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A novel divergence measure and its based TOPSIS method for multi criteria decision-making under single-valued neutrosophic environment

Nancy and Harish Garg*
School of Mathematics, Thapar Institute of Engineering & Technology (Deemed University), Patiala, Punjab, India

Abstract. The theme of this work is to present an axiomatic definition of divergence measure for single-valued neutrosophic sets (SVNSs). The properties of the proposed divergence measure have been studied. Further, we develop a novel technique for order preference by similarity to ideal solution (TOPSIS) method for solving single-valued neutrosophic multi-criteria decision-making with incomplete weight information. Finally, a numerical example is presented to verify the proposed approach and to present its effectiveness and practicality.

Keywords: Single-valued Neutrosophic set, Divergence measure, TOPSIS, multi criteria decision-making

1. Introduction

Intuitionistic fuzzy (IF) set (IFS) developed by [1], is a useful tool characterized by the membership and the non-membership degrees to describe the data more accurately. Since IFS allows two degrees of freedom into a set description, therefore, it renders us an additional possibility to represent the uncertain data when trying to solve decision-making problems. Under this environment, [2, 3] presented distance and similarity measures between the IFSs. [4] developed the distance measures between the type-2 intuitionistic fuzzy sets. [5] developed some series of the distance measures while [6] presented IF cross-entropy for IFSs. [7] presented generalized directed divergence measures under IFS environment. [8] extended the IFS to the complex IFS and hence presented some series of distance measures for it. [9] presented some new similarity measures of IFS based on the set pair analysis theory. Apart from them, several other algorithms namely, score functions [10–12], aggregation operators [13, 14], ranking method [15–17] etc., under IFS environment have gained much attention by the researchers.

From this survey, it is remarked that neither the fuzzy set (FS) nor IFS theory is able to deal with indeterminate and inconsistent data. For instance, consider an expert which gives their opinion about a certain object in such a way that 0.5 being the "possibility that the statement is true", 0.7 being the "possibility that the statement is false" and 0.2 being the "possibility that he or she is not sure". To resolve this, [18] introduced a new component called the "indeterminacy-membership function" and added to the "truth membership function" and "falsity membership function", all which are independent of each

*Corresponding author. Harish Garg, School of Mathematics, Thapar Institute of Engineering & Technology (Deemed University), Patiala, Punjab, India. Tel.: +91 8699031147; E-mail: harish.garg@thapar.edu, http://sites.google.com/site/harishg58iitr/

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other and lying in $]0^-, 1^+[$, and the corresponding set is known as a neutrosophic set (NS). Since it is hard to apply the concept of NSs to the practical problems, therefore, [19] introduced the concept of a single-valued neutrosophic set (SVNS), which is the particular class of NS. After they came into existence, researchers have made their efforts to enrich the concept of information measure, used to reflect the decision maker's imperative and nature of individual choice, in the neutrosophic environment. [20] introduced the distance measures while [21, 22] presented the correlation for single-valued neutrosophic numbers(SVNNs). Also, [23] improved the concept of cosine similarity for SVNSs which was firstly introduced by [24] in a neutrosophic environment. [25] presented an improved score function for ranking the SVNNs. Apart from them, various authors incorporated the idea of SVNS theory into information measures [26–29], aggregation operators [31–33] and applied them to solve the decision-making problems.

However, TOPSIS developed by [15] is well-known decision-making approach to find the best alternative(s) based on its ideal values. The chief advantages of the TOPSIS are to consider positive and negative ideal solutions as anchor points to reflect the contrast of the currently achievable criterion performances. In TOPSIS, the preferred alternative should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. The difference between the TOPSIS and the neutrosophic TOPSIS is in their rating approaches. The merit of neutrosophic TOPSIS is to designate the importance of attributes and the performance of alternatives with respect to various attributes by using SVNNs instead of precise numbers. Under this environment, [34–36] presented the TOPSIS method for decision-making problems based on the distance measure to rank the alternatives.

Consequently, keeping the flexibility and efficiency of SVNS, the theme of this work is to present a novel divergence measure for SVNSs to find discrimination between them. Further, based on it, a TOPSIS method for solving neutrosophic decision-making problem has been presented. As far we know, there is no investigation on divergence measure in the neutrosophic environment. The proposed measure has elegant properties, which are expressed and tested in the paper to enhance the employability of this measure. In contrast to the classical TOPSIS method, which is based on distance measure, this paper applies the proposed divergence measure to establish the comparative index of closeness coefficient. The strength of this extension has been demonstrated by an example of the decision-making process. Finally, through an example, the superiority of divergence based TOPSIS method over classical TOPSIS has been shown.

The rest of the text has been summarized as follows: Section 2 introduces the divergence measure in the neutrosophic environment and various properties have also been investigated in details. Section 3 describes the TOPSIS approach for solving the decision-making problems based on the divergence measure followed by an illustrative example. Further, various test criteria are applied to check its applicability and explore its effectiveness. Finally, paper arrives at conclusion in Section 4.

2. Proposed divergence measure for single-valued neutrosophic set

In this section, we present a divergence measure between the two SVNSs to rank it. Further, we discuss its various properties apart from their basic axioms.

2.1. Basic concepts

Firstly, some basic definitions related to NS, SVNS on the universal set X have been discussed.

Definition 2.1. [18] A neutrosophic set (NS) A in X is

$$A = \Big\{ \langle x_j, \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j) | x_j \in X \rangle \Big\} \quad (1)$$

where $\zeta_A(x_j)$, $\rho_A(x_j)$, $\vartheta_A(x_j)$ represents the truth, indeterminacy and falsity-membership functions respectively, and are real standard or non-standard subsets of $]0^-, 1^+[$ such that $0^- \leq \sup \zeta_A(x_j) + \sup \rho_A(x_j) + \sup \vartheta_A(x_j) \leq 3^+$. Here, sup represents the supremum of the set.

Definition 2.2. [18, 19] A single-valued neutrosophic set (SVNS) A in X is defined as

$$A = \Big\{ \langle x_j, \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j) | x_j \in X \rangle \Big\} \quad (2)$$

where $\zeta_A(x_j)$, $\rho_A(x_j)$, $\vartheta_A(x_j) \in [0, 1]$ such that $0 \leq \zeta_A(x_j) + \rho_A(x_j) + \vartheta_A(x_j) \leq 3$ for all $x_j \in X$. For convenience, we denote these pairs as $A = \langle \zeta_A, \rho_A, \vartheta_A \rangle$, throughout this article, and called as single-valued neutrosophic number (SVNN).

Definition 2.3. Let $A = \{x_j, \langle \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j) \mid x_j \in X \rangle\}$ and $B = \{\langle x_j, \zeta_B(x_j), \rho_B(x_j), \vartheta_B(x_j) \rangle | x_j \in$

$X\}$ be two SVNSs. Then the following operations are defined as [19]

(i) $A \subseteq B$ if $\zeta_A(x_j) \leq \zeta_B(x_j)$, $\rho_A(x_j) \geq \rho_B(x_j)$ and $\vartheta_A(x_j) \geq \vartheta_B(x_j)$ for all x_j in X;
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(iii) $A^c = \{x_j, \langle \vartheta_A(x_j), \rho_A(x_j), \zeta_A(x_j) | x_j \in X \rangle\}$;
(iv) $A \cap B = \left\{ \langle x_j, \min(\zeta_A(x_j), \zeta_B(x_j)), \max(\rho_A(x_j), \rho_B(x_j)), \max(\vartheta_A(x_j), \vartheta_B(x_j)) \rangle | x_j \in X \right\}$;
(v) $A \cup B = \left\{ \langle x_j, \max(\zeta_A(x_j), \zeta_B(x_j)), \min(\rho_A(x_j), \rho_B(x_j)), \min(\vartheta_A(x_j), \vartheta_B(x_j)) \rangle | x_j \in X \right\}$.

2.2. Proposed divergence measure

Let $\Phi(X)$ be the class of SVNSs over the universal set X. Then for any $A, B \in$ SVNSs, a real function $D : \Phi(X) \times \Phi(X) \to R^+$ is called a divergence measure, denoted by $D(A|B)$, if it satisfies the following axioms:

(P1) $D(A|B) \geq 0$
(P2) $D(A|B) = D(B|A)$
(P3) $D(A|B) = 0$ if $A = B$
(P4) $D(A|B) = D(A^c|B^c)$

Definition 2.4. For two SVNSs $A = \langle \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j) | x_j \in X \rangle$ and $B = \langle \zeta_B(x_j), \rho_B(x_j), \vartheta_B(x_j) \mid x_j \in X \rangle$, the divergence measure of A against B is to measure the degree of discrimination between the pair is defined as:

$$D(A|B) = \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_B(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_B(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_B(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_B(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_B(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_B(x_j)}{2} \end{bmatrix} \quad (3)$$

Theorem 2.1. *The divergence measure $D(A|B)$, as defined in Definition 2.4, for two SVNSs A and B satisfies the following four axioms (P1)-(P4)*

(P1) $D(A|B) \geq 0$,
(P2) $D(A|B) = D(B|A)$,
(P3) $D(A|B) = 0$ if $A = B$,
(P4) $D(A|B) = D(A^c|B^c)$

Proof 2.1. For two SVNSs $A = \langle \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j) \mid x_j \in X \rangle$ and $B = \langle \zeta_B(x_j), \rho_B(x_j), \vartheta_B(x_j) | x_j \in X \rangle$, we have

(P1) Since $0 \leq \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j) \leq 1$ and $0 \leq \zeta_B(x_j), \rho_B(x_j), \vartheta_B(x_j) \leq 1$ which implies that $\sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_B(x_j))^2}{2}} \geq \frac{\zeta_A(x_j) + \zeta_B(x_j)}{2}$, $\sqrt{\frac{(\rho_A(x_j))^2 + (\rho_B(x_j))^2}{2}} \geq \frac{\rho_A(x_j) + \rho_B(x_j)}{2}$ and $\sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_B(x_j))^2}{2}} \geq \frac{\vartheta_A(x_j) + \vartheta_B(x_j)}{2}$ for each j. Therefore, from Equation (3), we get $D(A|B) \geq 0$.
(P2) It is trivial from the Equation (3).
(P3) Assume that $A = B$ which implies $\zeta_A(x_j) = \zeta_B(x_j)$, $\rho_A(x_j) = \rho_B(x_j)$ and $\vartheta_A(x_j) = \vartheta_B(x_j)$ for each j, which implies that $D(A|B) = \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n} \Big[\zeta_A(x_j) - \zeta_A(x_j) + \rho_A(x_j) - \rho_A(x_j) + \vartheta_A(x_j) - \vartheta_A(x_j) \Big] = 0$.
(P4) For SVNSs A and B, we have $A^c = \langle \vartheta_A(x_j), \rho_A(x_j), \zeta_A(x_j) \rangle$ and $B^c = \langle \vartheta_B(x_j), \rho_B(x_j), \zeta_B(x_j) \rangle$, so by Equation (3), we have

$$\begin{aligned} D(A^c|B^c) &= \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n} \begin{bmatrix} \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_B(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_B(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_B(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_B(x_j)}{2} \\ + \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_B(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_B(x_j)}{2} \end{bmatrix} \\ &= D(A|B) \end{aligned}$$

Hence, $D(A|B)$ is a valid divergence measure.

Also, it is observed that $D(A|B)$ satisfies certain properties which are stated as below:

Theorem 2.2. *For SVNS $A = \langle \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j) \mid x_j \in X \rangle$, $D(A|A^c) = 0$ if and only if $\zeta_A(x_j) = \vartheta_A(x_j)$ for each $x_j \in X$.*

Proof 2.2. For SVNS $A = \langle \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j) | x_j \in X \rangle$, we have $A^c = \langle \vartheta_A(x_j), \rho_A(x_j), \zeta_A(x_j) \, | x_j \in X \rangle$. Thus, from Equation (3), we get

$$\begin{aligned} & D(A|A^c) = 0 \\ & \Leftrightarrow \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n} 2 \left[\sqrt{\frac{(\zeta_A(x_j))^2 + (\vartheta_A(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \vartheta_A(x_j)}{2} \right] = 0 \\ & \Leftrightarrow \sqrt{\frac{(\zeta_A(x_j))^2 + (\vartheta_A(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \vartheta_A(x_j)}{2} = 0 \quad \textit{for each } x_j \in X \\ & \Leftrightarrow \left(\frac{\zeta_A(x_j)}{\sqrt{2}} - \frac{\vartheta_A(x_j)}{\sqrt{2}} \right)^2 = 0 \quad \textit{for each } x_j \in X \\ & \Leftrightarrow \zeta_A(x_j) = \vartheta_A(x_j) \quad \textit{for each } x_j \in X \end{aligned}$$

Hence, $D(A|A^c)=0$ if and only if $\zeta_A(x_j)=\vartheta_A(x_j)$ for each $x_j \in X$.

Theorem 2.3. *For SVNS* $A=\langle \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j)|\ x_j \in X\rangle$, $D(A|A^c)=1$ *if and only if either* $\zeta_A(x_j)=0$, $\vartheta_A(x_j)=1$ *or* $\zeta_A(x_j)=1$, $\vartheta_A(x_j)=0$.

Proof 2.3. For SVNS A, we have $D(A|A^c)=1 \Leftrightarrow \frac{1}{n(\sqrt{2}-1)}\sum_{j=1}^{n} 2\left[\sqrt{\frac{(\zeta_A(x_j))^2+(\vartheta_A(x_j))^2}{2}}-\frac{\zeta_A(x_j)+\vartheta_A(x_j)}{2}\right]=1 \quad \Leftrightarrow \sum_{j=1}^{n}\left[\sqrt{\frac{(\zeta_A(x_j))^2+(\vartheta_A(x_j))^2}{2}}-\frac{\zeta_A(x_j)+\vartheta_A(x_j)}{2}\right]=\frac{n(\sqrt{2}-1)}{2} \quad \Leftrightarrow \sqrt{\frac{(\zeta_A(x_j))^2+(\vartheta_A(x_j))^2}{2}}-\frac{\zeta_A(x_j)+\vartheta_A(x_j)}{2}=\frac{(\sqrt{2}-1)}{2}$ for each $x_j \Leftrightarrow (\zeta_A(x_j))^2+(\vartheta_A(x_j))^2=1$ and $\zeta_A(x_j)+\vartheta_A(x_j)=1$ which implies that $\zeta_A(x_j)\vartheta_A(x_j)=0$ and hence $D(A|A^c)=1$ if and only if either $\zeta_A(x_j)=0$, $\vartheta_A(x_j)=1$ or $\zeta_A(x_j)=1$, $\vartheta_A(x_j)=0$.

Theorem 2.4. *For SVNSs* A *and* B, *we have* $D(A|B^c)=D(A^c|B)$.

Proof 2.4. As $A^c=\langle \vartheta_A(x_j), \rho_A(x_j), \zeta_A(x_j)|x_j \in X\rangle$ and $B^c=\langle \vartheta_B(x_j), \rho_B(x_j), \zeta_B(x_j)|x_j \in X\rangle$ be the complement of the SVNSs A and B, then from Equation (3) we have

$$
\begin{aligned}
&D(A^c|B)\\
&=\frac{1}{n(\sqrt{2}-1)}\sum_{j=1}^{n}\begin{bmatrix}\sqrt{\frac{(\vartheta_A(x_j))^2+(\zeta_B(x_j))^2}{2}}-\frac{\vartheta_A(x_j)+\zeta_B(x_j)}{2}\\+\sqrt{\frac{(\rho_A(x_j))^2+(\rho_B(x_j))^2}{2}}-\frac{\rho_A(x_j)+\rho_B(x_j)}{2}\\+\sqrt{\frac{(\zeta_A(x_j))^2+(\vartheta_B(x_j))^2}{2}}-\frac{\zeta_A(x_j)+\vartheta_B(x_j)}{2}\end{bmatrix}\\
&=\frac{1}{n(\sqrt{2}-1)}\sum_{j=1}^{n}\begin{bmatrix}\sqrt{\frac{(\zeta_A(x_j))^2+(\vartheta_B(x_j))^2}{2}}-\frac{\zeta_A(x_j)+\vartheta_B(x_j)}{2}\\+\sqrt{\frac{(\rho_A(x_j))^2+(\rho_B(x_j))^2}{2}}-\frac{\rho_A(x_j)+\rho_B(x_j)}{2}\\+\sqrt{\frac{(\vartheta_A(x_j))^2+(\zeta_B(x_j))^2}{2}}-\frac{\vartheta_A(x_j)+\zeta_B(x_j)}{2}\end{bmatrix}\\
&=D(A|B^c)
\end{aligned}
$$

Divide the universe X into two disjoint parts X_1 and X_2, where $X_1=\{x_j : x_j \in X, A(x_j) \subseteq B(x_j)\}$ and $X_2=\{x_j : x_j \in X, B(x_j) \subseteq A(x_j)\}$. Based on these considerations, we further propose some properties of the divergence measure which are explained as follows:

Theorem 2.5. *If* A *and* B *be two SVNSs defined on universal set* X, *then*

$$D(A \cap B|A \cup B)=D(A|B).$$

Proof 2.5. Let $A=\langle \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j)|x_j \in X\rangle$ and $B=\langle \zeta_B(x_j), \rho_B(x_j), \vartheta_B(x_j)|x_j \in X\rangle$ be two SVNSs defined on the universal set X, then by Equation (3), we have

$$
\begin{aligned}
&D(A \cap B|A \cup B)=\frac{1}{n(\sqrt{2}-1)}\sum_{j=1}^{n}\\
&\begin{bmatrix}\sqrt{\frac{(\zeta_{A\cap B}(x_j))^2+(\zeta_{A\cup B}(x_j))^2}{2}}-\frac{\zeta_{A\cap B}(x_j)+\zeta_{A\cup B}(x_j)}{2}\\+\sqrt{\frac{(\rho_{A\cap B}(x_j))^2+(\rho_{A\cup B}(x_j))^2}{2}}-\frac{\rho_{A\cap B}(x_j)+\rho_{A\cup B}(x_j)}{2}\\+\sqrt{\frac{(\vartheta_{A\cap B}(x_j))^2+(\vartheta_{A\cup B}(x_j))^2}{2}}-\frac{\vartheta_{A\cap B}(x_j)+\vartheta_{A\cup B}(x_j)}{2}\end{bmatrix}\\
&=\frac{1}{n(\sqrt{2}-1)}\sum_{x_j\in X_1}\begin{bmatrix}\sqrt{\frac{(\zeta_A(x_j))^2+(\zeta_B(x_j))^2}{2}}-\frac{\zeta_A(x_j)+\zeta_B(x_j)}{2}\\+\sqrt{\frac{(\rho_A(x_j))^2+(\rho_B(x_j))^2}{2}}-\frac{\rho_A(x_j)+\rho_B(x_j)}{2}\\+\sqrt{\frac{(\vartheta_A(x_j))^2+(\vartheta_B(x_j))^2}{2}}-\frac{\vartheta_A(x_j)+\vartheta_B(x_j)}{2}\end{bmatrix}\\
&+\frac{1}{n(\sqrt{2}-1)}\sum_{x_j\in X_2}\begin{bmatrix}\sqrt{\frac{(\zeta_B(x_j))^2+(\zeta_A(x_j))^2}{2}}-\frac{\zeta_B(x_j)+\zeta_A(x_j)}{2}\\+\sqrt{\frac{(\rho_B(x_j))^2+(\rho_A(x_j))^2}{2}}-\frac{\rho_B(x_j)+\rho_A(x_j)}{2}\\+\sqrt{\frac{(\vartheta_B(x_j))^2+(\vartheta_A(x_j))^2}{2}}-\frac{\vartheta_B(x_j)+\vartheta_A(x_j)}{2}\end{bmatrix}\\
&=D(A|B)
\end{aligned}
$$

Thus, the result holds.

Theorem 2.6. *For two SVNSs* A *and* B, *we have*

(i) $D(A|A \cup B)=D(B|A \cap B)$
(ii) $D(A|A \cap B)=D(B|A \cup B)$
(iii) $D(A|A \cup B)+D(A|A \cap B)=D(A|B)$
(iv) $D(B|A \cup B)+D(B|A \cap B)=D(A|B)$

Proof 2.6. We will prove the first part and rest can be proved in the same manner. By the definition of divergence, we have

$$
\begin{aligned}
&D(A|A \cup B)=\frac{1}{n(\sqrt{2}-1)}\sum_{j=1}^{n}\\
&\begin{bmatrix}\sqrt{\frac{(\zeta_A(x_j))^2+(\zeta_{A\cup B}(x_j))^2}{2}}-\frac{\zeta_A(x_j)+\zeta_{A\cup B}(x_j)}{2}\\+\sqrt{\frac{(\rho_A(x_j))^2+(\rho_{A\cup B}(x_j))^2}{2}}-\frac{\rho_A(x_j)+\rho_{A\cup B}(x_j)}{2}\\+\sqrt{\frac{(\vartheta_A(x_j))^2+(\vartheta_{A\cup B}(x_j))^2}{2}}-\frac{\vartheta_A(x_j)+\vartheta_{A\cup B}(x_j)}{2}\end{bmatrix}
\end{aligned}
$$

$$= \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_1} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_B(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_B(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_B(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_B(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_B(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_B(x_j)}{2} \end{bmatrix}$$

$$+ \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_2} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_A(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_A(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_A(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_A(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_A(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_A(x_j)}{2} \end{bmatrix}$$

$$= \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_1} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_B(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_B(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_B(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_B(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_B(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_B(x_j)}{2} \end{bmatrix} \quad (4)$$

Also,

$$D(B|A \cap B) = \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n} \begin{bmatrix} \sqrt{\frac{(\zeta_B(x_j))^2 + (\zeta_{A\cap B}(x_j))^2}{2}} - \frac{\zeta_B(x_j) + \zeta_{A\cap B}(x_j)}{2} \\ + \sqrt{\frac{(\rho_B(x_j))^2 + (\rho_{A\cap B}(x_j))^2}{2}} - \frac{\rho_B(x_j) + \rho_{A\cap B}(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_B(x_j))^2 + (\vartheta_{A\cap B}(x_j))^2}{2}} - \frac{\vartheta_B(x_j) + \vartheta_{A\cap B}(x_j)}{2} \end{bmatrix}$$

$$= \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_1} \begin{bmatrix} \sqrt{\frac{(\zeta_B(x_j))^2 + (\zeta_A(x_j))^2}{2}} - \frac{\zeta_B(x_j) + \zeta_A(x_j)}{2} \\ + \sqrt{\frac{(\rho_B(x_j))^2 + (\rho_A(x_j))^2}{2}} - \frac{\rho_B(x_j) + \rho_A(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_B(x_j))^2 + (\vartheta_A(x_j))^2}{2}} - \frac{\vartheta_B(x_j) + \vartheta_A(x_j)}{2} \end{bmatrix}$$

$$+ \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_2} \begin{bmatrix} \sqrt{\frac{(\zeta_B(x_j))^2 + (\zeta_B(x_j))^2}{2}} - \frac{\zeta_B(x_j) + \zeta_B(x_j)}{2} \\ + \sqrt{\frac{(\rho_B(x_j))^2 + (\rho_B(x_j))^2}{2}} - \frac{\rho_B(x_j) + \rho_B(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_B(x_j))^2 + (\vartheta_B(x_j))^2}{2}} - \frac{\vartheta_B(x_j) + \vartheta_B(x_j)}{2} \end{bmatrix}$$

$$= \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_1} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_B(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_B(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_B(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_B(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_B(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_B(x_j)}{2} \end{bmatrix} \quad (5)$$

Then, from Equations (4) and (5), we get $D(A|A \cup B) = D(B|A \cap B)$.

Theorem 2.7. *If A and B be two SVNSs defined on the universal sets X, then*

(i) $D(A|C) + D(B|C) - D(A \cup B|C) \geq 0$,
(ii) $D(A|C) + D(B|C) - D(A \cap B|C) \geq 0$

Proof 2.7. In this property, we prove only the first part because of having analogously similar proofs.

$$D(A|C) = \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

$$D(B|C) = \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n} \begin{bmatrix} \sqrt{\frac{(\zeta_B(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_B(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_B(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_B(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_B(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_B(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

and

$$D(A \cup B|C) = \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n} \begin{bmatrix} \sqrt{\frac{(\zeta_{A\cup B}(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_{A\cup B}(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_{A\cup B}(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_{A\cup B}(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_{A\cup B}(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_{A\cup B}(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

$$= \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_1} \begin{bmatrix} \sqrt{\frac{(\zeta_B(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_B(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_B(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_B(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_B(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_B(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

$$+ \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_2} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

Then,

$$D(A|C) + D(B|C) - D(A \cup B|C)$$

$$= \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_1} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

$$+ \frac{1}{n(\sqrt{2}-1)} \sum_{x_j \in X_2} \begin{bmatrix} \sqrt{\frac{(\zeta_B(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_B(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_B(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_B(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_B(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_B(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

Since, $\zeta(x_j), \rho(x_j), \vartheta(x_j) \in [0, 1]$ for all $x_j \in X$. Thus, we get $D(A|C) + D(B|C) - D(A \cup B|C) \geq 0$.

Theorem 2.8. *For three SVNSs A, B and C, we have*

$$D(A \cap B|C) + D(A \cup B|C) = D(A|C) + D(B|C)$$

Proof 2.8. For three SVNSs $A = \langle \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j)|x_j \in X\rangle$, $B = \langle \zeta_B(x_j), \rho_B(x_j), \vartheta_B(x_j)|x_j \in X\rangle$ and $C = \langle \zeta_C(x_j), \rho_C(x_j), \vartheta_C(x_j)|x_j \in X\rangle$ and by definition of the divergence measure, we have

$$D(A \cap B|C) = \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n}$$

$$\begin{bmatrix} \sqrt{\frac{(\zeta_{A\cap B}(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_{A\cap B}(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_{A\cap B}(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_{A\cap B}(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_{A\cap B}(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_{A\cap B}(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

$$= \frac{1}{n(\sqrt{2}-1)} \sum_{j \in X_1} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

$$+ \frac{1}{n(\sqrt{2}-1)} \sum_{j \in X_2} \begin{bmatrix} \sqrt{\frac{(\zeta_B(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_B(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_B(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_B(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_B(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_B(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

Also,

$$D(A \cup B|C) = \frac{1}{n(\sqrt{2}-1)} \sum_{j=1}^{n}$$

$$\begin{bmatrix} \sqrt{\frac{(\zeta_{A\cup B}(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_{A\cup B}(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_{A\cup B}(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_{A\cup B}(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_{A\cup B}(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_{A\cup B}(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

$$= \frac{1}{n(\sqrt{2}-1)} \sum_{j \in X_1} \begin{bmatrix} \sqrt{\frac{(\zeta_B(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_B(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_B(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_B(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_B(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_B(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

$$+ \frac{1}{n(\sqrt{2}-1)} \sum_{j \in X_2} \begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_C(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_C(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_C(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_C(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_C(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_C(x_j)}{2} \end{bmatrix}$$

Thus, by adding these equations, we get $D(A \cap B|C) + D(A \cup B|C) = D(A|C) + D(B|C)$.

Theorem 2.9. *For two SVNSs A and B, we have*

(i) $D(A|B) = D(A^c|B^c)$
(ii) $D(A|B^c) = D(A^c|B)$
(iii) $D(A|B) + D(A^c|B) = D(A^c|B^c) + D(A|B^c)$

Proof 2.9. First and second parts are similar and third part can be proved by adding first and second one. Proof of the part (ii) follows from Theorem 2.4.

Definition 2.5. For two SVNSs $A = \langle \zeta_A(x_j), \rho_A(x_j), \vartheta_A(x_j)|\ x_j \in X\rangle$ and $B = \langle \zeta_B(x_j), \rho_B(x_j), \vartheta_B(x_j)|\ x_j \in X\rangle$, the weighted divergence measure of A against B is to measure the degree of discrimination between the pair is defined as:

$$D_w(A|B) = \frac{1}{(\sqrt{2}-1)} \sum_{j=1}^{n} w_j$$

$$\begin{bmatrix} \sqrt{\frac{(\zeta_A(x_j))^2 + (\zeta_B(x_j))^2}{2}} - \frac{\zeta_A(x_j) + \zeta_B(x_j)}{2} \\ + \sqrt{\frac{(\rho_A(x_j))^2 + (\rho_B(x_j))^2}{2}} - \frac{\rho_A(x_j) + \rho_B(x_j)}{2} \\ + \sqrt{\frac{(\vartheta_A(x_j))^2 + (\vartheta_B(x_j))^2}{2}} - \frac{\vartheta_A(x_j) + \vartheta_B(x_j)}{2} \end{bmatrix} \quad (6)$$

3. TOPSIS approach based on proposed divergence measure for MCDM problems

In this section, a TOPSIS method based on the proposed divergence measure for SVNSs has been presented succeeded by an illustrative example to demonstrate the approach.

3.1. Maximizing divergence method for determine the weights

In this subsection, we construct a nonlinear programming model by maximizing the divergence value $D(w)$ to find the optimal weights of the criteria. For this, let Δ be the set of the partially known weight information. For j^{th} criteria, the divergence of the i^{th} alternative to all the other alternatives can be defined as follows:

$$D_{ij} = \sum_{q=1}^{m} D(A_{ij}|A_{qj})$$

where $D(A_{ij}|A_{qj})$ denotes the divergence measure between the two alternatives A_{ij} and A_{qj} defined in Definition 2.4.

Let $D_j = \sum\limits_{i=1}^{m} D_{ij} = \sum\limits_{i=1}^{m}\sum\limits_{q=1}^{m} D(A_{ij}|A_{qj})$ represents the total divergence values of all the alternatives to other alternatives for j^{th} attributes and hence

$$D(w) = \sum_{j=1}^{n} w_j D_j = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j D(A_{ij}|A_{qj})$$

represents the total divergence values of all the alternatives with respect to all criteria.

Under these, a nonlinear optimization model has been constructed to determine the optimal weight vector as follow:

$$\left.\begin{aligned} &\max D(w) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j D(A_{ij}|A_{qj}) \\ &\text{subject to } w_j \in \Delta,\, w_j \geq 0, \\ &\sum_{j=1}^{n} w_j = 1 \end{aligned}\right\} \quad (7)$$

By solving this model, we get the optimal weight $w = (w_1, w_2, \ldots, w_n)^T$ of the criteria.

On the other hand, if information about the criteria weights are completely unknown, then we establish another nonlinear optimization model as

$$\begin{aligned} &\max D(w) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j D(A_{ij}|A_{qj}) \\ &\text{subject to} \quad w_j \geq 0, \qquad (8) \\ &\sum_{j=1}^{n} w_j^2 = 1 \end{aligned}$$

Solve the above model by using the Lagrangian multiplier method and hence get the normalized weight vector as

$$w_j = \frac{\sum\limits_{i=1}^{m}\sum\limits_{q=1}^{m} D(A_{ij}|A_{qj})}{\sum\limits_{j=1}^{n}\sum\limits_{i=1}^{m}\sum\limits_{q=1}^{m} D(A_{ij}|D_{qj})} \qquad (9)$$

3.2. Proposed TOPSIS approach based on divergence measure

TOPSIS method [15] is a simple and effective tool to solve the decision-making problems which aims to pick out the best alternative(s) with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). In this section, instead of using the distance measures to figure the relative closeness coefficient, the concept of divergence measure is appropriately applied to the main structure of the TOPSIS method. For it, assume that there are 'm' alternatives denoted by $A_1, A_2, \ldots, A_m$ which are being evaluated with respect to 'n' criteria namely, $C_1, C_2, \ldots, C_n$ by an expert. The preferences related to each alternative $A_i (i = 1, 2, \ldots, m)$ are represented under the SVNS environment which are denoted by $\alpha_{ij} = \langle \zeta_{ij}, \rho_{ij}, \vartheta_{ij} \rangle$, $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$, where $\zeta_{ij}, \rho_{ij}, \vartheta_{ij}$ represents the degree of satisfaction, indeterminacy, and dissatisfaction, respectively, of A_i corresponding to criteria C_j such that $0 \leq \zeta_{ij}, \rho_{ij}, \vartheta_{ij} \leq 1$ and $\zeta_{ij} + \rho_{ij} + \vartheta_{ij} \leq 3$. Then, the following steps of the TOPSIS method have been summarized for solving the decision-making problems under SVNN information by using the proposed measures as:

Step 1: *Arrange the collective information:* The information related to each alternative $A_i (i = 1, 2, \dots, m)$ with respect to the different criteria $C_j (j = 1, 2, \dots, n)$ are arranged in the form of the neutrosophic decision-matrix D which is represented as

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \dots & \alpha_{mn} \end{pmatrix} \end{array}$$

Step 2: *Normalize the decision matrix:* Since there are two types in criteria, one is benefit type and the contrary is cost type. Therefore, to balance the physical dimensions and eliminate the influence of the criterion types, the matrix $D = (\alpha_{ij})_{m \times n}$ is converted to standard matrix $R = (r_{ij})_{m \times n}$ by transforming the rating values of cost type into benefit type criteria, if any, by using the normalization formula:

$$r_{ij} = \begin{cases} \langle \zeta_{ij}, \rho_{ij}, \vartheta_{ij} \rangle \text{ ; for benefit type criteria} \\ \langle \vartheta_{ij}, \rho_{ij}, \zeta_{ij} \rangle \text{ ; for cost type criteria} \end{cases}$$

Step 3: *Determine the attribute weights:* The weights of the different criteria $w = (w_1, w_2, \dots, w_n)^T$ are determined by solving the optimization model 7 or 8 accordingly whether the information about the criteria weights are partially known and completely unknown.

Step 4: *Determine the discrimination of each alternative from ideal and anti-ideal alternatives:* As $0 \leq \zeta_{ij}, \rho_{ij}, \vartheta_{ij} \leq 1$ and hence all rating values of the alternative as given in decision matrix $R = (r_{ij})_{m \times n}$ are SVNNs. Therefore, accordingly the membership degrees of relative positive ideal ideal solution (RPIS) may be expressed as $A^+ = \langle 1, 0, 0 \rangle_{1 \times n}$. Similarly, the membership degrees of the relative negative ideal solution (RNIS) may be summarized as $A^- = \langle 0, 1, 1 \rangle_{1 \times n}$. From these, it has been seen that A^+ and A^- are complement to each other. Furthermore, instead of fixing the degree of A^+ to be 1, 0 and 0, the decision maker may varying it by defining A^+ and A^- respectively as $\langle \zeta_j^+, \rho_j^+, \vartheta_j^+ \rangle)_{1 \times n}$, and $\langle \zeta_j^-, \rho_j^-, \vartheta_j^- \rangle)_{1 \times n}$ where $\zeta_j^+ = \max_i \{\zeta_{ij} | i = 1, 2, \dots, m\}$, $\rho_j^+ = \min_i \{\rho_{ij} | i = 1, 2, \dots, m\}$, $\vartheta_j^+ = \min_i \{\vartheta_{ij} | i = 1, 2, \dots, m\}$, $\zeta_j^- = \min_i \{\zeta_{ij} | i = 1, 2, \dots, m\}$, $\rho_j^- = \max_i \{\rho_{ij} | i = 1, 2, \dots, m\}$, $\vartheta_j^- = \max_i \{c_{ij} | i = 1, 2, \dots, m\}$. From this, it has been clearly seen that $(\langle \zeta_j^-, \rho_j^-, \vartheta_j^- \rangle) \subseteq (\langle \zeta_{ij}, \rho_{ij}, \vartheta_{ij} \rangle) \subseteq (\langle \zeta_j^+, \rho_j^+, \vartheta_j^+ \rangle)$. In order to compare the different alternatives, the divergence measure defined in Equation (6) is used to measures the degree of discrimination between an alternative A_i and the RPIS A^+ as well as the RNIS A^- as follows:

$$D_w(A_i|A^+) = \frac{1}{(\sqrt{2}-1)} \sum_{j=1}^{n} w_j \left[\begin{array}{l} \sqrt{\frac{(\zeta_{ij}(r_{ij}))^2 + (\zeta_j^+(r_{ij}))^2}{2}} - \frac{\zeta_{ij}(r_{ij}) + \zeta_j^+(r_{ij})}{2} \\ + \sqrt{\frac{(\rho_{ij}(r_{ij}))^2 + (\rho_j^+(r_{ij}))^2}{2}} - \frac{\rho_{ij}(r_{ij}) + \rho_j^+(r_{ij})}{2} \\ + \sqrt{\frac{(\vartheta_{ij}(r_{ij}))^2 + (\vartheta_j^+(r_{ij}))^2}{2}} - \frac{\vartheta_{ij}(r_{ij}) + \vartheta_j^+(r_{ij})}{2} \end{array} \right] \quad (10)$$

and

$$D_w(A_i|A^-) = \frac{1}{(\sqrt{2}-1)} \sum_{j=1}^{n} w_j \left[\begin{array}{l} \sqrt{\frac{(\zeta_{ij}(r_{ij}))^2 + (\zeta_j^-(r_{ij}))^2}{2}} - \frac{\zeta_{ij}(r_{ij}) + \zeta_j^-(r_{ij})}{2} \\ + \sqrt{\frac{(\rho_{ij}(r_{ij}))^2 + (\rho_j^-(r_{ij}))^2}{2}} - \frac{\rho_{ij}(r_{ij}) + \rho_j^-(r_{ij})}{2} \\ + \sqrt{\frac{(\vartheta_{ij}(r_{ij}))^2 + (\vartheta_j^-(r_{ij}))^2}{2}} - \frac{\vartheta_{ij}(r_{ij}) + \vartheta_j^-(r_{ij})}{2} \end{array} \right] \quad (11)$$

Step 5: *Compute the relative-closeness coefficient:* Based on Equations (10) and (11), the relative-closeness coefficient of the alternative $A_i (i = 1, 2, \dots, m)$ with respect to A^+ and A^- is defined as follows:

$$R_i = \frac{D_w(A_i|A^-)}{D_w(A_i|A^-) + D_w(A_i|A^+)} \quad (12)$$

provided $D_w(A_i|A^+) \neq 0$. It has been seen that $0 \leq D(A_i|A^-) \leq D(A_i|A^-) + D(A_i|A^+)$ and hence $0 \leq R_i \leq 1$.

1. *Rank the alternative:* Based on the descending order of the values of R_i, we rank the alternatives $A_i (i = 1, 2, \ldots, m)$ and select the best alternative(s).

3.3. Illustrative Example

A travel agency naming, Marricot Trip mate , has excelled in providing travel related services to domestic and Inbound tourists . Agency wants to provide more facilities like detailed information, online booking capabilities, allow to book and sell airline tickets, car rentals, hotels, and other travel related services etc. to their customers. For this purpose, agency intends to find an appropriate information technology (IT) software development company that delivers affordable solutions through software development. To complete this motive, agency forms a set of five companies (alternatives), namely, Zensar Tech (A_1), NIIT Tech (A_2), HCL Tech(A_3), Hexaware Tech(A_4), and Tech Mahindra (A_5) and the selection is held on the basis of the different criteria, namely, Technology Expertise (C_1), Service quality (C_2), Project Management (C_3), Industry Experience (C_4). Now, we can obtain the evaluation of an alternative $A_i (i = 1, 2, 3, 4, 5)$ with respect to the criterion $C_j (j = 1, 2, 3, 4)$ from the questionnaire of a domain expert. For instance, corresponding to alternative A_1 under criterion C_1, when we ask the opinion of an expert about the alternative A_1 with respect to the criterion C_1, he or she may that the "possibility degree in which the statement is good" is 0.5, "the statement is false" is 0.4 and "the degree in which he or she is unsure" is 0.3. In this case, the evaluation of this alternative is represented as SVNN $\alpha_{11} = \langle 0.5, 0.3, 0.4 \rangle$. In the similar manner, we can obtain all the evaluations of the alternatives A_i with respect to criterion C_j. Then, the following steps of the proposed approach have been executed to find the most desirable alternative(s).

Step 1: The complete rating values of all the alternatives under single-valued neutrosophic information under above four criteria are summarized are listed in the following single-valued neutrosophic decision matrix $D = (\alpha_{ij})$ as

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad & \quad C_4 \quad \end{array} \\ \begin{pmatrix} \langle 0.5, 0.3, 0.4 \rangle & \langle 0.5, 0.2, 0.5 \rangle & \langle 0.2, 0.2, 0.6 \rangle & \langle 0.3, 0.2, 0.4 \rangle \\ \langle 0.7, 0.1, 0.3 \rangle & \langle 0.7, 0.2, 0.3 \rangle & \langle 0.6, 0.3, 0.2 \rangle & \langle 0.6, 0.4, 0.2 \rangle \\ \langle 0.5, 0.3, 0.4 \rangle & \langle 0.6, 0.2, 0.4 \rangle & \langle 0.6, 0.1, 0.2 \rangle & \langle 0.5, 0.1, 0.3 \rangle \\ \langle 0.7, 0.3, 0.2 \rangle & \langle 0.7, 0.2, 0.2 \rangle & \langle 0.4, 0.5, 0.2 \rangle & \langle 0.5, 0.2, 0.2 \rangle \\ \langle 0.4, 0.1, 0.3 \rangle & \langle 0.5, 0.1, 0.2 \rangle & \langle 0.4, 0.1, 0.5 \rangle & \langle 0.4, 0.3, 0.6 \rangle \end{pmatrix} \end{array}$$

Step 2: Since all the criteria are of the benefit types, so there is no need of normalizing process.

Step 3: Consider the partial information about the criterion weights given by $\Delta = \{0.10 \leq w_1 \leq 0.2, 0.2 \leq w_2 \leq 0.3, 0.2 \leq w_3 \leq 0.25, 0.15 \leq w_4 \leq 0.25, w_j \geq 0, \sum_{j=1}^{4} w_j = 1\}$ and hence the optimization model (7) is constructed as follows:

$$\begin{aligned} \text{maximize } D(w) &= 0.3344w_1 + 0.2372w_2 \\ &\quad + 0.7670w_3 + 0.4519w_4 \\ \text{subject to} \quad & w \in \Delta \end{aligned}$$

By solving this model, we get the optimal weight vector of criteria $w = (0.2, 0.3, 0.25, 0.25)^T$.

Step 4: The RPIS and RNIS is calculated from the given information as $A^+ = \{(C_1, \langle 0.7, 0.1, 0.2 \rangle), (C_2, \langle 0.7, 0.1, 0.2 \rangle), (C_3, \langle 0.6, 0.1, 0.2 \rangle), (C_4, \langle 0.6, 0.1, 0.2 \rangle)\}$ and $A^- = \{(C_1, \langle 0.4, 0.3, 0.4 \rangle), (C_2, \langle 0.5, 0.2, 0.5 \rangle), (C_3, \langle 0.2, 0.5, 0.6 \rangle), (C_4, \langle 0.3, 0.4, 0.6 \rangle)\}$. Thus, the degree of discrimination between the alternative $A_i (i = 1, 2, 3, 4, 5)$ to A^+ and A^- are computed by using Equations (10) and (11) and get as

$$
\begin{aligned}
&D_w(A_1|A^+)\\
&=\frac{1}{(\sqrt{2}-1)}\left[\begin{array}{l}
0.2\left\{\sqrt{\frac{0.5^2+0.7^2}{2}}-\frac{0.5+0.7}{2}\right\}+0.3\left\{\sqrt{\frac{0.5^2+0.7^2}{2}}-\frac{0.5+0.7}{2}\right\}\\
+0.25\left\{\sqrt{\frac{0.2^2+0.6^2}{2}}-\frac{0.2+0.6}{2}\right\}+0.25\left\{\sqrt{\frac{0.3^2+0.6^2}{2}}-\frac{0.3+0.6}{2}\right\}\\
+0.2\left\{\sqrt{\frac{0.3^2+0.1^2}{2}}-\frac{0.3+0.1}{2}\right\}+0.3\left\{\sqrt{\frac{0.2^2+0.1^2}{2}}-\frac{0.2+0.1}{2}\right\}\\
+0.25\left\{\sqrt{\frac{0.2^2+0.1^2}{2}}-\frac{0.2+0.1}{2}\right\}+0.25\left\{\sqrt{\frac{0.2^2+0.1^2}{2}}-\frac{0.2+0.1}{2}\right\}\\
+0.2\left\{\sqrt{\frac{0.4^2+0.2^2}{2}}-\frac{0.4+0.2}{2}\right\}+0.3\left\{\sqrt{\frac{0.5^2+0.2^2}{2}}-\frac{0.5+0.2}{2}\right\}\\
+0.25\left\{\sqrt{\frac{0.6^2+0.2^2}{2}}-\frac{0.6+0.2}{2}\right\}+0.25\left\{\sqrt{\frac{0.4^2+0.2^2}{2}}-\frac{0.4+0.2}{2}\right\}
\end{array}\right]\\
&=0.1487
\end{aligned}
$$

and

$$
\begin{aligned}
&D_w(A_1|A^-)\\
&=\frac{1}{(\sqrt{2}-1)}\left[\begin{array}{l}
0.2\left\{\sqrt{\frac{0.5^2+0.4^2}{2}}-\frac{0.5+0.4}{2}\right\}+0.3\left\{\sqrt{\frac{0.5^2+0.5^2}{2}}-\frac{0.5+0.5}{2}\right\}\\
+0.25\left\{\sqrt{\frac{0.2^2+0.2^2}{2}}-\frac{0.2+0.2}{2}\right\}+0.25\left\{\sqrt{\frac{0.3^2+0.3^2}{2}}-\frac{0.3+0.3}{2}\right\}\\
+0.2\left\{\sqrt{\frac{0.3^2+0.3^2}{2}}-\frac{0.3+0.3}{2}\right\}+0.3\left\{\sqrt{\frac{0.2^2+0.2^2}{2}}-\frac{0.2+0.2}{2}\right\}\\
+0.25\left\{\sqrt{\frac{0.2^2+0.5^2}{2}}-\frac{0.2+0.5}{2}\right\}+0.25\left\{\sqrt{\frac{0.2^2+0.4^2}{2}}-\frac{0.2+0.4}{2}\right\}\\
+0.2\left\{\sqrt{\frac{0.4^2+0.4^2}{2}}-\frac{0.4+0.4}{2}\right\}+0.3\left\{\sqrt{\frac{0.5^2+0.5^2}{2}}-\frac{0.5+0.5}{2}\right\}\\
+0.25\left\{\sqrt{\frac{0.6^2+0.6^2}{2}}-\frac{0.6+0.6}{2}\right\}+0.25\left\{\sqrt{\frac{0.4^2+0.6^2}{2}}-\frac{0.4+0.6}{2}\right\}
\end{array}\right]\\
&=0.0357
\end{aligned}
$$

Similarly, we can calculate the others and get $D_w(A_2|A^+) = 0.0512$; $D_w(A_2|A^-) = 0.1453$; $D_w(A_3|A^+) = 0.0466$; $D_w(A_3|A^-) = 0.1457$; $D_w(A_4|A^+) = 0.0661$; $D_w(A_4|A^-) = 0.1298$; $D_w(A_5|A^+) = 0.0914$ and $D_w(A_5|A^-) = 0.0933$.

Step 5: The closeness coefficients of i^{th} alternative $A_i (i = 1, 2, 3, 4, 5)$ is calculated by using Equation (12) and are given as $R_1 = 0.1936$; $R_2 = 0.7396$; $R_3 = 0.7577$; $R_4 = 0.6628$; and $R_5 = 0.5052$.

Step 6: Therefore, the optimal ranking of these five alternatives are $A_3 \succ A_2 \succ A_4 \succ A_5 \succ A_1$, and thus, the best alternative is A_3 namely HCL Tech.

3.4. Test Criteria for proposed approach

Some decision-making methods deals with irregularities which may lead to give some undesirable results when some of the information in the given decision matrix is changed. Therefore, their approach is not acceptable to rank the alternatives and hence a validity of the newly proposed methods is necessary. [37] established the following testing criteria to evaluate the validity of such types of methods.

Test criterion 1: "An effective decision-making method should not change the indication of the best alternative on replacing a non-optimal alternative by another worse alternative without changing the relative importance of each decision criteria."

Test criterion 2: "An effective decision-making method should follow transitive property."

Test criterion 3: "When a decision-making problem is decomposed into smaller problems and same method is applied on smaller problems to rank the alternatives, combined ranking of the alternatives should be identical to the original ranking of undecomposed problem."

The validity of the proposed TOPIS method is tested using these test criteria.

3.4.1. Validity test under test criterion 1

Under this test criterion, the rating values of the non-optimal alternative A_1 and the worse alternative A_5 is replaced with the another ones and hence their updated decision matrix is summarized as

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad & \quad C_4 \quad \end{array} \\ \begin{pmatrix} \langle 0.4, 0.3, 0.5\rangle & \langle 0.5, 0.2, 0.5\rangle & \langle 0.6, 0.2, 0.2\rangle & \langle 0.4, 0.2, 0.3\rangle \\ \langle 0.7, 0.1, 0.3\rangle & \langle 0.7, 0.2, 0.3\rangle & \langle 0.6, 0.3, 0.2\rangle & \langle 0.6, 0.4, 0.2\rangle \\ \langle 0.5, 0.3, 0.4\rangle & \langle 0.6, 0.2, 0.4\rangle & \langle 0.6, 0.1, 0.2\rangle & \langle 0.5, 0.1, 0.3\rangle \\ \langle 0.7, 0.3, 0.2\rangle & \langle 0.7, 0.2, 0.2\rangle & \langle 0.4, 0.5, 0.2\rangle & \langle 0.5, 0.2, 0.2\rangle \\ \langle 0.3, 0.1, 0.4\rangle & \langle 0.2, 0.1, 0.5\rangle & \langle 0.5, 0.1, 0.4\rangle & \langle 0.6, 0.3, 0.4\rangle \end{pmatrix} \end{array}$$

Based on this information, by applying the proposed approach, we compute the divergence measures of each alternative $A_i (i = 1, 2, \ldots, 5)$ to RPIS (A^+) and RNIS (A^-) are $D_w(A_1|A^+) = 0.0889$, $D_w(A_1|A^-) = 0.0703$, $D_w(A_2|A^+) = 0.0512$, $D_w(A_2|A^-) = 0.1307$, $D_w(A_3|A^+) = 0.0466$, $D_w(A_3|A^-) = 0.1247$, $D_w(A_4 \;|A^+) = 0.0661$, $D_w(A_4|A^-) = 0.1337$, $D_w(A_5|A^+) = 0.1309$ and $D_w(A_5|A^-) = 0.0650$. Thus, the optimal values of the relative closeness coefficient of each alternative are computed by using Equation (12) as $R_1 = 0.4416$; $R_2 = 0.7187$; $R_3 = 0.7279$ $R_4 = 0.6693$ and $R_5 = 0.3317$. According to the descending order of these values, the alternatives are ranked as $A_3 \succ A_2 \succ A_4 \succ A_1 \succ A_5$. Thus, the best alternative is remain unchanged i.e., A_3. Hence the proposed method is valid under *test criterion 1*.

3.4.2. Validity test using test criterion 2 and 3

Under it, if we decomposed the original problem into a set of $\{A_1, A_2, A_4\}$, $\{A_1, A_3, A_5\}$, $\{A_2, A_4, A_5\}$, $\{A_2, A_3, A_5\}$ and $\{A_1, A_3, A_4\}$, then the proposed approach has been applied to these subproblems. The ranking orders of the alternatives is $A_2 \succ A_4 \succ A_1$, $A_3 \succ A_5 \succ A_1$, $A_2 \succ A_4 \succ A_5$, $A_3 \succ A_2 \succ A_5$ and $A_3 \succ A_4 \succ A_1$ respectively and hence the combined ranking order is $A_3 \succ A_2 \succ A_4 \succ A_5 \succ A_1$, which is identical to original problem. Therefore, it displays transitive property and hence the proposed method is valid under the *test criterion 2 and 3*.

3.5. Superiority of the proposed approach over the existing approaches

In this section, we have presented some counterexamples to show that the existing approaches [20, 23, 26] under SVNS environment fails to rank the given alternatives while the proposed divergence measure can overcome their shortcoming.

Example 3.1. Consider a decision-making problem in which there are two alternatives, namely, A_1 and A_2 which are evaluated by a decision maker under the three different criteria denoted by C_1, C_2 and C_3. The preference values of each alternative given by the decision maker are summarized under SVNS environment as follows:

$$K = \begin{array}{c c} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{pmatrix} \langle 0.5, 0.2, 0.3\rangle & \langle 0.6, 0.3, 0.1\rangle & \langle 0.3, 0.2, 0.4\rangle \\ \langle 0.4, 0.3, 0.2\rangle & \langle 0.6, 0.2, 0.2\rangle & \langle 0.4, 0.3, 0.3\rangle \end{pmatrix} \end{array} \tag{13}$$

In order to find the best alternative, we utilize the existing TOPSIS approach as proposed by [36]. For it, the following steps of their approach are followed as

- Step 1: The information of each alternative in SVNS environment is represented in the form of decision matrix as given in Equation (13).
- Step 2: The positive and negative ideal solutions are obtained as $A^+ = \{\langle 0.5, 0.2, 0.2\rangle, \langle 0.6, 0.2, 0.1\rangle, \langle 0.4, 0.2, 0.3\rangle\}$ and $A^- = \{\langle 0.4, 0.3, 0.3\rangle, \langle 0.6, 0.3, 0.2\rangle, \langle 0.3, 0.3, 0.4\rangle\}$, respectively.
- Step 3: Based on these values, the hamming distance measures between the alternatives $A_i (i = 1, 2)$ and ideals solutions are evaluted as $d(A_1, A^+) = 0.0444$, $d(A_1, A^-) = 0.0444$, $d(A_2, A^+) = 0.0444$ and $d(A_2, A^-) = 0.0444$.
- Step 4: Thus, the optimal values of the relative closeness coefficient $R_i = \frac{d(A_i, A^-)}{d(A_i, A^+)+d(A_i, A^-)}$ of each alternative $A_i (i = 1, 2)$ are computed as $R_1 = 0.5$ and $R_2 = 0.5$. Hence, this method is unable to rank the given alternatives.

On the other hand, if we utilize the proposed divergence measure to compute the discrimination between the alternatives and the ideal solutions then their measurement values are $D(A_1|A^+) = 0.0137$, $D(A_2|A^+) = 0.0167$, $D(A_1|A^-) = 0.0167$ and $D(A_2|A^-) = 0.0137$. Thus, the relative closeness coefficient $R_i (i = 1, 2)$ of the alternative A_i $(i = 1, 2)$ is computed by using Equation (12) as $R_1 = 0.5500$ and $R_2 = 0.4500$. Since $R_1 > R_2$ which gives us that A_1 is better alternative than A_2. Therefore, proposed divergence based TOPSIS approach is able to rank the alternatives in the situation when existing approach fails.

Example 3.2. Consider two set of SVNSs A_1 and A_2 defined over the universal set $X = \{x_1, x_2, \ldots, x_5\}$ as

$$A_1 = \left\{ \begin{array}{l} \langle x_1, 0.5, 0.3, 0.2\rangle, \langle x_2, 0.5, 0.2, 0.3\rangle, \langle x_3, 0.9, \\ 0.0, 0.1\rangle, \langle x_4, 0.5, 0.4, 0.1\rangle, \langle x_5, 0.7, 0.1, 0.2\rangle \end{array} \right\}$$

and

$$A_2 = \left\{ \begin{array}{l} \langle x_1, 0.7, 0.2, 0.1\rangle, \langle x_2, 0.5, 0.4, 0.1\rangle, \langle x_3, 0.9, \\ 0.1, 0.0\rangle, \langle x_4, 0.6, 0.3, 0.1\rangle, \langle x_5, 0.8, 0.0, 0.2\rangle \end{array} \right\}$$

Then the aim of this problem is to classify the unknown pattern $B \in \text{SVNS}(X)$ which is defined as

$$B = \left\{ \begin{array}{l} \langle x_1, 0.7, 0.1, 0.2\rangle, \langle x_2, 0.6, 0.3, 0.1\rangle, \langle x_3, 0.7, \\ 0.1, 0.2\rangle, \langle x_4, 0.5, 0.4, 0.1\rangle, \langle x_5, 0.4, 0.5, 0.1\rangle \end{array} \right\}$$

in one of the class of A_1 and A_2. For it, if we apply the existing normalized Hamming (d_N) and Euclidean (q_N) distances [20] as defined in Equations (14) and (15) respectively, between the alternatives $A_i (i = 1, 2)$ and the unknown pattern B

$$d_N(A_i, B) = \frac{1}{3n} \sum_{j=1}^{n} \left\{ \begin{array}{l} |\zeta_{A_i}(x_j) - \zeta_B(x_j)| \\ + |\rho_{A_i}(x_j) - \rho_B(x_j)| \\ + |\vartheta_{A_i}(x_j) - \vartheta_B(x_j)| \end{array} \right\} \tag{14}$$

and

$$q_N(A_i, B) = \sqrt{\frac{1}{3n} \sum_{j=1}^{n} \left\{ \begin{array}{l} \left(\zeta_{A_i}(x_j) - \zeta_B(x_j)\right)^2 \\ + \left(\rho_{A_i}(x_j) - \rho_B(x_j)\right)^2 \\ + \left(\vartheta_{A_i}(x_j) - \vartheta_B(x_j)\right)^2 \end{array} \right\}} \tag{15}$$

then, we get the measurement values corresponding to the set A_1 and A_2 as $d_N(A_1, B) =$

$d_N(A_2, B) = 0.1333$ and $q_N(A_1, B) = q_N(A_2, B) = 0.1751$. Thus, this existing approach is unable to classify the pattern B with any one of the class of A_1 and A_2. On the other hand, if we apply the proposed divergence measure between the alternative $A_i (i = 1, 2)$ and B, then we get their values as $D(A_1|B) = 0.0901$ and $D(A_2|B) = 0.1061$. Hence, we conclude that unknown pattern B is most likely to belong to the class of A_2. Therefore, the proposed divergence measure is successfully work in those cases where the existing measure fails.

Example 3.3. Consider two SVNSs $A_1 = \{\langle x_1, 0.6, 0.1, 0.2\rangle, \langle x_2, 0.5, 0.1, 0.3\rangle, \langle x_3, 0.4, 0.1, 0.1\rangle\}$ and $A_2 = \{\langle x_1, 0.5, 0.1 ,0.2\rangle, \langle x_2, 0.4, 0.1, 0.3\rangle, \langle x_3, 0.4, 0.1, 0.1\rangle\}$ defined over the universal set $X = \{x_1, x_2, x_3\}$. Let the ideal solution considered by decision maker is $B = \{\langle x_1, 0.6, 0.1, 0.2\rangle, \langle x_2, 0.4, 0.1, 0.3\rangle, \langle x_3, 0.5, 0.1, 0.1\rangle\}$. In order to rank these alternatives, we apply the existing improved cosine similarity measures [23], defined in Equations (16) and (17), as

$$SC_1(A_i, B) = \frac{1}{n}\sum_{j=1}^{n} \cos\left[\frac{\pi}{2}\max\begin{pmatrix} |\zeta_{A_i}(x_j) - \zeta_B(x_j)|, \\ |\rho_{A_i}(x_j) - \rho_B(x_j)|, \\ |\vartheta_{A_i}(x_j) - \vartheta_B(x_j)| \end{pmatrix}\right] \quad (16)$$

and

$$SC_2(A_i, B) = \frac{1}{n}\sum_{j=1}^{n} \cos\left[\frac{\pi}{6}\begin{pmatrix} |\zeta_{A_i}(x_j) - \zeta_B(x_j)| \\ + |\rho_{A_i}(x_j) - \rho_B(x_j)| \\ + |\vartheta_{A_i}(x_j) - \vartheta_B(x_j)| \end{pmatrix}\right] \quad (17)$$

and hence obtain their corresponding measurement values are $SC_1(A_1, B) = SC_1(A_2, B) = 0.3292$ and $SC_2(A_1, B) = SC_2(A_2, B) = 0.3315$. Thus, their approach is unable to rank the alternatives. On the other hand, if we compute the proposed divergence measurement values for these two alternatives, then we get their respective values are $D(A_1, B) = 0.0045$ and $D(A_2, B) = 0.0041$ and hence conclude that A_1 is the best alternative than A_3.

Hence, we can say that the proposed divergence measures, as well as their corresponding TOPSIS approach, are able to solve the decision-making problem in a better way under SVNS environment where the existing studies fail to rank the alternatives.

4. Conclusion

In this manuscript, the theory of SVNS has been enriched by proposing a new information measure, called as divergence measure, to evaluate the discrimination between the two SVNSs. Some properties and the correlations of the measure have been investigated in detail. A maximizing divergence method has been presented to determine the optimal criterion weights under SVN environment. Then, a single-valued neutrosophic TOPSIS is proposed to solve the decision-making problems. A practical example is given to validate its effectiveness and practicality. From the study, we resolve that the proposed measure shows its superiority in those cases also where the existing measures fail to classify the objects. In the future, we will extend the proposed approach to the Pythagorean fuzzy set environment [38–40] and uncertain and fuzzy environment [41–43].

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