Research Article

A New Single-Valued Neutrosophic Rough Sets and Related Topology

Qiu Jin, Kai Hu, Chunxin Bo, and Lingqiang Li

School of Mathematical Sciences, Liaocheng University, Liaocheng 252059, China

Correspondence should be addressed to Kai Hu; hukai80@126.com and Lingqiang Li; lilingqiang@lcu.edu.cn

Received 1 March 2021; Revised 19 June 2021; Accepted 5 July 2021; Published 17 July 2021

1. Introduction

The original notion of neutrosophic set was proposed by Smarandache [1]. For the convenience of application, Wang et al. [2] investigated the single-valued neutrosophic set (Svns). In Svns, three independent membership functions (truth, indeterminacy, and falsity) are considered; hence, it can be regarded as extensions of fuzzy set [3] and intuitionistic fuzzy set [4]. There are many works on the theory and application of Svns (see Abdel-Basset [5], Ye [6, 7], Samant [8], Yang [9, 10], Zhang [11–13], Zavadskas [14], and Xu [15] as well as Peng’s review paper [16]).

The fusion of neutrosophic sets with rough sets theory [17] is an important research direction. According to Li’s review paper [18], there exists two fundamental combinations of rough sets and neutrosophic sets: Broumi’s rough neutrosophic sets [19] and Sweety’s neutrosophic rough sets [20]. Many other models can be regarded as their extensions [12, 21–24].

(i) Broumi’s rough neutrosophic sets [19]: let R be an equivalent relation (can be easily extended for an arbitrary binary relation) on U. Then, for each neutrosophic set A on U, a pair of neutrosophic sets \( \overline{R}(A) \) and \( \overline{R}(A) \) on U are defined as the lower and upper approximations of A w.r.t. (U, R).

(ii) Sweety’s neutrosophic rough sets [20]: let R be a neutrosophic relation on U. Then, for each neutrosophic set A on U, a pair of neutrosophic sets \( \overline{R}(A) \) and \( \overline{R}(A) \) on U are defined as the lower and upper approximations of A w.r.t. (U, R). Yang [10] defined a similar model by considering the single-valued neutrosophic relation and single-valued neutrosophic set on U.

In this paper, we shall introduce a new model of rough sets fusion with neutrosophic sets under the framework of single-valued neutrosophic approximation space (U, R) (i.e., a nonempty set U together with a single-valued neutrosophic relation R on U). For each ordinary subset A of U, we shall...
define a pair of single-valued neutrosophic sets $R(A)$ and $\bar{R}(A)$ on $U$ as the lower and upper approximations of $A$ with respect to $(U, R)$. Obviously, our model is different from Broumi–Sweety–Yang’s models, since, in our model, the original sets are ordinary subsets of $U$ and their approximations are single-valued neutrosophic sets, but, in Broumi–Sweety–Yang’s models, the original sets and their approximations are all (single-valued) neutrosophic sets. Hence, our rough sets will be called ordinary single-valued neutrosophic rough sets.

(Fuzzy) rough sets are closely related to (fuzzy) topology [25–42]. The well-known result may be that there is a one-to-one correspondence between reflexive and transitive (fuzzy) approximation spaces and quasidiscrete (fuzzy) topological spaces [26, 37, 38]. Under the framework of single-valued neutrosophic sets, two kinds of neutrosophic topological spaces are discussed (for more general neutrosophic topology, refer to Al-Omeri [43] and Lupianez [44]).

(i) Yang’s single-valued neutrosophic topological spaces [45]: for a nonempty set $U$, Yang defined the single-valued neutrosophic topology on $U$ as a subset $\tau$ of $\text{Svns}(U)$ (the set of all single-valued neutrosophic sets on $U$) with some conditions. Yang’s space can be regarded as an extension of Lowen’s fuzzy topological space [46]. Yang also proved that there is a one-to-one correspondence between reflexive and transitive single-valued neutrosophic approximation spaces and his single-valued neutrosophic rough topological spaces.

(ii) Kim’s ordinary single-valued neutrosophic topological spaces [47]: for a nonempty set $U$, Kim defined the ordinary single-valued neutrosophic topology on $U$ as a neutrosophic set $\tau$ on $P(U)$ (the power set of $U$) with some conditions. Kim’s space can be regarded as an extension of Sostak’s fuzzy topology [48] (or Ying’s fuzzifying topology [49]).

In this paper, we shall prove that there are close relationships between our ordinary single-valued neutrosophic rough sets and Kim’s ordinary single-valued neutrosophic topological spaces. The close relationships exhibit that it is meaningful to investigate the new rough sets model.

The method of this paper and the comparison with related literature can be summarized in Table 1.

The remainder of this paper is organized as follows. In Section 2, we will recall some knowledge about neutrosophic sets and rough sets. In Section 3, we shall give the notion of ordinary single-valued neutrosophic upper and lower approximation operators and discuss their properties. Then we will explore the further properties of the proposed approximations corresponding to reflexive (transitive) single-valued neutrosophic approximation space. In Section 4, we will prove that each single-valued neutrosophic approximation space induces an ordinary single valued neutrosophic topological space via our defined lower approximation. In Section 5, we shall verify that each ordinary single-valued neutrosophic topological space induces a single-valued neutrosophic approximation space. In Section 6, we will show that there is a one-to-one correspondence between transitive single-valued neutrosophic approximation spaces and quasidiscrete ordinary single-valued neutrosophic topological spaces.

2. Preliminaries

In this section, we recall some knowledge about neutrosophic rough sets and neutrosophic topologies used in this paper.

Unless otherwise stated, we always assume that $U$ is a nonempty infinite set. We denote $P(U)$ as the power set of $U$ and define $A^c = U - A$ for $A \in P(U)$.

Definition 1 (see [2]). An $\text{Svns} = (A_T, A_I, A_F)$ on $U$ is defined as three membership functions $A_T, A_I, A_F$: $U \rightarrow [0, 1]$, which are interpreted as truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively. All $\text{Svns}$ are denoted by $\text{Svns}(U)$.

Each $\alpha = (\alpha_1, \alpha_2, \alpha_3) \in [0, 1]^3$ is called a single-valued neutrosophic number, and its complement is defined as $\alpha^c = (\alpha_3, 1 - \alpha_2, \alpha_1)$. We denote the single-valued neutrosophic numbers $\top = (1, 0, 0)$ and $\bot = (0, 1, 1)$. Obviously, $\top^c = \bot$ and $\bot^c = \top$.

Remark 1. Pythagorean fuzzy set [50] is also an important extension of intuitionistic fuzzy set. We can observe that when restricting $0 \leq (A_T(x))^2 + (A_F(x))^2 \leq 1$ and $A_T(x) = \sqrt{1 - (A_T(x))^2 - (A_F(x))^2}$, an $\text{Svns}$ becomes a Pythagorean fuzzy set.

For $A \in P(U)$, we define $\tau_A \in \text{Svns}(U)$ as follows: $\forall x \in U, \tau_A(x) = \top$ if $x \in A$ and $\tau_A(x) = \bot$ if $x \notin A$.

Definition 2 (see [2, 6, 10]). Let $A, B, A_j (j \in J) \in \text{Svns}(U)$.

1. We denote $A \subseteq B$ if, for any $x \in U$, $A_T(x) \leq B_T(x)$, $A_I(x) \geq B_I(x)$, and $A_F(x) \geq B_F(x)$. By $= B$, we mean $A \subseteq B$ and $B \subseteq A$.

2. We define $A^c \in \text{Svns}(U)$ as $\forall x \in U, A^c(x) = (A(x))^c = (A_T(x), 1 - A_I(x), A_T(x))$.

3. We define $\cup_{j \in J} A_j, \cap_{j \in J} A_j \in \text{Svns}(U)$ by $\forall x \in U$, $(\cup_{j \in J} A_j)(x) = \vee_{j \in J} (A_j)(x)$, $(\cap_{j \in J} A_j)(x) = \wedge_{j \in J} (A_j)(x)$.

(1)
Rough set

<table>
<thead>
<tr>
<th>Rough neutrosophic sets [19]</th>
<th>R</th>
<th>A</th>
<th>( \bar{R}(A), \bar{R}(A) )</th>
<th>Topology ( \tau_B )</th>
<th>Bijection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our neutrosophic rough sets</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>Yang's topology [45]</td>
<td>√</td>
</tr>
</tbody>
</table>

Notes: “+” represents that the set is a single-valued neutrosophic set, and “−” represents that it is not; “√” represents that there is a bijection between the considered rough sets and topologies, and “×” represents that there is no bijection.

\[(i)\] Reflexive if \( \forall x \in U, \ R(x, x) = 1, R_f(x, x) = 0, R_f(x, x) = 0 \]

\[(ii)\] Transitive if \( R_f, R_f, R_f \) are all transitive fuzzy relations, that is, \( \forall x, y, z \in U \)

\[
R_f(x, y) \land R_f(y, z) \leq R_f(x, z), R_f(x, y) \lor R_f(y, z) \geq R_f(x, z), R_f(x, y) \lor R_f(y, z) \geq R_f(x, z). \tag{2}
\]

\[\text{The pair } (U, \tau) \text{ is said to be an ordinary single-valued neutrosophic topological space (\text{OSvnts}).}\]

\[\text{For examples and more results about \text{OSvnts}, refer to [47].}\]

The following lemma can be easily observed. We will use it without mentioning again.

\[\text{Lemma 1. Let } a, b \in [0, 1]. \text{ Then the following conditions are equivalent:}\]

\[(1)\] \( a \leq b \)
\[(2)\] For all \( y \in [0, 1], y \leq a \Rightarrow y \leq b \)
\[(3)\] For all \( y \in [0, 1], y \leq a \Rightarrow y \leq b \)
\[(4)\] For all \( y \in [0, 1], y \leq a \Rightarrow y \leq b \)
\[(5)\] For all \( y \in [0, 1], y \leq a \Rightarrow y \leq b \)

\[\text{3. Ordinary Single-Valued Neutrosophic Rough Sets for \text{Svns}}\]

In this section, we present the notions and properties of ordinary single-valued neutrosophic upper and lower approximation operators.

\[\text{Definition 6. Let } (U, R) \text{ be an } \text{Svns}. \text{ For } A \in P(U), \text{ the upper and lower approximations of } A, \text{ denoted by } \bar{R}(A), \bar{R}(A) \in \text{Svns}(U), \text{ are defined as follows: } \forall x \in U, \]

\[
\bar{R}(A)_T(x) = \bigvee_{y \in A} R_T(x, y), \\
\bar{R}(A)_I(x) = \bigwedge_{y \in A} R_I(x, y), \\
\bar{R}(A)_F(x) = \bigvee_{y \in A} R_F(x, y). \tag{5}
\]

\[\text{for any } A \in P(U).\]
The pair \((\mathcal{R}(A), \overline{\mathcal{R}}(A))\) is referred to the ordinary single-valued neutrosophic rough sets of \(A\). \(\mathcal{R}\) and \(\overline{\mathcal{R}}\) are said to be the ordinary single-valued neutrosophic upper and lower approximation operators, respectively.

Remark 2

(1) The definition of \(\overline{\mathcal{R}}(A^r_T)(x)\) is an interpretation of the fact that "the join of \(R_T^r(x)\) and \(A\) is not empty," and the definition of \(\overline{\mathcal{R}}(A^r_T)(x)\) is an interpretation of the fact that "\(R_T^r(x)\) is contained in \(A\) (or equivalent, \(A^c\) is contained in \(R_T^r(x)\))."

(2) For a fuzzy relation \(r\) on \(U\), it is easily observed that \(r\) induces an \(\text{Synr}\) \(R_r\) on \(U\) defined as follows: \(\forall (x, y) \in U \times U, (R_r)^T(x, y) = r(x, y), (R_r)^U(x, y) = 0, (R_r)^F(x, y) = 1 - r(x, y)\). For \(A \in P(U)\), we have \(\overline{\mathcal{R}}_r^T = \bigwedge_{y \in A} \overline{\mathcal{R}}_r^T(x, y) = \overline{\mathcal{R}}(A)\), \(\overline{\mathcal{R}}_r^U = \bigvee_{y \notin A} \overline{\mathcal{R}}_r^U(x, y)\) and \(\overline{\mathcal{R}}_r^F\) are the fuzzy approximations of ordinary subset w.r.t. fuzzy relation in the work of Yao [51]. Therefore, the single-valued neutrosophic approximations in this paper are a generalization of Yao’s fuzzy approximations.

(3) Obviously, the single-valued neutrosophic approximation operators in this paper are different from the single-valued neutrosophic approximation operators in the work of Yang [10], since our operators are defined from \(P(U)\) to \(\text{Syns}(U)\) and Yang’s operators are defined from \(\text{Syns}(U)\) to \(\text{Syns}(U)\).

Example 1. Let \((U, R)\) be an \(\text{Synas}\) with \(U = \{x_1, x_2, x_3\}\) and let \(R\) be defined as in Table 2.

Taking \(A = \{x_1, x_2\}\), we have
\[
\begin{align*}
\mathcal{R}(A)^r_T(x_1) &= R_T(x_1, x_1) = 0.4, \\
\mathcal{R}(A)^r_I(x_1) &= 1 - R_I(x_1, x_3) = 1, \\
\mathcal{R}(A)^r_F(x_1) &= R_F(x_1, x_1) = 1, \\
\overline{\mathcal{R}}(A)^r_T(x_2) &= R_T(x_2, x_2) = 1, \\
\overline{\mathcal{R}}(A)^r_I(x_2) &= 1 - R_I(x_2, x_3) = 1, \\
\overline{\mathcal{R}}(A)^r_F(x_2) &= R_F(x_2, x_2) = 0.6, \\
\mathcal{R}(A)^r_T(x_3) &= R_T(x_3, x_3) = 0, \\
\mathcal{R}(A)^r_I(x_3) &= 1 - R_I(x_3, x_3) = 1, \\
\mathcal{R}(A)^r_F(x_3) &= R_F(x_3, x_3) = 1,
\end{align*}
\]

\[
\begin{align*}
\overline{\mathcal{R}}(A)^r_T(x_1) &= R_T(x_1, x_1) \lor R_T(x_1, x_2) = 0.3 \lor 0.3 = 0.6, \\
\overline{\mathcal{R}}(A)^r_I(x_1) &= R_I(x_1, x_1) \land R_I(x_1, x_2) = 0 \land 0 = 0, \\
\overline{\mathcal{R}}(A)^r_F(x_1) &= R_F(x_1, x_1) \land R_F(x_1, x_2) = 1 \land 0.6 = 0.6, \\
\overline{\mathcal{R}}(A)^r_T(x_2) &= R_T(x_2, x_2) \lor R_T(x_2, x_3) = 0.6 \lor 0.6 = 0.6, \\
\overline{\mathcal{R}}(A)^r_I(x_2) &= R_I(x_2, x_1) \land R_I(x_2, x_2) = 0.2 \land 0.5 = 0.2, \\
\overline{\mathcal{R}}(A)^r_F(x_2) &= R_F(x_2, x_1) \land R_F(x_2, x_2) = 0 \land 0.4 = 0.4, \\
\overline{\mathcal{R}}(A)^r_T(x_3) &= R_T(x_3, x_3) \lor R_T(x_3, x_2) = 1 \lor 1 = 1, \\
\overline{\mathcal{R}}(A)^r_I(x_3) &= R_I(x_3, x_1) \land R_I(x_3, x_2) = 0 \land 0 = 0, \\
\overline{\mathcal{R}}(A)^r_F(x_3) &= R_F(x_3, x_1) \land R_F(x_3, x_2) = 1 \land 1 = 1.
\end{align*}
\]

Hence, we obtain \(\mathcal{R}(A)\) and \(\overline{\mathcal{R}}(A)\) as in Table 3.

Theorem 1. Let \((U, R)\) be an \(\text{Synas}\). Then we have the following:

(1) \(\mathcal{R}(U) = \tau_U; \overline{\mathcal{R}}(\emptyset) = \tau_\emptyset\)

(2) If \(A \subseteq B\), then \(\mathcal{R}(A) \subseteq \mathcal{R}(B)\) and \(\overline{\mathcal{R}}(A) \subseteq \overline{\mathcal{R}}(B)\)

(3) For all \(A_j (j \in J) \in P(U)\), \(\mathcal{R}(\bigcap_{j \in J} A_j) = \bigcap_{j \in J} \mathcal{R}(A_j)\)

(4) For \(A \in P(U)\), \(\mathcal{R}(A) = (\mathcal{R}(A^c))^c\) and \(\overline{\mathcal{R}}(A) = (\overline{\mathcal{R}}(A^c))^c\)

Proof. For (1)–(3), we prove only the results for lower approximation. The proofs for upper approximation are similar and hence are omitted.

(1) For any \(x \in U\), we have \(\mathcal{R}(U)^r_T(x) = \bigwedge_{y \in U} R_F(x, y) = 0, \mathcal{R}(U)^r_I(x) = \bigvee_{y \notin U} (1 - R_I(x, y)) = 0, \mathcal{R}(U)^r_F(x) = \bigvee_{y \in U} R_F(x, y) = \tau_U\).

(2) For any \(x \in U\) and \(A \subseteq B\), we obtain

\[
\begin{align*}
\mathcal{R}(A)^r_T(x) &= \bigwedge_{y \in A} R_F(x, y) \leq \bigwedge_{y \notin B} R_F(x, y) = \mathcal{R}(B)^r_T(x), \\
\mathcal{R}(A)^r_I(x) &= \bigvee_{y \in A} (1 - R_I(x, y)) \geq \bigvee_{y \notin B} (1 - R_I(x, y)) = \mathcal{R}(B)^r_I(x), \\
\mathcal{R}(A)^r_F(x) &= \bigvee_{y \in A} R_F(x, y) \geq \bigvee_{y \notin B} R_F(x, y) = \mathcal{R}(B)^r_F(x).
\end{align*}
\]
Theorem 2. Let \((U, R)\) be an Svnas. Then the following three are equivalent:

1. \(R\) is reflexive
2. \(R(A) \subseteq \top_A\) for each \(A \in P(U)\)
3. For any \(x \in U\),

\[
\left( \bigcap_{j \in J} \mathcal{R}(A_j) \right)_T(x) = \bigwedge_{j \in J} \mathcal{R}(A_j)_T(x)
\]

\[
\left( \bigcap_{j \in J} \mathcal{R}(A_j) \right)_I(x) = \bigvee_{j \in J} \mathcal{R}(A_j)_I(x)
\]

\[
\left( \bigcap_{j \in J} \mathcal{R}(A_j) \right)_F(x) = \bigvee_{j \in J} \mathcal{R}(A_j)_F(x)
\]

Hence, \(\mathcal{R}(\bigcap_{j \in J} A_j) = \bigcap_{j \in J} \mathcal{R}(A_j)\).

\[
\left( R(A^c) \right)_T(x) = \bigvee_{y \in A} R_T(x, y) = \mathcal{R}(A)_T(x),
\]

\[
\left( R(A^c) \right)_I(x) = 1 - \left( R(A^c) \right)_I(x) = \bigwedge_{y \in A} R_I(x, y) = \mathcal{R}(A)_I(x),
\]

\[
\left( R(A^c) \right)_F(x) = \left( R(A^c) \right)_F(x) = \bigvee_{y \in A} R_F(x, y) = \mathcal{R}(A)_F(x).
\]

Hence, \(\mathcal{R}(A) = (\mathcal{R}(A^c))^c\). That is, \(\mathcal{R}(A) = (\mathcal{R}(A^c))^c\)

\[
\top_A \subseteq \mathcal{R}(A)\text{ for each } A \in P(U)
\]

Proof. (1) \(\Rightarrow\) (2). If \(x \in A\), then

\[
R_T(A)(x) = (\top_A)_T(x) = 1,
\]

\[
R_I(A)(x) \geq (\top_A)_I(x) = 0,
\]

\[
R_F(A)(x) \geq (\top_A)_F(x) = 0.
\]

(10)

(2) \(\Rightarrow\) (1). For any \(x \in U\), by (2), we have

\[
R_T(A)(x) = \bigwedge_{y \notin A} R_F(x, y) \leq R_F(x, x)^{\uparrow} = 0 = (\top_A)_F(x),
\]

\[
R_I(A)(x) = \bigvee_{y \notin A} (1 - R_I(x, y)) \geq 1 - R_I(x, x)^{\uparrow} = 1 - 0 = 1 = (\top_A)_I(x),
\]

\[
R_F(A)(x) = \bigvee_{y \notin A} R_T(x, y) \geq R_T(x, x)^{\downarrow} = 1 = (\top_A)_F(x).
\]

Hence, \(\mathcal{R}(A) \subseteq \top_A\).
Theorem 3. Let \((U, R)\) be an Svnas. Then the following three are equivalent:

1. \(R\) is transitive.
2. For each \(A \in P(U)\) and \(x \in U\),
   \[
   R(A)_T(x) = \bigvee_{B \subseteq A} \left( R(B)_T(x) \bigwedge \bigwedge_{y \in B} R(B)_T(y) \right),
   \]
   \[
   R(A)_F(x) = \bigwedge_{B \subseteq A} \left( R(B)_F(x) \bigvee \bigvee_{y \in B} R(B)_F(y) \right),
   \]
3. For each \(A \in P(U)\) and \(x \in U\),
   \[
   \overline{R(A)}_T(x) = \bigwedge_{B \subseteq A} \left( \overline{R(B)}(x) \bigvee \bigvee_{y \in B} \overline{R(B)}(y) \right),
   \]
   \[
   \overline{R(A)}_F(x) = \bigvee_{B \subseteq A} \left( \overline{R(B)}_F(x) \bigwedge \bigwedge_{y \notin B} \overline{R(B)}_F(y) \right).
   \]

Proof. \((1) \Rightarrow (2)\). Let \(A \in P(U)\) and \(x \in U\).

1. For any \(B \subseteq A\), we have \(R(B)_T(x) \leq R(A)_T(x)\) and so
   \[
   \bigvee_{B \subseteq A} \left( R(B)_T(x) \bigwedge \bigwedge_{y \in B} R(B)_T(y) \right) \leq \bigvee_{B \subseteq A} \left( R(B)_T(x) \right) = R(A)_T(x).
   \]
2. Conversely, let \(\alpha = R(A)_T(x) = \bigvee_{y \in A} R_F(x, y);\) then \(R_F(x, y) \geq \alpha\) for any \(y \notin A\). Take \(B_x = \{z \in U\mid R_F(x, z) < \alpha\}\); then \(B_x \subseteq A\). It follows that
   \[
   \bigwedge_{y \in B} R_F(x, y) \geq \bigwedge_{y \notin B} R_F(x, y, z) \geq \alpha = R(A)_T(x).
   \]

Note that, for any \(y \in B_x,\ z \notin B_x\), \(1 \leq R_F(x, y, z) \geq \alpha\). Since \(R\) is transitive, we have \(R_F(x, y) \forall \forall R_F(x, z) \geq \alpha\), which means that \(R_F(x, z) \geq \alpha\). So,
   \[
   \bigwedge_{y \in B_x} R_F(x, y) \geq \bigwedge_{y \notin B_x} R_F(x, y, z) \geq \alpha = R(A)_T(x).
   \]

Hence,
\[
\bigvee_{B \subseteq A} \left( R(B)_T(x) \bigwedge \bigwedge_{y \in B} R(B)_T(y) \right) = R(A)_T(x).
\]

(ii) For any \(B \subseteq A\), we have \(R(B)_T(x) \geq R(A)_T(x)\) and so
   \[
   \bigwedge_{B \subseteq A} \left( R(B)_T(x) \bigvee \bigvee_{y \in B} R(B)_T(y) \right) \leq \bigwedge_{B \subseteq A} \left( R(B)_T(x) \right) = R(A)_T(x).
   \]

Conversely, let \(\alpha = R(A)_T(x) = \bigvee_{y \in A} R_F(x, y);\) then \(1 \leq R_F(x, y) \forall \forall \alpha\) for any \(y \notin A\). Take \(B_x = \{z \in U\mid 1 - R_F(x, y) > \alpha\}\); then \(B_x \subseteq A\). It follows that
   \[
   R(B_x)_T(x) = \bigwedge_{y \in B_x} R(B)(x, y) \leq \alpha \leq \bigwedge_{y \notin B_x} R(B)(x, y) \leq \bigwedge_{B \subseteq A} \left( R(B)_T(x) \right).
   \]

Note that, for any \(y \in B_x,\ z \notin B_x\), \(1 \leq R_F(x, y, z) \leq \alpha\). Since \(R\) is transitive, we have \(1 \leq \bigvee_{y \in B_x} R_F(x, y) \forall \forall \left( 1 - R_F(x, y) \right) \leq \alpha\), which means that \(1 \leq \alpha\). So,
\[ \bigvee_{y \in B_x} R(B_x)_f(y) = \bigvee_{y \in B_x, z \notin B_x} (1 - R_f(y, z)) \leq \alpha, \quad (22) \]

and then
\[
\bigwedge_{B \subseteq A} \left( \bigvee_{y \in B} R(B)_f(x) \bigwedge \bigvee_{y \in B} R(B)_f(y) \right) \leq R(B)_f(x) \bigwedge \bigvee_{y \in B} R(B)_f(y) \leq \alpha = R(A)_f(x). \]

Hence,
\[
\bigwedge_{B \subseteq A} \left( \bigvee_{y \in B} R(B)_f(x) \bigwedge \bigvee_{y \in B} R(B)_f(y) \right) = R(A)_f(x). \quad (24) \]

(iii) For any \(B \subseteq A\), we have \(R(B)_f(x) \geq R(A)_f(x)\) and so
\[
\bigwedge_{B \subseteq A} \left( \bigvee_{y \in B} R(B)_f(x) \bigwedge \bigvee_{y \in B} R(B)_f(y) \right) \geq \bigwedge_{B \subseteq A} \left( \bigvee_{y \in B} R(B)_f(x) \right) = R(A)_f(x). \quad (25) \]

Conversely, let \(\alpha = R(A)_f(x) = \bigvee_{y \in A} R_T(x, y)\); then \(R_T(x, y) \leq \alpha\) for any \(y \notin A\). Take \(B_x = \{ z \in U | R_T(x, z) > \alpha \}\); then \(B_x \subseteq A\). It follows that
\[
R(B)_f(x) = \bigvee_{y \notin B_x} R_T(x, y) \leq \alpha, \quad (26) \]
\[
\bigvee_{y \in B_x} R(B)_f(x) = \bigvee_{y \in B_x, z \notin B_x} R_T(y, z). \quad (27) \]

Note that, for any \(y \in B_x, z \notin B_x\), we have \(R_T(x, y) > \alpha\). Since \(R\) is transitive, we have \(R_T(x, y) \land \neg R_f(y, z) \leq R_T(x, z) \leq \alpha\), which means that \(R_T(y, z) \leq \alpha\). So,
\[
\bigwedge_{B \subseteq A} \left( \bigvee_{y \in B} R(B)_f(x) \bigwedge \bigvee_{y \in B} R(B)_f(y) \right) \leq \bigwedge_{B \subseteq A} \left( \bigvee_{y \in B} R(B)_f(x) \right) = R(A)_f(x). \quad (28) \]

Hence,
\[
\bigwedge_{B \subseteq A} \left( \bigvee_{y \in B} R(B)_f(x) \bigwedge \bigvee_{y \in B} R(B)_f(y) \right) = R(A)_f(x). \quad (29) \]

(2) \(\Rightarrow\) (1). Let \(x, y, z \in U\).

(i) Note that
\[
R_T(x, z) = R(U - \{z\})_f(x) \quad \text{(2)} \]
\[
\bigwedge_{A \subseteq U - \{z\}} \left( \bigvee_{u \in A} R(A)_f(x) \bigwedge \bigvee_{v \in A} R(A)_f(u) \right). \quad (30) \]

Take any \(A \subseteq U - \{z\}\); then \(y \in A\) or \(y \notin A\).

Case 1: if \(y \in A\), then
\[
R(A)_f(x) \bigvee \bigwedge_{u \in A} R(A)_f(u) \geq \bigvee_{u \in A} R(A)_f(u), \text{ by } y \in A \]
\[
= R(A)_f(y) \bigvee \bigwedge_{u \in A} R(A)_f(u), \text{ by } A \subseteq U - \{z\} \]
\[
\geq \bigwedge_{C \subseteq U - \{z\}} \left( \bigvee_{C} R(C)_f(y) \bigwedge \bigvee_{u \in C} R(C)_f(u) \right) \bigwedge \bigwedge_{C \subseteq U - \{z\}} \left( \bigvee_{C} R(C)_f(y) \bigwedge \bigvee_{u \in C} R(C)_f(u) \right). \quad (31) \]

Case 2: if \(y \notin A\), then \(A \subseteq U - \{y\}\) and so
\[
R(A)_f(x) \bigvee \bigwedge_{u \in A} R(A)_f(u), \text{ by } A \subseteq U - \{y\} \]
\[
\geq \bigwedge_{B \subseteq U - \{y\}} \left( \bigvee_{B} R(B)_f(x) \bigwedge \bigvee_{v \in B} R(B)_f(v) \right) \bigwedge \bigwedge_{B \subseteq U - \{y\}} \left( \bigvee_{B} R(B)_f(x) \bigwedge \bigvee_{v \in B} R(B)_f(v) \right). \quad (32) \]

By a combination of Cases 1 and 2, we obtain
\[
\bigwedge_{A \subseteq U - \{z\}} \left( \bigvee_{B \subseteq U - \{z\}} R(A)_f(x) \bigwedge \bigvee_{u \in A} R(A)_f(u) \right) \bigwedge \bigwedge_{B \subseteq U - \{y\}} \left( \bigvee_{B} R(B)_f(x) \bigwedge \bigvee_{v \in B} R(B)_f(v) \right) \bigwedge \bigwedge_{C \subseteq U - \{z\}} \left( \bigvee_{C} R(C)_f(y) \bigwedge \bigvee_{u \in C} R(C)_f(u) \right), \quad (33) \]

that is, \(R_T(x, z) \geq R_T(x, y) \land R_T(y, z)\), as desired.

(ii) Note that
1 − R₁(x, z) = R(U − {z})₁(x)  \tag{2}

\begin{align*}
&= \bigwedge_{A \subseteq U − {z}} \left( R\left(A\right)_₁(x) \right) \bigvee_{u \in A} \left( R\left(A\right)_₁(u) \right), \\
&= R(U − {y})₁(x)  \tag{2}

\begin{align*}
&= \bigwedge_{B \subseteq U − {y}} \left( R\left(B\right)_₁(x) \right) \bigvee_{v \in B} \left( R\left(B\right)_₁(v) \right),

&= R(U − {z})₁(y)
\end{align*}

\begin{align*}
&= \bigwedge_{C \subseteq U − {z}} \left( R\left(C\right)_₁(y) \right) \bigvee_{w \in C} \left( R\left(C\right)_₁(w) \right).
\end{align*}

(34)

By a combination of Cases 1 and 2, we obtain

\begin{align*}
&= \bigvee_{A \subseteq U − {z}} \left( R\left(A\right)_₁(x) \right) \bigwedge_{u \in A} \left( R\left(A\right)_₁(u) \right),

&\leq \bigwedge_{B \subseteq U − {y}} \left( R\left(B\right)_₁(x) \right) \bigvee_{v \in B} \left( R\left(B\right)_₁(v) \right)

&\text{scale}190\%\bigvee_{C \subseteq U − {z}} \left( R\left(C\right)_₁(y) \right) \bigwedge_{w \in C} \left( R\left(C\right)_₁(w) \right),
\end{align*}

that is, \( R_F(x, z) \leq R_F(x, y) \vee R_F(y, z) \), as desired.

From (i)–(iii), we know that \( R \) is transitive.

(2) \(\implies\) (3). It can be concluded from Theorem 1 (4). \(\Box\)

(3)

4. Ordinary Single-Valued Neutrosophic Topological Space Induced by Single-Valued Neutrosophic Approximation Space

In this section, we shall consider the \textit{OSvnt} induced by \textit{Svns} through the ordinary single-valued neutrosophic lower approximation operator.

At first, we fix a subclass of ordinary single-valued neutrosophic topological spaces.

\textbf{Definition 7.} An \textit{OSvnts} \((U, \tau)\) is said to be quasidiscrete if it fulfills the following:

\begin{align*}
\text{(OSvnt2s)} &\text{for any } A_j \in P(U) \text{ (} j \in J \text{)},

&\tau_T(\bigcap_{j \in J} A_j) \supseteq \bigwedge_{j \in J} \tau_T(A_j),

&\tau_T(\bigcap_{j \in J} A_j) \supseteq \bigvee_{j \in J} \tau_T(A_j),

&\tau_T(\bigcap_{j \in J} A_j) \supseteq \bigvee_{j \in J} \tau_T(A_j).
\end{align*}

(39)

It is not difficult to see that quasidiscrete \textit{OSvnts} is an extension of quasidiscrete topological space [10].

\textbf{Theorem 4.} Let \((U, R)\) be an \textit{Svns}. Then the \textit{Svns} \(\tau_R\) on \(P(U)\) is defined as follows: for any \( A \in P(U) \),

\begin{align*}
&\tau_R(\bigcap_{x \in U} R(A)_T(x)),

&\tau_R(\bigvee_{x \in U} R(A)_F(x)),

&\text{is a quasidiscrete \textit{OSvnt} on } U.
\end{align*}

\textbf{Proof.} \textit{OSvnt1}: it follows that

\begin{align*}
&\tau_R(\bigcap_{x \in U} R(\emptyset)_T(x)) = 1,

&\tau_R(\bigvee_{x \in U} R(\emptyset)_F(x)) = 0,

&\tau_R(\bigcap_{x \in U} R(\emptyset)_F(x)) = 0,

&\tau_R(\bigvee_{x \in U} R(\emptyset)_T(x)) = 1,

&\tau_R(\bigvee_{x \in U} R(\emptyset)_F(x)) = 0.
\end{align*}

(41)
OSvnt2s: let \( A_j \in \mathcal{P}(U) \) \((j \in J)\). Then
\[
\bigwedge_{j \in J} (\tau_R)_{\mathcal{T}}(A_j) = \bigwedge_{j \in J} \bigwedge_{x \in A_j} \mathcal{R}(A_j)(x_j)
\leq \bigwedge_{j \in J} \bigwedge_{x \in A_j} \mathcal{R}(A_j)(x_j), \tag{43}
\]
by Theorem 1(3)
\[
= \bigwedge_{x \in \bigcup_{j \in J} A_j} \mathcal{R}(\bigcup_{j \in J} A_j)(x) = (\tau_R)_{\mathcal{T}}(\bigcup_{j \in J} A_j),
\]
\[
\bigvee_{j \in J} (\tau_R)_{\mathcal{T}}(A_j) = \bigvee_{j \in J} \bigvee_{x \in A_j} \mathcal{R}(A_j)(x_j)
\geq \bigvee_{j \in J} \bigvee_{x \in A_j} \mathcal{R}(A_j)(x_j), \tag{42}
\]
by Theorem 1(3)
\[
= \bigvee_{x \in \bigcup_{j \in J} A_j} \mathcal{R}(\bigcup_{j \in J} A_j)(x) = (\tau_R)_{\mathcal{T}}(\bigcup_{j \in J} A_j).
\]
Similarly, we can prove that \( \bigvee_{j \in J} (\tau_R)_{\mathcal{T}}(A_j) \geq (\tau_R)_{\mathcal{T}}(\bigcap_{j \in J} A_j) \).

OSvnt3: let \( A_j \in \mathcal{P}(U) \) \((j \in J)\). Then it follows by Theorem 1(2) that
\[
\bigwedge_{j \in J} (\tau_R)_{\mathcal{T}}(A_j) = \bigwedge_{j \in J} \bigwedge_{x \in A_j} \mathcal{R}(A_j)(x_j),
\leq \bigwedge_{j \in J} \bigwedge_{x \in A_j} \mathcal{R}(\bigcup_{j \in J} A_j)(x_j)
\]
\[
= \bigwedge_{x \in \bigcup_{j \in J} A_j} \mathcal{R}(\bigcup_{j \in J} A_j)(x) = (\tau_R)_{\mathcal{T}}(\bigcup_{j \in J} A_j),
\]
\[
\bigvee_{j \in J} (\tau_R)_{\mathcal{T}}(A_j) = \bigvee_{j \in J} \bigvee_{x \in A_j} \mathcal{R}(A_j)(x_j)
\geq \bigvee_{j \in J} \bigvee_{x \in A_j} \mathcal{R}(\bigcup_{j \in J} A_j)(x_j)
\]
\[
= \bigvee_{x \in \bigcup_{j \in J} A_j} \mathcal{R}(\bigcup_{j \in J} A_j)(x) = (\tau_R)_{\mathcal{T}}(\bigcup_{j \in J} A_j).
\]
Similarly, we can prove that \( \bigvee_{j \in J} (\tau_R)_{\mathcal{T}}(A_j) = (\tau_R)_{\mathcal{T}}(\bigcap_{j \in J} A_j) \).

\[\square\]

**Remark 3.** The definition of \( \tau_{\mathcal{T}}(A) \) is an interpretation of the fact that “\( A \) is contained in its lower approximation.”

### 5. Single-Valued Neutrosophic Approximation Space Induced by Ordinary Single-Valued Neutrosophic Topological Space

In this section, we shall consider the **Svns** induced by OSvnt.

**Theorem 5.** Let \((U, \tau)\) be an **OSvnt**. Then the **Svnr** \( R_{\mathcal{T}} \) on \( U \) is defined as follows: for any \((x, y) \in U \times U \),
\[
(R_{\mathcal{T}})(x, y) = \bigwedge_{(x, y) \in A} \tau_{\mathcal{T}}(A),
\]
\[
(R_{\mathcal{T}})(x, y) = \bigvee_{(x, y) \in A} (1 - \tau_{\mathcal{T}}(A)),
\]
\[
(R_{\mathcal{T}})(x, y) = \bigvee_{(x, y) \in A} \tau_{\mathcal{T}}(A),
\]
is reflexive and transitive.

**Proof.** Reflexivity: it follows that
\[
(R_{\mathcal{T}})(x, x) = \bigwedge_{(x, x) \in A} \tau_{\mathcal{T}}(A) = 1,
\]
\[
(R_{\mathcal{T}})(x, x) = \bigvee_{(x, x) \in A} (1 - \tau_{\mathcal{T}}(A)) = 0, \tag{45}
\]
\[
(R_{\mathcal{T}})(x, x) = \bigvee_{(x, x) \in A} \tau_{\mathcal{T}}(A) = 0.
\]

Transitivity: let \( x, y, z \in U \).

(i) Note that
\[
(R_{\mathcal{T}})(x, y) = \bigwedge_{(x, y) \in A} \tau_{\mathcal{T}}(A),
\]
\[
(R_{\mathcal{T}})(y, z) = \bigwedge_{(y, z) \in B} \tau_{\mathcal{T}}(B), \tag{46}
\]
\[
(R_{\mathcal{T}})(x, z) = \bigwedge_{(x, z) \in D} \tau_{\mathcal{T}}(D).
\]

Take any \( D \in \mathcal{P}(U) \) with \((x, z) \in D \times D^c\); then \( y \in D \) or \( y \in D^c \).

Case 1: if \( y \in D \), then \((y, z) \in D \times D^c\). So,
\[
(R_{\mathcal{T}})(x, y) \land (R_{\mathcal{T}})(y, z)
\leq (R_{\mathcal{T}})(x, z) = \bigwedge_{(y, z) \in B} \tau_{\mathcal{T}}(B) \leq \tau_{\mathcal{T}}(D). \tag{47}
\]

Case 2: if \( y \in D^c \), then \((x, y) \in D \times D^c\). So,
\[
(R_{\mathcal{T}})(x, y) \land (R_{\mathcal{T}})(y, z)
\leq (R_{\mathcal{T}})(x, z) = \bigwedge_{(x, z) \in D \times D^c} \tau_{\mathcal{T}}(D). \tag{48}
\]

By a combination of Cases 1 and 2, we obtain that
\[
(R_{\mathcal{T}})(x, y) \land (R_{\mathcal{T}})(y, z)
\leq (R_{\mathcal{T}})(x, z) = \bigwedge_{(x, z) \in D \times D^c} \tau_{\mathcal{T}}(D). \tag{49}
\]

(ii) Note that
\[
(R_{\mathcal{T}})(x, y) = \bigvee_{(x, y) \in A} (1 - \tau_{\mathcal{T}}(A)),
\]
\[
(R_{\mathcal{T}})(y, z) = \bigvee_{(y, z) \in B} (1 - \tau_{\mathcal{T}}(B)), \tag{50}
\]
\[
(R_{\mathcal{T}})(x, z) = \bigvee_{(x, z) \in D} (1 - \tau_{\mathcal{T}}(D)).
\]

Take any \( D \in \mathcal{P}(U) \) with \((x, z) \in D \times D^c\); then \( y \in D \) or \( y \in D^c \).

Case 1: if \( y \in D \), then \((y, z) \in D \times D^c\). So,
\[
(R_{\mathcal{T}})(x, y) \lor (R_{\mathcal{T}})(y, z)
\geq (R_{\mathcal{T}})(x, z) = \bigvee_{(y, z) \in B} (1 - \tau_{\mathcal{T}}(B)) \geq (1 - \tau_{\mathcal{T}}(D)). \tag{51}
\]

Case 2: if \( y \in D^c \), then \((x, y) \in D \times D^c\). So,
\[
(R_{\mathcal{T}})(x, y) \lor (R_{\mathcal{T}})(y, z)
\geq (R_{\mathcal{T}})(x, z) = \bigvee_{(x, z) \in D \times D^c} (1 - \tau_{\mathcal{T}}(A)) \geq (1 - \tau_{\mathcal{T}}(D)). \tag{52}
\]
By a combination of Cases 1 and 2, we obtain that
\[
(R_τ)_1(x, y) \vee (R_τ)_1(y, z) \geq \bigvee_{(x, z) \in D \times D'} (1 - r_I(D))
\]
\[
= (R_τ)_1(x, z).
\]
(iii) Similar to (ii), one can prove that
\[
(R_τ)_F(x, y) \vee (R_τ)_F(y, z) \geq (R_τ)_F(x, z).
\]

Remark 4. Note that neither of the topological conditions (OSvnt1)-(OSvnt3) is used in the above theorem. Hence, it can be extended to any single-valued neutrosophic relation on \( P(U) \).

6. One-to-One Correspondence between Reflexive and Transitive Single-Valued Neutrosophic Approximation Spaces and Quasidiscrete Ordinary Single-Valued Neutrosophic Topological Spaces

In this section, we prove that there is a one-to-one correspondence between reflexive and transitive \( SVna \) and quasidiscrete \( OSvnts \).

Theorem 6. Let \((U, R)\) be an \( SVna \). Then \( R_{RA} \supseteq R \), and \( R_{RA} = R \) if \( R \) is reflexive and transitive.

Proof. (1) For \( x, y \in U \),
\[
(R_{RA})_1(x, y) = \bigwedge_{(x, y) \in A \times A'} (r_A)_1(A)
\]
\[
= \bigwedge_{(x, y) \in A \times A'} \bigvee_{z \in A} R_1(z)
\]
\[
= \bigwedge_{(x, y) \in A \times A'} \bigvee_{z \in A, w \neq A} R_1(z, w)
\]
\[\text{taking } z = x, w = y\]
\[\geq \bigvee_{(x, y) \in A \times A'} \bigvee_{z \in A} R_1(z, w), \text{ taking } z = x, w = y\]
\[= (R_{RA})_1(x, y) \Rightarrow \forall \alpha \in [0, 1), \quad \alpha < (R_{RA})_1(x, y) \text{ implies } \alpha < R_1(x, y).\]  

We assume that there is an \( \alpha_0 \in [0, 1) \) such that \( \alpha_0 < (R_{RA})_1(x, y) \) but \( \alpha_0 \geq R_1(x, y) \). Putting \( A_0 = \{ z \in U \mid R_1(x, z) > \alpha_0 \} \), by reflexivity of \( R \), we have \( R_1(x, x) = 1 > \alpha_0 \), so \( x \in A_0 \), and by \( \alpha_0 \geq R_1(x, y) \) we have \( y \in (A_0)^c \). This means that \( (x, y) \in A_0 \times (A_0)^c \). From
\[
\alpha_0 < (R_{RA})_1(x, y) = \bigwedge_{(x, y) \in A \times A'} \bigvee_{z, w \in A} R_1(z, w),
\]
\[\text{we know that there exists } (z, w) \in A_0 \times (A_0)^c \text{ such that } R_1(z, w) > \alpha_0; \text{that is, } R_1(x, z) > \alpha_0 \text{ and } R_1(x, w) \leq \alpha_0. \text{ It follows by the transitivity that}
\]
\[\alpha_0 < R_1(x, z) \land R_1(z, w) \leq R_1(x, w) \leq \alpha_0, \quad \text{a contradiction! Therefore, } \alpha < (R_{RA})_1(x, y) \text{ always implies that } \alpha < R_1(x, y). \quad \text{Hence,}
\]
\[(R_{RA})_1(x, y) \leq R_1(x, y).
\]

(ii) Note that
\[(R_{RA})_F(x, y) = \bigvee_{(x, y) \in A \times A'} (r_A)_F(A)
\]
\[= \bigvee_{(x, y) \in A \times A'} \bigwedge_{z \in A} R_1(z)
\]
\[= \bigvee_{(x, y) \in A \times A'} \bigwedge_{z \in A, w \neq A} R_1(z, w)
\]
\[\leq \bigvee_{(x, y) \in A \times A'} R_1(x, y) \Rightarrow \forall \alpha \in (0, 1], \quad \alpha \leq R_1(x, y) \text{ implies } \alpha \leq (R_{RA})_F(x, y). \]  

\[\text{Hence, } R_{RA} \supseteq R.
\]
We assume that there is an \( a_0 \in (0, 1) \) such that 
\[
\alpha_0 \leq R_\tau(x, y) \quad \text{but} \quad \alpha_0 > (R_\tau)_T(x, y).
\]
Putting \( A_0 = \{ z \in U | R_\tau(x, z) < \alpha_0 \} \), by reflexivity of \( R \), we have 
\[ R_\tau(x, z) = 0 < \alpha_0, \] 
so \( x \in A_0 \), and by 
\[ \alpha_0 \leq R_\tau(x, y) \] 
we have \( y \in (A_0)^c \). This means that \((x, y) \in A_0 \times (A_0)^c \). From 
\[
\alpha_0 > (R_\tau)_T(x, y) = \bigvee_{(x, y) \in A_0 \times (A_0)^c} R_\tau(z, w), 
\]
(59) we know that there exists \((z, w) \in A_0 \times (A_0)^c \)
such that 
\[ R_\tau(z, w) < \alpha_0; \] that is, \[ R_\tau(x, z) < \alpha_0 \] and 
\[ R_\tau(x, w) > \alpha_0. \] 
It follows by the transitivity that 
\[ \alpha_0 > R_\tau(x, z) \lor R_\tau(x, w) \geq R_\tau(x, y) \geq \alpha_0, \] 
(60)
a contradiction! Therefore, \( \alpha \leq R_\tau(x, y) \) always implies that \( \alpha \leq (R_\tau)_T(x, y) \). Hence, 
\((R_\tau)_T(x, y) \geq R_\tau(x, y) \).

(iii) Similar to (ii), we can prove that 
\((R_\tau)_T(x, y) \geq R_\tau(x, y) \).

(i)–(iii) show that \( R \not\equiv R_\tau \) and so \( R_\tau = R \) by (1).

(3) If \( R_\tau = R \), then it follows by Theorems 4 and 5 that \( R \) is reflexive and transitive. \( \Box \)

**Theorem 7.** Let \((U, R)\) be an OSvnt. Then \( \tau_{R^}\succeq \tau \) and \( \tau_{R^}\succeq \tau \), if \( \tau \) is quasidiscrete.

**Proof**

(1) Let \( A \in P(U) \). Then 
\[
\left( \tau_{R^}\right)_T(A) = \bigwedge_{x \in A} R_\tau(\tau(A)_T(x)) 
= \bigwedge_{x \in A} \bigwedge_{y \in A^c} R_\tau(B), \quad \text{taking } B = A 
\]
\[
\geq \bigwedge_{(x, y) \in A \times A^c} \tau_\tau(B), \quad \text{taking } B = A 
\]
\[
\geq \bigwedge_{(x, y) \in A \times A^c} \tau_\tau(A) = \tau_\tau(A), 
\]
(61)
Similarly, we can prove that \( \left( \tau_{R^}\right)_T(A) \leq \tau_\tau(A) \).

(2) Let \( A \in P(U) \).

(i) Note that 
\[
\left( \tau_{R^}\right)_T(A) \leq \tau_\tau(A) \iff \forall \alpha \in (0, 1), \quad \alpha < \left( \tau_{R^}\right)_T(A) \implies \alpha \leq \tau_\tau(A). \] 
(62)

z \in \cap_{y \in A} B_y; hence, \( A \subseteq \cap_{y \in A} B_y \); if \( z \notin A \), then, for any \( x \in A \), we have \( (x, z) \in A \times A^c \), and then 
\[ z \notin B_x \] 
which means that 
\[ z \notin \cap_{y \in A} B_y \] 
\( \cap_{y \in A} B_y \subseteq A \); then it follows by OSvnt3 that 
\[ \tau_\tau(A) = \tau_\tau(\cap_{y \in A} B_y) \geq \bigwedge_{y \in A} \tau_\tau(B_y) \geq \alpha. \] 
(65)
Therefore, \( \left( \tau_{R^}\right)_T(A) \leq \tau_\tau(A) \).

(ii) Note that 
\[
\left( \tau_{R^}\right)_T(A) \geq \tau_\tau(A) \iff \forall \alpha \in (0, 1), \quad \alpha > \left( \tau_{R^}\right)_T(A) \implies \alpha \geq \tau_\tau(A). \] 
(66)

Then, for any \( (x, y) \in A \times A^c \), there is 
\[ B_{xy} \in P(U) \text{ such that } (x, y) \in B_{xy} \times (B_{xy})^c \] 
and 
\[ \alpha > \tau_\tau(B_{xy}). \] Putting \( B_y = \cup_{x \in A} B_{xy}, \) by OSvnt3, we have 
\[
\left( \tau_{R^}\right)_T(A) \geq \tau_\tau(A) \iff \forall \alpha \in (0, 1), \quad \alpha > \left( \tau_{R^}\right)_T(A) \implies \alpha \geq \tau_\tau(A). \] 
(67)
Remark 5. We can give a similar discussion on the same underlying set. No data were used to support this study.

7. Conclusions

In this paper, we presented a new model of neutrosophic rough sets. The difference between this model and the existing models is that, in our model, the original sets are ordinary subsets of $U$ and their approximations are single-valued neutrosophic sets; however, in the existing models, the original sets and their approximations are all (single-valued) neutrosophic sets. We also discussed the basic properties of the proposed rough sets and gave their relationships with Kim’s ordinary single-valued neutrosophic topology. Particularly, we proved by our lower approximation operator that there is a one-to-one correspondence between reflexive and transitive single-valued neutrosophic rough sets related to the new single-valued neutrosophic topology. Furthermore, from Remark 1, we know that when restricting single-valued neutrosophic sets to Pythagorean fuzzy sets, we can define a model of Pythagorean fuzzy rough sets. It is well known that Pythagorean fuzzy sets and (fuzzy) rough sets have been applied in many fields, particularly in multiple attribute decision-making [9, 16, 52–55]. Therefore, in the future, we will also consider the potential application of Pythagorean fuzzy rough sets.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares no conflicts of interest.

Authors’ Contributions

Qiu Jin and Kai Hu contributed the central idea, and all authors contributed to the writing and revisions.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (nos. 11801248 and 11501278) and Natural Science Foundation of Shandong Province (no. ZR2020MA042), and the KeYan Foundation of Liaocheng University (318012030).

References


