A Study of Distance Measures for Interval Neutrosophic Sets with Numerical Example

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Abstract— In this paper, we introduce an inclusion measure for interval neutrosophic sets, which is one of information measures of interval neutrosophic theory. Using the concept of inclusion measure based on various distance measure, we develop a simple inclusion measure for ranking the interval neutrosophic sets. Though having a simple measure for calculation, the inclusion measure presents a new approach for handling the interval neutrosophic information.

Key words: Single Valued Neutrosophic Set (SVNS), Interval Neutrosophic Set (INS), Normalized Hamming Distance, Hamming Distance, Normalized Euclidean Distance, Hausdorff Distance Measure, Normalized Geometric Distance, Euclidean Distance, Geometric Distance

I. INTRODUCTION

The concept of the neutrosophic set developed by Smarandache [12] is a set model which generalizes the classic set, fuzzy set [21], interval fuzzy set [14] intuitionistic fuzzy set [1] and interval valued intuitionistic fuzzy set [2]. In contrast to intuitionistic fuzzy sets and also interval valued intuitionistic fuzzy sets, indeterminacy degree of an element in a universe of discourse is expressed explicitly in the neutrosophic set. There are three membership functions such that truth membership, indeterminacy membership and falsity membership in a neutrosophic set, and they are independent. However, the neutrosophic set generalizes the above mentioned sets from philosophical point of view and its functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $[0,1]$ and are defined by $T_A(x) : X \rightarrow [0,1], I_A(x) : X \rightarrow [0,1]$ and $F_A(x) : X \rightarrow [0,1]$. That is, its components $T_A(x), I_A(x), F_A(x)$ are non-standard subsets included in the unitary nonstandard interval $[0,1]$ or standard subsets included in the unitary standard interval $[0,1]$ as in the intuitionistic fuzzy set. Furthermore, the connectors in the intuitionistic fuzzy set are only defined by $T(x)$ and $F(x)$ (i.e. truth-membership and falsity-membership), hence the indeterminacy $I(x)$ is what is left from 1, while in the neutrosophic set, they can be defined by any of them (no restriction) [12]. However, the neutrosophic set is to be difficult to use in real scientific or engineering applications. So Wang et al. [5], [6] defined the concepts of single valued neutrosophic set (SVNS) and interval neutrosophic set (INS) which is an instance of a neutrosophic set. At present, studies on the SVNSs and INSs are progressing rapidly in many different aspects [4], [9], [13], [17], [18], [19]. Recently, Şahin and Küçük [13] proposed the subsethood (inclusion) measure for single valued neutrosophic sets and applied it to a multi criteria decision making problem with information of single valued neutrosophic sets.

Fuzzy entropy, distance measure and similarity measure are three basic concepts used in fuzzy sets theory. Usually subsethood measures are constructed using implication operators, t-norms or t-conorms, entropy measures or cardinalities. In classical theory, it is said that a set $A$ is a subset of $B$ and is denoted by $A \subseteq B$ if every element of $A$ is an element of $B$, whenever $X$ is a universal set and $A$, $B$ are two sets in $X$. Therefore, inclusion measure should be two valued for crisp sets. That is, either $A$ is precisely subset of $B$ or vice versa. But since an element $x$ in universal set $X$ can belong to a fuzzy set $A$ to varying degrees, it is notable to consider situations describing as being "more and less" a subset of another set and to measure the degree of this inclusion. Fuzzy inclusion allows a given fuzzy set to contain another to some degree between 0 and 1. According to Zadeh’s fuzzy set containment, a fuzzy set $B$ contains a fuzzy set $A$ if $m_A(x) \leq m_B(x)$, for all $x$ in $X$, in which $m_A(x)$ and $m_B(x)$ are the membership functions of $A$ and $B$, respectively.

In this paper, we firstly review the systems of axioms of Young’s fuzzy inclusion measure. Then we extend the inclusion measure of single valued neutrosophic sets to interval neutrosophic environment and give a new system of axioms for inclusion measure of interval neutrosophic sets. Moreover, we utilize the neutrosophic inclusion measure to rank the interval neutrosophic sets. To demonstrate the effectiveness of the proposed inclusion measure, we consider a multi attribute decision-making problem.

A. Preliminaries

In the following we give a brief review of some preliminaries.

1) Single valued neutrosophic sets

A single valued neutrosophic set has been defined in [5] as follows:

a) Definition 1.1 [5]

Let $X$ be a universe of discourse. A single valued neutrosophic set $A$ over $X$ is an object having the form

$$A = \{(x, u_A(x), p_A(x), v_A(x)) : x \in X\},$$

Where

$$u_A(x) : X \rightarrow [0,1], p_A(x) : X \rightarrow [0,1]$$

and

$$v_A(x) : X \rightarrow [0,1]$$

with

$$0 \leq u_A(x) + p_A(x) + v_A(x) \leq 3$$

for all $x \in X$. All rights reserved by www.ijsrd.com 924
The values $u_A(x)$, $p_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of $x$ to $A$, respectively.

**B. Interval neutrosophic sets**

An interval neutrosophic set is a model of a neutrosophic set, which can be used to handle uncertainty in fields of scientific, environment and engineering. We introduce the definition of an interval neutrosophic set as follows.

1) **Definition 1.2** [6]

Let $X$ be a space of points (objects) and $\text{Int}[0,1]$ be the set of all closed subsets of $[0,1]$. An interval neutrosophic $A$ in $X$ is defined with the form

$$A = \{ (x, u_A(x), p_A(x), v_A(x)) : x \in X \}$$

Where $u_A(x) : X \rightarrow \text{Int}[0,1]$, $p_A(x) : X \rightarrow \text{Int}[0,1]$ and $v_A(x) : X \rightarrow \text{Int}[0,1]$ with $0 \leq \sup u_A(x) + \sup p_A(x) + \sup v_A(x) \leq 3$ for all $x \in X$.

The intervals $u_A(x)$, $p_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of $x$ to $A$, respectively.

For convenience, if let

$$u_A(x) = [u_A^L(x), u_A^U(x)]$$

$$p_A(x) = [p_A^L(x), p_A^U(x)]$$

$$v_A(x) = [v_A^L(x), v_A^U(x)]$$

then

$$A = \{ (x, u_A(x), u_A(x), [p_A(x), p_A(x)], [v_A(x), v_A(x)]) : x \in X \}$$

with the condition,

$$0 \leq \sup u_A^U(x) + \sup p_A^U(x) + \sup v_A^U(x) \leq 3 \text{ for all } x \in X$$

Here, we only take the sub-unitary interval of $[0,1]$.

Therefore, an interval neutrosophic set is clearly neutrosophic set.

2) **Definition 1.3** [6]

Let $\text{INS}(X)$ denote the family of all the interval neutrosophic sets in universe $X$, assume $A, B \in \text{INS}(X)$ such that

$$A = \{ (x, u_A(x), u_A(x), [p_A(x), p_A(x)], [v_A(x), v_A(x)]) : x \in X \}$$

$$B = \{ (x, u_B(x), u_B(x), [p_B(x), p_B(x)], [v_B(x), v_B(x)]) : x \in X \}$$

then some operations can be defined as follows:

1. **Definition 2.1** [23]

   **I. The Hamming distance measure**

   $$d_H(A, B) = \frac{1}{6} \sum_{x \in X} [u_A^L(x) - u_B^L(x)]^2 + [p_A^L(x) - p_B^L(x)]^2 + [v_A^L(x) - v_B^L(x)]^2$$

   $$+ [u_A^U(x) - u_B^U(x)]^2 + [p_A^U(x) - p_B^U(x)]^2 + [v_A^U(x) - v_B^U(x)]^2$$

   **II. The Euclidean distance measure**

   $$d_E(A, B) = \frac{1}{6} \sum_{x \in X} [u_A(x) - u_B(x)]^2 + [p_A(x) - p_B(x)]^2 + [v_A(x) - v_B(x)]^2$$

   $$+ [u_A(x) - u_B(x)]^2 + [p_A(x) - p_B(x)]^2 + [v_A(x) - v_B(x)]^2$$

   $$\leq \text{INS} \subseteq X$$. 

**A. Sets with Numerical Example**

**B. Distance measures for interval neutrosophic set**

Distance measure is a term that describes the difference between interval neutrosophic sets and can be considered as a dual concept of inclusion measure. We make use of the various distance measures proposed in [23, 24, 27, 28, 29] between interval neutrosophic sets, which were partly based on the geometric interpretation of interval neutrosophic sets, and have some good geometric properties.
Inclusion measures based on the distance measure

The Hausdorff distance measure

\[ d_H(A, B) = \frac{1}{4} \sum_{i=1}^{n} \left( \max \left\{ p_v^L (x_i) - p_v^L (x_j), p_v^U (x_i) - p_v^U (x_j) \right\} + \max \left\{ p_u^L (x_i) - p_u^L (x_j), p_u^U (x_i) - p_u^U (x_j) \right\} \right) \]

Inclusion measures for interval neutrosophic sets

H. Definition 2.8 [18]

1) Inclusion measures based on the distance measure

In this section, we give a formal definition of inclusion measure for interval neutrosophic sets.

Assume that \( d : INS (X) \times INS (X) \rightarrow R^+ \cup \{0\} \) is a distance between interval neutrosophic sets in \( X \). To establish the inclusion indicator expressing the degree to which \( A \) belongs to \( B \), we use the distance between interval neutrosophic sets \( A \) and \( A \cap B \). If it is considered the inclusion measure based on distance measure, we have the formal given by

\[ I_H (A, B) = 1 - d (A, A \cap B) \]

I. Definition 2.9 [18]

1) The Inclusion measure for interval neutrosophic sets

A mapping \( I : INS (X) \times INS (X) \rightarrow [0,1] \) is called an inclusion measure for interval neutrosophic sets, if \( I \) satisfies the following properties (for all \( A, B, C \in INS (X) \)).

1) \( I (A, B) = 1 \) if \( A \subseteq B \).
2) \( I (A, A^C) = 1 \leq \forall x \in X \), where \( A \) is the interval absolute neutrosophic set and \( 0 \) is the interval empty neutrosophic set.
3) \( I (1, 0) = 0 \), where \( 1 \) and \( 0 \) are interval absolute neutrosophic set.
4) \( A \subseteq B \subseteq C \rightarrow I (C, A) \leq I (B, A) \) and \( I (C, A) \leq I (C, B) \)

III. THE INCLUSION MEASURE TO MULTIATTRIBUTE NEUTROSOPHIC DECISION-MAKING METHOD BASED ON VARIOUS DISTANCE MEASURES

In the following, we apply the above inclusion measure to multi-attribute decision making problem based on INSs.

A. Numerical Example

Let us consider the following pattern recognition problem. Assume \( A_1, A_2, A_3 \) and \( A_4 \) are given four known patterns which correspond to four decision alternatives \( d_1, d_2, d_3 \) and \( d_4 \) respectively. The patterns are denoted by the following INSs in \( X = \{x_1, x_2, x_3\} \).

\( A_1 = \{x_1, 0.3, 0.9\}, \{x_2, 0.3, 0.4\}, \{x_3, 0.2, 0.3\} \)
\( A_2 = \{x_1, 0.5, 0.6\}, \{x_2, 0.1, 0.4\}, \{x_3, 0.4, 0.6\} \)
\( A_3 = \{x_1, 0.4, 0.7\}, \{x_2, 0.1, 0.3\}, \{x_3, 0.1, 0.2\} \)
\( A_4 = \{x_1, 0.5, 0.6\}, \{x_2, 0.1, 0.2\}, \{x_3, 0.1, 0.4\} \)

Given an unknown sample (i.e., the positive ideal solution of decision).

\[ A^* = \left\{ \left\{ x_1, 0.5, 0.6 \right\}, \left\{ x_2, 0.1, 0.2 \right\}, \left\{ x_3, 0.1, 0.4 \right\} \right\} \]

Our aim is to classify pattern \( A^* \) to one of the decision alternatives \( A_1, A_2, A_3, \) and \( A_4 \).

First we have to find \( A_1 \bigcup \cdots \bigcup_{i=1}^{4} A_i \) as follows:
Similarly we compute
\[
A^+_1 A_2 = \left\{ \left\{ x_1, [0.5, 0.6], [0.1, 0.4], [0.3, 0.6] \right\}, \left\{ x_2, [0.4, 0.7], [0.1, 0.2], [0.7, 0.7] \right\} \right\}
\]
\[
A^+_1 A_3 = \left\{ \left\{ x_1, [0.4, 0.5], [0.2, 0.4], [0.3, 0.4] \right\}, \left\{ x_2, [0.5, 0.2], [0.5, 0.3], [0.4, 0.7] \right\} \right\},
\]
and
\[
A^+_1 A_4 = \left\{ \left\{ x_1, [0.5, 0.6], [0.1, 0.3], [0.3, 0.4] \right\}, \left\{ x_2, [0.4, 0.7], [0.1, 0.3], [0.4, 0.7] \right\} \right\}.
\]
Using the above mentioned various distance measures, we can compute the inclusion measure for INSs as follows:

**B. Based on normalized Hamming distance measure:**
\[
d_m(A^+, A_i) = \frac{1}{6n} \sum_{i=1}^{n} \left[ \frac{1}{6} \sum_{i=1}^{4} \right] \left( u^+_i(x) - u^-_i(x) \right) + \left( p^+_i(x) - p^-_i(x) \right) + \left( v^+_i(x) - v^-_i(x) \right)
\]

First we have to compute the distance between \( A^+ \) and \( A_i \), based on the normalized Hamming distance measure as follows:

Thus we rank the decision alternatives according to inclusion measure based on the normalized Hamming distance measure as
\[ A_1 f A_2 f A_3 f A_4 \]

**C. Based on normalized Euclidean distance measure:**
\[
d_e(A^+, A_i) = \frac{1}{6n} \sum_{i=1}^{n} \left[ \frac{1}{6} \sum_{i=1}^{4} \right] \left( u^+_i(x) - u^-_i(x) \right)^2 + \left( p^+_i(x) - p^-_i(x) \right)^2 + \left( v^+_i(x) - v^-_i(x) \right)^2
\]

First we have to compute the distance between \( A^+ \) and \( A_i \), based on the normalized Euclidean distance measure as follows:

Thus we rank the decision alternatives according to inclusion measure based on the normalized Euclidean distance measure as
\[ A_4 f A_2 f A_1 f A_3 \]
D. Based on normalized Geometric distance measure:

\[ d_{\omega}(A, B) = \frac{1}{4} \sum_{j=1}^{4} \sum_{i=1}^{6} \left[ (u_i^A(x) - u_i^B(x))^2 + (p_i^A(x) - p_i^B(x))^2 \right] \]

First we have to compute the distance between \( A^+ \) and \( A_i \) based on the Geometric distance measure as follows:

\[ d_{\omega}(A^+, A_i) = \frac{1}{4} \sum_{j=1}^{4} \sum_{i=1}^{6} \left[ (u_i^A(x) - u_i^{A^+}(x))^2 + (p_i^A(x) - p_i^{A^+}(x))^2 \right] \]

Thus the best distance measures that gives us the more accurate results.

E. Based on Hausdorff distance measure:

\[ d_{H}(A, B) = \frac{1}{4} \sum_{j=1}^{4} \sum_{i=1}^{6} \left[ \max \{ |u_i^A(x) - u_i^B(x)|, |u_i^A(x) - u_i^B(x)| \} + \max \{ |p_i^A(x) - p_i^B(x)|, |p_i^A(x) - p_i^B(x)| \} \right] \]

First we have to compute the distance between \( A^+ \) and \( A_i \) based on the Hausdorff distance measure as follows:

\[ d_{H}(A^+, A_i) = \frac{1}{4} \sum_{j=1}^{4} \sum_{i=1}^{6} \left[ \max \{ |u_i^A(x) - u_i^{A^+}(x)|, |u_i^A(x) - u_i^{A^+}(x)| \} + \max \{ |p_i^A(x) - p_i^{A^+}(x)|, |p_i^A(x) - p_i^{A^+}(x)| \} \right] \]
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