A novel approach to neutrosophic sets in UP-algebras

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A novel approach to neutrosophic sets in UP-algebras

Metawee Songsaeng, Aiyared Iampan

Abstract

The notion of neutrosophic sets in UP-algebras was introduced by Songsaeng and Iampan [M. Songsaeng, A. Iampan, Eur. J. Pure Appl. Math., 12 (2019), 1382–1409]. In this paper, the notions of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras are introduced, and several properties are investigated. Conditions for neutrosophic sets to be special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras are provided. Relations between special neutrosophic UP-subalgebras (resp., special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, special neutrosophic strong UP-ideals) and their level subsets are considered.

Keywords: UP-algebra, special neutrosophic UP-subalgebra, special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal, special neutrosophic strong UP-ideal.

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1. Introduction and preliminaries

Smarandache [19] introduced the notion of neutrosophic sets in 1999 which is a more general platform that extends the notions of classic sets, (intuitionistic) fuzzy sets and interval valued (intuitionistic) fuzzy sets (see [19, 20]). Neutrosophic set theory is applied to various part which is referred to the site http://fs.gallup.unm.edu/neutrosophy.htm


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Sunsongsaeng and Iampan [23] introduced the notions of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strong UP-ideals of UP-algebras.

In this paper, the notions of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras are introduced, and several properties are investigated. Conditions for neutrosophic sets to be special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras are provided. Relations between special neutrosophic UP-subalgebras (resp., special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, special neutrosophic strong UP-ideals) and their level subsets are considered.

Before we begin our study, we will give the definition and useful properties of UP-algebras. An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a UP-algebra, where $X$ is a nonempty set, $\cdot$ is a binary operation on $X$, and $0$ is a fixed element of $X$ (i.e., a nullary operation) if it satisfies the following axioms:

\begin{align*}
\text{(UP-1)} & \quad (\forall x, y, z \in X) ((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0); \\
\text{(UP-2)} & \quad (\forall x \in X) (0 \cdot x = x); \\
\text{(UP-3)} & \quad (\forall x \in X) (x \cdot 0 = 0); \quad \text{and} \\
\text{(UP-4)} & \quad (\forall x, y \in X) (x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y).
\end{align*}


Example 1.1 ([18]). Let $X$ be a universal set and let $\Omega \in \mathcal{P}(X)$, where $\mathcal{P}(X)$ means the power set of $X$. Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$, where $A^C$ means the complement of a subset $A$. Then $(\mathcal{P}_\Omega(X), \cdot, 0)$ is a UP-algebra and we shall call it the generalized power UP-algebra of type 1 with respect to $\Omega$. Let $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $\circ$ on $\mathcal{P}^\Omega(X)$ by putting $A \circ B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(X)$. Then $(\mathcal{P}^\Omega(X), \circ, \Omega)$ is a UP-algebra and we shall call it the generalized power UP-algebra of type 2 with respect to $\Omega$. In particular, $(\mathcal{P}(X), \cdot, 0)$ is a UP-algebra and we shall call it the power UP-algebra of type 1, and $(\mathcal{P}(X), \circ, X)$ is a UP-algebra and we shall call it the power UP-algebra of type 2.

Example 1.2 ([3]). Let $\mathbb{N}$ be the set of all natural numbers with two binary operations $\circ$ and $\cdot$ defined by

\begin{align*}
(\forall x, y \in \mathbb{N}) & \quad (x \circ y = \begin{cases} y, & \text{if } x < y, \\
0, & \text{otherwise,} \end{cases}) \\
(\forall x, y \in \mathbb{N}) & \quad (x \cdot y = \begin{cases} y, & \text{if } x > y \text{ or } x = 0, \\
0, & \text{otherwise,} \end{cases}).
\end{align*}

Then $(\mathbb{N}, \circ, 0)$ and $(\mathbb{N}, \cdot, 0)$ are UP-algebras.
**Example 1.3 ([15]).** Let \( X = \{0,1,2,3,4,5\} \) be a set with a binary operation \( \cdot \) defined by the following Cayley table:

\[
\begin{array}{c|cccccc}
\cdot & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & 0 & 1 & 2 & 3 & 4 & 5 \\
1 & 0 & 0 & 2 & 3 & 4 & 5 \\
2 & 0 & 1 & 0 & 3 & 1 & 5 \\
3 & 0 & 1 & 2 & 0 & 4 & 5 \\
4 & 0 & 0 & 0 & 3 & 0 & 5 \\
5 & 0 & 0 & 2 & 0 & 2 & 0 \\
\end{array}
\]

Then \( (X, \cdot, 0) \) is a UP-algebra.

For more examples of UP-algebras, see [1, 2, 6, 17, 18].

In a UP-algebra \( X = (X, \cdot, 0) \), the following assertions are valid (see [5, 6]).

\[
(\forall x \in X)(x \cdot x = 0), \quad (1.1)
\]
\[
(\forall x, y, z \in X)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0), \quad (1.2)
\]
\[
(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0), \quad (1.3)
\]
\[
(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0), \quad (1.4)
\]
\[
(\forall x, y \in X)(x \cdot (y \cdot x) = 0), \quad (1.5)
\]
\[
(\forall x, y \in X)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x), \quad (1.6)
\]
\[
(\forall x, y \in X)(x \cdot (y \cdot y) = 0), \quad (1.7)
\]
\[
(\forall a, x, y, z \in X)(x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0, \quad (1.8)
\]
\[
(\forall a, x, y, z \in X)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0), \quad (1.9)
\]
\[
(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0), \quad (1.10)
\]
\[
(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0), \quad (1.11)
\]
\[
(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0), \quad (1.12)
\]
\[
(\forall a, x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0). \quad (1.13)
\]

From [5], the binary relation \( \leq \) on a UP-algebra \( X = (X, \cdot, 0) \) defined as follows:

\[
(\forall x, y \in X)(x \leq y \iff x \cdot y = 0).
\]

**Definition 1.4 ([4, 5, 7, 21]).** A nonempty subset \( S \) of a UP-algebra \( (X, \cdot, 0) \) is called

1. a UP-subalgebra of \( X \) if \( (\forall x, y \in S)(x \cdot y \in S) \);
2. a near UP-filter of \( X \) if
   
   (i) the constant 0 of \( X \) is in \( S \), and
   (ii) \( (\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S) \);
3. a UP-filter of \( X \) if
   
   (i) the constant 0 of \( X \) is in \( S \), and
   (ii) \( (\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S) \);
4. a UP-ideal of \( X \) if
   
   (i) the constant 0 of \( X \) is in \( S \), and
   (ii) \( (\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S) \);
(5) a strong UP-ideal (renamed from a strongly UP-ideal) of \( X \) if

(i) the constant 0 of \( X \) is in \( S \), and

(ii) \( (\forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S) \).

Guntasow et al. [4] and Iampan [7] proved that the notion of UP-subalgebras is a generalization of near UP-filters, near UP-filters is a generalization of UP-filters, UP-filters is a generalization of UP-ideals, and UP-ideals is a generalization of strong UP-ideals. Moreover, they also proved that the only strong UP-ideal of a UP-algebra \( X \) is \( X \).

2. NSs of special types in UP-algebras

The notion of a fuzzy set in a nonempty set was first considered by Zadeh [29] in 1965. A fuzzy set (briefly, FS) in a nonempty set \( X \) (or a fuzzy subset of \( X \)) is an arbitrary function \( f : X \to [0, 1] \), where \([0, 1]\) is the unit segment of the real line, and the fuzzy set \( f \) defined by \( f(x) = 1 - f(x) \) for all \( x \in X \) is said to be the complement of \( f \) in \( X \). In 1999, Smarandache [19] introduced the notion of neutrosophic sets as the following definition. A neutrosophic set (briefly, NS) in a nonempty set \( X \) is a structure of the form:

\[
\Lambda = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\},
\]

where \( \lambda_T : X \to [0, 1] \) is a truth membership function, \( \lambda_I : X \to [0, 1] \) is an indeterminate membership function, and \( \lambda_F : X \to [0, 1] \) is a false membership function. For our convenience, we will denote a NS as \( \Lambda = (X, \lambda_T, \lambda_I, \lambda_F) = (X, \lambda_{T, I, F}) = ((x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\).

**Definition 2.1** ([19]). Let \( \Lambda \) be a NS in a nonempty set \( X \). The NS \( \overline{\Lambda} = (X, \overline{\lambda}_{T, I, F}) \) in \( X \) defined by

\[
\begin{align*}
\overline{\lambda}_T(x) &= 1 - \lambda_T(x) \\
\overline{\lambda}_I(x) &= 1 - \lambda_I(x) \\
\overline{\lambda}_F(x) &= 1 - \lambda_F(x)
\end{align*}
\]

is called the complement of \( \Lambda \) in \( X \).

**Remark** ([23]). For all NS \( \Lambda \) in a nonempty set \( X \), we have \( \Lambda = \overline{\Lambda} \).

**Lemma 2.3** ([27]). Let \( a, b, c \in \mathbb{R} \). Then the following statements hold:

1. \( a - \min\{b, c\} = \max\{a - b, a - c\} \), and

2. \( a - \max\{b, c\} = \min\{a - b, a - c\} \).

**Lemma 2.4** ([23]). Let \( f \) be a fuzzy set in a nonempty set \( X \). Then the following statements hold:

1. \( (\forall x, y, z \in X)(f(x) \geq \min\{f(y), f(z)\} \Leftrightarrow f(x) \leq \max\{f(y), f(z)\}) \);

2. \( (\forall x, y, z \in X)(f(x) \leq \min\{f(y), f(z)\} \Leftrightarrow f(x) \geq \max\{f(y), f(z)\}) \);

3. \( (\forall x, y, z \in X)(f(x) \geq \max\{f(y), f(z)\} \Leftrightarrow f(x) \leq \min\{f(y), f(z)\}) \); and

4. \( (\forall x, y, z \in X)(f(x) \leq \max\{f(y), f(z)\} \Leftrightarrow f(x) \geq \min\{f(y), f(z)\}) \).

In what follows, let \( X \) denote a UP-algebra \((X, \cdot, 0)\) unless otherwise specified.

**Definition 2.5** ([23]). A NS \( \Lambda \) in \( X \) is called a neutrosophic UP-subalgebra of \( X \) if it satisfies the following conditions:

\[
(\forall x, y \in X)(\lambda_T(x \cdot y) \geq \min\{\lambda_T(x), \lambda_T(y)\}), \quad (2.3)
\]

\[
(\forall x, y \in X)(\lambda_I(x \cdot y) \leq \max\{\lambda_I(x), \lambda_I(y)\}), \quad (2.4)
\]

\[
(\forall x, y \in X)(\lambda_F(x \cdot y) \geq \min\{\lambda_F(x), \lambda_F(y)\}). \quad (2.5)
\]
**Definition 2.6** ([23]). A NS $\Lambda$ in $X$ is called a *neutrosophic near UP-filter* of $X$ if it satisfies the following conditions:

\[
(\forall x \in X)(\lambda_T(0) \geq \lambda_T(x)), \tag{2.6}
\]
\[
(\forall x \in X)(\lambda_I(0) \leq \lambda_I(x)), \tag{2.7}
\]
\[
(\forall x \in X)(\lambda_F(0) \geq \lambda_F(x)), \tag{2.8}
\]
\[
(\forall x, y \in X)(\lambda_T(x \cdot y) \geq \lambda_T(y)), \tag{2.9}
\]
\[
(\forall x, y \in X)(\lambda_I(x \cdot y) \leq \lambda_I(y)), \tag{2.10}
\]
\[
(\forall x, y \in X)(\lambda_F(x \cdot y) \geq \lambda_F(y)). \tag{2.11}
\]

**Definition 2.7** ([23]). A NS $\Lambda$ in $X$ is called a *neutrosophic UP-filter* of $X$ if it satisfies the following conditions: (2.6), (2.7), (2.8), and

\[
(\forall x, y \in X)(\lambda_T(y) \geq \min[\lambda_T(x \cdot y), \lambda_T(x)]), \tag{2.12}
\]
\[
(\forall x, y \in X)(\lambda_I(y) \leq \max[\lambda_I(x \cdot y), \lambda_I(x)]), \tag{2.13}
\]
\[
(\forall x, y \in X)(\lambda_F(y) \geq \min[\lambda_F(x \cdot y), \lambda_F(x)]). \tag{2.14}
\]

**Definition 2.8** ([23]). A NS $\Lambda$ in $X$ is called a *neutrosophic UP-ideal* of $X$ if it satisfies the following conditions: (2.6), (2.7), (2.8), and

\[
(\forall x, y, z \in X)(\lambda_T(x \cdot z) \geq \min[\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)]), \tag{2.15}
\]
\[
(\forall x, y, z \in X)(\lambda_I(x \cdot z) \leq \max[\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)]), \tag{2.16}
\]
\[
(\forall x, y, z \in X)(\lambda_F(x \cdot z) \geq \min[\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)]). \tag{2.17}
\]

**Definition 2.9** ([23]). A NS $\Lambda$ in $X$ is called a *neutrosophic strong UP-ideal* (renamed from a neutrosophic strongly UP-ideal) of $X$ if it satisfies the following conditions: (2.6), (2.7), (2.8), and

\[
(\forall x, y, z \in X)(\lambda_T(x) \geq \min[\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)]), \tag{2.18}
\]
\[
(\forall x, y, z \in X)(\lambda_I(x) \leq \max[\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)]), \tag{2.19}
\]
\[
(\forall x, y, z \in X)(\lambda_F(x) \geq \min[\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)]). \tag{2.20}
\]

**Definition 2.10** ([23]). A NS $\Lambda$ in $X$ is said to be *constant* if $\Lambda$ is a constant function from $X$ to $[0, 1]^3$. That is, $\lambda_T, \lambda_I$, and $\lambda_F$ are constant functions from $X$ to $[0, 1]$.

**Theorem 2.11** ([23]). A NS $\Lambda$ in $X$ is constant if and only if it is a neutrosophic strong UP-ideal of $X$.

Songsaeng and Iampan [23] proved that the notion of neutrosophic UP-subalgebras is a generalization of neutrosophic near UP-filters, neutrosophic near UP-filters is a generalization of neutrosophic UP-filters, neutrosophic UP-filters is a generalization of neutrosophic UP-ideals, and neutrosophic UP-ideals is a generalization of neutrosophic strong UP-ideals.

Now, we introduce the notions of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

**Definition 2.12.** A NS $\Lambda$ in $X$ is called an *special neutrosophic UP-subalgebra* of $X$ if it satisfies the following conditions:

\[
(\forall x, y \in X)(\lambda_T(x \cdot y) \leq \max[\lambda_T(x), \lambda_T(y)]), \tag{2.21}
\]
\[
(\forall x, y \in X)(\lambda_I(x \cdot y) \geq \min[\lambda_I(x), \lambda_I(y)]), \tag{2.22}
\]
\[
(\forall x, y \in X)(\lambda_F(x \cdot y) \leq \max[\lambda_F(x), \lambda_F(y)]). \tag{2.23}
\]
Example 2.13. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

\[
\begin{array}{c|ccccc}
\cdot & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 2 & 3 & 4 \\
1 & 0 & 0 & 1 & 0 & 4 \\
2 & 0 & 0 & 0 & 0 & 4 \\
3 & 0 & 1 & 1 & 0 & 4 \\
4 & 0 & 3 & 3 & 3 & 0 \\
\end{array}
\]

We define a NS $\Lambda$ in $X$ as follows:

\[\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.2 & 0.3 & 0.5 & 0.7 & 0.8 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.7 & 0.6 & 0.5 & 0.2 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.4 & 0.6 & 0.7 & 0.9 \end{pmatrix}.\]

Hence, $\Lambda$ is a special neutrosophic UP-subalgebra of $X$.

Definition 2.14. A NS $\Lambda$ in $X$ is called an special neutrosophic near UP-filter of $X$ if it satisfies the following conditions:

\[(\forall x \in X)(\lambda_T(0) \leq \lambda_T(x)), \quad (2.24)\]
\[(\forall x \in X)(\lambda_I(0) \geq \lambda_I(x)), \quad (2.25)\]
\[(\forall x \in X)(\lambda_F(0) \leq \lambda_F(x)), \quad (2.26)\]
\[(\forall x, y \in X)(\lambda_T(x \cdot y) \leq \lambda_T(y)), \quad (2.27)\]
\[(\forall x, y \in X)(\lambda_I(x \cdot y) \geq \lambda_I(y)), \quad (2.28)\]
\[(\forall x, y \in X)(\lambda_F(x \cdot y) \leq \lambda_F(y)). \quad (2.29)\]

Example 2.15. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

\[
\begin{array}{c|ccccc}
\cdot & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 2 & 3 & 4 \\
1 & 0 & 0 & 2 & 2 & 4 \\
2 & 0 & 1 & 0 & 1 & 4 \\
3 & 0 & 0 & 0 & 0 & 4 \\
4 & 0 & 1 & 2 & 3 & 0 \\
\end{array}
\]

We define a NS $\Lambda$ in $X$ as follows:

\[\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.3 & 0.5 & 0.6 & 0.2 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.8 & 0.7 & 0.3 & 0.4 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.2 & 0.6 & 0.7 & 0.5 \end{pmatrix}.\]

Hence, $\Lambda$ is a special neutrosophic near UP-filter of $X$.

Definition 2.16. A NS $\Lambda$ in $X$ is called an special neutrosophic UP-filter of $X$ if it satisfies the following conditions: (2.24), (2.25), (2.26), and

\[(\forall x, y \in X)(\lambda_T(y) \leq \max(\lambda_T(x \cdot y), \lambda_T(x))), \quad (2.30)\]
\[(\forall x, y \in X)(\lambda_I(y) \geq \min(\lambda_I(x \cdot y), \lambda_I(x))), \quad (2.31)\]
\[(\forall x, y \in X)(\lambda_F(y) \leq \max(\lambda_F(x \cdot y), \lambda_F(x))). \quad (2.32)\]

Example 2.17. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

\[
\begin{array}{c|ccccc}
\cdot & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 2 & 3 & 4 \\
1 & 0 & 0 & 2 & 2 & 4 \\
2 & 0 & 1 & 0 & 1 & 4 \\
3 & 0 & 0 & 0 & 0 & 4 \\
4 & 0 & 1 & 1 & 1 & 0 \\
\end{array}
\]
We define a NS \( \Lambda \) in \( X \) as follows:

\[
\lambda_T = \begin{pmatrix}
0 & 1 & 2 & 3 & 4 \\
0.1 & 0.5 & 0.4 & 0.5 & 0.8
\end{pmatrix},
\lambda_I = \begin{pmatrix}
0 & 1 & 2 & 3 & 4 \\
0.8 & 0.3 & 0.5 & 0.3 & 0.4
\end{pmatrix},
\lambda_F = \begin{pmatrix}
0 & 1 & 2 & 3 & 4 \\
0.2 & 0.6 & 0.4 & 0.6 & 0.3
\end{pmatrix}.
\]

Hence, \( \Lambda \) is a special neutrosophic UP-filter of \( X \).

**Definition 2.20.** A NS \( \Lambda \) in \( X \) is called an *special neutrosophic strong UP-ideal* of \( X \) if it satisfies the following conditions: (2.24), (2.25), (2.26), and

\[
(\forall x, y, z \in X)(\lambda_T(x \cdot z) \leq \max(\lambda_T(x \cdot (y \cdot z)), \lambda_T(y))), \tag{2.33}
\]

\[
(\forall x, y, z \in X)(\lambda_I(x \cdot z) \geq \min(\lambda_I(x \cdot (y \cdot z)), \lambda_I(y))), \tag{2.34}
\]

\[
(\forall x, y, z \in X)(\lambda_F(x \cdot z) \leq \max(\lambda_F(x \cdot (y \cdot z)), \lambda_F(y))). \tag{2.35}
\]

**Example 2.19.** Let \( X = \{0, 1, 2, 3, 4\} \) be a UP-algebra with a fixed element 0 and a binary operation \( \cdot \) defined by the following Cayley table:

<table>
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<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

We define a NS \( \Lambda \) in \( X \) as follows:

\[
\lambda_T = \begin{pmatrix}
0 & 1 & 2 & 3 & 4 \\
0.3 & 0.5 & 0.4 & 0.6 & 0.7
\end{pmatrix},
\lambda_I = \begin{pmatrix}
0 & 1 & 2 & 3 & 4 \\
1 & 0.7 & 0.4 & 0.7 & 0.3
\end{pmatrix},
\lambda_F = \begin{pmatrix}
0 & 1 & 2 & 3 & 4 \\
0.1 & 0.2 & 0.7 & 0.3 & 0.9
\end{pmatrix}.
\]

Hence, \( \Lambda \) is a special neutrosophic UP-ideal of \( X \).

**Definition 2.20.** A NS \( \Lambda \) in \( X \) is called an *special neutrosophic strong UP-ideal* of \( X \) if it satisfies the following conditions: (2.24), (2.25), (2.26), and

\[
(\forall x, y, z \in X)(\lambda_T((z \cdot y) \cdot (z \cdot x))) = \max(\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y))), \tag{2.36}
\]

\[
(\forall x, y, z \in X)(\lambda_I((z \cdot y) \cdot (z \cdot x))) \geq \min(\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y))), \tag{2.37}
\]

\[
(\forall x, y, z \in X)(\lambda_F((z \cdot y) \cdot (z \cdot x))) \leq \max(\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y))). \tag{2.38}
\]

**Example 2.21.** Let \( X = \{0, 1, 2, 3, 4\} \) be a UP-algebra with a fixed element 0 and a binary operation \( \cdot \) defined by the following Cayley table:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

We define a NS \( \Lambda \) in \( X \) as follows:

\[
(\forall x \in X) \begin{pmatrix}
\lambda_T(x) = 0.5 \\
\lambda_I(x) = 0.4 \\
\lambda_F(x) = 0.7
\end{pmatrix}.
\]

Hence, \( \Lambda \) is a special neutrosophic strong UP-ideal \( X \).
Theorem 2.22. Every special neutrosophic UP-subalgebra of $X$ satisfies the conditions (2.24), (2.25), and (2.26).

Proof. Assume that $\Lambda$ is a special neutrosophic UP-subalgebra of $X$. Then for all $x \in X$,

$$
\lambda_T(0) = \lambda_T(x \cdot x) \leq \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x) \quad \text{by (1.1) and (2.21)}
$$

$$
\lambda_I(0) = \lambda_I(x \cdot x) \geq \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x) \quad \text{by (1.1) and (2.22)}
$$

$$
\lambda_F(0) = \lambda_F(x \cdot x) \leq \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x) \quad \text{by (1.1) and (2.23)}.
$$

Hence, $\Lambda$ satisfies the conditions (2.24), (2.25), and (2.26). □

By Lemma 2.4 (1) and (4), we have the following five theorems.

Theorem 2.23. A NS $\Lambda$ in $X$ is a neutrosophic UP-subalgebra of $X$ if and only if $\overline{\Lambda}$ is a special neutrosophic UP-subalgebra of $X$.

Theorem 2.24. A NS $\Lambda$ in $X$ is a neutrosophic near UP-filter of $X$ if and only if $\overline{\Lambda}$ is a special neutrosophic near UP-filter of $X$.

Theorem 2.25. A NS $\Lambda$ in $X$ is a neutrosophic UP-filter of $X$ if and only if $\overline{\Lambda}$ is a special neutrosophic UP-filter of $X$.

Theorem 2.26. A NS $\Lambda$ in $X$ is a neutrosophic UP-ideal of $X$ if and only if $\overline{\Lambda}$ is a special neutrosophic UP-ideal of $X$.

Theorem 2.27. A NS $\Lambda$ in $X$ is a neutrosophic strong UP-ideal of $X$ if and only if $\overline{\Lambda}$ is a special neutrosophic strong UP-ideal of $X$.

Theorem 2.28. A NS $\Lambda$ in $X$ is constant if and only if it is a special neutrosophic strong UP-ideal of $X$.

Proof. It is straightforward by Remark 2.2 and Theorems 2.11 and 2.27. □

By Remark 2.2 and Theorems 2.23, 2.24, 2.25, 2.26, and 2.27, we have that the notion of special neutrosophic UP-subalgebras is a generalization of special neutrosophic near UP-filters, special neutrosophic near UP-filters is a generalization of special neutrosophic UP-filters, special neutrosophic UP-filters is a generalization of special neutrosophic UP-ideals, and special neutrosophic UP-ideals is a generalization of special neutrosophic strong UP-ideals. Moreover, by Theorem 2.28, we obtain that special neutrosophic strong UP-ideals and constant neutrosophic set coincide.

Theorem 2.29. If $\Lambda$ is a special neutrosophic UP-subalgebra of $X$ satisfying the following condition:

$$
(\forall x, y \in X) \left( x \cdot y \neq 0 \Rightarrow \begin{cases} 
\lambda_T(x) \leq \lambda_T(y) \\
\lambda_I(x) \geq \lambda_I(y) \\
\lambda_F(x) \leq \lambda_F(y)
\end{cases} \right),
$$

then $\Lambda$ is a special neutrosophic near UP-filter of $X$.

Proof. Assume that $\Lambda$ is a special neutrosophic UP-subalgebra of $X$ satisfying the condition (2.39). By Theorem 2.22, we have $\Lambda$ satisfies the conditions (2.24), (2.25), and (2.26). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$
\lambda_T(x \cdot y) = \lambda_T(0) \leq \lambda_T(y) \quad \text{by (2.24)}
$$

$$
\lambda_I(x \cdot y) = \lambda_I(0) \geq \lambda_I(y) \quad \text{by (2.25)}
$$

$$
\lambda_F(x \cdot y) = \lambda_F(0) \leq \lambda_F(y) \quad \text{by (2.26)}.
$$
Hence, \( \Lambda \) satisfies the conditions (2.24), (2.25), and (2.26). Next, let \( x \in X \).

**Proof.** Assume that \( \Lambda \) satisfies the condition (2.41). Then \( \Lambda \) satisfies the conditions (2.24), (2.25), and (2.26). Next, let \( x, y, z \in X \). Then

\[
\max\{\lambda_T(x \cdot y), \lambda_T(x)\} = \max\{\lambda_1(x \cdot y), \lambda_1(x)\} \quad \text{by (2.40)}
\]

\[
\geq \max\{\lambda_1(y), \lambda_T(x)\} \quad \text{by (2.28)}
\]

\[
= \max\{\lambda_T(y), \lambda_T(x)\} \quad \text{by (2.40)}
\]

\[
\geq \lambda_T(y),
\]

\[
\min\{\lambda_1(x \cdot y), \lambda_1(x)\} = \min\{\lambda_T(x \cdot y), \lambda_1(x)\} \quad \text{by (2.40)}
\]

\[
\leq \min\{\lambda_T(y), \lambda_1(x)\} \quad \text{by (2.27)}
\]

\[
= \min\{\lambda_1(y), \lambda_1(x)\} \quad \text{by (2.40)}
\]

\[
\leq \lambda_1(y),
\]

\[
\max\{\lambda_F(x \cdot y), \lambda_F(x)\} = \max\{\lambda_1(x \cdot y), \lambda_F(x)\} \quad \text{by (2.40)}
\]

\[
\geq \max\{\lambda_1(y), \lambda_F(x)\} \quad \text{by (2.28)}
\]

\[
= \max\{\lambda_F(y), \lambda_F(x)\} \quad \text{by (2.40)}
\]

\[
\geq \lambda_F(y).
\]

Hence, \( \Lambda \) is a special neutrosophic UP-filter of \( X \). □

**Theorem 2.30.** If \( \Lambda \) is a special neutrosophic near UP-filter of \( X \) satisfying the following condition:

\[
\lambda_T = \lambda_1 = \lambda_F,
\]

then \( \Lambda \) is a special neutrosophic UP-filter of \( X \).

**Proof.** Assume that \( \Lambda \) is a special neutrosophic near UP-filter of \( X \) satisfying the condition (2.40). Then \( \Lambda \) satisfies the conditions (2.24), (2.25), and (2.26). Next, let \( x, y, z \in X \). Then

\[
\max\{\lambda_T(x \cdot y), \lambda_T(x)\} = \max\{\lambda_1(x \cdot y), \lambda_1(x)\} \quad \text{by (2.40)}
\]

\[
\geq \max\{\lambda_1(y), \lambda_T(x)\} \quad \text{by (2.28)}
\]

\[
= \max\{\lambda_T(y), \lambda_T(x)\} \quad \text{by (2.40)}
\]

\[
\geq \lambda_T(y),
\]

\[
\min\{\lambda_1(x \cdot y), \lambda_1(x)\} = \min\{\lambda_T(x \cdot y), \lambda_1(x)\} \quad \text{by (2.40)}
\]

\[
\leq \min\{\lambda_T(y), \lambda_1(x)\} \quad \text{by (2.27)}
\]

\[
= \min\{\lambda_1(y), \lambda_1(x)\} \quad \text{by (2.40)}
\]

\[
\leq \lambda_1(y),
\]

\[
\max\{\lambda_F(x \cdot y), \lambda_F(x)\} = \max\{\lambda_1(x \cdot y), \lambda_F(x)\} \quad \text{by (2.40)}
\]

\[
\geq \max\{\lambda_1(y), \lambda_F(x)\} \quad \text{by (2.28)}
\]

\[
= \max\{\lambda_F(y), \lambda_F(x)\} \quad \text{by (2.40)}
\]

\[
\geq \lambda_F(y).
\]

Hence, \( \Lambda \) is a special neutrosophic UP-filter of \( X \). □

**Theorem 2.31.** If \( \Lambda \) is a special neutrosophic UP-filter of \( X \) satisfying the following condition:

\[
(\forall x, y, z \in X) \left\{ \begin{array}{l}
\lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z)) \\
\lambda_1(y \cdot (x \cdot z)) = \lambda_1(x \cdot (y \cdot z)) \\
\lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z))
\end{array} \right. \quad (2.41)
\]

then \( \Lambda \) is a special neutrosophic UP-ideal of \( X \).

**Proof.** Assume that \( \Lambda \) is a special neutrosophic UP-filter of \( X \) satisfying the condition (2.41). Then \( \Lambda \) satisfies the conditions (2.24), (2.25), and (2.26). Next, let \( x, y, z \in X \). Then

\[
\lambda_T(x \cdot z) \leq \max\{\lambda_T(y \cdot (x \cdot z)), \lambda_T(y)\} \quad \text{by (2.30)}
\]

\[
= \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\} \quad \text{by (2.41) for} \lambda_T
\]

\[
\lambda_1(x \cdot z) \geq \min\{\lambda_1(y \cdot (x \cdot z)), \lambda_1(y)\} \quad \text{by (2.31)}
\]

\[
= \min\{\lambda_1(x \cdot (y \cdot z)), \lambda_1(y)\} \quad \text{by (2.41) for} \lambda_1
\]

\[
\lambda_F(x \cdot z) \leq \max\{\lambda_F(y \cdot (x \cdot z)), \lambda_F(y)\} \quad \text{by (2.32)}
\]

\[
= \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\} \quad \text{by (2.41) for} \lambda_F.
\]

Hence, \( \Lambda \) is a special neutrosophic UP-ideal of \( X \). □
Theorem 2.32. If Λ is a NS in X satisfying the following condition:

\[
(\forall x, y, z \in X) \left( z \leq x \cdot y \Rightarrow \begin{cases} 
\lambda_T(z) \leq \max(\lambda_T(x), \lambda_T(y)) \\
\lambda_1(z) \geq \min(\lambda_1(x), \lambda_1(y)) \\
\lambda_F(z) \leq \max(\lambda_F(x), \lambda_F(y)) 
\end{cases} \right),
\]

then Λ is a special neutrosophic UP-subalgebra of X.

Proof. Assume that Λ is a NS in X satisfying the condition (2.42). Let \( x, y \in X \). By (1.1), we have \( x \cdot y = 0 \), that is, \( x \cdot y \geq x \cdot y \). It follows from (2.42) that

\[
\lambda_T(x \cdot y) \leq \max(\lambda_T(x), \lambda_T(y)), \quad \lambda_1(x \cdot y) \geq \min(\lambda_1(x), \lambda_1(y)), \quad \lambda_F(x \cdot y) \leq \max(\lambda_F(x), \lambda_F(y)).
\]

Hence, Λ is a special neutrosophic UP-subalgebra of X.

Theorem 2.33. If Λ is a NS in X satisfying the following condition:

\[
(\forall x, y, z \in X) \left( z \leq x \cdot y \Rightarrow \begin{cases} 
\lambda_T(z) \leq \lambda_T(y) \\
\lambda_1(z) \geq \lambda_1(y) \\
\lambda_F(z) \leq \lambda_F(y) 
\end{cases} \right),
\]

then Λ is a special neutrosophic near UP-filter of X.

Proof. Assume that Λ is a NS in X satisfying the condition (2.43). Let \( x \in X \). By (UP-2) and (1.1), we have \( 0 \cdot (x \cdot x) = 0 \), that is, \( 0 \leq x \cdot x \). It follows from (2.43) that \( \lambda_T(0) \leq \lambda_T(x), \lambda_1(0) \geq \lambda_1(x) \), and \( \lambda_F(0) \leq \lambda_F(x) \). Next, let \( x, y \in X \). By (1.1), we have \( (x \cdot y) \cdot (x \cdot y) = 0 \), that is, \( x \cdot y \geq x \cdot y \). It follows from (2.43) that

\[
\lambda_T(x \cdot y) \leq \lambda_T(y), \quad \lambda_1(x \cdot y) \geq \lambda_1(y), \quad \lambda_F(x \cdot y) \leq \lambda_F(y).
\]

Hence, Λ is a special neutrosophic near UP-filter of X.

Theorem 2.34. If Λ is a NS in X satisfying the following condition:

\[
(\forall x, y, z \in X) \left( z \leq x \cdot y \Rightarrow \begin{cases} 
\lambda_T(y) \leq \max(\lambda_T(z), \lambda_T(x)) \\
\lambda_1(y) \geq \min(\lambda_1(z), \lambda_1(x)) \\
\lambda_F(y) \leq \max(\lambda_F(z), \lambda_F(x)) 
\end{cases} \right),
\]

then Λ is a special neutrosophic UP-filter of X.

Proof. Assume that Λ is a NS in X satisfying the condition (2.44). Let \( x \in X \). By (UP-3), we have \( x \cdot (x \cdot 0) = 0 \), that is, \( x \leq x \cdot 0 \). It follows from (2.44) that

\[
\lambda_T(0) \leq \max(\lambda_T(x), \lambda_T(x)) = \lambda_T(x), \quad \lambda_1(0) \geq \min(\lambda_1(x), \lambda_1(x)) = \lambda_1(x), \quad \lambda_F(0) \leq \max(\lambda_F(x), \lambda_F(x)) = \lambda_F(x).
\]

Next, let \( x, y \in X \). By (1.1), we have \( (x \cdot y) \cdot (x \cdot y) = 0 \), that is, \( x \cdot y \geq x \cdot y \). It follows from (2.44) that

\[
\lambda_T(y) \leq \max(\lambda_T(x \cdot y), \lambda_T(x)), \quad \lambda_1(y) \geq \min(\lambda_1(x \cdot y), \lambda_1(x)), \quad \lambda_F(y) \leq \max(\lambda_F(x \cdot y), \lambda_F(x)).
\]

Hence, Λ is a special neutrosophic UP-filter of X.

Theorem 2.35. If Λ is a NS in X satisfying the following condition:

\[
(\forall a, x, y, z \in X) \left( a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} 
\lambda_T(x \cdot z) \leq \max(\lambda_T(a), \lambda_T(y)) \\
\lambda_1(x \cdot z) \geq \min(\lambda_1(a), \lambda_1(y)) \\
\lambda_F(x \cdot z) \leq \max(\lambda_F(a), \lambda_F(y)) 
\end{cases} \right),
\]

then Λ is a special neutrosophic UP-ideal of X.
Proof. Assume that $\Lambda$ is a NS in $X$ satisfying the condition \eqref{2.45}. Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from \eqref{2.45} that
\begin{align*}
\lambda_T(0) &= \lambda_T(0 \cdot 0) \leq \max(\lambda_T(x), \lambda_T(x)) = \lambda_T(x), \\
\lambda_I(0) &= \lambda_I(0 \cdot 0) \geq \min(\lambda_I(x), \lambda_I(x)) = \lambda_I(x), \\
\lambda_F(0) &= \lambda_F(0 \cdot 0) \leq \max(\lambda_F(x), \lambda_F(x)) = \lambda_F(x).
\end{align*}
(UP-2)

Next, let $x, y, z \in X$. By \eqref{1.1}, we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \geq x \cdot (y \cdot z)$. It follows from \eqref{2.45} that
\begin{align*}
\lambda_T(x \cdot z) &= \max(\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)), \\
\lambda_I(x \cdot z) &= \min(\lambda_I(x \cdot (y \cdot z)), \lambda_I(x)), \\
\lambda_F(x \cdot z) &= \max(\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)) [\text{UP-2}].
\end{align*}

Hence, $\Lambda$ is a special neutrosophic UP-ideal of $X$. \hfill $\square$

For any fixed numbers $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0, 1]$ such that $\alpha^+ < \alpha^-, \beta^+ < \beta^-, \gamma^+ < \gamma^-$ and a nonempty subset $G$ of $X$, a NS $\Lambda_G^{\alpha^+, \beta^+, \gamma^+, \gamma^-} = (X, \lambda_{G_T}[\alpha^+], \lambda_{G_I}[\beta^+], \lambda_{G_F}[\gamma^+])$ in $X$, where $\lambda_{G_T}[\alpha^+], \lambda_{G_I}[\beta^+], \lambda_{G_F}[\gamma^+]$ are functions on $X$ which are given as follows:
\begin{align*}
\lambda_{G_T}[\alpha^+](x) &= \begin{cases} 
\alpha^+, & \text{if } x \in G, \\
\alpha^-, & \text{otherwise},
\end{cases} \\
\lambda_{G_I}[\beta^+](x) &= \begin{cases} 
\beta^-, & \text{if } x \in G, \\
\beta^+, & \text{otherwise},
\end{cases} \\
\lambda_{G_F}[\gamma^+](x) &= \begin{cases} 
\gamma^+, & \text{if } x \in G, \\
\gamma^-, & \text{otherwise}.
\end{cases}
\end{align*}

A NS $G\lambda_{G_T}[\alpha^+, \beta^+, \gamma^+, \gamma^-] = (X, G\lambda_{G_T}[\alpha^+], G\lambda_{G_I}[\beta^+], G\lambda_{G_F}[\gamma^+])$ in $X$, where $G\lambda_{G_T}[\alpha^+], G\lambda_{G_I}[\beta^+], \text{ and } G\lambda_{G_F}[\gamma^-]$ are functions on $X$ which are given as follows:
\begin{align*}
G\lambda_{G_T}[\alpha^+](x) &= \begin{cases} 
\alpha^-, & \text{if } x \in G, \\
\alpha^+, & \text{otherwise},
\end{cases} \\
G\lambda_{G_I}[\beta^+](x) &= \begin{cases} 
\beta^+, & \text{if } x \in G, \\
\beta^-, & \text{otherwise},
\end{cases} \\
G\lambda_{G_F}[\gamma^+](x) &= \begin{cases} 
\gamma^-, & \text{if } x \in G, \\
\gamma^+, & \text{otherwise}.
\end{cases}
\end{align*}

Lemma 2.36. Let $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0, 1]$. Then the following statements hold:

(1) $\Lambda_G^{\alpha^+, \beta^+, \gamma^+, \gamma^-} = \Lambda_G^{1-\alpha^+, 1-\beta^-, 1-\gamma^-, \gamma^-}$, and

(2) $G\Lambda_{G_T}[\alpha^+, \beta^+, \gamma^+, \gamma^-] = \Lambda_G^{1-\alpha^-, 1-\beta^+, 1-\gamma^-, \gamma^-}$.

Proof.

(1). Let $\Lambda_G^{\alpha^+, \beta^+, \gamma^+, \gamma^-}$ be a NS in $X$. Then $\Lambda_G^{\alpha^+, \beta^+, \gamma^+, \gamma^-} = (X, \lambda_{G_T}[\alpha^+], \lambda_{G_I}[\beta^+], \lambda_{G_F}[\gamma^+])$. Since
\begin{align*}
\lambda_{G_T}[\alpha^+](x) &= \begin{cases} 
\alpha^+, & \text{if } x \in G, \\
\alpha^-, & \text{otherwise},
\end{cases} \\
\lambda_{G_I}[\beta^+](x) &= \begin{cases} 
\beta^-, & \text{if } x \in G, \\
\beta^+, & \text{otherwise},
\end{cases} \\
\lambda_{G_F}[\gamma^+](x) &= \begin{cases} 
\gamma^+, & \text{if } x \in G, \\
\gamma^-, & \text{otherwise}.
\end{cases}
\end{align*}

Thus
\begin{align*}
\lambda_{G_T}[\alpha^+](x) &= \begin{cases} 
1 - \alpha^+, & \text{if } x \in G, \\
1 - \alpha^-, & \text{otherwise},
\end{cases} = G\lambda_{G_T}[1-\alpha^+](x), \\
\lambda_{G_I}[\beta^+](x) &= \begin{cases} 
1 - \beta^-, & \text{if } x \in G, \\
1 - \beta^+, & \text{otherwise},
\end{cases} = G\lambda_{G_I}[1-\beta^+](x), \\
\lambda_{G_F}[\gamma^+](x) &= \begin{cases} 
1 - \gamma^+, & \text{if } x \in G, \\
1 - \gamma^-, & \text{otherwise},
\end{cases} = G\lambda_{G_F}[1-\gamma^+](x).
\end{align*}

Hence, $(X, G\lambda_{G_T}[1-\alpha^+], G\lambda_{G_I}[1-\beta^+], G\lambda_{G_F}[1-\gamma^+]) = \Lambda_G^{1-\alpha^-, 1-\beta^+, 1-\gamma^-, \gamma^-}$. \\


(2). Let \( G \Lambda_{\alpha^+, \beta^+, \gamma^-} \) be a NS in \( X \). Then \( G \Lambda_{\alpha^+, \beta^+, \gamma^-}(x) = (X, G \lambda_T(\alpha^+), G \lambda_{I}(\beta^+), G \lambda_{F}(\gamma^-)). \) Since

\[
G \lambda_T(\alpha^+)(x) = \begin{cases} 
\alpha^-, & \text{if } x \in G, \\
\alpha^+, & \text{otherwise},
\end{cases}
\]

\[
G \lambda_I(\beta^+)(x) = \begin{cases} 
\beta^+, & \text{if } x \in G, \\
\beta^-, & \text{otherwise},
\end{cases}
\]

\[
G \lambda_{F}(\gamma^-)(x) = \begin{cases} 
\gamma^-, & \text{if } x \in G, \\
\gamma^+, & \text{otherwise}.
\end{cases}
\]

Thus

\[
G \lambda_T(\alpha^+)(x) = \begin{cases} 
1 - \alpha^-, & \text{if } x \in G, \\
1 - \alpha^+, & \text{otherwise}
\end{cases} = \lambda_T(1 - \alpha^-)(x),
\]

\[
G \lambda_I(\beta^+)(x) = \begin{cases} 
1 - \beta^+, & \text{if } x \in G, \\
1 - \beta^- & \text{otherwise}
\end{cases} = \lambda_I(1 - \beta^+)(x),
\]

\[
G \lambda_{F}(\gamma^-)(x) = \begin{cases} 
1 - \gamma^-, & \text{if } x \in G, \\
1 - \gamma^+ & \text{otherwise}
\end{cases} = \lambda_{F}(1 - \gamma^-)(x).
\]

Hence, \( (X, \lambda_T(1 - \alpha^-), \lambda_I(1 - \beta^+), \lambda_{F}(1 - \gamma^-)) = \Lambda_{1 - \alpha^-} \Lambda_{1 - \beta^+} \Lambda_{1 - \gamma^-} \).

\[\square\]

**Lemma 2.37.** If the constant 0 of \( X \) is in a nonempty subset \( G \) of \( X \), then a NS \( G \Lambda_{\alpha^+, \beta^+, \gamma^-} \) in \( X \) satisfies the conditions (2.24), (2.25), and (2.26).

**Proof.** If \( 0 \in G \), then \( G \lambda_T(\alpha^+)(0) = \alpha^- \), \( G \lambda_I(\beta^+)(0) = \beta^+ \), and \( G \lambda_{F}(\gamma^-)(0) = \gamma^- \). Thus

\[
(\forall x \in X) \begin{cases} 
G \lambda_T(\alpha^+)(0) = \alpha^- = G \lambda_I(\alpha^-)(x), \\
G \lambda_I(\beta^+)(0) = \beta^- = G \lambda_I(\beta^-)(x), \\
G \lambda_{F}(\gamma^-)(0) = \gamma^- = G \lambda_{F}(\gamma^-)(x)
\end{cases}
\]

Hence, \( G \Lambda_{\alpha^+, \beta^+, \gamma^-} \) satisfies the conditions (2.24), (2.25), and (2.26).

\[\square\]

**Lemma 2.38.** If a NS \( G \Lambda_{\alpha^+, \beta^+, \gamma^-} \) in \( X \) satisfies the condition (2.24) (resp., (2.25), (2.26)), then the constant 0 of \( X \) is in a nonempty subset \( G \) of \( X \).

**Proof.** Assume that a NS \( G \Lambda_{\alpha^+, \beta^+, \gamma^-} \) in \( X \) satisfies the condition (2.24). Then \( G \lambda_T(\alpha^-)(x) \leq G \lambda_T(\alpha^-)(x) \) for all \( x \in X \). Since \( G \) is nonempty, there exists \( g \in G \). Thus \( G \lambda_T(\alpha^-)(g) = \alpha^- \), so \( G \lambda_T(\alpha^-)(0) \leq G \lambda_T(\alpha^-)(g) = \alpha^- \), that is, \( G \lambda_T(\alpha^-)(0) = \alpha^- \). Hence, \( 0 \in G \).

\[\square\]

**Theorem 2.39.** A NS \( G \Lambda_{\alpha^+, \beta^+, \gamma^-} \) in \( X \) is a special neutrosophic UP-subalgebra of \( X \) if and only if a nonempty subset \( G \) of \( X \) is a UP-subalgebra of \( X \).

**Proof.** Assume that \( G \Lambda_{\alpha^+, \beta^+, \gamma^-} \) is a special neutrosophic UP-subalgebra of \( X \). Let \( x, y \in G \). Then \( G \lambda_T(\alpha^-)(x) = \alpha^- = G \lambda_T(\alpha^-)(y) \). Thus

\[
G \lambda_T(\alpha^-)(x \cdot y) \leq \max\{G \lambda_T(\alpha^-)(x), G \lambda_T(\alpha^-)(y)\} = \alpha^- \leq G \lambda_T(\alpha^-)(x \cdot y) \text{ by (2.21)}
\]

and so \( G \lambda_T(\alpha^-)(x \cdot y) = \alpha^- \). Thus \( x \cdot y \in G \). Hence, \( G \) is a UP-subalgebra of \( X \). Conversely, assume that \( G \) is a UP-subalgebra of \( X \). Let \( x, y \in X \).
Case 1: $x, y \in G$. Then

$$G_{\lambda^T[\alpha^-]}(x) = \alpha^- = G_{\lambda^T[\alpha^+]}(y), G_{\lambda_1[\beta^-]}(x) = \beta^+ = G_{\lambda_1[\beta^+]}(y), \quad G_{\lambda^F[\gamma^-]}(x) = \gamma^- = G_{\lambda^F[\gamma^+]}(y).$$

Thus

$$\max\{G_{\lambda^T[\alpha^-]}(x), G_{\lambda^T[\alpha^+]}(y)\} = \alpha^-,$$

$$\min\{G_{\lambda_1[\beta^-]}(x), G_{\lambda_1[\beta^+]}(y)\} = \beta^+,$$

$$\max\{G_{\lambda^F[\gamma^-]}(x), G_{\lambda^F[\gamma^+]}(y)\} = \gamma^-.$$

Since $G$ is a UP-subalgebra of $X$, we have $x \cdot y \in G$ and so $G_{\lambda^T[\alpha^-]}(x \cdot y) = \alpha^-$, $G_{\lambda_1[\beta^+]}(x \cdot y) = \beta^+$, and $G_{\lambda^F[\gamma^-]}(x \cdot y) = \gamma^-$. Hence,

$$G_{\lambda^T[\alpha^-]}(x \cdot y) = \alpha^- \leq \alpha^- = \max\{G_{\lambda^T[\alpha^-]}(x), G_{\lambda^T[\alpha^+]}(y)\},$$

$$G_{\lambda_1[\beta^+]}(x \cdot y) = \beta^+ \geq \beta^+ = \min\{G_{\lambda_1[\beta^-]}(x), G_{\lambda_1[\beta^+]}(y)\},$$

$$G_{\lambda^F[\gamma^-]}(x \cdot y) = \gamma^- \leq \gamma^- = \max\{G_{\lambda^F[\gamma^-]}(x), G_{\lambda^F[\gamma^+]}(y)\}.$$

Case 2: $x \not\in G$ or $y \not\in G$. Then

$$G_{\lambda^T[\alpha^+]}(x) = \alpha^- \text{ or } G_{\lambda^T[\alpha^-]}(y) = \alpha^-,$$

$$G_{\lambda_1[\beta^+]}(x) = \beta^+ \text{ or } G_{\lambda_1[\beta^-]}(y) = \beta^-,$$

$$G_{\lambda^F[\gamma^+]}(x) = \gamma^- \text{ or } G_{\lambda^F[\gamma^-]}(y) = \gamma^-.$$

Thus

$$\max\{G_{\lambda^T[\alpha^-]}(x), G_{\lambda^T[\alpha^-]}(y)\} = \alpha^-,$$

$$\min\{G_{\lambda_1[\beta^-]}(x), G_{\lambda_1[\beta^-]}(y)\} = \beta^-,$$

$$\max\{G_{\lambda^F[\gamma^-]}(x), G_{\lambda^F[\gamma^-]}(y)\} = \gamma^-.$$

Therefore,

$$G_{\lambda^T[\alpha^+]}(x \cdot y) \geq \alpha^- = \max\{G_{\lambda^T[\alpha^-]}(x), G_{\lambda^T[\alpha^-]}(y)\},$$

$$G_{\lambda_1[\beta^+]}(x \cdot y) \leq \beta^+ = \min\{G_{\lambda_1[\beta^-]}(x), G_{\lambda_1[\beta^+]}(y)\},$$

$$G_{\lambda^F[\gamma^+]}(x \cdot y) \geq \gamma^- = \max\{G_{\lambda^F[\gamma^-]}(x), G_{\lambda^F[\gamma^-]}(y)\}.$$

Hence, $G_{\lambda^T[\alpha^-], \lambda_1^T[\beta^-], \lambda^F[\gamma^-]}$ is a special neutrosophic UP-subalgebra of $X$. \qed

**Theorem 2.40.** A NS $G_{\lambda^T[\alpha^-], \lambda_1^T[\beta^-], \lambda^F[\gamma^-]}$ in $X$ is a special neutrosophic near UP-filter of $X$ if and only if a nonempty subset $G$ of $X$ is a near UP-filter of $X$.

**Proof.** Assume that $G_{\lambda^T[\alpha^-], \lambda_1^T[\beta^-], \lambda^F[\gamma^-]}$ is a special neutrosophic near UP-filter of $X$. Since $G_{\lambda^T[\alpha^-], \lambda_1^T[\beta^-], \lambda^F[\gamma^-]}$ satisfies the condition (2.24), it follows from Lemma 2.38 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then $G_{\lambda^T[\alpha^-]}(y) = \alpha^-$. Thus

$$G_{\lambda^T[\alpha^-]}(x \cdot y) \leq \alpha^- = \max\{G_{\lambda^T[\alpha^-]}(x), G_{\lambda^T[\alpha^-]}(y)\} \quad \text{by (2.27)}$$

and so $G_{\lambda^T[\alpha^-]}(x \cdot y) = \alpha^-$. Thus $x \cdot y \in G$. Hence, $G$ is a near UP-filter of $X$.

Conversely, assume that $G$ is a near UP-filter of $X$. Since $0 \in G$, it follows from Lemma 2.37 that $G_{\lambda^T[\alpha^-], \lambda_1^T[\beta^-], \lambda^F[\gamma^-]}$ satisfies the conditions (2.24), (2.25), and (2.26). Next, let $x, y \in X$. 

Case 1: \( y \in G \). Then \( G \lambda_T[\alpha_\alpha^-](y) = \alpha^- \), \( G \lambda_I[\beta_\beta^+](y) = \beta^+ \), and \( G \lambda_F[\gamma_{\gamma_Y}^-](y) = \gamma^- \). Since \( G \) is a near UP-filter of \( X \), we have \( x \cdot y \in G \) and so \( G \lambda_T[\alpha_\alpha^-](x \cdot y) = \alpha^- \), \( G \lambda_I[\beta_\beta^+](x \cdot y) = \beta^+ \), and \( G \lambda_F[\gamma_{\gamma_Y}^-](x \cdot y) = \gamma^- \). Thus
\[
G \lambda_T[\alpha_\alpha^-](x \cdot y) = \alpha^- \leq \alpha^- = G \lambda_T[\alpha_\alpha^-](y),
\]
\[
G \lambda_I[\beta_\beta^+](x \cdot y) = \beta^+ \geq \beta^+ = G \lambda_I[\beta_\beta^+](y),
\]
\[
G \lambda_F[\gamma_{\gamma_Y}^-](x \cdot y) = \gamma^- \leq \gamma^- = G \lambda_F[\gamma_{\gamma_Y}^-](y).
\]

Case 2: \( y \notin G \). Then \( G \lambda_T[\alpha_\alpha^-](y) = \alpha^+ \), \( G \lambda_I[\beta_\beta^+](y) = \beta^- \), and \( G \lambda_F[\gamma_{\gamma_Y}^-](y) = \gamma^+ \). Thus
\[
G \lambda_T[\alpha_\alpha^-](x \cdot y) \leq \alpha^+ = G \lambda_T[\alpha_\alpha^-](y),
\]
\[
G \lambda_I[\beta_\beta^+](x \cdot y) \geq \beta^- = G \lambda_I[\beta_\beta^+](y),
\]
\[
G \lambda_F[\gamma_{\gamma_Y}^-](x \cdot y) \leq \gamma^+ = G \lambda_F[\gamma_{\gamma_Y}^-](y).
\]

Hence, \( G \Lambda[\alpha^-_\alpha, \beta^+_\beta, \gamma^-_{\gamma_Y}] \) is a special neutrosophic near UP-filter of \( X \).

\[\square\]

**Theorem 2.41.** A NS \( G \Lambda[\alpha^-_\alpha, \beta^+_\beta, \gamma^-_{\gamma_Y}] \) in \( X \) is a special neutrosophic UP-filter of \( X \) if and only if a nonempty subset \( G \) of \( X \) is a UP-filter of \( X \).

**Proof.** Assume that \( G \Lambda[\alpha^-_\alpha, \beta^+_\beta, \gamma^-_{\gamma_Y}] \) is a special neutrosophic UP-filter of \( X \). Since \( G \Lambda[\alpha^-_\alpha, \beta^+_\beta, \gamma^-_{\gamma_Y}] \) satisfies the condition (2.24), it follows from Lemma 2.38 that \( 0 \in G \). Next, let \( x, y \in X \) be such that \( x \cdot y \in G \) and \( x \in G \). Then \( G \lambda_T[\alpha_\alpha^-](x \cdot y) = \alpha^- \leq G \lambda_T[\alpha_\alpha^-](y) \) by (2.30) and so \( G \lambda_T[\alpha_\alpha^-](y) = \alpha^- \). Thus \( y \in G \). Hence, \( G \) is a UP-filter of \( X \).

Conversely, assume that \( G \) is a UP-filter of \( X \). Since \( 0 \in G \), it follows from Lemma 2.37 that \( G \Lambda[\alpha^-_\alpha, \beta^+_\beta, \gamma^-_{\gamma_Y}] \) satisfies the conditions (2.24), (2.25), and (2.26). Next, let \( x, y \in X \).

**Case 1:** \( x \cdot y \in G \) and \( x \in G \). Then
\[
G \lambda_T[\alpha_\alpha^-](x \cdot y) = \alpha^- = G \lambda_T[\alpha_\alpha^-](x),
\]
\[
G \lambda_I[\beta_\beta^+](x \cdot y) = \beta^+ = G \lambda_I[\beta_\beta^+](x),
\]
\[
G \lambda_F[\gamma_{\gamma_Y}^-](x \cdot y) = \gamma^- = G \lambda_F[\gamma_{\gamma_Y}^-](x).
\]
Since \( G \) is a UP-filter of \( X \), we have \( y \in G \) and so \( G \lambda_T[\alpha_\alpha^-](y) = \alpha^- \), \( G \lambda_I[\beta_\beta^+](y) = \beta^+ \), and \( G \lambda_F[\gamma_{\gamma_Y}^-](y) = \gamma^- \). Thus
\[
G \lambda_T[\alpha_\alpha^-](y) = \alpha^- \leq \alpha^- = \max\{G \lambda_T[\alpha_\alpha^-](x \cdot y), G \lambda_T[\alpha_\alpha^-](x)\},
\]
\[
G \lambda_I[\beta_\beta^+](y) = \beta^+ \geq \beta^+ = \min\{G \lambda_I[\beta_\beta^+](x \cdot y), G \lambda_I[\beta_\beta^+](x)\},
\]
\[
G \lambda_F[\gamma_{\gamma_Y}^-](y) = \gamma^- \leq \gamma^- = \max\{G \lambda_F[\gamma_{\gamma_Y}^-](x \cdot y), G \lambda_F[\gamma_{\gamma_Y}^-](x)\}.
\]

**Case 2:** \( x \cdot y \notin G \) or \( x \notin G \). Then
\[
G \lambda_T[\alpha_\alpha^-](x \cdot y) = \alpha^+ \text{ or } G \lambda_T[\alpha_\alpha^-](x) = \alpha^+,
\]
\[
G \lambda_I[\beta_\beta^+](x \cdot y) = \beta^- \text{ or } G \lambda_I[\beta_\beta^+](x) = \beta^-,
\]
\[
G \lambda_F[\gamma_{\gamma_Y}^-](x \cdot y) = \gamma^+ \text{ or } G \lambda_F[\gamma_{\gamma_Y}^-](x) = \gamma^+.
\]
Thus
\[
\begin{align*}
\max\{G\lambda_T^{[\alpha^+]}(x \cdot y), G\lambda_T^{[\alpha^-]}(x)\} &= \alpha^+, \\
\min\{G\lambda_I^{[\beta^+]}(x \cdot y), G\lambda_I^{[\beta^-]}(x)\} &= \beta^-,
\end{align*}
\]
\[
\max\{G\lambda_F^{[\gamma^+]}(x \cdot y), G\lambda_F^{[\gamma^-]}(x)\} = \gamma^+.
\]

Therefore,
\[
\begin{align*}
G\lambda_T^{[\alpha^+]}(x) &\leq \alpha^+ = \max\{G\lambda_T^{[\alpha^+]}(x \cdot y), G\lambda_T^{[\alpha^-]}(x)\}, \\
G\lambda_I^{[\beta^+]}(x) &\geq \beta^- = \min\{G\lambda_I^{[\beta^+]}(x \cdot y), G\lambda_I^{[\beta^-]}(x)\}, \\
G\lambda_F^{[\gamma^+]}(x) &\leq \gamma^+ = \max\{G\lambda_F^{[\gamma^+]}(x \cdot y), G\lambda_F^{[\gamma^-]}(x)\}.
\end{align*}
\]

Hence, \(G\lambda^{[\alpha^-, \beta^+, \gamma^-]}_X\) is a special neutrosophic UP-filter of \(X\).

**Theorem 2.42.** A NS \(G\lambda^{[\alpha^-, \beta^+, \gamma^-]}_X\) in \(X\) is a special neutrosophic UP-ideal of \(X\) if and only if a nonempty subset \(G\) of \(X\) is a UP-ideal of \(X\).

**Proof.** Assume that \(G\lambda^{[\alpha^-, \beta^+, \gamma^-]}_X\) is a special neutrosophic UP-ideal of \(X\). Since \(G\lambda^{[\alpha^-, \beta^+, \gamma^-]}_X\) satisfies the condition (2.24), it follows from Lemma 2.38 that \(0 \in G\). Next, let \(x, y, z \in X\) be such that \(x \cdot (y \cdot z) \in G\) and \(y \in G\). Then \(G\lambda_T^{[\alpha^+]}(x \cdot (y \cdot z)) = \alpha^- = G\lambda_T^{[\alpha^-]}(y)\). Thus
\[
G\lambda_T^{[\alpha^+]}(x \cdot z) \leq \max\{G\lambda_T^{[\alpha^+]}(x \cdot (y \cdot z)), G\lambda_T^{[\alpha^-]}(y)\} = \alpha^- \leq G\lambda_T^{[\alpha^+]}(x \cdot z) \text{ by (2.33)}
\]
and so \(G\lambda_T^{[\alpha^+]}(x \cdot z) = \alpha^-\). Thus \(x \cdot z \in G\). Hence, \(G\) is a UP-ideal of \(X\).

Conversely, assume that \(G\) is a UP-ideal of \(X\). Since \(0 \in G\), it follows from Lemma 2.37 that \(G\lambda^{[\alpha^-, \beta^+, \gamma^-]}_X\) satisfies the conditions (2.24), (2.25), and (2.26). Next, let \(x, y, z \in X\).

**Case 1:** \(x \cdot (y \cdot z) \in G\) and \(y \in G\). Then
\[
\begin{align*}
G\lambda_T^{[\alpha^+]}(x \cdot (y \cdot z)) &= \alpha^- = G\lambda_T^{[\alpha^-]}(y), \\
G\lambda_I^{[\beta^+]}(x \cdot (y \cdot z)) &= \beta^+ = G\lambda_I^{[\beta^-]}(y), \\
G\lambda_F^{[\gamma^+]}(x \cdot (y \cdot z)) &= \gamma^- = G\lambda_F^{[\gamma^-]}(y).
\end{align*}
\]

Thus
\[
\begin{align*}
\max\{G\lambda_T^{[\alpha^+]}(x \cdot (y \cdot z)), G\lambda_T^{[\alpha^-]}(y)\} &= \alpha^-,
\min\{G\lambda_I^{[\beta^+]}(x \cdot (y \cdot z)), G\lambda_I^{[\beta^-]}(y)\} &= \beta^+,
\max\{G\lambda_F^{[\gamma^+]}(x \cdot (y \cdot z)), G\lambda_F^{[\gamma^-]}(y)\} &= \gamma^-.
\end{align*}
\]

Since \(G\) is a UP-ideal of \(X\), we have \(x \cdot z \in G\) and so \(G\lambda_T^{[\alpha^+]}(x \cdot z) = \alpha^-, G\lambda_I^{[\beta^+]}(x \cdot z) = \beta^+, \) and \(G\lambda_F^{[\gamma^+]}(x \cdot z) = \gamma^-\). Thus
\[
\begin{align*}
G\lambda_T^{[\alpha^+]}(x \cdot z) &= \alpha^- \leq \alpha^- = \max\{G\lambda_T^{[\alpha^+]}(x \cdot (y \cdot z)), G\lambda_T^{[\alpha^-]}(y)\}, \\
G\lambda_I^{[\beta^+]}(x \cdot z) &= \beta^+ \geq \beta^+ = \min\{G\lambda_I^{[\beta^+]}(x \cdot (y \cdot z)), G\lambda_I^{[\beta^-]}(y)\}, \\
G\lambda_F^{[\gamma^+]}(x \cdot z) &= \gamma^- \leq \gamma^- = \max\{G\lambda_F^{[\gamma^+]}(x \cdot (y \cdot z)), G\lambda_F^{[\gamma^-]}(y)\}.
\end{align*}
\]
Case 2: \( x \cdot (y \cdot z) \not\in G \) or \( y \not\in G \). Then

\[
\begin{align*}
G\lambda_T^{[\alpha^+]}(x \cdot (y \cdot z)) &= \alpha^+ \text{ or } G\lambda_T^{[\alpha^-]}(y) = \alpha^+ , \\
G\lambda_I^{[\beta^+]}(x \cdot (y \cdot z)) &= \beta^- \text{ or } G\lambda_I^{[\beta^-]}(y) = \beta^- , \\
G\lambda_F^{[\gamma^+]}(x \cdot (y \cdot z)) &= \gamma^+ \text{ or } G\lambda_F^{[\gamma^-]}(y) = \gamma^+ .
\end{align*}
\]

Thus

\[
\begin{align*}
\max[G\lambda_T^{[\alpha^+]}(x \cdot (y \cdot z)), G\lambda_T^{[\alpha^-]}(y)] &= \alpha^+ , \\
\min[G\lambda_I^{[\beta^+]}(x \cdot (y \cdot z)), G\lambda_I^{[\beta^-]}(y)] &= \beta^- , \\
\max[G\lambda_F^{[\gamma^+]}(x \cdot (y \cdot z)), G\lambda_F^{[\gamma^-]}(y)] &= \gamma^+ .
\end{align*}
\]

Therefore,

\[
\begin{align*}
G\lambda_T^{[\alpha^+]}(x \cdot z) &\leq \alpha^+ = \max[G\lambda_T^{[\alpha^+]}(x \cdot (y \cdot z)), G\lambda_T^{[\alpha^-]}(y)] , \\
G\lambda_I^{[\beta^+]}(x \cdot z) &\geq \beta^- = \min[G\lambda_I^{[\beta^+]}(x \cdot (y \cdot z)), G\lambda_I^{[\beta^-]}(y)] , \\
G\lambda_F^{[\gamma^+]}(x \cdot z) &\leq \gamma^+ = \max[G\lambda_F^{[\gamma^+]}(x \cdot (y \cdot z)), G\lambda_F^{[\gamma^-]}(y)] .
\end{align*}
\]

Hence, \( G\Lambda^{[\alpha^+, \beta^+, \gamma^-]} \) is a special neutrosophic UP-ideal of \( X \).

Theorem 3.24. A NS \( G\Lambda^{[\alpha^-, \beta^+, \gamma^-]} \) in \( X \) is a special neutrosophic strong UP-ideal of \( X \) if and only if a nonempty subset \( G \) of \( X \) is a strong UP-ideal of \( X \).

Proof. Assume that \( G\Lambda^{[\alpha^-, \beta^+, \gamma^-]} \) is a special neutrosophic strong UP-ideal of \( X \). By Theorem 2.28, we have \( G\lambda_T^{[\alpha^+]} \) is constant, that is, \( G\lambda_T^{[\alpha^-]} \) is constant. Since \( G \) is nonempty, we have \( G\lambda_T^{[\alpha^-]}(x) = \alpha^- \) for all \( x \in X \). Thus \( G = X \). Hence, \( G \) is a strong UP-ideal of \( X \).

Conversely, assume that \( G \) is a strong UP-ideal of \( X \). Then \( G = X \), so

\[
(\forall x \in X) \begin{cases} 
G\lambda_T^{[\alpha^+]}(x) = \alpha^- \\
G\lambda_I^{[\beta^+]}(x) = \beta^+ \\
G\lambda_F^{[\gamma^+]}(x) = \gamma^-
\end{cases}
\]

Thus \( G\lambda_T^{[\alpha^-]} \), \( G\lambda_I^{[\beta^+]} \), and \( G\lambda_F^{[\gamma^+]} \) are constant, that is, \( G\Lambda^{[\alpha^-, \beta^+, \gamma^-]} \) is constant. By Theorem 2.28, we have \( G\Lambda^{[\alpha^-, \beta^+, \gamma^-]} \) is a special neutrosophic strong UP-ideal of \( X \).

3. Level subsets of a NS of special types

In this paper, we discuss the relationships among special neutrosophic UP-subalgebras (resp., special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, special neutrosophic strong UP-ideals) of UP-algebras and their level subsets.

Definition 3.1 ([21]). Let \( f \) be a fuzzy set in \( X \). For any \( t \in [0, 1] \), the sets

\[
\begin{align*}
U(f; t) &= \{ x \in X \mid f(x) \geq t \} , \\
L(f; t) &= \{ x \in X \mid f(x) \leq t \} , \\
E(f; t) &= \{ x \in X \mid f(x) = t \}
\end{align*}
\]

are called an upper \( t \)-level subset, a lower \( t \)-level subset, and an equal \( t \)-level subset of \( f \), respectively.

Theorem 3.2. A NS \( \Lambda \) in \( X \) is a special neutrosophic UP-subalgebra of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [0, 1] \), the sets \( L(\lambda_T; \alpha), U(\lambda_I; \beta), \) and \( L(\lambda_F; \gamma) \) are UP-subalgebras of \( X \) if \( L(\lambda_T; \alpha), U(\lambda_I; \beta), \) and \( L(\lambda_F; \gamma) \) are nonempty.
Proof. Assume that $\Lambda$ is a special neutrosophic UP-subalgebra of $X$. Let $\alpha, \beta, \gamma \in [0, 1]$ be such that $L(\lambda; \alpha), U(\lambda; \beta),$ and $L(\lambda; \gamma)$ are nonempty.

Let $x, y \in L(\lambda; \alpha)$. Then $\lambda_T(x) \leq \alpha$ and $\lambda_T(y) \leq \alpha$, so $\alpha$ is a upper bound of $\{\lambda_T(x), \lambda_T(y)\}$. By (2.21), we have $\lambda_T(x \cdot y) \leq \max\{\lambda_T(x),\lambda_T(y)\} \leq \alpha$. Thus $x \cdot y \in L(\lambda; \alpha)$.

Let $x, y \in U(\lambda; \beta)$. Then $\lambda_I(x) \geq \beta$ and $\lambda_I(y) \geq \beta$, so $\beta$ is an lower bound of $\{\lambda_I(x), \lambda_I(y)\}$. By (2.22), we have $\lambda_I(x \cdot y) \geq \min\{\lambda_I(x),\lambda_I(y)\} \geq \beta$. Thus $x \cdot y \in U(\lambda; \beta)$.

Let $x, y \in L(\lambda; \gamma)$. Then $\lambda_F(x) \leq \gamma$ and $\lambda_F(y) \leq \gamma$, so $\gamma$ is a upper bound of $\{\lambda_F(x), \lambda_F(y)\}$. By (2.23), we have $\lambda_F(x \cdot y) \leq \max\{\lambda_F(x),\lambda_F(y)\} \leq \gamma$. Thus $x \cdot y \in L(\lambda; \gamma)$.

Hence, $L(\lambda; \alpha), U(\lambda; \beta)$, and $L(\lambda; \gamma)$ are UP-subalgebras of $X$.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0, 1]$, the set $L(\lambda; \alpha), U(\lambda; \beta)$, and $L(\lambda; \gamma)$ are UP-subalgebras if $L(\lambda; \alpha), U(\lambda; \beta)$, and $L(\lambda; \gamma)$ are nonempty.

Let $x, y \in X$. Then $\lambda_T(x), \lambda_I(x) \in [0, 1]$. Choose $\alpha = \max\{\lambda_T(x),\lambda_I(x)\}$. Thus $\lambda_T(x) \leq \alpha$ and $\lambda_I(x) \leq \alpha$, so $x, y \in L(\lambda; \alpha)$. By assumption, we have $L(\lambda; \alpha)$ is a UP-subalgebra of $X$ and so $x, y \in L(\lambda; \alpha)$. Thus $\lambda_T(x \cdot y) \leq \alpha = \max\{\lambda_T(x),\lambda_I(x)\}$.

Let $x, y \in X$. Then $\lambda_I(x), \lambda_I(y) \in [0, 1]$. Choose $\beta = \min\{\lambda_I(x),\lambda_I(y)\}$. Thus $\lambda_I(x) \geq \beta$ and $\lambda_I(y) \geq \beta$, so $x, y \in U(\lambda; \beta)$. By assumption, we have $U(\lambda; \beta)$ is a UP-subalgebra of $X$ and so $x, y \in U(\lambda; \beta)$. Thus $\lambda_I(x \cdot y) \geq \beta = \min\{\lambda_I(x),\lambda_I(y)\}$.

Let $x, y \in X$. Then $\lambda_F(x), \lambda_F(y) \in [0, 1]$. Choose $\gamma = \max\{\lambda_F(x),\lambda_F(y)\}$. Thus $\lambda_F(x) \leq \gamma$ and $\lambda_F(y) \leq \gamma$, so $x, y \in L(\lambda; \gamma)$. By assumption, we have $L(\lambda; \gamma)$ is a UP-subalgebra of $X$ and so $x, y \in L(\lambda; \gamma)$. Thus $\lambda_F(x \cdot y) \leq \gamma = \max\{\lambda_F(x),\lambda_F(y)\}$.

Therefore, $\Lambda$ is a special neutrosophic UP-subalgebra of $X$. □

Theorem 3.3. A NS $\Lambda$ in $X$ is a special neutrosophic near UP-filter of $X$ if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $L(\lambda; \alpha), U(\lambda; \beta)$, and $L(\lambda; \gamma)$ are near UP-filters of $X$ if $L(\lambda; \alpha), U(\lambda; \beta)$, and $L(\lambda; \gamma)$ are nonempty.

Proof. Assume that $\Lambda$ is a special neutrosophic near UP-filter of $X$. Let $\alpha, \beta, \gamma \in [0, 1]$ be such that $L(\lambda; \alpha), U(\lambda; \beta)$, and $L(\lambda; \gamma)$ are nonempty.

Let $x \in L(\lambda; \alpha)$. Then $\lambda_T(x) \leq \alpha$. By (2.24), we have $\lambda_T(0) \leq \lambda_T(x) \leq \alpha$. Thus $0 \in L(\lambda; \alpha)$. Next, let $y \in L(\lambda; \alpha)$. Then $\lambda_T(y) \leq \alpha$. By (2.27), we have $\lambda_T(x \cdot y) \leq \lambda_T(y) \leq \alpha$. Thus $x \cdot y \in L(\lambda; \alpha)$.

Let $x \in U(\lambda; \beta)$. Then $\lambda_I(x) \geq \beta$. By (2.25), we have $\lambda_I(0) \geq \lambda_I(x) \geq \beta$. Thus $0 \in U(\lambda; \beta)$. Next, let $y \in U(\lambda; \beta)$. Then $\lambda_I(y) \geq \beta$. By (2.28), we have $\lambda_I(x \cdot y) \geq \lambda_I(y) \geq \beta$. Thus $x \cdot y \in U(\lambda; \beta)$.

Let $x \in L(\lambda; \gamma)$. Then $\lambda_F(x) \leq \gamma$. By (2.26), we have $\lambda_F(0) \leq \lambda_F(x) \leq \gamma$. Thus $0 \in L(\lambda; \gamma)$. Next, $y \in L(\lambda; \gamma)$. Then $\lambda_F(y) \leq \gamma$. By (2.28), we have $\lambda_F(x \cdot y) \leq \lambda_F(y) \leq \gamma$. Thus $x \cdot y \in L(\lambda; \gamma)$.

Hence, $L(\lambda; \alpha), U(\lambda; \beta)$, and $L(\lambda; \gamma)$ are near UP-filters of $X$.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0, 1]$, the set $L(\lambda; \alpha), U(\lambda; \beta)$, and $L(\lambda; \gamma)$ are near UP-filters if $L(\lambda; \alpha), U(\lambda; \beta)$, and $L(\lambda; \gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(0) \in [0, 1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \leq \alpha$, so $x \in L(\lambda; \alpha) \neq \emptyset$. By assumption, we have $L(\lambda; \alpha)$ is a UP-filter of $X$ and so $0 \in L(\lambda; \alpha)$. Thus $\lambda_T(0) \leq \alpha = \lambda_T(x)$. Next, let $y \in X$. Then $\lambda_T(y) \in [0, 1]$. Choose $\alpha = \lambda_T(y)$. Thus $\lambda_T(y) \leq \alpha$, so $y \in L(\lambda; \alpha) \neq \emptyset$. By assumption, we have $L(\lambda; \alpha)$ is a UP-filter of $X$, and so $x \cdot y \in L(\lambda; \alpha)$. Thus $\lambda_T(x \cdot y) \leq \alpha = \lambda_T(y)$.

Let $x \in X$. Then $\lambda_I(0) \in [0, 1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \geq \beta$, so $x \in U(\lambda; \beta) \neq \emptyset$. By assumption, we have $U(\lambda; \beta)$ is a UP-filter of $X$, and so $x \cdot y \in U(\lambda; \beta)$. Thus $\lambda_I(x \cdot y) \geq \beta = \lambda_I(x)$.

Let $x \in X$. Then $\lambda_F(0) \in [0, 1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \leq \gamma$, so $x \in L(\lambda; \gamma) \neq \emptyset$. By assumption, we have $L(\lambda; \gamma)$ is a UP-filter of $X$, and so $x \cdot y \in L(\lambda; \gamma)$. Thus $\lambda_F(x \cdot y) \leq \gamma = \lambda_F(x)$.

Therefore, $\Lambda$ is a special neutrosophic near UP-filter of $X$. □
Theorem 3.4. A NS $\Lambda$ in $X$ is a special neutrosophic UP-filter of $X$ if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-filters of $X$ if $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Proof. Assume that $\Lambda$ is a special neutrosophic UP-filter of $X$. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x \in L(\lambda_T; \alpha)$. Then $\lambda_T(x) \leq \alpha$. By (2.24), we have $\lambda_T(0) \leq \lambda_T(x) \leq \alpha$. Thus $0 \in L(\lambda_T; \alpha)$. Next, let $x \cdot y \in L(\lambda_T; \alpha)$ and $x \in L(\lambda_T; \alpha)$. Then $\lambda_T(x \cdot y) \leq \alpha$ and $\lambda_T(x) \leq \alpha$, so $\alpha$ is a upper bound of $\{\lambda_T(x \cdot y), \lambda_T(x)\}$. By (2.30), we have $\lambda_T(y) \leq \max(\lambda_T(x \cdot y), \lambda_T(x)) \leq \alpha$. Thus $y \in L(\lambda_T; \alpha)$.

Let $x \in U(\lambda_I; \beta)$. Then $\lambda_I(x) \geq \beta$. By (2.25), we have $\lambda_I(0) \geq \lambda_I(x) \geq \beta$. Thus $0 \in U(\lambda_I; \beta)$. Next, let $x \cdot y \in U(\lambda_I; \beta)$ and $x \in U(\lambda_I; \beta)$. Then $\lambda_I(x \cdot y) \geq \beta$ and $\lambda_I(x) \geq \beta$, so $\beta$ is an lower bound of $\{\lambda_I(x \cdot y), \lambda_I(x)\}$. By (2.31), we have $\lambda_I(y) \geq \min(\lambda_I(x \cdot y), \lambda_I(x)) \geq \beta$. Thus $y \in U(\lambda_I; \beta)$.

Let $x \in L(\lambda_F; \gamma)$. Then $\lambda_F(x) \leq \gamma$. By (2.26), we have $\lambda_F(0) \leq \lambda_F(x) \leq \gamma$. Thus $0 \in L(\lambda_F; \gamma)$. Next, let $x \cdot y \in L(\lambda_F; \gamma)$ and $x \in L(\lambda_F; \gamma)$. Then $\lambda_F(x \cdot y) \leq \gamma$ and $\lambda_F(x) \leq \gamma$, so $\gamma$ is a upper bound of $\{\lambda_F(x \cdot y), \lambda_F(x)\}$. By (2.32), we have $\lambda_F(y) \leq \max(\lambda_F(x \cdot y), \lambda_F(x)) \leq \gamma$. Thus $y \in L(\lambda_F; \gamma)$.

Hence, $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-filters of $X$.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the set $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-ideals if $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x \in L(\lambda_T; \alpha)$. Then $\lambda_T(x) \leq \alpha$. By (2.24), we have $\lambda_T(0) \leq \lambda_T(x) \leq \alpha$. Thus $0 \in L(\lambda_T; \alpha)$. Next, let $x \cdot y \cdot z \in L(\lambda_T; \alpha)$ and $y \in L(\lambda_T; \alpha)$. Then $\lambda_T(x \cdot y \cdot z) \leq \alpha$ and $\lambda_T(y) \leq \alpha$, so $\alpha$ is an upper bound of $\{\lambda_T(x \cdot y \cdot z), \lambda_T(y)\}$. By (2.33), we have $\lambda_T(y) \leq \max(\lambda_T(x \cdot y \cdot z), \lambda_T(y)) \leq \alpha$. Thus $x \cdot z \in L(\lambda_T; \alpha)$.

Let $x \in U(\lambda_I; \beta)$. Then $\lambda_I(x) \geq \beta$. By (2.25), we have $\lambda_I(0) \geq \lambda_I(x) \geq \beta$. Thus $0 \in U(\lambda_I; \beta)$. Next, let $x \cdot y \cdot z \in U(\lambda_I; \beta)$ and $y \in U(\lambda_I; \beta)$. Then $\lambda_I(x \cdot y \cdot z) \geq \beta$ and $\lambda_I(y) \geq \beta$, so $\beta$ is a lower bound of $\{\lambda_I(x \cdot y \cdot z), \lambda_I(y)\}$. By (2.34), we have $\lambda_I(x \cdot y \cdot z) \leq \min(\lambda_I(x \cdot y \cdot z), \lambda_I(y)) \geq \beta$. Thus $x \cdot z \in U(\lambda_I; \beta)$.

Let $x \in L(\lambda_F; \gamma)$. Then $\lambda_F(x) \leq \gamma$. By (2.26), we have $\lambda_F(0) \leq \lambda_F(x) \leq \gamma$. Thus $0 \in L(\lambda_F; \gamma)$. Next, let $x \cdot y \cdot z \in L(\lambda_F; \gamma)$ and $y \in L(\lambda_F; \gamma)$. Then $\lambda_F(x \cdot y \cdot z) \leq \gamma$ and $\lambda_F(y) \leq \gamma$, so $\gamma$ is a upper bound of $\{\lambda_F(x \cdot y \cdot z), \lambda_F(y)\}$. By (2.35), we have $\lambda_F(x \cdot z) \leq \max(\lambda_F(x \cdot y \cdot z), \lambda_F(y)) \leq \gamma$. Thus $x \cdot z \in L(\lambda_F; \gamma)$.

Hence, $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-ideals of $X$.

Therefore, $\Lambda$ is a special neutrosophic UP-filter of $X$. □

Theorem 3.5. A NS $\Lambda$ in $X$ is a special neutrosophic UP-ideals of $X$ if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-ideals of $X$ if $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Proof. Assume that $\Lambda$ is a special neutrosophic UP-ideal of $X$. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x \in L(\lambda_T; \alpha)$. Then $\lambda_T(x) \leq \alpha$. By (2.24), we have $\lambda_T(0) \leq \lambda_T(x) \leq \alpha$. Thus $0 \in L(\lambda_T; \alpha)$. Next, let $x \cdot y \cdot z \in L(\lambda_T; \alpha)$ and $y \in L(\lambda_T; \alpha)$. Then $\lambda_T(x \cdot y \cdot z) \leq \alpha$ and $\lambda_T(y) \leq \alpha$, so $\alpha$ is an upper bound of $\{\lambda_T(x \cdot y \cdot z), \lambda_T(y)\}$. By (2.33), we have $\lambda_T(y) \leq \max(\lambda_T(x \cdot y \cdot z), \lambda_T(y)) \leq \alpha$. Thus $x \cdot z \in L(\lambda_T; \alpha)$.

Let $x \in U(\lambda_I; \beta)$. Then $\lambda_I(x) \geq \beta$. By (2.25), we have $\lambda_I(0) \geq \lambda_I(x) \geq \beta$. Thus $0 \in U(\lambda_I; \beta)$. Next, let $x \cdot y \cdot z \in U(\lambda_I; \beta)$ and $y \in U(\lambda_I; \beta)$. Then $\lambda_I(x \cdot y \cdot z) \geq \beta$ and $\lambda_I(y) \geq \beta$, so $\beta$ is a lower bound of $\{\lambda_I(x \cdot y \cdot z), \lambda_I(y)\}$. By (2.34), we have $\lambda_I(x \cdot y \cdot z) \leq \min(\lambda_I(x \cdot y \cdot z), \lambda_I(y)) \geq \beta$. Thus $x \cdot z \in U(\lambda_I; \beta)$.

Let $x \in L(\lambda_F; \gamma)$. Then $\lambda_F(x) \leq \gamma$. By (2.26), we have $\lambda_F(0) \leq \lambda_F(x) \leq \gamma$. Thus $0 \in L(\lambda_F; \gamma)$. Next, let $x \cdot y \cdot z \in L(\lambda_F; \gamma)$ and $y \in L(\lambda_F; \gamma)$. Then $\lambda_F(x \cdot y \cdot z) \leq \gamma$ and $\lambda_F(y) \leq \gamma$, so $\gamma$ is a upper bound of $\{\lambda_F(x \cdot y \cdot z), \lambda_F(y)\}$. By (2.35), we have $\lambda_F(x \cdot z) \leq \max(\lambda_F(x \cdot y \cdot z), \lambda_F(y)) \leq \gamma$. Thus $x \cdot z \in L(\lambda_F; \gamma)$.

Hence, $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-ideals of $X$.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the set $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-ideals if $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.
Let \( x \in X \). Then \( \lambda_T(x) \in [0,1] \). Choose \( \alpha = \lambda_T(x) \). Thus \( \lambda_T(x) \leq \alpha \), so \( x \in L(\lambda_T; \alpha) \neq \emptyset \). By assumption, we have \( L(\lambda_T; \alpha) \) is a UP-ideal of \( X \) and so \( 0 \in L(\lambda_T; \alpha) \). Thus \( \lambda_T(0) \leq \alpha = \lambda_T(x) \). Next, let \( x, y, z \in X \). Then \( \lambda_T(x \cdot (y \cdot z)), \lambda_T(y) \in [0,1] \). Choose \( \alpha = \max(\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)) \). Thus \( \lambda_T(x \cdot (y \cdot z)) \leq \alpha \) and \( \lambda_T(y) \leq \alpha \), so \( x \cdot (y \cdot z), y \in L(\lambda_T; \alpha) \neq \emptyset \). By assumption, we have \( L(\lambda_T; \alpha) \) is a UP-ideal of \( X \) and so \( x \cdot z \in L(\lambda_T; \alpha) \). Thus \( \lambda_T(x \cdot z) \leq \alpha = \max(\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)) \).

Let \( x \in X \). Then \( \lambda_1(x) \in [0,1] \). Choose \( \beta = \lambda_1(x) \). Thus \( \lambda_1(x) \geq \beta \), so \( x \in U(\lambda_1; \beta) \neq \emptyset \). By assumption, we have \( U(\lambda_1; \beta) \) is a UP-ideal of \( X \) and so \( 0 \in U(\lambda_1; \beta) \). Thus \( \lambda_1(0) \geq \beta = \lambda_1(x) \). Next, let \( x, y, z \in X \). Then \( \lambda_1(x \cdot (y \cdot z)), \lambda_1(y) \in [0,1] \). Choose \( \beta = \min(\lambda_1(x \cdot (y \cdot z)), \lambda_1(y)) \). Thus \( \lambda_1(x \cdot (y \cdot z)) \geq \beta \) and \( \lambda_1(y) \geq \beta \), so \( x \cdot (y \cdot z), y \in U(\lambda_1; \beta) \neq \emptyset \). By assumption, we have \( U(\lambda_1; \beta) \) is a UP-ideal of \( X \) and so \( x \cdot z \in U(\lambda_1; \beta) \). Thus \( \lambda_1(x \cdot z) \geq \beta = \min(\lambda_1(x \cdot (y \cdot z)), \lambda_1(y)) \).

Therefore, \( \Lambda \) is a special neutrosophic UP-ideal of \( X \).

\[\square\]

**Definition 3.6 ([23]).** Let \( \Lambda \) be a NS in \( X \). For \( \alpha, \beta, \gamma \in [0,1] \), the sets

\[
U \cup U(\alpha, \beta, \gamma) = \{ x \in X \mid \lambda_T \geq \alpha, \lambda_1 \leq \beta, \lambda_F \geq \gamma \},
\]

\[
L \cup L(\alpha, \beta, \gamma) = \{ x \in X \mid \lambda_T \leq \alpha, \lambda_1 \geq \beta, \lambda_F \leq \gamma \},
\]

\[
E(\alpha, \beta, \gamma) = \{ x \in X \mid \lambda_T = \alpha, \lambda_1 = \beta, \lambda_F = \gamma \}
\]

are called a \( U \cup U \)-\( (\alpha, \beta, \gamma) \)-level subset, an \( L \cup L \)-\( (\alpha, \beta, \gamma) \)-level subset, and an \( E \)-\( (\alpha, \beta, \gamma) \)-level subset of \( \Lambda \), respectively. Then we see that

\[
U \cup U(\alpha, \beta, \gamma) = U(\lambda_T; \alpha) \cap L(\lambda_1; \beta) \cap U(\lambda_F; \gamma),
\]

\[
L \cup L(\alpha, \beta, \gamma) = L(\lambda_T; \alpha) \cap U(\lambda_1; \beta) \cap L(\lambda_F; \gamma),
\]

\[
E(\alpha, \beta, \gamma) = E(\lambda_T; \alpha) \cap E(\lambda_1; \beta) \cap E(\lambda_F; \gamma).
\]

**Corollary 3.7.** A NS \( \Lambda \) in \( X \) is a special neutrosophic UP-subalgebra of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [0,1] \), \( L \cup L(\alpha, \beta, \gamma) \) is a UP-subalgebra of \( X \), where \( L \cup L(\alpha, \beta, \gamma) \) is nonempty.

**Proof.** It is straightforward by Theorem 3.2. \[\square\]

**Corollary 3.8.** A NS \( \Lambda \) in \( X \) is a special neutrosophic near UP-filter of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [0,1] \), \( L \cup L(\alpha, \beta, \gamma) \) is a near UP-filter of \( X \), where \( L \cup L(\alpha, \beta, \gamma) \) is nonempty.

**Proof.** It is straightforward by Theorem 3.3. \[\square\]

**Corollary 3.9.** A NS \( \Lambda \) in \( X \) is a special neutrosophic UP-filter of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [0,1] \), \( L \cup L(\alpha, \beta, \gamma) \) is a UP-filter of \( X \), where \( L \cup L(\alpha, \beta, \gamma) \) is nonempty.

**Proof.** It is straightforward by Theorem 3.4. \[\square\]

**Corollary 3.10.** A NS \( \Lambda \) in \( X \) is a special neutrosophic UP-ideal of \( X \) if and only if for all \( \alpha, \beta, \gamma \in [0,1] \), \( L \cup L(\alpha, \beta, \gamma) \) is a UP-ideal of \( X \), where \( L \cup L(\alpha, \beta, \gamma) \) is nonempty.

**Proof.** It is straightforward by Theorem 3.5. \[\square\]
4. Conclusions

In this paper, we have introduced the notions of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras and investigated some of their important properties. Then, we have the diagram of generalization of NSs of special types in UP-algebras as shown in Figure 1.

\[
\begin{align*}
(2.42) & \quad \text{Special neutrosophic UP-subalgebra} \\
(2.43) & \quad \text{Special neutrosophic near-UP-filter} \\
(2.44) & \quad \text{Special neutrosophic UP-filter} \\
(2.45) & \quad \text{Special neutrosophic UP-ideal} \\
& \quad \text{Special neutrosophic strongly UP-ideal} \\
& \quad \text{Constant neutrosophic set}
\end{align*}
\]

Figure 1: NSs of special types in UP-algebras.

In our future study, we will apply this notion/results to other type of NSs in UP-algebras. Also, we will study the soft set theory/cubic set theory of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals.

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