

# A Novel Approach Of Computing With Words By Using Neutrosophic Information

Ling Xin, Bin Zhou, Haitao Lin, Arindam Dey

**Abstract**—The computing with Words (CW) is a well known soft computing method to find the solutions of many decision making problems in real life scenarios which consists of selective information used in natural language. CW is characterized by the human potentiality to do different types of physical and mental works or jobs without making any calculations or computations. Words are less accurate than numbers and words are used if exact numbers are unknown. Same word means different items to different person. There exists uncertainties associated with any word. Fuzzy set is generally used to model the words in the CW technique. The neutrosophic set is an extended version of fuzzy set. Here, we introduce a new idea of CW to model the words using neutrosophic set. In our proposed method, computation are done by words and words are translated to a mathematical model using neutrosophic set. The main objective of this paper is on CW based on neutrosophic set for taking subjective judgments. We call it as perceptual neutrosophic computing. An architecture is introduced for perceptual neutrosophic computing which we call perceptual neutrosophic computer (PNC). PNC has three components: encoder, CW engine for neutrosophic set and decoder. We have introduced a modified Kruskal's algorithm to compute the minimum spanning tree (MST) of a neutrosophic graph. Human being describe those edge weights in real world problems using some terms, like, 'some', 'small', 'big', 'large', etc terms which do not provide any numbers. Those English terms are described here as words. A model of PNC is proposed to employ in our proposed method for solving this problem. The PNC model that is related with minimum spanning tree is defined as *minimum spanning tree advisor* (MSTA) and we have described the design of MSTA in detail in this study. We use a numerical example to describe the efficiency of our proposed algorithm.

**Index Terms**—Computing with Words, Kruskal's algorithm, neutrosophic set

Computing with Words (CW) is a soft computing technique where the main computation objects/items are generally natural words and those words are received from our natural languages. We have several remarkable capacities. Among those capacities, two are very significant. First, human being can able to communicate, converse, reasoning and take good decisions in an uncertainty scenarios. Second, human being can perform many mental and physical jobs without any computations and measurements. The idea of CW is inspired by those two important capacities.

In our classical computing system, the real numbers and symbols are used in computational purpose. But CW uses the natural words for computation. CW introduces the idea to use

the natural languages in many engineering systems. There, CW can minimize the difference between the capability of our brain and the artificial machine. The idea of CW helps the computers to deal with uncertain real life problems. The main reason of CW introduces based on the idea that most of our knowledge comes from the natural language. A natural language is a nothing but a way to describe the perceptions of any scenarios. Shifting from computing with real numbers/symbols to CW is a simple paradigm change for computing industry. It has many purposes for this changes. First, we generally describe and make out wide varieties of information and knowledge based on our language, i.e., a collection of words. Second, a word is naturally less exact than a real number. We use words if the exact numbers are unknown for us or words are used to summarize many numbers. CW can be considered as a very powerful technique for working in uncertain environment. And third, precision is used to carry a cost. We can exploit through the usage of natural words instate of real numbers if there is an allowance for uncertainties. Same word can describe different measurement/object/thing to different persons.

Uncertainties [1], [2] exist in almost every words in any natural language. Uncertainty of any natural words in CW paradigm can be deal by classical fuzzy set (CFS) or type 2 fuzzy set (T2FS) or interval T2FS (IT2FS). Many researchers e.g., [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16] have used T1FS. The membership function of any T1FS is crisp set and it does not able to handle for various types of uncertainties that presents in the linguistic description of any words. The idea of type-2 fuzzy set (T2FS) has been introduced by L. A. Zadeh [17]. T2FS is an extended version of a classical fuzzy set, i.e., T1FS. Since the membership grade of any T1FS is totally real number. However, the membership grade of a T2FSs is not real number or crisp value. It is nothing but a CFS [18], [17]. So, the membership values of T2FS are between the interval [0,1] instead of crisp or real value. The idea of T2FS extends the degrees of freedom to mathematical representation of uncertainties of any information and T2FSs have greater capability than CFS to work with uncertainties in logically proper way. But the T2FS needs high computational times, several researches have considered the IT2FSs to handle the uncertainties in real life situations [19], [20], [21], [22]. IT2FSs are the especial type of T2FSs. Computations in IT2FS are low compared to classical T2FS [23], [24]. A Perceptual Computer model (Per-CM) is used to take decision of any decision making problem in CWW technique. It is presented in Fig. 1. The Per-CM has 3 components: encoder, CW engine and decoder. First, the encoder receives the linguistic perceptions, i.e., single word, phase, granulated term, collection of words, etc as an input and produces the output as a single IT2FS or collection of IT2FSs. The output

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of the encoder activates the CWW engine. The CWW engine takes IT2FSs as an input. The CWW engine can perform several operations on any IT2FSs. The CWW engine converts its input IT2FSs to its output IT2FS. This operation can be performed in several different possible manners, e.g., the average, addition, perceptual reasoning, linguistic weighted mean, using by some rules, etc. Last, the decoder produces a recommendation based on the outputs of the CWW engine. The recommendation of a decoder may be a rank, word or phase. In any decision making in real life scenarios, various alternatives/situations are equated at a time to determine the most beneficial one(s). To determine the exact situation(s), the decoder needs to use the rank method.

However the fuzzy set has been developed and generally used in CW, but it is unable to deal with many kinds of uncertainties in the meaning of the words. There exists some kinds of uncertainties e.g., indeterminate and inconsistent in the information which can not be properly handled by fuzzy set. For e.g., if a person wants to recognize the experience of a decision maker regarding any specific word, decision maker may give his opinion that the possibility of the truthfulness of the word is 0.7, the degree of sure is 0.2 and the degree of false is 0.5. We cannot handle this type problems using fuzzy sets. We require new theories to deal with this problems.

In 1995, F. Smarandache [25], [26], [27] presented a new idea of neutrosophic set (NS) and neutrosophic logic. In a neutrosophic set, each element has 3 membership degree: truth, indeterminacy, and falsity. Those 3 membership degree are lied in the interval of  $0^-$ ,  $1^+$  and it is an interval of nonstandard unit.  $0^-$ ,  $1^+$  is a modified version of standard interval between 0 and 1. The uncertainty due to indeterminacy is not dependent of false and true degree. In this study, we have modeled the words in the CW technique using neutrosophic set. Recently, several researchers have studied on graph theory in neutrosophic environment for instance, Ye [28], Yang et al. [29], Naz et al.[30], Broumi[31], and Arkam [32], [33], [34].

The finding MST of connected weighted graph [35], [36] is a well known classical optimization problems [37], [1] in the area of operation research. Several real life problems, e.g., logistics, transportation, image processing, computer networks, data storage, numerical cluster analysis, speech recognition [38], etc can be model as MST problem. Many researchers [39], [40], [41], [42], [43] have studied lots to develop an efficient algorithm for classical MST problem. In simple MST problem, we consider some fixed/crisp values to describe the edge weights of a graph. Decision maker assumes that we know the exact edge weight. However, in our daily life, we get several imprecise and incomplete information. It creates a very difficult scenarios to present the proper edge weight by a specific real number. For e.g., the edge weight of a graph can describe the travel time between two villages. This edge weights [44], [45] should not be defined exactly because travel time may change due to the traffic frequency, weather condition and any other unexpected reasons however the distance between the two villages is same. In such real life situation, we (decision makers) generally give some possible travel times in approximate intervals. The arc length is evaluated by human perception. CFS and IT2FS are unable to handle the indeterminate and inconsistent uncertainties. It can be

tackled by NS. Numerous papers have been published to determine the MST in fuzzy environment. In all those papers, different CFSs are considered to describe the arc length of the fuzzy graph. The paper by Itoh and Ishii [46] is one of the first paper on this subject and studied the fuzzy MST problem. They used the chance constrained program method to solve the MST problem. Following that, 3 methods based on the existence ranking index for the ranking the arcs of the spanning tree. The ranking methods were presented by Chang and Lee [47]. In [48], Almeida *et al.* formulated the MST problem with fuzzy arc lengths and they introduced an algorithm to solve this problem. They also proposed a genetic algorithm to determine the MST problem with fuzzy parameters. Janiak and Kasperski [49] have applied the idea of possibility theory to sort the arcs of a fuzzy graph. They used this idea to determine the MST whose edge weights are described as interval of fuzzy set. Liu [50], [51] presented the concept of credibility theory which consists of credibility measurement, pessimistic value and expected value. Gao and Lu [52] presented the fuzzy quadratic MST problem using the credibility theory. They have modeled the spanning tree as expected model, chance constrained programming model and dependent chance programming model.

In this study, we introduce a new idea of CW to model the words using neutrosophic set. In our proposed method, computation are done by words and the words are represented by neutrosophic set. The main objective of this paper is on CW based neutrosophic set for taking subjective judgments. We call it as perceptual neutrosophic computing. A computational model is introduced for perceptual neutrosophic computing which we call perceptual neutrosophic computer (PNC). PNC has 3 components: encoder, CWW engine for neutrosophic set and decoder. In this work, we present the MST problem of a neutrosophic graph where arc costs are represented by words, e.g., low, very high, Small, Medium, etc. We call this MST problem as computing the minimum spanning tree with word (CMSTW). We propose a PNC model to solve this MST problem. The PNC that is related with neutrosophic MST problem is called a *minimum spanning tree advisor* (MSTA). We have presented the design of MSTA in this study. MSTA has of 3 components: encoder, CW engine for MST and decoder. In a PNC, the encoder takes one and all the arcs, i.e., words of the spanning tree and transfer those words to NSs. The CW engine for MST is used as adder. It adds all those NSs and produces a NS which represents the cost of the spanning tree. The output of CWW engine is the input of the decoder and decoder determines the ranking value of the corresponding spanning tree. We use this rank to find the MST. A linear programming model for this CMSTW problem has been presented. Kruskal's algorithm is modified to find the solution this CMSTW problem. A numerical example is described to illustrate the proposed method. The primary objectives of this paper are as follows.

- (i) A new idea of CW is proposed to model the words using neutrosophic set. We introduce a computational model to solve the problem. This computational model is called PNC.
- (ii) The MST problem with natural word as arc length of a neutrosophic graph is described. We define this problem as CMSTW.
- (iii) A linear programming model for this CMSTW problem

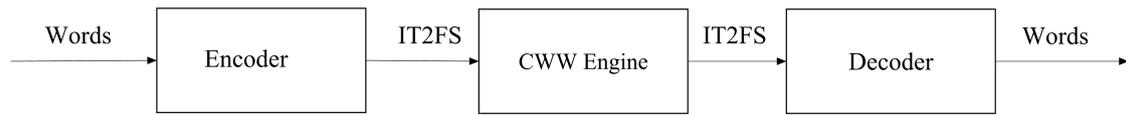


Fig. 1. The Perceptual computer model.

has been presented for the first time. The standard optimization solver LINGO is used to obtain the MST of the fuzzy graph.

- (iv) The classical Kruskal's algorithm is extended to solve the CMSTW problem.
- (v) The MSTW and its cost are the primary information for the decision makers in a MST problem. This paper aims at finding the MSTW and its corresponding cost.
- (vi) We have presented a numerical example to illustrate the performance of our algorithm.

### I. PRELIMINARIES

We have described ideas of intuitionistic fuzzy set, NS, SVN-number, SVTNN, SVTRN, addition operation of SVTNs and ranking method of SVTN number concerned with the paper. These concepts are very much needed to have a good understanding of the proposed algorithms described in the paper.

*Definition 1:* Let  $\xi$  represents an universe set and an intuitionistic fuzzy set  $P$  over the  $\xi$ . Then,  $P$  is described by

$$P = \{ \langle p, \mu_P(p), \gamma_P(p) \rangle : p \in \xi \} \quad (1)$$

Here,  $\mu_P : \xi \rightarrow [0, 1]$  and  $\gamma_P : \xi \rightarrow [0, 1]$  such that  $0 \leq \mu_P(p) + \gamma_P(p) \leq 1, \forall x \in \xi$ . The  $\mu_P(p)$  and  $\gamma_P(p)$  are used to describe the membership and non membership grades of an element  $p$  respectively.

*Definition 2:* Let  $\xi$  represents an universe set and a NS [25]  $Q$  over the  $\xi$ . Then, the NS  $Q$  can be described as follow.

$$Q = \{ \langle p, (T_Q(p), I_Q(p), Q_Q(p)) \rangle : p \in \xi \} \quad (2)$$

Where  $T_Q(p)$ ,  $I_Q(p)$  and  $F_Q(p)$  are represented three membership functions: truth, indeterminacy and false of element  $x$  respectively. Those membership functions are respectively described by

$$T_Q : \xi \rightarrow ]^{-0}, 1^{+}[ , I_Q : \xi \rightarrow ]^{-0}, 1^{+}[ , F_Q : \xi \rightarrow ]^{-0}, 1^{+}[$$

such that  $0^- \leq T_Q(p) + I_Q(p) + F_Q(p) \leq 3^+$ .

*Definition 3:* Any single valued NS [25] is a NS over  $\xi$ . It has 3 membership functions: truth, indeterminacy and false. Those are described as follows.

$$T_Q : \xi \rightarrow [0, 1], I_Q : \xi \rightarrow [0, 1], F_Q : \xi \rightarrow [0, 1]$$

Such that  $0 \leq T_Q(p) + I_Q(p) + F_Q(p) \leq 3$ .

*Definition 4:* Let  $w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \in [0, 1]$  represents the real numbers,  $\alpha_j, \beta_j, \delta_j, \gamma_j \in \mathbb{R}$  and  $\alpha_j, \beta_j, \delta_j, \gamma_j (j = 1, 2, 3)$ . Let  $\tilde{A}$  is a single valued neutrosophic number (SVNN). It is a specific type of NS over  $\mathbb{R}$ , whose truth membership

function  $\mu_{\tilde{A}}$ , indeterminacy membership function  $v_{\tilde{A}}$  and falsity membership function  $\lambda_{\tilde{A}}$  are described by

$$\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0, w_{\tilde{A}}], \quad \mu_{\tilde{A}}(p) = \begin{cases} f_{\mu}^l(p), & \alpha_1 \leq p < \beta_1 \\ w_{\tilde{A}}, & \beta_1 \leq p < \delta_1 \\ f_{\mu}^r(p), & \delta_1 \leq p < \gamma_1 \\ 0, & \text{otherwise} \end{cases}$$

$$v_{\tilde{A}} : \mathbb{R} \rightarrow [u_{\tilde{A}}, 1], \quad v_{\tilde{A}}(p) = \begin{cases} f_v^l(p), & \alpha_2 \leq p < \beta_2 \\ u_{\tilde{A}}, & \beta_2 \leq p < \delta_2 \\ f_v^r(p), & \delta_2 \leq p < \gamma_2 \\ 1, & \text{otherwise} \end{cases}$$

$$\lambda_{\tilde{A}} : \mathbb{R} \rightarrow [y_{\tilde{A}}, 1], \quad \lambda_{\tilde{A}}(p) = \begin{cases} f_{\lambda}^l(p), & \alpha_3 \leq p < \beta_3 \\ u_{\tilde{A}}, & \beta_3 \leq p < \delta_3 \\ f_{\lambda}^r(p), & \delta_3 \leq p < \gamma_3 \\ 1, & \text{otherwise} \end{cases}$$

Where the function  $f_{\mu}^l(p) : [\alpha_1, \beta_1] \rightarrow [0, w_{\tilde{A}}]$ ,  $f_v^r(p) : [\delta_2, \gamma_2] \rightarrow [u_{\tilde{A}}, 1]$ ,  $f_{\lambda}^r(x) : [\delta_3, \gamma_3] \rightarrow [y_{\tilde{A}}, 1]$ . Those functions are non-decreasing and continuous. They will satisfy the following conditions:  $f_{\mu}^l(\alpha_1) = 0$ ,  $f_{\mu}^l(\beta_1) = w_{\tilde{A}}$ ,  $f_v^r(\delta_2) = u_{\tilde{A}}$ ,  $f_v^r(\gamma_2) = 1$ ,  $f_v^r(\delta_3) = y_{\tilde{A}}$ , and  $f_{\lambda}^r(\gamma_3) = 1$ ; the functions  $f_{\mu}^r : [\delta_1, \gamma_1] \rightarrow [0, w_{\tilde{A}}]$ ,  $f_v^l : [\alpha_2, \beta_2] \rightarrow [u_{\tilde{A}}, 1]$  and  $f_{\lambda}^l : [\alpha_3, \beta_3] \rightarrow [y_{\tilde{A}}, 1]$  are non-increasing and continuous function. Those function will satisfy the following considerations:  $f_{\mu}^r(\delta_1) = w_{\tilde{A}}$ ,  $f_{\mu}^r(\gamma_1) = 0$ ,  $f_v^l(\alpha_2) = 1$ ,  $f_v^l(\beta_2) = u_{\tilde{A}}$ ,  $f_{\lambda}^l(\alpha_3) = 1$ , and  $f_{\lambda}^l(\beta_3) = y_{\tilde{A}}$ .  $[\beta_1, \delta_1]$ ,  $\alpha_1$  and  $\gamma_1$  are used to represent the average of interval, upper and lower limit values of the function truth membership of a simple neutrosophic number  $\tilde{A}$ .  $[\beta_2, \delta_2]$ ,  $\alpha_2$  and  $\gamma_2$  are used to represent the average of interval and the upper and lower limit values of the function of indeterminacy-membership for the simple neutrosophic number  $\tilde{A}$ , respectively.  $[\beta_3, \delta_3]$ ,  $\alpha_3$  and  $\gamma_3$  are used to represent the average of interval and the upper and lower limit values of the function of the falsity membership of a neutrosophic number  $\tilde{A}$  respectively.  $w_{\tilde{A}}$  is said to be the highest truth membership grade of  $\tilde{A}$ ,  $u_{\tilde{A}}$  is said to be the lowest indeterminacy membership grade of  $\tilde{A}$  and  $y_{\tilde{A}}$  is said to be the lowest falsity-membership degree of  $\tilde{A}$ .

*Definition 5:* Let  $\tilde{A}$  represents any single valued trapezoidal neutrosophic number (SVTNN) [53] where  $\tilde{A} = \langle (\alpha, \beta, \delta, \gamma); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ . Single valued triangular neutrosophic number is special type of SVTNN. It consists of four points. SVTNN is a special type of NS on the  $\mathbb{R}$ , whose truth, indeterminacy and false degree of membership are described as follows:

$$\mu_{\tilde{A}}(p) = \begin{cases} \frac{(p-\alpha)w_{\tilde{A}}}{(\beta-\alpha)} & (\alpha \leq p < \beta) \\ w_{\tilde{A}} & (\beta \leq p \leq \delta) \\ \frac{(\gamma-p)w_{\tilde{A}}}{(\gamma-\delta)} & (\delta < p \leq \gamma) \\ 0 & \text{otherwise} \end{cases}$$

$$v_{\tilde{A}}(p) = \begin{cases} \frac{(\beta-p+u_{\tilde{A}}(p-\alpha))}{(\beta-\alpha)} & (\alpha \leq x < \beta) \\ u_{\tilde{A}} & (\beta \leq p \leq \delta) \\ \frac{(p-\delta+u_{\tilde{A}}(\gamma-p))}{(\gamma-\delta)} & (\delta < x \leq \gamma) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\lambda_{\tilde{A}}(p) = \begin{cases} \frac{(\beta-p+y_{\tilde{A}}(p-\alpha))}{(\beta-\alpha)} & (\alpha \leq p < \delta) \\ y_{\tilde{A}} & (\beta \leq p \leq \delta) \\ \frac{(x-\delta+y_{\tilde{A}}(\gamma-p))}{(\gamma-\delta)} & (\delta < p \leq \gamma) \\ 0 & \text{otherwise} \end{cases}$$

respectively.

**Definition 6:** Let  $\tilde{A}$  is a single valued triangular neutrosophic number (SVTrN-number) [53] where  $\tilde{A} = \langle (\alpha, \beta, \delta, \gamma); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ . It is a specific type of NS on the  $\mathbb{R}$ . The truth degree of membership, indeterminacy degree of membership and falsity degree of membership are described as follows.

$$\mu_{\tilde{A}}(p) = \begin{cases} \frac{(p-\alpha)w_{\tilde{A}}}{(\beta-\alpha)}, & (\alpha \leq p < \beta) \\ \frac{(\delta-p)w_{\tilde{A}}}{(\delta-\beta)}, & (\beta \leq p < \delta) \\ 0, & \text{otherwise} \end{cases}$$

$$v_{\tilde{A}}(p) = \begin{cases} \frac{(\beta-p+u_{\tilde{A}}(p-\alpha))}{(\beta-\alpha)}, & (\alpha \leq p < \beta) \\ \frac{(p-\beta+u_{\tilde{A}}(\delta-p))}{(\delta-\beta)}, & (\beta \leq p < \delta) \\ 0, & \text{otherwise} \end{cases}$$

$$\lambda_{\tilde{A}}(p) = \begin{cases} \frac{(\beta-p+y_{\tilde{A}}(p-\alpha))}{(\beta-\alpha)}, & (\alpha \leq p < \delta) \\ \frac{(p-\beta+y_{\tilde{A}}(\delta-p))}{(\delta-\beta)}, & (\beta \leq x \leq \delta) \\ 0, & \text{otherwise} \end{cases}$$

If  $\alpha \geq 0$  and at least  $c > 0$  then  $\tilde{A} = \langle (\alpha, \beta, \delta, \gamma); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$  is said to be a positive SVTrN-number, denoted by  $\tilde{A} > 0$ .

**Definition 7:** Let  $\tilde{A} = \langle (\alpha_1, \beta_1, \delta_1, \gamma_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$  and  $\tilde{B} = \langle (\alpha_2, \beta_2, \delta_2, \gamma_2); w_{\tilde{B}}, u_{\tilde{B}}, y_{\tilde{B}} \rangle$  be two SVTrN-number and  $\gamma \neq 0$  be any real number. Then, the addition operation between  $\tilde{A}$  and  $\tilde{B}$

$$\tilde{A} + \tilde{B} = (\alpha_1 + \alpha_2, \beta_1 + \beta_2, \delta_1 + \delta_2, \gamma_1 + \gamma_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \quad (3)$$

**Definition 8:** Let  $\tilde{A} = \langle (\alpha_1, \beta_1, \delta_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$  and  $\tilde{B} = \langle (\alpha_2, \beta_2, \delta_2); w_{\tilde{B}}, u_{\tilde{B}}, y_{\tilde{B}} \rangle$  are two SVTrN-numbers and  $\gamma \neq 0$  is a real number. The addition operation between  $\tilde{A}$  and  $\tilde{B}$  are described as follows.

$$\tilde{A} + \tilde{B} = (\alpha_1 + \alpha_2, \beta_1 + \beta_2, \delta_1 + \delta_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \quad (4)$$

**Definition 9:** Let  $\tilde{A} = \langle (\alpha, \beta, \delta); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$  is a SVN-number. The value of  $\theta$  lies in  $\in [0, 1]$ .

1. Let  $X_\theta$  be the  $\theta$  weighted value of  $\tilde{A}$ . Then,  $X_\theta$  can be described as follows.

$$X_\theta(\tilde{A}) = \theta X_\mu(\tilde{A}) + (1 - \theta) X_v(\tilde{A}) + (1 - \theta) X_\lambda(\tilde{A}) \quad (5)$$

2. Let  $Y_\theta$  be the  $\theta$  weighted value of  $\tilde{A}$ . Then,  $Y_\theta$  can be

described as follows.

$$Y_\theta(\tilde{A}) = \theta Y_\mu(\tilde{A}) + (1 - \theta) Y_v(\tilde{A}) + (1 - \theta) Y_\lambda(\tilde{A}) \quad (6)$$

**Corollary 3.11** Let  $\tilde{A}$  is an arbiter SVTrN-number where  $\tilde{A} = \langle (\alpha, \beta, \delta, \gamma); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ . Then,

1. The weighted value  $X_\theta(\tilde{A})$  is calculated as;

$$V_\theta(\tilde{A}) = \frac{\alpha + 2\beta + 2\delta + \gamma}{6} \times [\theta w_{\tilde{A}}^2 + (1 - \theta)(1 - u_{\tilde{A}})^2 + (1 - \theta)(1 - y_{\tilde{A}})^2] \quad (7)$$

2. Let  $Y_\theta(\tilde{A})$  be the weighted ambiguity of  $\tilde{A}$  and it is computed as follows.

$$Y_\theta(\tilde{A}) = \frac{\gamma - \alpha + 2\delta - 2\beta}{6} \times [\theta w_{\tilde{A}}^2 + (1 - \theta)(1 - u_{\tilde{A}})^2 + (1 - \theta)(1 - y_{\tilde{A}})^2] \quad (8)$$

**Definition 10:** Let  $\tilde{A}$  is an arbiter SVTrN-number where  $\tilde{A} = \langle (\alpha, \beta, \delta, \gamma); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ . Then,

1. The weighted value  $X_\theta(\tilde{A})$  is calculated as follows.

$$X_\theta(\tilde{A}) = \frac{\alpha + 4\beta + \gamma}{6} \times [\theta w_{\tilde{A}}^2 + (1 - \theta)(1 - u_{\tilde{A}})^2 + (1 - \theta)(1 - y_{\tilde{A}})^2] \quad (9)$$

2. Let  $Y_\theta(\tilde{A})$  be the weighted ambiguity of  $\tilde{A}$  and it is computed as follows.

$$Y_\theta(\tilde{A}) = \frac{\delta - \alpha}{6} \times [\theta w_{\tilde{A}}^2 + (1 - \theta)(1 - u_{\tilde{A}})^2 + (1 - \theta)(1 - y_{\tilde{A}})^2] \quad (10)$$

**Definition 11:** Let  $\tilde{A}$  and  $\tilde{B}$  are two SVN-numbers [53] and  $\theta \in [0, 1]$ .

1. If  $X_\theta(\tilde{A}) > X_\theta(\tilde{B})$ , then the value of  $\tilde{A}$  is bigger than the value of  $\tilde{B}$ .

2. If  $X_\theta(\tilde{A}) < X_\theta(\tilde{B})$ , then the value of  $\tilde{A}$  is smaller than the value of  $\tilde{B}$ .

3. If  $X_\theta(\tilde{A}) = X_\theta(\tilde{B})$ , then

(a) If  $Y_\theta(\tilde{A}) = Y_\theta(\tilde{B})$ , then the value of  $\tilde{A}$  is equal to the value of  $\tilde{B}$ .

(b) If  $Y_\theta(\tilde{A}) > Y_\theta(\tilde{B})$ , then the value of  $\tilde{A}$  is bigger than  $\tilde{B}$ .

(c) If  $Y_\theta(\tilde{A}) < Y_\theta(\tilde{B})$ , then the value of  $\tilde{A}$  is smaller than the value of  $\tilde{B}$ .

## II. CW IN NEUTROSOPHIC ENVIRONMENT

L. A. Zadeh [17], [16] introduced the fuzzy set with CW. CW technique is not that computers are computed based single word, words or phrase rather than boolean number. It is method that activates the computers using the words. Word in CW paradigm are converted to a numerical representation based fuzzy set and that these fuzzy set are mapped some other fuzzy set by using CW engine.

A single word can describe different items to different persons, so it is very essential to deal word uncertainties properly. To the best of our information, there exists two

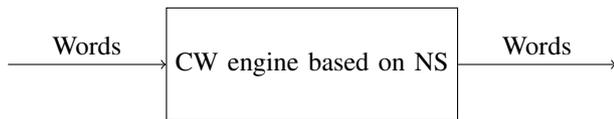


Fig. 2. CW in neutrosophic environment

possible choices: T1FS and IT2FS. Fuzzy set is unable to deal with many kinds of uncertainties in the meaning of the words. There exists some kinds of uncertainties e.g., indeterminate and inconsistent in the information which can not be properly handled by fuzzy set. In this study, we have modeled the words in the CW technique using neutrosophic set. Neutrosophic set can handle the uncertainties due to indeterminate and inconsistent.

#### A. Perceptual neutrosophic computer

In this study, we have introduced a model of computer for CW using neutrosophic set. This computer model is called as Perceptual neutrosophic computer (PNC). It consists of three part: encoder, CW engine based on NS and decoder. The PNC is activated by words, phases or granulated terms and the output of the PNC is also words. We can interact with the PNC using a set of vocabulary.

First, we have to establish a set of vocabulary for a specific application. This helps to us to interact with PNC in a user friendly environment. The encoder converts the word into a NS and construct to a codebook of words with corresponding NS. The output of an encoder in PNC is a NS that activates the CW engine. The output of CW engine is an other NS. It is then converted in rank/similar word of a vocabulary by the decoder. The figure of a PNC is Figure 3.

### III. PROBLEM FORMULATION

Let  $\tilde{G} = (V, \tilde{E})$  is a connected undirected weighted neutrosophic graph, where  $V$  represents a collection of nodes of  $G$  and  $\tilde{E}$  is a collection of arcs. A neutrosophic tree  $\tilde{T}$  is called a neutrosophic spanning tree of a neutrosophic graph  $\tilde{G}$  if and only  $\tilde{T}$  is a sub graph of  $\tilde{G}$  and  $\tilde{T}$  consists of all the nodes of  $\tilde{G}$ . Any spanning tree of neutrosophic graph  $\tilde{G}$  has always just  $n - 1$  edges, where  $n$  denotes the total number of nodes in  $\tilde{G}$ . A neutrosophic graph may have more than one spanning tree. Neutrosophic MST (NMST) is a neutrosophic spanning tree whose cost is lowest among all the spanning tree. In general, real numbers are used to represent the arc length of the network. However, we are unable to model many real life application using classical graph. The arc lengths/edge costs are inexact because poor evidence, incomplete information, imperfect statistical analysis, etc. Neutrosophic network can handle those uncertainties. We can describe the neutrosophic graph in five possible types in which a simple graph can be neutrosophic graph.

- 1: A graph is a neutrosophic graph if it consists of neutrosophic set of simple crisp graphs.
- 2: A graph is a neutrosophic graph if it consists of neutrosophic arc set and crisp node set.
- 3: A graph is a neutrosophic graph if it consists of crisp nodes and edges with neutrosophic connectivity.
- 4: A graph is a neutrosophic graph if it consists of crisp graph with neutrosophic arc lengths.

5: A graph is a neutrosophic graph if it consists of neutrosophic nodes and crisp edges.

In this work, we consider an undirected weighted connected neutrosophic graph. This neutrosophic graph is belonged to Type 4 class. The neutrosophic arc lengths are considered as words instead of real number or fuzzy numbers. This MST is defined as minimum spanning tree with words (MSTW). The MSTW problem can be formulated as the following linear programming form:

$$\text{Min } \tilde{Z} = \sum_{\tilde{e} \in \tilde{E}} c_{\tilde{e}}^w x_{\tilde{e}} \quad (11)$$

Subject to

$$\sum_{\tilde{e} \in \tilde{E}} x_{\tilde{e}} = n - 1 \quad (12)$$

$$\sum_{\tilde{e} \in \delta(s)} x_{\tilde{e}} \geq 1 \quad \forall s \subset V, \quad \emptyset \neq s \neq V \quad (13)$$

$$x_{\tilde{e}} \in \{0, 1\} \quad \forall \tilde{e} \in \tilde{E} \quad (14)$$

Here, each edge is represented by  $\tilde{e}$ . The edge  $\tilde{e}$  is nothing but an order pair of  $(v_i, v_j)$ , where the vertices  $v_i, v_j \in V$  and  $v_i \neq v_j$ . If any edge  $\tilde{e}$  is absent in the MSTW then  $x_{\tilde{e}} = 0$ , otherwise  $x_{\tilde{e}} = 1$ .  $c_{\tilde{e}}^w$  denotes a word related to arc  $\tilde{e}$  which is arc length of the arc  $\tilde{e}$ . The Equation (12) provides that the total number of edges in the MSTW is  $n - 1$ . The Equation (13) ensures that the cut set of a subset of the nodes  $s$ , i.e., a edge that have one node in the set  $s$  and the other outside  $s$ .

### IV. PROPOSED ALGORITHM FOR THE MSTW AND ITS COST

In this section, the NMST of a neutrosophic graph is described where edge weights are represented by words, e.g., Low, Not low, Very low, High, Not high, Very high, Fairly low etc. The PNC that is related with MSTW problem is called a *minimum spanning tree advisor* (MSTA). The design of MSTA is presented in Section IV-A. We introduce the modified Kruskal's algorithm for this in Section IV-B.

#### A. Minimum spanning tree Judgment Advisor

In any application of CW technique, first decision maker establishes a list of vocabularies of specific application related words. First, we have established a vocabulary for minimum spanning tree problem. The user can interact with the PNC easily using those vocabulary. It creates a user friendly environment. The set of all those words, i.e.,  $W_i$ , in the vocabulary and their SVN-numbers, i.e.,  $SVN(W_i)$ , creates a Codebook for NMST problem, i.e., Codebook =  $\{(W_i, SVN(W_i))\}$ . Here, a spanning tree is a set of edges and edge costs are expressed by the words. So, spanning tree is a collection of words. A PNC model is introduced to determine the cost of a neutrosophic spanning tree of a neutrosophic graph. The PNC that is related with NMST is the *minimum spanning tree advisor* (MSTA). It is shown in Fig. 4. It has of 3 components:

- 1: First, the encoder takes all the words as input which present in a spanning tree and the encoder transforms all those words into SVN-numbers. CW uses the Codebook of words with their SVN-numbers.

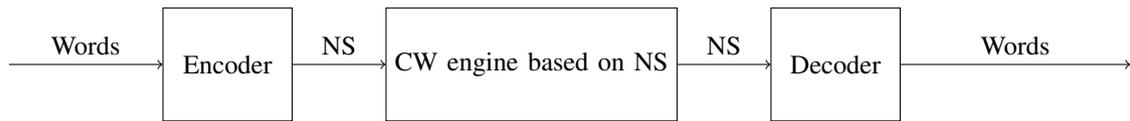


Fig. 3. An architecture for CW in neutrosophic environment the perceptual neutrosophic computer model.

- 2: : The output of encoder is a SVN-number that activates Step 4. Repeat the Step 2 until  $n - 1$  edges have been considered in MSTW.
- the CW engine of NS. The output of CW is an another SVN number. We propose a CW engine for NS. If there exists a single arc/word in the spanning tree then it provides the corresponding SVN-number. Otherwise, it sums all SVN-numbers as described in (7) and returns a SVN-number for the corresponding spanning tree.
- 3: Last, the decoder takes the SVN-number. This SVN number is the output of CW engine. We determine the corresponding  $X_{\theta}(\tilde{A})$  value as defined in Definition 8. The decoder produces the rank value of the spanning tree. So, PNC receives a spanning tree in the collection of words and produces the rank of the spanning tree.

### B. Proposed Kruskal's algorithm for MSTW problem

Our proposed algorithm is the extended version of classical Kruskal's algorithm for solving NMST problem. We have integrated the idea of uncertainties in classical Kruskal's algorithmic method using natural word as an arc length. The classical Kruskal's algorithm is a popular algorithm for minimum spanning tree problem which finds an edge with lowest cost that joins any 2 trees in the graph. This Kruskal's algorithm is an example of greedy algorithm in operation research as it finds MST of graph adding increasing arc length at each and every step. There are 3 important subjects require to consider to extend the Kruskal's algorithmic approach to figure out the MSTW. One is how determine the NS for any word. Second issue is how to determine the cost of the spanning tree. In this problem, it is nothing but a collection of words. The third one is how to do the comparison between the lengths of two different spanning trees. We use *minimum spanning tree advisor* to solve those problem.

In our proposed algorithm, we use the notation  $T^w$  to denote the MSTW of the graph. The variable  $A$  represents the set of total unvisited arc that are needed to be taken out.  $n$  denotes the number of nodes in the graph  $\tilde{G}$ . An another notation list-word  $[T^w]$  is used to store the total words related to the neutrosophic spanning tree, i.e.,  $T^w$ . We construct a basic function  $PNC()$ . This function takes the spanning tree using list-word  $[T]$  and it returns the rank of the spanning tree. The main steps of the *Kruskal's algorithm with words as arc lengths* are as follows:

- Step 1. All the edges  $\tilde{e} \in \tilde{E}$  of the neutrosophic graph  $\tilde{G}$  are passed though the function  $PNC()$ .
- Step 2. The rank values are determined for all the edge of the neutrosophic graph  $\tilde{G}$ .
- Step 3. Sort all the edges of neutrosophic graph  $\tilde{G}$  based on the corresponding ranking value: least ranking value first and heaviest ranking value last.
- Step 3. Select the lightest not verified edge of the  $\tilde{G}$ . Then, add this selected edge to the MSTW, only if doing so will not create any cycle.

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**Algorithm 1** Pseudocode of the proposed algorithm for designing MSTW

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**Input:** The undirected connected weighted neutrosophic graph  $\tilde{G}$  with words as arc lengths

**Output:** MSTW, i.e.,  $T^w$

- 1: **Begin**
  - 2:  $T^w \leftarrow \{\emptyset\}$   $\triangleright T^w$  describes a set of arcs, which represents the MSTW.
  - 3: **for** each edge  $\tilde{e} \in \tilde{G}$  **do**
  - 4:   rank $[e] \leftarrow PCN(\text{list-word}[e])$   $\triangleright$  The function  $PCN(\text{list-word}[e])$  returns the rank of arc  $e$ .
  - 5:   insert  $\tilde{e}$  into  $A$  with its corresponding value rank $[e]$
  - 6: **end for**
  - 7: **while**  $|T^w| < n$  **do**
  - 8:   Find an edge  $\tilde{e}$  form  $A$  with lowest value of rank.
  - 9:   **if**  $T^w \cup \tilde{e}$  is not a cycle **then**
  - 10:      $T^w \leftarrow T^w \cup \tilde{e}$
  - 11:     list-word $[T] \leftarrow \text{list-word}[T] \cup \text{list-word}[\tilde{e}]$
  - 12:   **end if**
  - 13:    $A = A \setminus \tilde{e}$
  - 14: **end while**
  - 15: cost $[T^w] \leftarrow PCN(\text{list-word}[T^w])$
  - 16: **return**  $T^w$
  - 17: **End**
- 

We have presented the pseudocode of the proposed algorithm in Algorithm 1.

### V. NUMERICAL ILLUSTRATIONS

In this section, we have presented a numerical example of the MSTW problem to describe our proposed algorithm. This problem contains 6 vertices and 9 edges. Our algorithm determines the MSTW of a graph, whose edge weights are denoted by natural word. The eight words, presented in Table I are considered as the codebook for the MSTW: low, not low, very low, high, not high, very high, fairly high and fairly low. The edge weights of the neutrosophic graph are represented by those words. For the neutrosophic graph, shown in Fig. 5, those words are assigned to the edges as edge weight of the graph randomly.

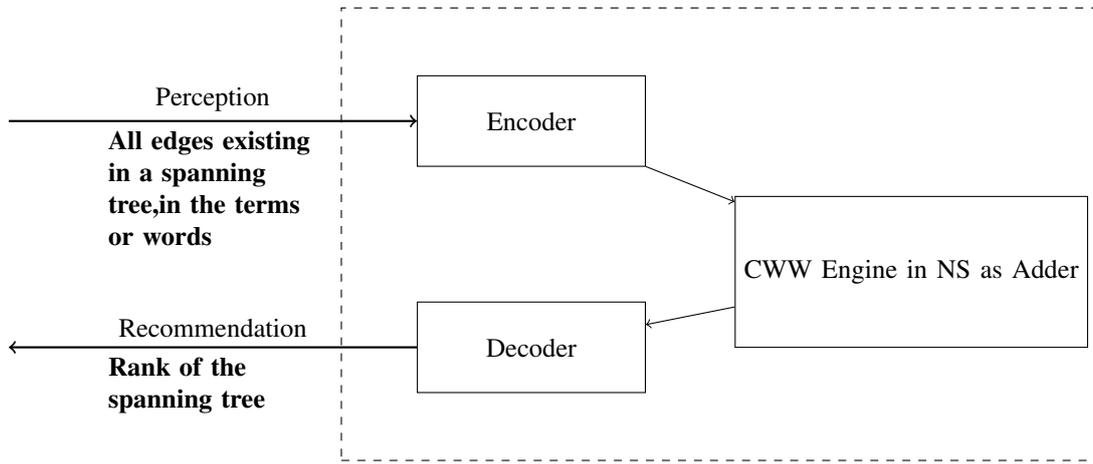


Fig. 4. PNC model for NMST

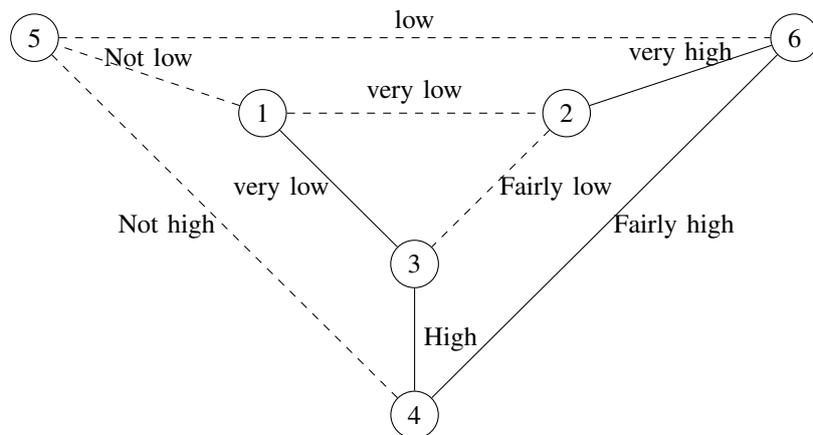


Fig. 5. A neutrosophic graph with natural words.

TABLE I  
WORD CODEBOOK FOR MSTW PROBLEM

Index	Word	SVN number
1	Low	$\langle(4.6, 5.5, 8.6), (0.4, 0.7, 0.2)\rangle$
2	Not low	$\langle(4.7, 6.9, 8.5), (0.7, 0.2, 0.6)\rangle$
3	Very low	$\langle(6.2, 7.6, 8.2), (0.4, 0.1, 0.3)\rangle$
4	High	$\langle(6.2, 8.9, 9.1), (0.6, 0.3, 0.5)\rangle$
5	Not high	$\langle(4.4, 5.9, 7.2), (0.7, 0.2, 0.3)\rangle$
6	Very high	$\langle(6.6, 8.8, 10), (0.6, 0.2, 0.2)\rangle$
7	Fairly high	$\langle(6.3, 7.5, 8.9), (0.7, 0.4, 0.6)\rangle$
8	Fairly low	$\langle(7.1, 7.7, 8.3), (0.5, 0.2, 0.4)\rangle$

- There exists 9 edges in the neutrosophic graph  $\tilde{G}$ . They are respectively:  
i. (1, 2) ii. (1, 3) iii. (2, 3) iv. (1, 5) v. (2, 6)  
vi. (3, 4) vii. (4, 5) viii. (4, 6) ix. (5, 6)  
The rank of the all the edges of graph  $\tilde{G}$  are computed by the passing corresponding word of the arc into the PCN().
- The edge (2, 3) is the smallest arc which has smallest among all the values of ranks of all the arcs in  $A$ . The edge (2, 3) is inserted in  $T$  and the (1, 2) is removed from the  $A$ . Now, the MSTW  $T^w$  is  $\{(2, 3)\}$ .

- The arc (1, 2) is the minimum arc which has lowest among all the values of ranks of all the arcs in  $A$ . The arc (1, 2) is inserted in  $T$  and the (1, 2) is removed from the  $A$ . Now, the MSTW  $T^w$  is  $\{(2, 3), (1, 2)\}$ .
- The arc (5, 6) is the lowest edge cost among all the values in  $A$ . The arc (5, 6) is inserted in  $T$  and the (1, 5) is removed from the  $A$ . Now, the MSTW  $T^w$  is  $\{(2, 3), (1, 3), (5, 6)\}$ .
- The arc (1, 3) is the lowest among all the values of rank of all the arcs in  $A$ . The arc (1, 3) is not inserted in  $T$  because it creates a cycle. The arc (1, 3) is removed from the  $A$ . The arc (1, 5) is the next smallest arc is lowest among all the values of score of all the arcs in  $A$  and the (1, 5) is removed from the  $A$ . Now, the MSTW  $T^w$  is  $\{(2, 3), (1, 2), (5, 6), (1, 5)\}$ .
- The arc (4, 5) is the arc with lowest value of all the arcs in  $A$ . The MSTW  $T^w$  is  $\{(2, 3), (1, 2), (5, 6), (1, 5), (4, 5)\}$ .

Jarnk's (Prim's) algorithm is a well known greedy algorithm that designs a MST for an undirected weighted classical graph. This algorithm selects a set of the arcs that constructs a tree which admits each and every nodes. This algorithmic approach finds by construing this MST one node at a time. This algorithm starts from an arbitrary node and each and every step includes the minimum possible arc from the tree to another node. We have also modified the Jarnik's algorithm to determine the MSTW. The result is shown in Table 2.

TABLE II  
OPTIMAL RESULT OF MSTW

Sol using LINGO	Sol using Kruskal's algorithm	Sol using Jarnik's algorithm	Sol using Boruvka's algorithm
Min $Z= 47.9$ $x_{23} = 1, x_{12} = 1, x_{56} = 1$ $x_{15} = 1, x_{45} = 1$	Cost of MSTW= 47.9 MSTW=(23)(12)(51)(45)(56)	Cost of MSTW= 47.9 MSTW=(23)(12)(51)(45)(56)	Cost of MSTW= 47.9 MSTW=(23)(12)(51)(45)(56)

TABLE III  
OPTIMAL RESULT OF MSTW

Sol using LINGO	Sol using Kruskal's algorithm	Sol using Jarnik's algorithm	Sol using Boruvka's algorithm
Min $Z= 60.76$ $x_{13} = 1, x_{34} = 1, x_{24} = 1$ $x_{45} = 1, x_{56} = 1, x_{58} = 1, x_{57} = 1$	Cost of MSTW= 60.76 MSTW=(1,3)(3,4)(2,4)(4,5) (5,6)(5,8)(5,7)	Cost of MSTW= 60.76 MSTW=(1,3)(3,4)(2,4)(4,5) (5,6)(5,8)(5,7)	Cost of MSTW= 60.76 MSTW=(1,3)(3,4)(2,4)(4,5) (5,6)(5,8)(5,7)

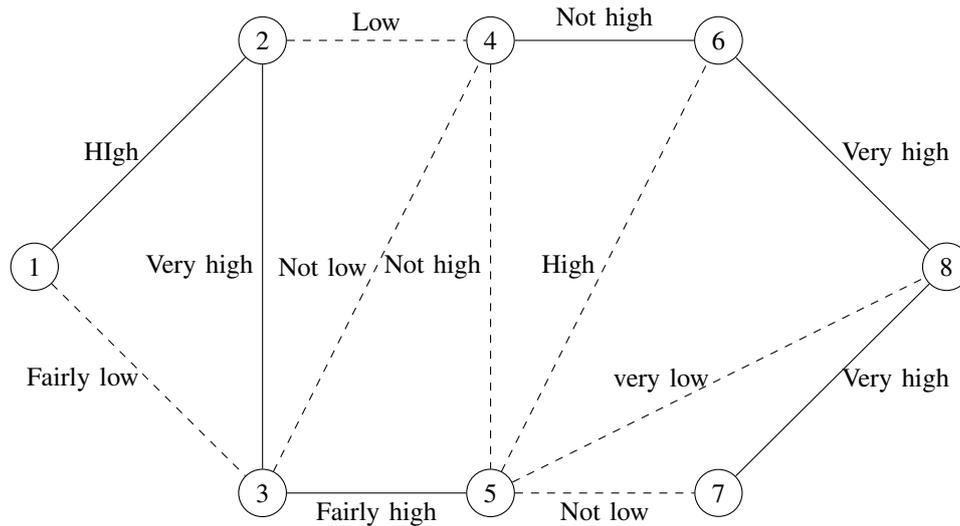


Fig. 6. The neutrosophic graph for Example 2.

The Boruvka's algorithm is an another greedy algorithm to find the MST in an undirected weighted classical graph. This algorithm can work for a graph whose all arc lengths are distinct, or a MST in the case of a undirected weighted classical graph that is disconnected. We have also modified the Boruvka's algorithm to determine the MSTW. The result is shown in Table 2.

We have used a LPP model to solve this problem. The solution is obtain using LINGO. Table 2 describes the result which is calculated by LINGO. A decision variable  $x_{i,j}=1$ , if any arc  $i, j$  is in the minimum spanning tree. The result of Kruskal's algorithm is also shown in Table 2. The solutions of LINGO and our proposed algorithm are identical.

A. Example 2

We have considered an another neutrosophic graph in Figure 6 with 8 vertices and 14 edges having words as edge weights. We have used those edge weights of the neutrosophic graph in the form of word, as described in Table I. The eight different triangular neutrosophic numbers are used to represent those eight words. We apply our proposed algorithm to explore its corresponding NMST of the neutrosophic graph. The edges of NMST is depicted as dashed lines in Fig. 6. The MST is  $\{(3, 4), (1, 3), (4, 2), (5, 6), (5, 7), (5, 4), (5, 8)\}$  and the cost of this spanning tree 60.76.

We have used a LPP model to solve this problem. The solution is obtain using LINGO. Table 3 describes the result

which is calculated by LINGO. A decision variable  $x_{i,j}=1$ , if any arc  $i, j$  is in the minimum spanning tree. The result of Kruskal's algorithm is also shown in Table 3. The solutions of LINGO and our proposed algorithm are identical.

VI. CONCLUSION

In this study, we propose a new idea of CW to model the simple words using neutrosophic set. The main purpose of this work is on CW based on neutrosophic set for taking subjective judgments. We defined this computing as perceptual neutrosophic computing. We have introduced a new architecture for perceptual neutrosophic computing. The MST problem with natural word as arc length of a neutrosophic graph is described. We define this problem as CMSTW. We use *minimum spanning tree advisor* to solve the problem. A linear programming model for this CMSTW problem has been presented for the first time. The standard optimization solver LINGO is used to obtain the minimum spanning tree of the neutrosophic graph. The classical Kruskal's algorithm is extended to solve the CMSTW problem. The MSTW and its corresponding cost are the main information for the decision makers in a MST problem. This paper aims at finding the MSTW and its corresponding cost. One numerical example is given to show the performance of the proposed algorithm. This proposed algorithm can be used in future to solve some decision making problems, e.g., supply chain management, telecommunication and assignment problem.

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