A Study of Distance Measures for Interval Neutrosophic Sets with Numerical Example

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Abstract— In this paper, we introduce an inclusion measure for interval neutrosophic sets, which is one of information measures of interval neutrosophic theory. Using the concept of inclusion measure based on various distance measure, we develop a simple inclusion measure for ranking the interval neutrosophic sets. Though having a simple measure for calculation, the inclusion measure presents a new approach for handling the interval neutrosophic information.

Key words: Single Valued Neutrosophic Set (SVNS), Interval Neutrosophic Set (INS), Normalized Hamming Distance, Hamming Distance, Normalized Euclidean Distance, Hausdorff Distance Measure, Normalized Geometric Distance, Euclidean Distance, Geometric Distance

I. INTRODUCTION

The concept of the neutrosophic set developed by Smarandache [12] is a set model which generalizes the classic set, fuzzy set [21], interval fuzzy set [14] intuitionistic fuzzy set [1] and interval valued intuitionistic fuzzy set [2]. In contrast to intuitionistic fuzzy sets and also interval valued intuitionistic fuzzy sets, indeterminacy degree of an element in a universe of discourse is expressed explicitly in the neutrosophic set. There are three membership functions such that truth membership, indeterminacy membership and falsity membership in a neutrosophic set, and they are independent. However, the neutrosophic set generalizes the above mentioned sets from philosophical point of view and its functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-,1^+[$ and are defined by $T_A(x): X \to]0^-,1^+[$, $I_A(x): X \to]0^-,1^+[$ and $F_A(x): X \to]0^-,1^+[$. That is, its components $T_A(x), I_A(x), \mathrm{F_A}(\mathrm{x})$ are non-standard subsets included in the unitary nonstandard interval $]0^-,1^+[$ or standard subsets included in the unitary standard interval $[0,1]$ as in the intuitionistic fuzzy set. Furthermore, the connectors in the intuitionistic fuzzy set are only defined by $T(x)$ and $\mathrm{F}(x)$ (i.e. truth-membership and falsity-membership), hence the indeterminacy $I(x)$ is what is left from 1, while in the neutrosophic set, they can be defined by any of them (no restriction) [12]. However, the neutrosophic set is to be difficult to use in real scientific or engineering applications. So Wang et al. [5],[6] defined the concepts of single valued neutrosophic set (SVNS) and interval neutrosophic set (INS) which is an instance of a neutrosophic set. At present, studies on the SVNSs and INSs are progressing rapidly in many different aspects [4],[9],[13],[17],[18],[19]. Recently, Şahin and Küçük [13] proposed the subsethood (inclusion) measure for single valued neutrosophic sets and applied it to a multi criteria decision making problem with information of single valued neutrosophic sets.

Fuzzy entropy, distance measure and similarity measure are three basic concepts used in fuzzy sets theory. Usually subsethood measures are constructed using implication operators, t-norms or t-conorms, entropy measures or cardinalities. In classical theory, it is said that a set A is a subset of B and is denoted by $A \subset B$ if every element of A is an element of B, whenever X is a universal set and A, B are two sets in X. Therefore, inclusion measure should be two valued for crisp sets. That is, either A is precisely subset of B or vice versa. But since an element x in universal set X can belong to a fuzzy set A to varying degrees, it is notable to consider situations describing as being "more and less" a subset of another set and to measure the degree of this inclusion. Fuzzy inclusion allows a given fuzzy set to contain another to some degree between 0 and 1. According to Zadeh's fuzzy set containment, a fuzzy set B contains a fuzzy set A if $m_A(x) \le m_B(x)$, for all x in X, in which $m_A(x)$ and $m_B(x)$ are the membership functions of A and B, respectively.

In this paper, we firstly review the systems of axioms of Young's fuzzy inclusion measure. Then we extend the inclusion measure of single valued neutrosophic sets to interval neutrosophic environment and give a new system of axioms for inclusion measure of interval neutrosophic sets. Moreover, we utilize the neutrosophic inclusion measure to rank the interval neutrosophic sets. To demonstrate the effectiveness of the proposed inclusion measure, we consider a multi attribute decision-making problem.

A. *Preliminaries*

In the following we give a brief review of some preliminaries.

1) *Single valued neutrosophic sets*

A single valued neutrosophic set has been defined in [5] as follows:

a) Definition 1.1 [5]

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form

$$A = \{\langle x, u_A(x), p_A(x), v_A(x)\rangle : x \in X\},$$

Where

$u_A(x): X \to [0,1]$, $p_A(x): X \to [0,1]$ and

$v_A(x): X \to [0,1]$ with

$0 \le u_A(x) + p_A(x) + v_A(x) \le 3$ for all $x \in X$.

The values $u_A(x)$, $p_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

B. *Interval neutrosophic sets*

An interval neutrosophic set is a model of a neutrosophic set, which can be used to handle uncertainty in fields of scientific, environment and engineering. We introduce the definition of an interval neutrosophic set as follows.

1) *Definition 1.2 [6]*

Let X be a space of points (objects) and $\text{Int}[0,1]$ be the set of all closed subsets of $[0,1]$. An interval neutrosophic A in X is defined with the form

$$A = \{\langle x, u_A(x), p_A(x), v_A(x)\rangle : x \in X\}$$

Where

$u_A(x): X \to \text{int}[0,1]$, $p_A(x): X \to \text{int}[0,1]$ and $v_A(x): X \to \text{int}[0,1]$

with $0 \le \sup u_A(x) + \sup p_A(x) + \sup v_A(x) \le 3$) for all $x \in X$.

The intervals $u_A(x)$, $p_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

For convenience, if let

$u_A(x) = [u_A^L(x), u_A^U(x)]$,

$p_A(x) = [p_A^L(x), p_A^U(x)]$ and

$v_A(x) = [v_A^L(x), v_A^U(x)]$,

then

$$A = \{\langle x, [u_A^L(x), u_A^U(x)], [p_A^L(x), p_A^U(x)], [v_A^L(x), v_A^U(x)]\rangle : x \in X\}$$

with the condition,

$0 \le \sup u_A^U(x) + \sup p_A^U(x) + \sup v_A^U(x) \le 3$ for all $x \in X$.

Here, we only take the sub-unitary interval of $[0,1]$. Therefore, an interval neutrosophic set is clearly neutrosophic set.

2) Definition 1.3 [6]

Let $INS(X)$ denote the family of all the interval neutrosophic sets in universe X, assume $A, B \in INS(X)$ such that

$$A = \{\langle x, [u_A^L(x), u_A^U(x)], [p_A^L(x), p_A^U(x)], [v_A^L(x), v_A^U(x)]\rangle : x \in X\}$$

$$B = \{\langle x, [u_B^L(x), u_B^U(x)], [p_B^L(x), p_B^U(x)], [v_B^L(x), v_B^U(x)]\rangle : x \in X\}$$

then some operations can be defined as follows:

(1) $$A \cup B = \left\{\left\langle \begin{matrix} x, [\max\{u_A^L(x), u_B^L(x)\}, \max\{u_A^U(x), u_B^U(x)\}] \\ [\min\{p_A^L(x), p_B^L(x)\}, \min\{p_A^U(x), p_B^U(x)\}] \\ [\min\{v_A^L(x), v_B^L(x)\}, \min\{v_A^U(x), v_B^U(x)\}] \end{matrix} \right\rangle : x \in X\right\}$$

(2) $$A \cap B = \left\{\left\langle \begin{matrix} x, [\min\{u_A^L(x), u_B^L(x)\}, \min\{u_A^U(x), u_B^U(x)\}] \\ [\max\{p_A^L(x), p_B^L(x)\}, \max\{p_A^U(x), p_B^U(x)\}] \\ [\max\{v_A^L(x), v_B^L(x)\}, \max\{v_A^U(x), v_B^U(x)\}] \end{matrix} \right\rangle : x \in X\right\}$$

(3) $$A^C = \left\{\left\langle \begin{matrix} x, [v_A^L(x), v_A^U(x)], \\ [1 - p_A^U(x), 1 - p_A^L(x)], \\ [u_A^L(x), u_A^U(x)] \end{matrix} \right\rangle : x \in X\right\}$$

(4) $A \subseteq B$, if $u_A^L(x) \le u_B^L(x)$, $u_A^U(x) \le u_B^U(x)$, $p_A^L(x) \ge p_B^L(x)$, $p_A^U(x) \ge p_B^U(x)$ and $v_A^L(x) \ge v_B^L(x)$, $v_A^U(x) \ge v_B^U(x)$ for all $x \in X$.

(5) $A = B$, if $A \subseteq B$ and $B \supseteq A$.

II. DISTANCE MEASURES FOR INTERVAL NEUTROSOPHIC SET

Distance measure is a term that describes the difference between interval neutrosophic sets and can be considered as a dual concept of inclusion measure. We make use of the various distance measures proposed in [23, 24, 27, 28, 29] between interval neutrosophic sets, which were partly based on the geometric interpretation of interval neutrosophic sets, and have some good geometric properties.

Let

$$A = \{\langle x, [u_A^L(x), u_A^U(x)], [p_A^L(x), p_A^U(x)], [v_A^L(x), v_A^U(x)]\rangle : x \in X\}$$

$$B = \{\langle x, [u_B^L(x), u_B^U(x)], [p_B^L(x), p_B^U(x)], [v_B^L(x), v_B^U(x)]\rangle : x \in X\}$$

be two INS in X.

A. *Definition 2.1 [23]*

1) *The Hamming distance measure*

$$d_H(A,B) = \frac{1}{6}\sum_{i=1}^{n}\left\{\begin{matrix} |u_A^L(x_i) - u_B^L(x_i)| + |p_A^L(x_i) - p_B^L(x_i)| + \\ |v_A^L(x_i) - v_B^L(x_i)| + |u_A^U(x_i) - u_B^U(x_i)| + \\ |p_A^U(x_i) - p_B^U(x_i)| + |v_A^U(x_i) - v_B^U(x_i)| \end{matrix}\right\}$$

B. *Definition 2.2 [23]*

1) *The Euclidean distance measure*

$$d_E(A,B) = \sqrt{\frac{1}{6}\sum_{i=1}^{n}\left\{\begin{matrix} (u_A^L(x_i) - u_B^L(x_i))^2 + (p_A^L(x_i) - p_B^L(x_i))^2 + \\ (v_A^L(x_i) - v_B^L(x_i))^2 + (u_A^U(x_i) - u_B^U(x_i))^2 + \\ (p_A^U(x_i) - p_B^U(x_i))^2 + (v_A^U(x_i) - v_B^U(x_i))^2 \end{matrix}\right\}}$$

C. Definition 2.3 [23]

1) *The normalized Hamming distance measure*

$$d_{nH}(A,B)=\frac{1}{6n}\sum_{i=1}^{n}\left\{\begin{array}{l}\left|u_A^L(x_i)-u_B^L(x_i)\right|+\left|p_A^L(x_i)-p_B^L(x_i)\right|+\\ \left|v_A^L(x_i)-v_B^L(x_i)\right|+\left|u_A^U(x_i)-u_B^U(x_i)\right|+\\ \left|p_A^U(x_i)-p_B^U(x_i)\right|+\left|v_A^U(x_i)-v_B^U(x_i)\right|\end{array}\right\}$$

D. Definition 2.4 [23]

1) *The normalized Euclidean distance measure*

$$d_{nE}(A,B)=\sqrt{\frac{1}{6n}\sum_{i=1}^{n}\left\{\begin{array}{l}\left(u_A^L(x_i)-u_B^L(x_i)\right)^2+\left(p_A^L(x_i)-p_B^L(x_i)\right)^2+\\ \left(v_A^L(x_i)-v_B^L(x_i)\right)^2+\left(u_A^U(x_i)-u_B^U(x_i)\right)^2+\\ \left(p_A^U(x_i)-p_B^U(x_i)\right)^2+\left(v_A^U(x_i)-v_B^U(x_i)\right)^2\end{array}\right\}}$$

E. Definition 2.5 [25]

1) *The Geometric distance measure*

$$d_r(A,B)=\left[\sum_{i=1}^{n}\left\{\begin{array}{l}\left(u_A^L(x_i)-u_B^L(x_i)\right)^r+\left(p_A^L(x_i)-p_B^L(x_i)\right)^r+\\ \left(v_A^L(x_i)-v_B^L(x_i)\right)^r+\left(u_A^U(x_i)-u_B^U(x_i)\right)^r+\\ \left(p_A^U(x_i)-p_B^U(x_i)\right)^r+\left(v_A^U(x_i)-v_B^U(x_i)\right)^r\end{array}\right\}\right]^{1/r}$$

F. Definition 2.6 [26]

1) *The normalized Geometric distance measure*

$$d_{nr}(A,B)=\frac{1}{4}\sum_{j=1}^{4}\frac{1}{6}\sum_{i=1}^{6}\left\{\begin{array}{l}\left(u_A^L(x_i)-u_B^L(x_i)\right)^2+\left(p_A^L(x_i)-p_B^L(x_i)\right)^2+\\ \left(v_A^L(x_i)-v_B^L(x_i)\right)^2+\left(u_A^U(x_i)-u_B^U(x_i)\right)^2+\\ \left(p_A^U(x_i)-p_B^U(x_i)\right)^2+\left(v_A^U(x_i)-v_B^U(x_i)\right)^2\end{array}\right\}$$

G. Definition 2.7 [26]

1) *The Hausdorff distance measure*

$$d_q(A,B)=\frac{1}{4}\sum_{j=1}^{4}\frac{1}{3}\sum_{i=1}^{3}\max\left\{\begin{array}{l}\left|u_A^L(x_i)-u_B^L(x_i)\right|,\left|u_A^U(x_i)-u_B^U(x_i)\right|+\\ \left|v_A^L(x_i)-v_B^L(x_i)\right|,\left|v_A^U(x_i)-v_B^U(x_i)\right|+\\ \left|p_A^L(x_i)-p_B^L(x_i)\right|,\left|p_A^U(x_i)-p_B^U(x_i)\right|\end{array}\right\}$$

Inclusion measures for interval neutrosophic sets

H. Definition 2.8 [18]

1) *Inclusion measures based on the distance measure*

In this section, we give a formal definition of inclusion measure for interval neutrosophic sets.

Assume that $d: INS(X)\times INS(X)\to R^{+}\cup\{0\}$ is a distance between interval neutro-sophic sets in X. To establish the inclusion indicator expressing the degree to which A belongs to B, we use the distance between interval neutrosophic sets A and $A\cap B$. If it is considered the inclusion measure based on distance measure, we have the formal given by

$$I_d(A,B)=1-d(A,A\cap B)$$

I. Definition 2.9 [18]

1) *The Inclusion measure for interval neutrosophic sets*

A mapping $I: INS(X)\times INS(X)\to[0,1]$ is called an inclusion measure for interval neutrosophic sets, if I satisfies the following properties (for all $A,B,C\in INS(X)$).

1) $I(A,B)=1$ if $A\subseteq B$.
2) $I(A,A^C)=1\Leftrightarrow\forall x\in X$, $\left[u_A^L(x),u_A^U(x)\right]\le\left[v_A^L(x),v_A^U(x)\right]$ and $\left[p_A^L(x),p_A^U(x)\right]\le[0.5,0.5]$.
3) $I(\underline{1},\underline{0})=0$, where $\underline{1}$ is the interval absolute neutrosophic set and $\underline{0}$ is the interval empty neutrosophic set.
4) $A\subseteq B\subseteq C\Rightarrow I(C,A)\le I(B,A)$ and $I(C,A)\le I(C,B)$

III. THE INCLUSION MEASURE TO MULTI-ATTRIBUTE NEUTROSOPHIC DECISION-MAKING METHOD BASED ON VARIOUS DISTANCE MEASURES

In the following, we apply the above inclusion measure to multi-attribute decision making problem based on INSs.

A. Numerical Example

Let us consider the following pattern recognition problem. Assume A_1, A_2, A_3 and A_4 are given four known patterns which correspond to four decision alternatives d_1, d_2, d_3 and d_4 respectively. The patterns are denoted by the following INSs in $X=\{x_1,x_2\}$.

$$A_1=\{\langle x_1,[0.8,0.9],[0.3,0.4],[0.2,0.3]\rangle,\langle x_2,[0.6,0.7],[0.5,0.3],[0.4,0.2]\rangle\}$$

$$A_2=\{\langle x_1,[0.5,0.8],[0.1,0.4],[0.3,0.6]\rangle,\langle x_2,[0.4,0.8],[0.1,0.2],[0.7,0.2]\rangle\}$$

$$A_3=\{\langle x_1,[0.4,0.5],[0.3,0.1],[0.1,0.4]\rangle,\langle x_2,[0.7,0.2],[0.5,0.3],[0.4,0.6]\rangle\}$$

$$A_4=\{\langle x_1,[0.5,0.6],[0.1,0.3],[0.3,0.4]\rangle,\langle x_2,[0.4,0.7],[0.1,0.3],[0.1,0.2]\rangle\}$$

Given an unknown sample (i.e., the positive ideal solution of decision).

$$A^{+}=\{\langle x_1,[0.5,0.6],[0.1,0.2],[0.3,0.4]\rangle,\langle x_2,[0.5,0.7],[0.1,0.2],[0.4,0.7]\rangle\}$$

Our aim is to classify pattern A^{+} to one of the decision alternatives A_1, A_2, A_3 and A_4.

First we have to find $A^{+}\bigcap_{i=1}^{4}A_i$ as follows:

$$A^+ \cap A_1 = \left\{ \left\langle \begin{array}{l} x_1, [\min\{0.5,0.8\}, \min\{0.6,0.9\}], \\ [\max\{0.1,0.3\}, \max\{0.2,0.4\}], \\ [\max\{0.3,0.2\}, \max\{0.4,0.3\}] \end{array} \right\rangle, \left\langle \begin{array}{l} x_2, [\min\{0.5,0.6\}, \min\{0.7,0.7\}], \\ [\max\{0.1,0.5\}, \max\{0.2,0.3\}], \\ [\max\{0.4,0.4\}, \max\{0.7,0.2\}] \end{array} \right\rangle \right\}$$

$$A^+ \cap A_1 = \{\langle x_1, [0.5, 0.6], [0.3, 0.4], [0.3, 0.4]\rangle, \langle x_2, [0.5, 0.7], [0.5, 0.3], [0.4, 0.7]\rangle\},$$

Similarly we compute

$$A^+ \cap A_2 = \{\langle x_1, [0.5, 0.6], [0.1, 0.4], [0.3, 0.6]\rangle, \langle x_2, [0.4, 0.7], [0.1, 0.2], [0.7, 0.7]\rangle\}$$

$$A^+ \cap A_3 = \{\langle x_1, [0.4, 0.5], [0.3, 0.2], [0.3, 0.4]\rangle, \langle x_2, [0.5, 0.2], [0.5, 0.3], [0.4, 0.7]\rangle\},$$ and

$$A^+ \cap A_4 = \{\langle x_1, [0.5, 0.6], [0.1, 0.3], [0.3, 0.4]\rangle, \langle x_2, [0.4, 0.7], [0.1, 0.3], [0.4, 0.7]\rangle\},$$

Using the above mentioned various distance measures, we can compute the inclusion measure for INSs as follows:

B. Based on normalized Hamming distance measure:

$$d_{nH}(A,B) = \frac{1}{6n}\sum_{i=1}^{n} \left\{ \begin{array}{l} |u_A^L(x_i) - u_B^L(x_i)| + |p_A^L(x_i) - p_B^L(x_i)| + \\ |v_A^L(x_i) - v_B^L(x_i)| + |u_A^U(x_i) - u_B^U(x_i)| + \\ |p_A^U(x_i) - p_B^U(x_i)| + |v_A^U(x_i) - v_B^U(x_i)| \end{array} \right\}$$

First we have to compute the distance between A^+ and $A \bigcap_{i=1}^{4} A_i$ based on the normalized Hamming distance measure as follows:

$$d_{nH}(A^+, A^+ \cap A_1) = \frac{1}{6 \times 4} \left\{ \begin{array}{l} |0.5 - 0.5| + |0.6 - 0.6| + |0.1 - 0.3| + |0.2 - 0.4| + |0.3 - 0.3| + |0.4 - 0.4| + \\ |0.5 - 0.5| + |0.7 - 0.7| + |0.1 - 0.5| + |0.2 - 0.3| + |0.4 - 0.4| + |0.7 - 0.7| \end{array} \right\}$$

$$d_{nH}(A^+, A^+ \cap A_1) = \frac{1}{6 \times 4}\{0.2 + 0.2 + 0.4 + 0.1\}$$

$$d_{nH}(A^+, A^+ \cap A_1) = 0.0375$$

$$I(A^+, A_1) = 1 - 0.0375 = 0.9625$$

Similarly we can compute

$$I(A^+, A_1) = 0.9625$$

$$I(A^+, A_2) = 0.9667$$

$$I(A^+, A_3) = 0.9417$$

$$I(A^+, A_4) = 0.9875$$

Thus we rank the decision alternatives according to inclusion measure based on the normalized Hamming distance measure as

$$A_4 \succ A_2 \succ A_1 \succ A_3$$

C. Based on normalized Euclidean distance measure:

$$d_{nE}(A,B) = \sqrt{\frac{1}{6n}\sum_{i=1}^{n} \left\{ \begin{array}{l} (u_A^L(x_i) - u_B^L(x_i))^2 + (p_A^L(x_i) - p_B^L(x_i))^2 + \\ (v_A^L(x_i) - v_B^L(x_i))^2 + (u_A^U(x_i) - u_B^U(x_i))^2 + \\ (p_A^U(x_i) - p_B^U(x_i))^2 + (v_A^U(x_i) - v_B^U(x_i))^2 \end{array} \right\}}$$

First we have to compute the distance between A^+ and $A \bigcap_{i=1}^{4} A_i$ based on the normalized Euclidean distance measure as follows:

$$d_{nE}(A^+, A^+ \cap A_1) = \sqrt{\frac{1}{6 \times 4} \left\{ \begin{array}{l} |0.5 - 0.5|^2 + |0.6 - 0.6|^2 + |0.1 - 0.3|^2 + |0.2 - 0.4|^2 + |0.3 - 0.3|^2 + |0.4 - 0.4|^2 + \\ |0.5 - 0.5|^2 + |0.7 - 0.7|^2 + |0.1 - 0.5|^2 + |0.2 - 0.3|^2 + |0.4 - 0.4|^2 + |0.7 - 0.7|^2 \end{array} \right\}}$$

$$d_{nE}(A^+, A^+ \cap A_1) = \sqrt{\frac{1}{6 \times 4}\{0.2^2 + 0.2^2 + 0.4^2 + 0.1^2\}}$$

$$d_{nE}(A^+, A^+ \cap A_1) = 0.10206$$

$$I(A^+, A_1) = 1 - 0.10206 = 0.89794$$

Similarly we can compute

$$I(A^+, A_1) = 0.89794$$

$$I(A^+, A_2) = 0.91340$$

$$I(A^+, A_3) = 0.84589$$

$$I(A^+, A_4) = 0.96464$$

Thus we rank the decision alternatives according to inclusion measure based on the normalized Euclidean distance measure as

$$A_4 \succ A_2 \succ A_1 \succ A_3$$

D. Based on normalized Geometric distance measure:

$$d_{nr}(A,B)=\frac{1}{4}\sum_{j=1}^{4}\frac{1}{2}\sum_{i=1}^{6}\sqrt{\begin{Bmatrix}\left(u_A^L(x_i)-u_B^L(x_i)\right)^2+\left(p_A^L(x_i)-p_B^L(x_i)\right)^2+\\ \left(v_A^L(x_i)-v_B^L(x_i)\right)^2+\left(u_A^U(x_i)-u_B^U(x_i)\right)^2+\\ \left(p_A^U(x_i)-p_B^U(x_i)\right)^2+\left(v_A^U(x_i)-v_B^U(x_i)\right)^2\end{Bmatrix}}$$

First we have to compute the distance between A^+ and $A\bigcap_{i=1}^{4} A_i$ based on the Geometric distance measure as follows:

$$d_{nr}\left(A^+,A^+\cap A_1\right)=\frac{1}{4}\sum_{j=1}^{4}\frac{1}{2}\begin{Bmatrix}\left\{\sqrt{\begin{bmatrix}(0.5-0.5)^2+(0.6-0.6)^2+(0.1-0.3)^2+\\(0.2-0.4)^2+(0.3-0.3)^2+(0.4-0.4)^2\end{bmatrix}}\right\}\\ \left\{\sqrt{\begin{bmatrix}(0.5-0.5)^2+(0.7-0.7)^2+(0.1-0.5)^2+\\(0.2-0.3)^2+(0.4-0.4)^2+(0.7-0.7)^2\end{bmatrix}}\right\}\end{Bmatrix}$$

$$d_{nr}\left(A^+,A^+\cap A_1\right)=\frac{1}{4}\sum_{j=1}^{4}\frac{1}{2}\begin{Bmatrix}\left\{\sqrt{0+0+0.2^2+0.2^2+0+0}\right\},\\ \left\{\sqrt{0+0+0.4^2+0.1^2+0+0}\right\}\end{Bmatrix}$$

$$d_{nr}\left(A^+,A^+\cap A_1\right)=\frac{1}{4}\{0.14142+0.20616\}$$

$$d_{nr}\left(A^+,A^+\cap A_1\right)=0.08690$$

$$I(A^+,A_1)=0.91311$$

Similarly we can compute

$$I\left(A^+,A_1\right)=0.91311$$

$$I\left(A^+,A_2\right)=0.92512$$

$$I\left(A^+,A_3\right)=0.88837$$

$$I\left(A^+,A_4\right)=0.96982$$

Thus we rank the decision alternatives according to inclusion measure based on the normalized Geometric distance measure as

$A_4 \succ A_2 \succ A_1 \succ A_3$

E. Based on Hausdorff distance measure:

$$d_q(A,B)=\frac{1}{4}\sum_{j=1}^{4}\frac{1}{2}\begin{Bmatrix}\max\left\{\left|u_A^L(x_i)-u_B^L(x_i)\right|,\left|u_A^U(x_i)-u_B^U(x_i)\right|\right\}+\\ \max\left\{\left|v_A^L(x_i)-v_B^L(x_i)\right|,\left|v_A^U(x_i)-v_B^U(x_i)\right|\right\}+\\ \max\left\{\left|p_A^L(x_i)-p_B^L(x_i)\right|,\left|p_A^U(x_i)-p_B^U(x_i)\right|\right\}\end{Bmatrix}$$

First we have to compute the distance between A^+ and $A\bigcap_{i=1}^{4} A_i$ based on the Hausdorff distance measure as follows:

$$d_q\left(A^+,A^+\cap A_1\right)=\frac{1}{4}\sum_{j=1}^{4}\begin{Bmatrix}\frac{1}{2}\begin{Bmatrix}\max\{|0.5-0.5|,|0.6-0.6|\}+\\ \max\{|0.1-0.3|,|0.2-0.4|\}+\\ \max\{|0.3-0.3|,|0.4-0.4|\}\end{Bmatrix},\\ \frac{1}{2}\begin{Bmatrix}\max\{|0.5-0.5|,|0.7-0.7|\}+\\ \max\{|0.1-0.5|,|0.2-0.3|\}+\\ \max\{|0.4-0.4|,|0.7-0.7|\}\end{Bmatrix}\end{Bmatrix}$$

$$d_q\left(A^+,A^+\cap A_1\right)=\frac{1}{4}\left\{\frac{1}{2}(0+0.2+0)+\frac{1}{2}(0+0.4+0)\right\}$$

$$d_q\left(A^+,A^+\cap A_1\right)=0.075$$

$$I\left(A^+,A_1\right)=0.925$$

Similarly we can compute

$$I\left(A^+,A_1\right)=0.925$$

$$I\left(A^+,A_2\right)=0.9$$

$$I\left(A^+,A_3\right)=0.85$$

$$I\left(A^+,A_4\right)=0.9625$$

Thus we rank the decision alternatives according to inclusion measure based on the Hausdorff distance measure as

$A_4 \succ A_2 \succ A_1 \succ A_3$

Since $I\left(A^+,A_4\right)=\max_{1\le i\le 4} I\left(A^+,A_i\right)$ then the pattern A^+ should be classified to A_2 according to the principle of inclusion measure between INSs. It means that the decision alternative A_2 is the optimal alternative which is the closest alternative to positive ideal solution.

IV. CONCLUSION

In this paper, we introduce an inclusion measure for interval neutrosophic sets. For this purpose, we first give some basic definitions of neutrosophic sets, single neutrosophic sets, interval neutrosophic sets. Moreover, we have proposed a simple and natural inclusion measure based on the various distance measure between interval neutrosophic sets.

Thus normalized Hamming distance measure gives us the more accurate results.The next accurate result for the crops cultivation was given by both normalized Euclidean and normalized Geometric distance measure. Finally the normalized Geometric distance measure gives us the least accurate result.

Thus the best distance measures that gives us the most accurate results for our problem in the field of cultivation of crops were normalized Euclidean and normalized Geometric distance measures.

We hope that the findings in this paper will help the researchers to enhance and promote the further study on inclusion measure to carry out general framework for the applications in practical life.

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