



# A Nonlinear Approach for Neutrosophic Linear Programming

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<b>Chronicle:</b> <i>Received: 09 June 2019</i> <i>Revised: 14 August 2019</i> <i>Accepted: 17 September 2019</i>	Traditional linearl programming usually handles optimization problems involving deterministic objective functions and/or constrained functions. However, uncertainty also exists in real problems. Hence, many researchers have proposed uncertain optimization methods, such as approaches using fuzzy and stochastic logics, interval numbers, or uncertain variables. However, in practical situations, we often have to
Keywords: Single valued neutrosophic number. Neutrosophic linear programming problem. Linear programming problem.	handle programming problems involving indeterminate information. The aim of this paper is to put forward a new algorithm for solving the Single-Valued Neutrosophic linear programming problem. A numerical example is reported to verify the effectiveness of the new algorithms.

# **1. Introduction**

Neutrosophy has been proposed by Smarandache [1] as a new branch of philosophy, with ancient roots, dealing with "the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra". The fundamental thesis of neutrosophy is that every idea has not only a certain degree of truth, as is generally assumed in many-valued logic contexts but also a falsity degree and an indeterminacy degree that have to be considered independently from each other. Smarandache seems to understand such "indeterminacy" both in a subjective and in an objective sense, i.e. as uncertainty as well as imprecision, vagueness, error, doubtfulness, etc.

Neutrosophic Set (NS) is a generalization of the fuzzy set [2] and intuitionistic fuzzy set [3] and can deal with uncertain, indeterminate, and incongruous information where the indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are completely independent. Moreover, some extensions of NSs, including interval neutrosophic set [4], bipolar neutrosophic set [5], single-valued neutrosophic set [6], multi-valued neutrosophic set [7], and neutrosophic linguistic set [8] have been proposed and applied to solve various problems [9-12].

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The main purpose of this paper is to propose a new model for linear programming, including neutrosophic variables and right-hand side and to present a solution method for this neutrosophic LP problem.

### 2. Preliminaries

In this section, we present some basic definitions and arithmetic operations single valued trapezoidal neutrosophic fuzzy numbers.

**Definition 1.** [4]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function  $T_A(x)$ , an indeterminacy membership function  $I_A(x)$ , and a falsity-membership function  $F_A(x)$ . If the functions  $T_A(x)$ ,  $I_A(x)$  and  $F_A(x)$  are singleton subintervals/subsets in the real standard [0, 1], that is  $T_A(x)$ :  $X \rightarrow [0,1]$ ,  $I_A(x)$ :  $X \rightarrow [0,1]$ . Then, a Single valued neutrosophic set A is denoted by  $A = \{(x, T_A(x), I_A(x), F_A(x)) | x \in X\}$  which is called a SVNS. Also, SVNS satisfies the condition  $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ .

**Definition 2.** [4]. For SVNSs A and B, A  $\subseteq$ B if and only if  $T_A(x) \leq T_B(x)$ ,  $I_A(x) \geq I_B(x)$ , and  $F_A(x) \geq F_B(x)$  for every x in X.

**Definition 3.** (Single valued trapezoidal neutrosophic number (SVTNN)) let  $T_{\tilde{p}}, I_{\tilde{p}}, F_{\tilde{p}} \in [0,1]$  then a Single valued trapezoidal neutrosophic number  $\tilde{p} = \langle [p_1, p_2, p_3, p_4], (T_{\tilde{p}}, I_{\tilde{p}}, F_{\tilde{p}}) \rangle$  is a special NS on the real number set R, whose truth membership function  $\mu_{\tilde{p}}(x)$ , indeterminacy-membership function  $\nu_{\tilde{p}}(x)$ , and falsity-membership function  $\lambda_{\tilde{p}}(x)$  are given as follows:

$$\begin{split} \mu_{\tilde{p}}(x) &= \begin{cases} \frac{T_{\tilde{p}}\left(x-p_{1}\right)}{\left(p_{2}-p_{1}\right)} & p_{1} \leq x \leq p_{2}, \\ T_{\tilde{p}} & p_{2} \leq x \leq p_{3}, \\ \frac{T_{\tilde{p}}\left(p_{4}-x\right)}{\left(p_{4}-p_{3}\right)} & p_{3} \leq x \leq p_{4}, \\ 0 & \text{otherwise.} \end{cases} \\ \nu_{\tilde{p}}(x) &= \begin{cases} \frac{\left(p_{2}-x+I_{\tilde{p}}\left(x-p_{1}\right)\right)}{\left(p_{2}-p_{1}\right)} & p_{1} \leq x \leq p_{2}, \\ I_{\tilde{p}} & p_{2} \leq x \leq p_{3}, \\ \frac{\left(x-p_{3}+I_{\tilde{p}}\left(p_{4}-x\right)\right)}{\left(p_{4}-p_{3}\right)} & p_{3} \leq x \leq p_{4}, \\ 1 & \text{otherwise.} \end{cases} \\ \lambda_{\tilde{p}}(x) &= \begin{cases} \frac{\left(p_{2}-x+F_{\tilde{p}}\left(x-p_{1}\right)\right)}{\left(p_{2}-p_{1}\right)} & p_{1} \leq x \leq p_{2}, \\ \frac{\left(x-p_{3}+F_{\tilde{p}}\left(x-p_{1}\right)\right)}{\left(p_{2}-p_{1}\right)} & p_{1} \leq x \leq p_{2}, \\ \frac{\left(x-p_{3}+F_{\tilde{p}}\left(p_{4}-x\right)\right)}{\left(p_{4}-p_{3}\right)} & p_{3} \leq x \leq p_{4}, \\ \frac{\left(x-p_{3}+F_{\tilde{p}}\left(p_{4}-x\right)\right)}{\left(p_{4}-p_{3}\right)} & p_{3} \leq x \leq p_{4}, \\ 1 & \text{otherwise.} \end{cases} \end{split}$$

Additionally, when  $p_1 \ge 0$ ,  $\tilde{p} = \langle [p_1, p_2, p_3, p_4], (T_{\tilde{p}}, I_{\tilde{p}}, F_{\tilde{p}}) \rangle$  is called a nonnegative SVTNN. Similarly, when  $p_4 < 0$ ,  $\tilde{p} = \langle [p_1, p_2, p_3, p_4], (T_{\tilde{p}}, I_{\tilde{p}}, F_{\tilde{p}}) \rangle$  becomes a negative SVTNN.

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**Definition 4.** (Arithmetic operation on SVTNNs). Let  $\tilde{r} = \langle [r_1, r_2, r_3, r_4], (T_{\bar{r}}, I_{\bar{r}}, F_{\bar{r}}) \rangle$  and  $\tilde{s} = \langle [s_1, s_2, s_3, s_4], (T_{\bar{s}}, I_{\bar{s}}, F_{\bar{s}}) \rangle$  be two arbitrary SVTNNs, and  $\psi \ge 0$ ; then operations are defined as follows:

 $- \quad \tilde{r} \oplus \tilde{s} = \left\langle \left[r_1 + s_1, r_2 + s_2, r_3 + s_3, r_4 + s_4\right], \left(T_{\tilde{r}} \wedge T_{\tilde{s}}, I_{\tilde{r}} \vee I_{\tilde{s}}, F_{\tilde{s}} \vee F_{\tilde{r}}\right) \right\rangle$ 

$$\begin{split} & - \quad \tilde{r} - \tilde{s} = \left\langle \left[ r_1 - s_4, r_2 - s_3, r_3 - s_2, r_4 - s_1 \right], \left( T_{\tilde{r}} \wedge T_{\tilde{s}}, I_{\tilde{r}} \vee I_{\tilde{s}}, F_{\tilde{s}} \vee F_{\tilde{r}} \right) \right\rangle \\ & - \quad \psi \tilde{r} = \begin{cases} < \left[ \psi r_1, \psi r_2, \psi r_3, \psi r_4 \right], T_{\tilde{r}}, I_{\tilde{r}}, F_{\tilde{r}} >, & \text{if} \psi > 0, \\ < \left[ \psi r_4, \psi r_3, \psi r_2, \psi r_1 \right], T_{\tilde{r}}, I_{\tilde{r}}, F_{\tilde{r}} >, & \text{if} \psi < 0. \end{cases}$$

**Definition 5.** (Comparison of any two random SVTNNs). Let  $\tilde{r} = \langle [r_1, r_2, r_3, r_4], (T_{\tilde{r}}, I_{\tilde{r}}, F_{\tilde{r}}) \rangle$  be a SVTNN and then the score function, accuracy function, and certainty function of SVTNN  $\tilde{r}$  is defined, as follows:

score(
$$\tilde{\mathbf{r}}$$
) =  $\frac{1}{16} [\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4] \times [2 + T_{\tilde{r}} - I_{\tilde{r}} - F_{\tilde{r}}]$   
accuracy( $\tilde{\mathbf{r}}$ ) =  $\frac{1}{16} [\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4] \times [2 + T_{\tilde{r}} - I_{\tilde{r}} + F_{\tilde{r}}]$ 

Let  $\tilde{\mathbf{r}} = \langle [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4], (\mathbf{T}_{\tilde{\mathbf{r}}}, \mathbf{I}_{\tilde{\mathbf{r}}}, \mathbf{F}_{\tilde{\mathbf{r}}}) \rangle$  and  $\tilde{\mathbf{s}} = \langle [\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4], (\mathbf{T}_{\tilde{\mathbf{s}}}, \mathbf{I}_{\tilde{\mathbf{s}}}, \mathbf{F}_{\tilde{\mathbf{s}}}) \rangle$  be two arbitrary SVTNNs, the ranking of  $\tilde{\mathbf{r}}$  and  $\tilde{\mathbf{s}}$  by score function is defined as follows:

- if score( $\tilde{r}$ ) < score( $\tilde{s}$ ) then  $\tilde{r} < \tilde{s}$
- if score( $\tilde{r}$ ) = score( $\tilde{s}$ ) and if
  - $\quad \ \ accuracy(\tilde{r}) < accuracy(\tilde{s}) \ then \ \tilde{r} < \tilde{s}$
  - $\quad \ \ \text{accuracy}(\tilde{r}) > \text{accuracy}(\tilde{s}) \text{ then } \tilde{r} > \tilde{s}$
  - $\operatorname{accuracy}(\tilde{r}) = \operatorname{accuracy}(\tilde{s})$  then  $\tilde{r} = \tilde{s}$

**Definition 6.** A ranking function of neutrosophic numbers is a function  $R : N(R) \rightarrow R$ , where N(R) is a set of neutrosophic numbers defined on set of real numbers, which maps each neutrosophic number into the real line, where a natural order exists. Let  $\tilde{r} = \langle [r_1, r_2, r_3, r_4], (T_{\tilde{r}}, I_{\tilde{r}}, F_{\tilde{r}}) \rangle$  be a SVTNN, then we define:  $R(\tilde{r}) = \text{score}(\tilde{r})$ .

**Definition 7.** Let  $\tilde{A}, \tilde{B}$  be two SVTN numbers, then

- $\quad \tilde{A} \leq \tilde{B} \text{ iff } R(\tilde{A}) \leq R(\tilde{B}),$
- $\quad \tilde{A} < \tilde{B} \text{ iff } R(\tilde{A}) < R(\tilde{B}).$



# 3. Proposed Method

Consider the following Trapezoidal Neutrosophic Linear Programming (TNLP) with m constraints and n variables;

$$\begin{array}{l}
\text{Max}\left(\text{Min}\right)\left(c^{t}\tilde{x}\right)\\
\text{subject to}\\
& A\tilde{x} \leq \tilde{b},
\end{array}$$
(1)

 $\tilde{x}$  is a non-negative trapezoidal neutrosophic number.

where,  $A = [a_{ij}]_{m \times n}$  is the coefficient matrix,  $\tilde{b} = [\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, ..., \tilde{b}_m]^t$  is the trapezoidal neutrosophic available resource vector,  $c = [c_1, c_2, c_3, ..., c_n]^t$  is the objective coefficient vector and  $\tilde{x} = [\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, ..., \tilde{x}_n]^t$  is the trapezoidal neutrosophic decision variable vector.

The steps of the proposed method are as follows:

**Step 1.** Assuming  $\tilde{b} = \langle b^{l}, b^{m}, b^{n}, b^{r}; T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}} \rangle$ ,  $\tilde{x} = \langle x^{l}, x^{m}, x^{n}, x^{r}; T_{\tilde{x}}, I_{\tilde{x}}, F_{\tilde{x}} \rangle$ , and using Definition 4, the LP problem (1) can be transformed into problem (2).

$$\begin{split} & \text{Max} (\text{Min}) \sum_{j=1}^{n} < c_{j} x_{j}^{l}, c_{j} x_{j}^{m}, c_{j} x_{j}^{n}, c_{j} x_{j}^{r}; T_{\tilde{x}_{j}}, I_{\tilde{x}_{j}}, F_{\tilde{x}_{j}} >, \\ & \text{subject to} \\ & \sum_{j=1}^{n} < a_{ij} x_{j}^{l}, a_{ij} x_{j}^{m}, a_{ij} x_{j}^{n}, a_{ij} x_{j}^{r}; T_{\tilde{x}_{j}}, I_{\tilde{x}_{j}}, F_{\tilde{x}_{j}} > \le < b_{i}^{l}, b_{i}^{m}, b_{i}^{n}, b_{i}^{r}; T_{\tilde{b}_{i}}, I_{\tilde{b}}, F_{\tilde{b}} >, \forall i \\ & < x_{j}^{l}, x_{j}^{m}, x_{j}^{n}, x_{j}^{r}; T_{\tilde{x}_{j}}, I_{\tilde{x}_{j}}, F_{\tilde{x}_{j}} > \ge 0, \quad \forall j. \end{split}$$

Step 2. Using definition 2 -4, the LP problem (2) can be transformed into problem (3).

$$\begin{aligned} &\text{Max} (\text{Min}) \sum_{j=l}^{n} < c_{j} x_{j}^{l}, c_{j} x_{j}^{m}, c_{j} x_{j}^{r}; \mathbf{T}, \mathbf{I}, \mathbf{F} >, \\ &\text{subject to} \\ &\sum_{j=l}^{n} < a_{ij} x_{j}^{l}, a_{ij} x_{j}^{m}, a_{ij} x_{j}^{n}, a_{ij} x_{j}^{r}; \mathbf{T}_{\tilde{x}_{j}}, \mathbf{I}_{\tilde{x}_{j}}, \mathbf{F}_{\tilde{x}_{j}} > \leq < b_{i}^{l}, b_{i}^{m}, b_{i}^{n}, b_{i}^{n}; \mathbf{T}_{\tilde{b}_{i}}, \mathbf{I}_{\tilde{b}}, \mathbf{F}_{\tilde{b}} >, \forall i \\ < x_{j}^{l}, x_{j}^{m}, x_{j}^{n}, x_{j}^{n}; \mathbf{T}_{\tilde{x}_{j}}, \mathbf{I}_{\tilde{x}_{j}}, \mathbf{F}_{\tilde{x}_{j}} > \geq 0. \end{aligned}$$

**Step 3.** Using Definition 6, the neutrosophic objective function and also the mentioned constraints of the Model (3), obtained in Step 2, can be converted into the crisp nonlinear programming problem as follows:

$$\begin{split} & \text{Max} \left(\text{Min}\right) R(\sum_{j=1}^{n} < c_{j}x_{j}^{l}, c_{j}x_{j}^{m}, c_{j}x_{j}^{n}, c_{j}x_{j}^{r}; T, I, F >), \\ & \text{subject to} \\ & R(\sum_{j=1}^{n} < a_{ij}x_{j}^{l}, a_{ij}x_{j}^{m}, a_{ij}x_{j}^{n}, a_{ij}x_{j}^{r}; T_{\tilde{x}_{j}}, I_{\tilde{x}_{j}}, F_{\tilde{x}_{j}} >) \leq R(< b_{i}^{l}, b_{i}^{m}, b_{i}^{n}, b_{i}^{r}; T_{\tilde{b}_{i}}, I_{\tilde{b}}, F_{\tilde{b}} >), \forall i \\ & R(< x_{j}^{l}, x_{j}^{m}, x_{j}^{n}, x_{j}^{n}; T_{\tilde{x}_{j}}, I_{\tilde{x}_{j}}, F_{\tilde{x}_{j}} >) \geq 0, \\ & T + I + F \leq 3, \\ & T_{\tilde{x}_{j}} + I_{\tilde{x}_{j}} + F_{\tilde{x}_{j}} \leq 3, \\ & 0 \leq T_{\tilde{x}_{j}} \leq T_{\tilde{b}_{i}}, 1 \geq I_{\tilde{x}_{j}}, 1 \geq F_{\tilde{x}_{j}} \geq F_{\tilde{b}}, \\ & 0 \leq T \leq T_{\tilde{x}_{j}} \leq 1, 1 \geq I \geq I_{\tilde{x}_{j}}, 1 \geq F \geq F_{\tilde{x}_{j}}, \\ & x_{j}^{m} \geq x_{j}^{l}, x_{j}^{n} \geq x_{j}^{m}, x_{j}^{r} \geq x_{j}^{n}, \\ & x_{j}^{l} \geq 0. \end{split}$$

**Step 4.** Find the optimal solution  $\tilde{x}$  by solving the crisp nonlinear programming problems obtained in problem (3) and find the neutrosophic optimal value by putting in the objective function.

#### 4. Numerical Example

In this section, a numerical example problem has been solved using the proposed method to illustrate the applicability and efficiency of it.

#### Example 1.

Max  $(\tilde{z}) = 5\tilde{x}_1 + 4\tilde{x}_2$ subject to  $6\tilde{x}_1 + 4\tilde{x}_2 \le \langle 3, 5, 6, 8; 0.6, 0.5, 0.6 \rangle,$   $\tilde{x}_1 + 2\tilde{x}_2 \le \langle 5, 8, 10, 14; 0.3, 0.6, 0.6 \rangle,$   $-\tilde{x}_1 + \tilde{x}_2 \le \langle 12, 15, 19, 22; 0.6, 0.4, 0.5 \rangle,$   $\tilde{x}_2 \le \langle 14, 17, 21, 28; 0.8, 0.2, 0.6 \rangle,$  $\tilde{x}_1, \tilde{x}_2 \ge 0.$ 

Now. To solve the problem with the proposed method we have the following steps:

**Step 1.** Assuming  $\tilde{x} = \langle x^1, x^m, x^n, x^r; T_{\tilde{x}}, I_{\tilde{x}}, F_{\tilde{x}} \rangle$ , and using Definition 4, the LP problem (4) can be transformed into problem (5).

$$\begin{split} & \text{Max}\,\tilde{z} = 5 < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{1}}, F_{\tilde{x}_{1}} > \oplus 4 < x_{2}^{1}, x_{2}^{m}, x_{2}^{n}, x_{2}^{r}; T_{\tilde{x}_{2}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & \text{subject to} \\ & 6 < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{1}}, F_{\tilde{x}_{1}} > \oplus 4 < x_{2}^{1}, x_{2}^{m}, x_{2}^{n}, x_{2}^{r}; T_{\tilde{x}_{2}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{1}}, F_{\tilde{x}_{1}} > \oplus 4 < x_{2}^{1}, x_{2}^{m}, x_{2}^{n}, x_{2}^{r}; T_{\tilde{x}_{2}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{1}}, F_{\tilde{x}_{1}} > \oplus 2 < x_{2}^{1}, x_{2}^{m}, x_{2}^{n}, x_{2}^{r}; T_{\tilde{x}_{2}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{1}}, F_{\tilde{x}_{1}} > \oplus 2 < x_{2}^{1}, x_{2}^{m}, x_{2}^{n}, x_{2}^{r}; T_{\tilde{x}_{2}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{1}}, F_{\tilde{x}_{1}} > \oplus 2 < x_{2}^{1}, x_{2}^{m}, x_{2}^{n}, x_{2}^{r}; T_{\tilde{x}_{2}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \oplus < x_{2}^{1}, x_{2}^{m}, x_{2}^{n}; X_{2}^{r}; I_{\tilde{x}_{2}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{2}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^{n}; x_{1}^{r}; T_{\tilde{x}_{1}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \\ & < x_{1}^{1}, x_{1}^{m}, x_{1}^{n}, x_{1}^$$

Step 2. Using definition 2 -4, the LP problem (5) can be transformed into the problem (6).

(4)

$$\begin{aligned} &\operatorname{Max} \tilde{z} = < 5x_{1}^{1} + 4x_{2}^{1}, 5x_{1}^{m} + 4x_{2}^{m}, 5x_{1}^{n} + 4x_{2}^{n}, 5x_{1}^{r} + 4x_{2}^{r}; T, I, F > \\ &\operatorname{subject to} \\ &< 6x_{1}^{1} + 4x_{2}^{1}, 6x_{1}^{m} + 4x_{2}^{m}, 6x_{1}^{n} + 4x_{2}^{n}; 6x_{1}^{r} + 4x_{2}^{r}; T, I, F > \leq < 3, 5, 6, 8; 0.6, 0.5, 0.6 >, \\ &< x_{1}^{1} + 2x_{2}^{1}, x_{1}^{m} + 2x_{2}^{m}, x_{1}^{n} + 2x_{2}^{n}, x_{1}^{r} + 2x_{2}^{r}; T, I, F > \leq < 5, 8, 10, 14; 0.3, 0.6, 0.6 >, \\ &< -x_{1}^{r} + x_{2}^{1}, -x_{1}^{n} + x_{2}^{m}, -x_{1}^{m} + x_{2}^{n}; T, I, F > \leq < 12, 15, 19, 22; 0.6, 0.4, 0.5 >, \\ &< x_{2}^{1}, x_{2}^{m}, x_{2}^{n}, x_{2}^{r}; T_{\tilde{x}_{2}}, I_{\tilde{x}_{2}}, F_{\tilde{x}_{2}} > \leq < 14, 17, 21, 28; 0.8, 0.2, 0.6 >, \\ &< x_{1}^{1}, x_{3}^{m}, x_{3}^{n}, x_{5}^{r}; T_{\tilde{x}_{3}}, I_{\tilde{x}_{3}}, F_{\tilde{x}_{3}} > \geq 0, \quad \forall j. \end{aligned}$$

**Step 3.** Using Definition 6, the neutrosophic objective function, and the mentioned constraints of the Model (6), obtained in Step 2, can be converted into the crisp nonlinear programming problem as follows:

$$\begin{split} & \text{Max}\,\tilde{z} = \frac{1}{16} (5x_1^1 + 4x_2^1 + 5x_1^m + 4x_2^m + 5x_1^n + 4x_2^n, 5x_1^r + 4x_2^r)(2 + T - I - F) \\ & \text{subject to} \\ & \frac{1}{16} (6x_1^1 + 4x_2^1 + 6x_1^m + 4x_2^m + 6x_1^n + 4x_2^n + 6x_1^r + 4x_2^r)(2 + T - I - F) \\ & \leq \frac{1}{16} (\frac{374}{10}), \\ & \frac{1}{16} (x_1^1 + 2x_2^1 + x_1^m + 2x_2^m + x_1^n + 2x_2^n + x_1^r + 2x_2^r)(2 + T - I - F) \\ & \leq \frac{1}{16} (\frac{407}{10}), \\ & \frac{1}{16} (-x_1^r + x_2^1 - x_1^n + x_2^m - x_1^m + x_2^n - x_1^1 + x_2^r)(2 + T - I - F) \\ & \leq \frac{1}{16} (\frac{1156}{10}), \\ & \frac{1}{16} (x_2^1 + x_2^m + x_2^n + x_2^n)(2 + T_{\tilde{x}_2} - I_{\tilde{x}_2} - F_{\tilde{x}_2}) \\ & \leq \frac{160}{16}, \\ & \frac{1}{16} (x_2^1 + x_2^m + x_1^n + x_1^r)(2 + T_{\tilde{x}_1} - I_{\tilde{x}_1} - F_{\tilde{x}_1}) \geq 0, \\ & \frac{1}{16} (x_2^1 + x_2^m + x_2^n + x_2^n)(2 + T_{\tilde{x}_2} - I_{\tilde{x}_2} - F_{\tilde{x}_2}) \geq 0, \\ & T + I + F \leq 3, \\ & T_{\tilde{x}_1} + I_{\tilde{x}_1} + F_{\tilde{x}_1} \leq 3, \\ & 0 \leq T_{\tilde{x}_1} \leq T_{\tilde{b}_1}, \\ & 1 \geq I_{\tilde{x}_1} \geq I_{\tilde{b}}, \\ & 1 \geq I \geq I_{\tilde{x}_1}, \\ & x_1^n \geq x_1^n, x_1^n \geq x_1^n, x_1^n \geq x_1^n, \\ & x_1^n \geq 0. \end{split}$$

Step 4. Using Matlab or any software, we can solve the optimal solution.

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