \rangle$	$\langle [0.9,1.0], [0.0,0.3], [0.5,0.7] \rangle$	0.133
$\langle [1.0,1.0], [0.3,0.6], [0.3,0.6] \rangle$	$\langle [1.0,1.0], [0.3,0.6], [0.3,0.6] \rangle$	0.087
$\langle [0.8,1.0], [0.4,0.7], [0.7,1.0] \rangle$	$\langle [0.8,1.0], [0.4,0.7], [0.7,1.0] \rangle$	0.130
$\langle [1.0,1.0], [0.2,0.4], [0.5,0.7] \rangle$	$\langle [1.0,1.0], [0.2,0.4], [0.5,0.7] \rangle$	0.125

**Subtraction of interval neutrosophic focal elements and their BPA**

Subtracting the Interval neutrosophic focal elements using ordinary and modified arithmetic operations, the resulting interval neutrosophic focal elements and their corresponding BPA are calculated and are given in the following table.

Focal Elements (Ordinary)	Focal Elements (Modified)	BPA
$\langle [0.1,0.3], [0.7,0.9], [0.5,0.7] \rangle$	$\langle [0.0,0.3], [0.0,0.4], [0.0,0.4] \rangle$	0.1038
$\langle [0.2,0.4], [0.6,0.8], [0.3,0.5] \rangle$	$\langle [0.1,0.9], [0.0,0.3], [0.0,0.3] \rangle$	0.0713
$\langle [0.2,0.4], [0.9,1.0], [0.4,0.6] \rangle$	$\langle [0.0,0.2], [0.2,0.5], [0.0,0.3] \rangle$	0.0750
$\langle [0.1,0.3], [0.7,0.8], [0.2,0.3] \rangle$	$\langle [0.0,0.25], [0.0,0.4], [0.0,0.2] \rangle$	0.1660

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$\langle [0.5,0.7], [0.6,0.8], [0.2,0.3] \rangle$	$\langle [0.25,0.5], [0.0,0.4], [0.0,0.2] \rangle$	0.1140
$\langle [0.3,0.4], [0.9,1.0], [0.4,0.6] \rangle$	$\langle [0.1,0.4], [0.0,0.2], [0.1,0.4] \rangle$	0.1200
$\langle [0.1,0.3], [0.7,0.9], [0.5,0.7] \rangle$	$\langle [0.3,0.7], [0.0,0.2], [0.2,0.5] \rangle$	0.1453
$\langle [0.5,0.8], [0.6,0.8], [0.2,0.3] \rangle$	$\langle [0.0,0.35], [0.0,0.2], [0.0,0.2] \rangle$	0.0998
$\langle [0.3,0.4], [0.9,1.0], [0.4,0.6] \rangle$	$\langle [0.2,0.5], [0.0,0.4], [0.1,0.4] \rangle$	0.1050

**Multiplication of interval neutrosophic focal elements and their BPA :**

Multiplying the Interval neutrosophic focal elements using ordinary and modified arithmetic operations, the resulting interval neutrosophic focal elements and their corresponding BPA are calculated and are given in the following table.

Focal Elements (Ordinary)	Focal Elements (Modified)	BPA
$\langle [0.60,0.82], [0.03,0.15], [0.03,0.15] \rangle$	$\langle [0.10,0.26], [0.03,0.13], [0.03,0.13] \rangle$	0.0425
$\langle [0.30,0.42], [0.03,0.20], [0.15,0.40] \rangle$	$\langle [0.04,0.11], [0.06,0.18], [0.15,0.37] \rangle$	0.1075
$\langle [0.52,0.76], [0.0,0.05], [0.09,0.20] \rangle$	$\langle [0.08,0.22], [0.0,0.04], [0.09,0.19] \rangle$	0.1000
$\langle [0.75,0.91], [0.0,0.06], [0.02,0.09] \rangle$	$\langle [0.25,0.47], [0.0,0.04], [0.02,0.08] \rangle$	0.0680
$\langle [0.60,0.79], [0.0,0.08], [0.10,0.24] \rangle$	$\langle [0.10,0.20], [0.0,0.06], [0.10,0.22] \rangle$	0.1720
$\langle [0.70,0.88], [0.0,0.02], [0.06,0.12] \rangle$	$\langle [0.20,0.40], [0.0,0.01], [0.06,0.11] \rangle$	0.1600
$\langle [0.80,0.94], [0.02,0.09], [0.02,0.09] \rangle$	$\langle [0.30,0.54], [0.02,0.08], [0.02,0.08] \rangle$	0.0595
$\langle [0.68,0.86], [0.04,0.12], [0.10,0.24] \rangle$	$\langle [0.13,0.23], [0.05,0.11], [0.1,0.23] \rangle$	0.1505
$\langle [0.76,0.92], [0.0,0.03], [0.03,0.12] \rangle$	$\langle [0.24,0.46], [0.0,0.03], [0.06,0.12] \rangle$	0.1400

**Division of interval neutrosophic focal elements and their BPA**

Dividing the Interval neutrosophic focal elements using ordinary and modified arithmetic operations, the resulting interval neutrosophic focal elements and their corresponding BPA are calculated and are given in the following table.

Focal Elements (Ordinary)	Focal Elements (Modified)	BPA
$\langle [0.40,0.60], [0.20,0.30], [0.20,0.30] \rangle$	$\langle [0.20,0.80], [0.20,0.80], [0.20,0.80] \rangle$	0.1378
$\langle [0.0,0.30], [0.13,0.16], [0.60,0.62] \rangle$	$\langle [0.60,1.00], [0.50,1.00], [0.40,0.80] \rangle$	0.0545
$\langle [0.50,0.66], [0.30,0.40], [0.0,0.16] \rangle$	$\langle [0.30,0.90], [0.00,0.25], [0.75,0.90] \rangle$	0.0585
$\langle [0.00,1.00], [0.00,0.60], [0.10,1.00] \rangle$	$\langle [0.70,1.00], [0.00,1.00], [0.30,1.00] \rangle$	0.2205
$\langle [0.30,0.60], [0.00,0.50], [0.60,0.71] \rangle$	$\langle [0.30,0.50], [0.00,0.60], [0.30,0.60] \rangle$	0.8718
$\langle [0.20,0.30], [0.00,0.10], [0.70,0.80] \rangle$	$\langle [0.60,1.00], [0.00,1.00], [0.50,0.90] \rangle$	0.0937
$\langle [0.20,0.30], [0.10,1.00], [0.10,1.00] \rangle$	$\langle [0.60,1.00], [0.30,1.00], [0.30,1.00] \rangle$	0.1929
$\langle [0.50,0.70], [0.00,0.70], [0.30,0.70] \rangle$	$\langle [0.25,0.60], [0.50,1.00], [0.30,0.60] \rangle$	0.7628
$\langle [0.30,0.50], [0.00,0.30], [0.60,0.80] \rangle$	$\langle [0.50,0.90], [0.00,0.40], [0.40,1.00] \rangle$	0.0785

**4. Conclusion**

Evidence theory can handle both aleatory and epistemic uncertainty. Three important functions in evidence theory are the basic probability assignment function, Belief function and Plausibility function which are used to quantify the given variable. However, due to the presence of uncertainty, focal elements can sometimes be treated as interval neutrosophic numbers. As already stated in this paper interval neutrosophic focal elements and their BPA of three variables are combined by arithmetic operations. From

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the numerical example, it is observed that modified arithmetic operations have the advantages of simple calculations and high accuracy in multiplication and division. Those results are promising and interesting as it being addressed for the first time.

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