

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/342714346>

A NEW TYPE OF QUASI OPEN FUNCTIONS IN NEUTROSOPHIC TOPOLOGICAL ENVIRONMENT

Preprint · July 2020

CITATIONS

0

READS

2

4 authors:



Rishwanth M. Parimala

Bannari Amman Institute of Technology

64 PUBLICATIONS 134 CITATIONS

[SEE PROFILE](#)



Cenap Özel

King Abdulaziz University

239 PUBLICATIONS 326 CITATIONS

[SEE PROFILE](#)



Florentin Smarandache

University of New Mexico Gallup

3,149 PUBLICATIONS 24,499 CITATIONS

[SEE PROFILE](#)



Karthika Muthusamy

Bannari Amman Institute of Technology

14 PUBLICATIONS 18 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:



Pablo Baisotti, editor: The Latin American Board. Political Sovereignty with China? [View project](#)



Neutrosophic Correlation [View project](#)

A NEW TYPE OF QUASI OPEN FUNCTIONS IN NEUTROSOPHIC TOPOLOGICAL ENVIRONMENT

M. Parimala¹, C. Ozel², F. Smarandache³, M. Karthika¹

¹Department of Mathematics, Bannari Amman Institute of Technology,
Sathyamangalam - 638 401, Tamil Nadu, India.

Emails: rishwanthpari@gmail.com, karthikamuthusamy1991@gmail.com.

² Department of Mathematics, King Abdulaziz University-21589, Jeddah KSA.
Email: cenap.ozel@gmail.com.

³Mathematics & Science Department, University of New Mexico, 705 Gurley Ave,
Gallup, NM 87301, USA.
Email: fsmarandache@gmail.com.

Abstract

Neutrosophic topological space is an generalization of intuitionistic topological space and each neutrosophic set in neutrosophic topological space is triplet set. Intuitionistic topological set deals membership and non-membership values of each variable in open and closed functions and in neutrosophic topology inderteterminacy of the same variable has not discussed in the previous work. This motivates us to propose the new type of quasi open and closed functions in neutrosophic topological space. In this paper, we introduce the notion of neutrosophic *quasi $\alpha\psi$ -open* and neutrosophic *quasi $\alpha\psi$ -closed* functions. Also we investigate some of its fundamental properties and its characterizations.

Keywords. neutrosophic *$\alpha\psi$ -closed* set, neutrosophic *$\alpha\psi$ -open* set and neutrosophic *quasi $\alpha\psi$ -function*.

AMS(2010) Subject classification : 54A05, 54D10, 54F65, 54G05.

1. Introduction

Atanassov [2] defined the notion of intuitionistic fuzzy sets, which is a generalized form of Zadeh's [19] fuzzy set. Using this concept so many research work are brought in the literature, such as intuitionistic fuzzy semi-generalized closed sets [15]. D. Coker [3] initiated the concept of intuitionistic fuzzy topological spaces. F. Smarandache [17, 18] introduced and studied neutrosophic sets (NS). Later, Salama et al.[13, 14] introduced and studied neutrosophic topology. This approach leads to many investigations in this area. Since then more research

have been identified in the field of neutrosophic topology [7, 10, 11, 12, 13, 14], neutrosophic ideals [6, 8, 9], etc.

Many different terms of open functions have been introduced over the course of years. Various interesting problems arise when one consider openness. Its importance is significant in various areas of Mathematics and related sciences.

The notion of neutrosophic $\alpha\psi$ -closed set was introduced and studied by M. Parimala et al. [7, 11, 12]. In this paper, we will continue the study of related functions by involving neutrosophic $\alpha\psi$ -open sets. We introduce and characterize the concept of neutrosophic *quasi $\alpha\psi$ -functions*.

Through out this paper, spaces means neutrosophic topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f : (X, \tau) \rightarrow (Y, \sigma)$ denotes a neutrosophic function f of a space (X, τ) into a space (Y, σ) . Let A be a subset of a space X . The neutrosophic closure and the neutrosophic interior of A are denoted by $ncl(A)$ and $nint(A)$, respectively.

2. Preliminaries

This sections contains the collection of some existing definition in [1, 3, 6, 7, 10, 11, 12, 13, 14, 17, 18] which are helpful for this work.

Definition 2.1. Let X and I are a non-empty set and the interval $[0, 1]$, respectively. An NS A is defined by

$$A = \{\langle e, \mu_A(e), \sigma_A(e), \nu_A(e) \rangle : e \in X\}$$

where the mappings of membership μ_A , indeterminacy σ_A and non-membership ν_A respectively defined from non-empty set X to I , $\forall e \in X$ to the set A and with condition that the sum of $\mu_A(e), \sigma_A(e), \nu_A(e)$ should not exceed 3 and greater than zero, $\forall e \in X$.

Definition 2.2. Let the two NSs be of the form $A = \{\langle e, \mu_A(e), \sigma_A(e), \nu_A(e) \rangle : e \in X\}$ and $B = \{\langle e, \mu_B(e), \sigma_B(e), \nu_B(e) \rangle : e \in X\}$. Then

- (i) A is a subset of B if and only if membership of A is less than or equal to membership of B , indeterminacy and non-membership of A are respectively, greater than or equal to indeterminacy and non-membership of B .
- (ii) $\bar{A} = \{\langle e, \nu_A(e), \sigma_A(e), \mu_A(e) \rangle : e \in X\}$;
- (iii) Union of two NS's A and B is set of all maximum of membership of A and B , minimum of indeterminacy and non-membership of A and B , for each $e \in X$.

- (iv) Intersection of two NS's A and B is set of all minimum of membership of A and B , maximum of indeterminacy and non-membership of A and B , for each $e \in X$.

Definition 2.3. A neutrosophic topology (NT) on X is a collection τ of NS's in X holds the following properties

- (i) 0 and 1 are in τ .
- (ii) Union of any NS of τ in τ .
- (iii) Intersection of $A, B \in \tau$ in τ ;

Note: Every NS in τ is a neutrosophic open sets (NOS) and its complements are neutrosophic closed sets (NCS).

Definition 2.4. Let A be an NS in NTS (X, τ) . Then neutrosophic interior of the given NS A is maximum of all NOS contained in A . Neutrosophic closure of the given NS A is minimum of all NOS contains A . Neutrosophic interior of A is denoted by $nint(A)$ and Neutrosophic closure of A is denoted by $ncl(A)$.

Definition 2.5. Let (X, τ) be a neutrosophic topological space and a subset A of (X, τ) is called

1. a neutrosophic pre-open set, if A subset of neutrosophic interior of neutrosophic closure of A and a neutrosophic pre-closed set if the complement is a neutrosophic pre-open set.
2. a neutrosophic semi-open set, if A is a subset of neutrosophic closure of neutrosophic interior of A and a neutrosophic semi-closed set if the complement is a neutrosophic semi-open set.
3. a neutrosophic α -open set, if A is a subset of neutrosophic interior of neutrosophic closure of neutrosophic interior of A and a neutrosophic α -closed set if complement is a neutrosophic α -open set.

Definition 2.6. A subset A of an neutrosophic topological space (X, τ) is called

1. an neutrosophic semi-generalized closed (briefly, *nsg*-closed) set if intersection of all neutrosophic semi closed sets which contains A is a subset of U whenever A is a subset of U and U is semi-open in (X, τ) .

2. an neutrosophic ψ -closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is Nsg -open in (X, τ) .
3. an neutrosophic $\alpha\psi$ -closed (briefly, $n\alpha\psi$ -closed) set if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\alpha$ -open in (X, τ) .

Definition 2.7. Let A be an NS in neutrosophic topological space (X, τ) . Then

1. $n\alpha\psi$ interior of A is the minimum of $n\alpha\psi$ OS in X contained in A and it is denoted by $n\alpha\psi int(A)$.
2. $n\alpha\psi$ -closure of A is the maximum of $n\alpha\psi$ CS in X which contains A . and it is denoted by $n\alpha\psi cl(A)$.

3. On Neutrosophic Quasi $\alpha\psi$ -Open Functions

We introduce the following definition.

Definition 3.1. A neutrosophic function $f : X \rightarrow Y$ is said to be neutrosophic *quasi $\alpha\psi$ -open* (briefly, *nq - $\alpha\psi$ -open*), if the image of every neutrosophic $\alpha\psi$ -open set in X is neutrosophic *open* in Y .

It is evident that, the the concepts of neutrosophic *quasi $\alpha\psi$ -openness* and neutrosophic $\alpha\psi$ -continuity coincide if the function is a bijection.

Theorem 3.2. A neutrosophic function $f : X \rightarrow Y$ is *nq - $\alpha\psi$ -open* if and only if for every subset U of X , $f(n\alpha\psi int(U)) \subset nint(f(U))$.

Proof. Let f be a *nq - $\alpha\psi$ -open* function. Now, we have $nint(U) \subset U$ and $n\alpha\psi int(U)$ is a $n\alpha\psi$ -open set. Hence, we obtain that $f(n\alpha\psi int(U)) \subset f(U)$. As $f(n\alpha\psi int(U))$ is open, $f(n\alpha\psi int(U)) \subset nint(f(U))$.

Conversely, assume that U is a $n\alpha\psi$ -open set in X . then, $f(U) = f(n\alpha\psi int(U)) \subset nint(f(U))$ but $nint(f(U)) \subset f(U)$. Consequently, $f(U) = nint(f(U))$ and hence f is *nq - $\alpha\psi$ -open*.

Theorem 3.3. If a neutrosophic function $f : X \rightarrow Y$ is *nq - $\alpha\psi$ -open*, then $n\alpha\psi int(f^{-1}(G)) \subset f^{-1}(nint(G))$ for every subset G of Y .

Proof. Let G be any arbitrary subset of Y . Then, $n\alpha\psi int(f^{-1}(G))$ is a $n\alpha\psi$ -open set in X and f is *nq - $\alpha\psi$ -open*, then $f(n\alpha\psi int(f^{-1}(G))) \subset nint(f(f^{-1}(G))) \subset nint(G)$. Thus, $n\alpha\psi int(f^{-1}(G)) \subset f^{-1}(nint(G))$.

Definition 3.4. A subset A is said to be an $n\alpha\psi$ -neighbourhood of a point x of X if there exists a $n\alpha\psi$ -open set U such that $x \in U \subset A$.

Theorem 3.5. For a neutrosophic function $f : X \rightarrow Y$, the following are equivalent

- (i) f is nq - $\alpha\psi$ -open;
- (ii) for each subset U of X , $f(n\alpha\psi\text{int}(U)) \subset n\text{int}(f(U))$;
- (iii) for each $x \in X$ and each $n\alpha\psi$ -neighbourhood U of x in X , there exists a neighbourhood V of $f(x)$ in Y such that $V \subset f(U)$.

Proof. (i) \Rightarrow (ii) It follows from Theorem 3.1.

(ii) \Rightarrow (iii) Let $x \in X$ and U be an arbitrary $n\alpha\psi$ -neighbourhood of $x \in X$. Then, there exists a $n\alpha\psi$ -open set V in X such that $x \in V \subset U$. Then by (ii), we have $f(V) = f(n\alpha\psi\text{int}(V)) \subset n\text{int}(f(V))$ and hence $f(V)$ is open in Y such that $f(x) \in f(V) \subset f(U)$.

(iii) \Rightarrow (i) Let U be an arbitrary $n\alpha\psi$ -open set in X . Then for each $y \in f(U)$, by (iii) there exists a neighbourhood V_y of y in Y such that $V_y \subset f(U)$. As V_y is a neighbourhood of y , there exists an open set W_y in Y such that $y \in W_y \subset V_y$. Thus $f(U) = \bigcup\{W_y : y \in f(U)\}$ which is an open set in Y . This implies that f is nq - $\alpha\psi$ -open function.

Theorem 3.6. A neutrosophic function $f : X \rightarrow Y$ is nq - $\alpha\psi$ -open if and only if for any subset B of Y and for any $n\alpha\psi$ -closed set F of X containing $f^{-1}(B)$, there exists a neutrosophic closed set G of Y containing B such that $f^{-1}(G) \subset F$.

Proof. Suppose f is nq - $\alpha\psi$ -open. Let $B \subset Y$ and F be a $n\alpha\psi$ -closed set of X containing $f^{-1}(B)$. Now, put $G = Y - f(X - F)$. It is clear that $f^{-1}(B) \subset F \Rightarrow B \subset G$. Since f is nq - $\alpha\psi$ -open, we obtain G as a neutrosophic closed set of Y . Moreover, we have $f^{-1}(G) \subset F$.

Conversely, let U be a $n\alpha\psi$ -open set of X and put $B = Y - f(U)$. Then $X - U$ is a $n\alpha\psi$ -closed set in X containing $f^{-1}(B)$. By hypothesis, there exists a neutrosophic closed set F of Y such that $B \subset F$ and $f^{-1}(F) \subset X - U$. Hence, we obtain $f(U) \subset Y - F$. On the other hand, it follows that $B \subset F$, $Y - F \subset Y - B = f(U)$. Thus we obtain $f(U) = Y - F$ which is neutrosophic open and hence f is a nq - $\alpha\psi$ -open function.

Theorem 3.7 A neutrosophic function $f : X \rightarrow Y$ is nq - $\alpha\psi$ -open if and only

if $f^{-1}(cl(B)) \subset n\alpha\psi cl(f^{-1}(B))$ for every subset B of Y .

Proof. Suppose that f is $nq\text{-}\alpha\psi\text{-open}$. For any subset B of Y , $f^{-1}(B) \subset \alpha\psi\text{-}cl(f^{-1}(B))$. Therefore, by Theorem 3.5 there exists a neutrosophic closed set F in Y such that $B \subset F$ and $(f^{-1}(F)) \subset n\alpha\psi cl(f^{-1}(B))$. Therefore, we obtain $f^{-1}(ncl(B)) \subset (f^{-1}(F)) \subset n\alpha\psi cl(f^{-1}(B))$.

Conversely, let $B \subset Y$ and F be a $n\alpha\psi\text{-closed}$ set of X containing $f^{-1}(B)$. Put $W = ncl_Y(B)$, then we have $B \subset W$ and W is neutrosophic closed and $f^{-1}(W) \subset n\alpha\psi\text{-}cl(f^{-1}(B)) \subset F$. Then by Theorem 3.6., f is $nq\text{-}\alpha\psi\text{-open}$.

Theorem 3.8. Two neutrosophic function $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ and $g \circ f : X \rightarrow Z$ is $nq\text{-}\alpha\psi\text{-open}$. If g is continuous injective function, then f is $nq\text{-}\alpha\psi\text{-open}$.

Proof. Let U be a $n\alpha\psi\text{-open}$ set in X , then $(g \circ f)(U)$ is open in Z , since $g \circ f$ is $nq\text{-}\alpha\psi\text{-open}$. Again g is an injective continuous function, $f(U) = g^{-1}(g \circ f(U))$ is open in Y . This shows that f is $nq\text{-}\alpha\psi\text{-open}$

4. On Neutrosophic Quasi $\alpha\psi\text{-Closed}$ Functions

Definition 4.1. A neutrosophic function $f : X \rightarrow Y$ is said to be neutrosophic quasi $\alpha\psi\text{-closed}$ (briefly, $nq\text{-}\alpha\psi\text{-closed}$), if the image of every neutrosophic $\alpha\psi\text{-closed}$ set in X is neutrosophic closed in Y .

Theorem 4.2. Every $nq\text{-}\alpha\psi\text{-closed}$ function is neutrosophic closed as well as neutrosophic $\alpha\psi\text{-closed}$.

Proof. It is obvious.

The converse of the above theorem need not be true by the following example.

Example 4.3. Let a neutrosophic function $f : X \rightarrow Y$.

Let $X = \{p, q, r\}$ and $\tau_{N1} = \{0, A, B, C, D, 1\}$ ia a neutrosophic topology on X , Where

$$\begin{aligned} A &= \langle x, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle, \\ B &= \langle x, (\frac{p}{0.2}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle, \\ C &= \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.1}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle, \\ D &= \langle x, (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.4}, \frac{r}{0.3}) \rangle \text{ and} \end{aligned}$$

Let $Y = \{p, q, r\}$ and $\tau_{N2} = \{0, E, F, G, H, 1\}$ is a neutrosophic topology on Y , Where

$$E = \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle,$$

$$\begin{aligned}
F &= \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle, \\
G &= \langle y, (\frac{p}{0.4}, \frac{q}{0.3}, \frac{r}{0.3}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}), (\frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}) \rangle, \\
H &= \langle y, (\frac{p}{0.1}, \frac{q}{0.2}, \frac{r}{0.2}), (\frac{p}{0.3}, \frac{q}{0.5}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.6}, \frac{r}{0.4}) \rangle.
\end{aligned}$$

Here $f(p) = p, f(q) = q, f(r) = r$. Then clearly f is $n\alpha\psi$ -closed as well as neutrosophic closed but not $nq\text{-}\alpha\psi$ -closed.

Lemma 4.4. If a neutrosophic function is $nq\text{-}\alpha\psi$ -closed, then $f^{-1}(nint(B)) \subset n\alpha\psi int(f^{-1}(B))$ for every subset B of Y .

Proof. Let B any arbitrary subset of Y . Then, $n\alpha\psi int(f^{-1}(G))$ is a $n\alpha\psi$ -closed set in X and f is $nq\text{-}\alpha\psi$ -closed, then $f(n\alpha\psi\text{-}nint(f^{-1}(B))) \subset nint(f(f^{-1}(B))) \subset nint(B)$. Thus, $f(n\alpha\psi int(f^{-1}(B))) \subset f^{-1}(nint(B))$.

Theorem 4.5. A neutrosophic function $f : X \rightarrow Y$ is $nq\text{-}\alpha\psi$ -closed if and only if for any subset B of Y and for any $n\alpha\psi$ -open set G of X containing $f^{-1}(B)$, there exists an open set U of Y containing B such that $f^{-1}(U) \subset G$.

Proof This proof is similar to that of theorem 3.6.

Definition 4.6. A neutrosophic function $f : X \rightarrow Y$ is called $n\alpha\psi^*$ -closed if the image of every $n\alpha\psi$ -closed subset of X is $n\alpha\psi$ -closed in Y .

Theorem 4.7. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be any $nq\text{-}\alpha\psi$ -closed functions, then $g \circ f : X \rightarrow Z$ is a $nq\text{-}\alpha\psi$ -closed function.

Proof. It is obvious.

Theorem 4.8. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be any two neutrosophic functions, then

- (i) If f is $n\alpha\psi$ -closed and g is $nq\text{-}\alpha\psi$ -closed, then $g \circ f$ is neutrosophic closed;
- (ii) If f is $nq\text{-}\alpha\psi$ -closed and g is $nq\text{-}\alpha\psi$ -closed, then $g \circ f$ is $n\alpha\psi^*$ -closed;
- (iii) If f is $n\alpha\psi^*$ -closed and g is $nq\text{-}\alpha\psi$ -closed, then $g \circ f$ is $nq\text{-}\alpha\psi$ -closed.

Proof. It is obvious.

Theorem 4.9. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be any two neutrosophic functions such that $g \circ f : X \rightarrow Z$ is $nq\text{-}\alpha\psi$ -closed.

- (i) If f is $n\alpha\psi$ -irresolute surjective, then g is neutrosophic closed;

(ii) If g is $n\alpha\psi$ -continuous injective, then f is $n\alpha\psi^*$ -closed.

Proof. (i) Suppose that F is an arbitrary neutrosophic closed set in Y . As f is $n\alpha\psi$ -irresolute, $f^{-1}(F)$ is $n\alpha\psi$ -closed in X . Since $g \circ f$ is nq - $\alpha\psi$ -closed and f is surjective, $(g \circ f)(f^{-1}(F)) = g(F)$, which is closed in Z . This implies that g is a neutrosophic closed function.

(ii) Suppose F is any $n\alpha\psi$ -closed set in X . Since $g \circ f$ is nq - $\alpha\psi$ -closed, $(g \circ f)(F)$ is neutrosophic closed in Z . Again g is a $n\alpha\psi$ -continuous injective function, $g^{-1}(g \circ f(F)) = f(F)$, which is $n\alpha\psi$ -closed in Y . This shows that f is $n\alpha\psi^*$ -closed.

Theorem 4.10. Let X and Y be neutrosophic topological spaces. Then the function $f : X \rightarrow Y$ is a nq - $\alpha\psi$ -closed if and only if $f(X)$ is neutrosophic closed in Y and $f(V) - f(X - V)$ is open in $f(X)$ whenever V is $n\alpha\psi$ -open in X .

Proof. Necessity: Suppose $f : X \rightarrow Y$ is a nq - $\alpha\psi$ -closed function. Since X is $n\alpha\psi$ -closed, $f(X)$ is neutrosophic closed in Y and $f(V) - f(X - V) = f(V) \cap f(X) - f(X - V)$ is neutrosophic open in $f(X)$ when V is $n\alpha\psi$ -open in X .

Sufficiency: Suppose $f(X)$ is neutrosophic closed in Y , $f(V) - f(X - V)$ is neutrosophic open in $f(X)$ when V is $n\alpha\psi$ -open in X and let C be neutrosophic closed in X . Then $f(C) = f(X) - (f(X - C) - f(C))$ is neutrosophic closed in $f(X)$ and hence neutrosophic closed in Y .

Corollary 4.11. Let X and Y be two neutrosophic topological spaces. Then a surjective function $f : X \rightarrow Y$ is nq - $\alpha\psi$ -closed if and only if $f(V) - f(X - V)$ is open in Y whenever U is $n\alpha\psi$ -open in X .

Proof. It is obvious.

Theorem 4.12. Let X and Y be neutrosophic topological spaces and let $f : X \rightarrow Y$ be $n\alpha\psi$ -continuous and nq - $\alpha\psi$ -closed surjective function. Then the topology on Y is $\{f(V) - f(X - V) : V \text{ is } n\alpha\psi\text{-open in } X\}$.

Proof. Let W be open in Y . Let $f^{-1}(W)$ is $n\alpha\psi$ -open in X , and $f(f^{-1}(W)) - f(X - f^{-1}(W)) = W$. Hence all open sets in Y are of the form $f(V) - f(X - V)$, V is $n\alpha\psi$ -open in X . On the other hand, all sets of the form $f(V) - f(X - V)$, V is $n\alpha\psi$ -open in X , are neutrosophic open in Y from corollary 4.11.

Definition 4.13. A neutrosophic topological space (X, τ) is said to be $n\alpha\psi$ -normal if for any pair of disjoint $n\alpha\psi$ -closed subsets F_1 and F_2 of X , there exists

disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Theorem 4.14. Let X and Y be a neutrosophic topological spaces with X is $n\alpha\psi$ -normal. If $f : X \rightarrow Y$ is $n\alpha\psi$ -continuous and nq - $\alpha\psi$ -closed surjective function. Then Y is normal.

Proof. Let K and M be disjoint neutrosophic closed subsets of Y . Then $f^{-1}(K)$, $f^{-1}(M)$ are disjoint $n\alpha\psi$ -closed subsets of X . Since X is $n\alpha\psi$ -normal, there exists disjoint neutrosophic open sets V and W such that $f^{-1}(K) \subset V$, $f^{-1}(M) \subset W$. Then $K \subset f(V) - f(X - V)$ and $M \subset f(W) - f(X - W)$, further by corollary 4.11, $f(V) - f(X - V)$ and $f(W) - f(X - W)$ are neutrosophic open sets in Y and clearly $(f(V) - f(X - V)) \cap (f(W) - f(X - W)) = n\phi$. This shows that Y is normal.

References

- [1] I. Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala, On some new notions and functions in neutrosophic topological spaces, *Neutrosophic Sets Syst.*, **16**, (2017), 16-19.
- [2] K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy sets and systems*, **20**, (1986), 87-96.
- [3] D. Coker, An introduction to fuzzy topological spaces, *Fuzzy sets and systems*, **88**, (1997), 81-89.
- [4] K. K. Dubey and O. S. Panwar, Some properties of s-connectedness between sets and s-connected mapping, *Indian J. pure Math.*, **154**, (1984), 343-354.
- [5] K. Kuratowski, *Topology Vol. II (transl.)*, Academic Press, New York, (1966).
- [6] M. Parimala, S. Jafari, and S. Murali, Nano Ideal Generalized Closed Sets in Nano Ideal Topological Spaces, *Annales Univ. Sci. Budapest.*, **60**, (2017), 3-11.
- [7] M. Parimala, R. Jeevitha, S. Jafari, F. Smarandache and R. Udhayakumar, Neutrosophic $\alpha\psi$ -Homeomorphism in Neutrosophic Topological Spaces, *Information*, **9**(103)(2018), 1-7.

- [8] M.Parimala, R.Jeevitha and A.Selvakumar, A New Type of Weakly Closed Set in Ideal Topological Spaces, *International Journal of Mathematics and its Applications*, **5**(4-C), (2017), 301-312.
- [9] M. Parimala and R. Perumal, Weaker form of open sets in nano ideal topological spaces, *Global Journal of Pure and Applied Mathematics*, **12**(1), (2016), 302-305.
- [10] M. Parimala, M. Karthika, R. Dhavaseelan, S. Jafari. On neutrosophic supra pre-continuous functions in neutrosophic topological spaces, *New Trends in Neutrosophic Theory and Applications* , **2**, (2018), 371-383.
- [11] M.Parimala, M. Karthika, S. Jafari, F.Smarandache, A.A El-Atik, Neutrosophic $\alpha\psi$ -connectedness, *Journal of Intelligent and Fuzzy Systems*, **38**(1), (2020), 853-857.
- [12] M. Parimala, F. Smarandache, S. Jafari and R. Udhayakumar, On Neutrosophic $\alpha\psi$ -Closed Sets, *Information*, **9**, (2018), 103, 1-7.
- [13] A. A. Salama, S. A. Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces, *IOSR J. Math.*, **3**, (2012), 31-35.
- [14] A. A. Salama, Florentin Smarandache and Valeri Kromov, Neutrosophic Closed Set and Neutrosophic Continuous Functions, *Neutrosophic Sets and Systems*, **4**, (2014),4-8.
- [15] R. Santhi and K. Arun Prakash, On intuitionistic fuzzy Semi-Generalized Closed Sets and its Applications, *Int. J. Contemp. Math. Sciences*, **5**(34), (2010), 1677 - 1688.
- [16] R. Santhi and N. Udhayarani, N_ω closed sets in neutrosophic topological spaces, *Neutrosophic Sets and Systems*, **12**, (2016),114-117.
- [17] F. Smarandache, *Neutrosophy and Neutrosophic Logic*, First International Conference on Neutrosophy, Neutrosophic Logic Set, Probability and Statistics; University of New Mexico, Gallup, NM, USA, (2002).
- [18] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability*, American Research Press, Rehoboth, NM, USA, (1999).
- [19] L. A. Zadeh, Fuzzy Sets, *Information and Control*, **18**, (1965), 338-353.