

A Study on Graph Labeling Problems

J.Lisy Bennet

Ph.D. Research Scholar, Part-Time Reg.No:17223162092022,

Department of Mathematics

Scott Christian College, (Autonomous), Nagercoil,
Affiliated to Manonmaniam Sundaranar University,
Abishekapatti, Tirunelveli, Tamilnadu, India.

Dr. S. Chandra Kumar,

Associate Professor, Department of Mathematics
Scott Christian College (Autonomous), Nagercoil,
Affiliated to Manonmaniam Sundaranar University,
Abishekapatti, Tirunelveli, Tamilnadu, India.

ABSTRACT

Graph theory has applications in many areas of the computing, social and natural science. The theory is also intimately related to many branches of mathematics, including matrix theory, numerical analysis, probability, topology and combinatory. The fact is that graph theory serves as a mathematical for any system involving a binary relation. Over the last 50 year graph theory has evolved into an important mathematical tool in the solution of a wide variety of problems in many areas of society. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions have been motivated by practical problems, labeled graphs serve useful mathematical models for a broad range of applications such as: coding theory, including the design of good types codes, synch-set codes, missile guidance codes and convolutional codes with optimal auto correlation properties. They facilitates the optimal nonstandard encodings of integer's, labeled graph have also been applied in determining ambiguities in x-ray crystallographic analysis to design a communication network addressing system, data base management in determining optimal circuit layouts and radio astronomy problems etc.

KEYWORDS: Graph Labeling, Problems, Graph theory, applications, mathematics

INTRODUCTION

A graph $G = (V, E)$ comprises of two finite sets: $V(G)$, the vertex set of the graph, regularly indicated by just V , which is a nonempty set of elements called vertices, and $E(G)$, the

edge set of the graph, frequently signified by just E , which is a set (potentially empty) of elements called edges. A graph, at that point, can be thought of as a drawing or diagram comprising of an assortment of vertices (spots or points) together with edges (lines) joining certain pairs of these vertices. Figure 1 gives a graph $G = (V,E)$ with $V(G) = \{ v_1, v_2, v_3, v_4, v_5 \}$ and $E(G) = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7 \}$.

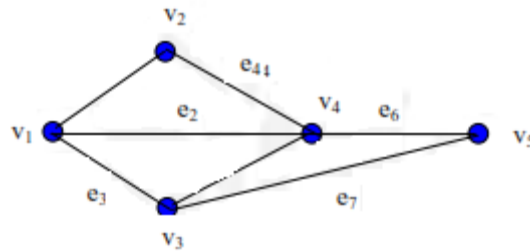


Figure 1: A graph G with five vertices and seven edges

Sometimes we speak to an edge by the two vertices that it interfaces. In Figure 1 we have $e_1 = (v_1, v_2)$, $e_2 = (v_1, v_4)$. An edge e of graph G is said to be episode with the vertex v if v is an end vertex of e . For example in Figure 1 an edge e_1 is episode with two vertices v_1 and v_2 . An edge e having indistinguishable end vertices called a loop. At the end of the day, in a loop a vertex v is joined to itself by an edge e . The level of a vertex v , composed $d(v)$, is the number of edges occurrence with v . In Figure 1.1 we have $d(v_1) = 3$, $d(v_2) = 2$, $d(v_3) = 3$, $d(v_4) = 4$ and $d(v_5) = 2$. On the off chance that for some positive whole number k , $d(v) = k$ for each vertex v of graph G , at that point G is called k -customary.

A graph G is called associated if there is a way between each pair of vertices. When there is no worry about the bearing of an edge the graph is called undirected. The graph in Figure 1 is an associated and undirected graph. In contrast to most different territories in Mathematics, the theory of graphs has a definite beginning stage, when the Swiss mathematician Leonard Euler (1707-1783) considered the problems of the seven Konigsberg spans. In the mid eighteenth century the city of Konigsberg (in Prussia) was isolated into four areas by the Pregel waterway. Seven scaffolds associated these districts as appeared in Figure 2 (a). Areas are appeared by A, B, C, D individually. It is said that the townfolk of Konigsberg delighted themselves by attempting to discover a course that crossed each extension just once (It was OK to go to a similar island any number of times).

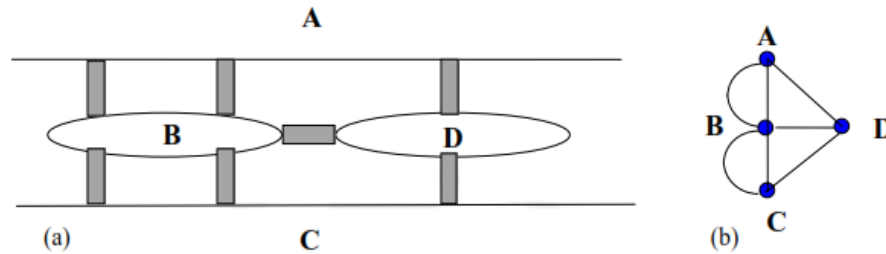


Figure 2: (a) A map of Königsberg (b) A graph representing the bridges of Königsberg

Euler examined whether it is conceivable to have such a course by utilizing the graph appeared in Figure 2 (b). He distributed the primary paper in graph theory in 1736 to show the difficulty of such a course and give the conditions which are important to allow such a walk. Graph theory was destined to consider problems of this sort.

Graph theory is one of the themes in a zone of mathematics portrayed as Discrete Mathematics. The problems just as the strategies for solution in discrete mathematics contrast on a very basic level from those in constant mathematics. In discrete mathematics we "check" the number of articles while in constant mathematics we "measure" their sizes. Albeit discrete mathematics started as right on time as man figured out how to check, it is ceaseless mathematics which has since quite a while ago ruled the historical backdrop of mathematics. This image started to change in twentieth century. The principal significant improvement was the change that occurred in the origination of mathematics. Its main issue transformed from the idea of a number to the idea of a set which was progressively reasonable to the techniques for discrete mathematics than to those of consistent mathematics. The second sensational point was the expanding utilization of PCs in the public eye. A great part of the theory of software engineering utilizes ideas of discrete mathematics.

Graph theory as an individual from the discrete mathematics family has an amazing number of applications, to software engineering as well as to numerous different sciences (physical, organic and social), designing and trade. A portion of the significant topics in graph theory are appeared in Figure 3.

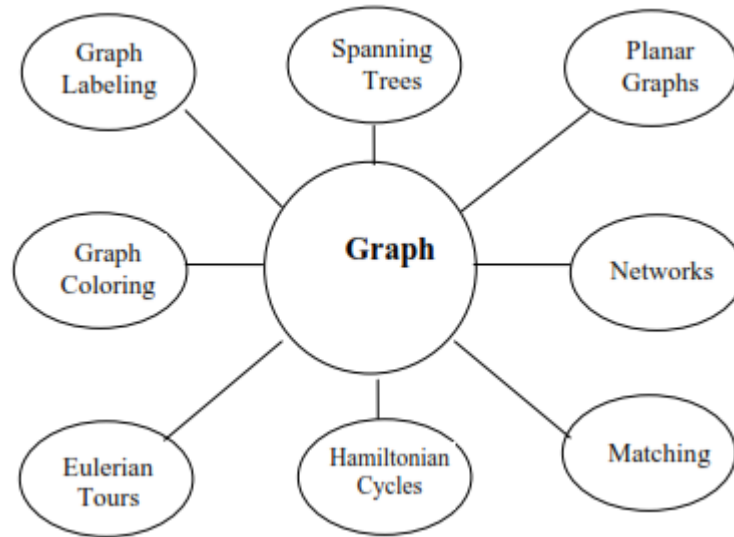


Figure 3: Some Graph Theory

The purpose of this study is to give a few outcomes in a class of problems arranged as Graph labeling. Let G be an undirected graph without loops or twofold associations between vertices. In labeling (valuation or numbering) of a graph G , we partner unmistakable nonnegative whole numbers to the vertices of G as vertex labels (vertex esteems or vertex numbers) so that each edge gets a particular positive whole number as an edge name (edge worth or edge number) contingent upon the vertex labels of vertices which are occurrence with this edge.

Enthusiasm for graph labeling started in mid-1960s with a guess by Kotzig-Ringel and a paper by Rosa[90]. In 1967, Rosa distributed a spearheading paper on graph labeling problems. He called a function f a β -labeling of a graph G with n edges (Golomb [45] along these lines called such labeling graceful and this term is presently the well known one) if f is an infusion from the vertices of G to the set $\{0, 1, \dots, n\}$ to such an extent that, when each edge is labeled with the supreme estimation of the contrast between the labels of the two end vertices, the subsequent edge labels are particular. This labeling gives a successive labeling of the edges from 1 to the number of edges. Any graph that can be gracefully labeled is a graceful graph.

Examples of graceful graphs are shown in Figure 1.4.

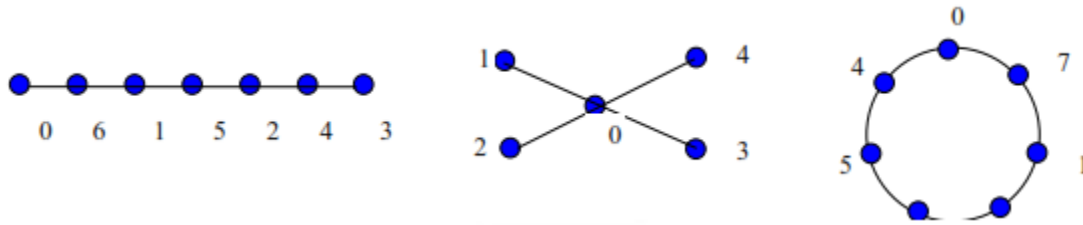


Figure 4: Examples of graceful labeling of graphs

Although numerous groups of graceful graphs are known, a general essential or adequate condition for gracefulness has not yet been found. Likewise It isn't known whether all tree graphs are graceful. Another significant labeling is a α -labeling or α -valuation which was additionally presented by Rosa [90]. A α -valuation of a graph G is a graceful valuation of G which likewise fulfills the accompanying condition: there exists a number γ ($0 \leq \gamma < E(G)$) to such an extent that, for any edge $e \in E(G)$ with the end vertices $u, v \in V(G)$, $\min \{ \text{vertex name } (v), \text{vertex name } (u) \} \leq \gamma < \max \{ \text{vertex mark } (v), \text{vertex name } (u) \}$

Obviously in the event that there exists a α -valuation of graph G , at that point G is a bipartite graph. The main graph in Figure 4 is a way with six edges and it has a α -labeling with $\gamma = 3$.

During the previous thirty years, more than 200 papers on this points have been showed up in diaries. Despite the fact that the guess that all trees are graceful has been the focal point of a large number of these papers, this guess is as yet unproved. Tragically there are hardly any broad outcomes in graph labeling. In reality in any event, for problems as barely engaged as the ones including the unique classes of graphs, the labelings have been hard-won and include a huge number of cases.

Finding a graph that has a α -labeling is another regular methodologies in numerous papers. The accompanying condition (because of Rosa) is known to be important and on account of cycles likewise adequate for a 2-ordinary graph $G = (V, E)$ to have a α -labeling: $|E(G)| \equiv 0 \pmod{4}$. In 1982, Kotzig guessed that this condition is likewise adequate for a 2-normal graph with parts.

Labeled graphs fill in as valuable apparatuses for an expansive scope of applications. Sprout and Golomb in two brilliant reviews have introduced deliberately a use of graph labeling in many research fields, for example, coding theory problems, X-beam crystallographic investigation, correspondence network structure, ideal circuit design, basic voltage generator, and added

substance number theory. Right now limit our conversation to applications of graceful labeling and its varieties in decomposition of graphs, ideal arrangement of distinction sets, and number groupings, for example, the Skolem succession:

A graph G is a constrained nonempty set of things gathered vertices with a ton of unordered pairs of unmistakable vertices of G which is called edges showed by $V(G)$ and $E(G)$, independently. In case $e = \{u, v\}$ is an edge, we form $e = uv$; we express that e joins the vertices u and v ; u and v are neighboring vertices; u and v are event with e . In case two vertices are not joined, by then we express that they are non-connecting. If two unmistakable edges are scene with a normal vertex, by then they are said to be coterminous each other.

GRAPH DECOMPOSITION

Definition 1: A decomposition of a graph G is a family $H = (H_1, H_2, \dots, H_n)$ of sub graphs of G such that each edge of G is contained in exactly one member of H . In fact G is the edge disjoint union of its sub graphs H_i

$$\begin{aligned}
 E(H_i) \cap E(H_j) &= \emptyset \quad \text{for } i \neq j; & E(G) \\
 &= \cup E(H_i) \quad i = 1, 2, \dots, n; & V(G) \\
 &= \cup V(H_i) \quad i = 1, 2, \dots, n.
 \end{aligned}$$

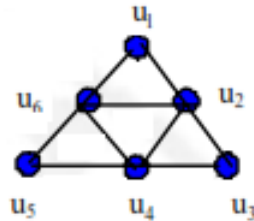
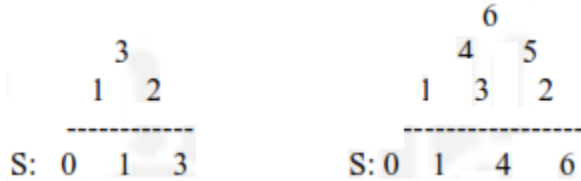


Figure 5: Decomposition of a graph

For example the graph G shown in Figure 1.5 has a decomposition $H = (H_1, H_2, H_3)$ into three K_3 : $E(H_1) = \{(u_1, u_2), (u_2, u_6), (u_1, u_6)\}$, $E(H_2) = \{(u_2, u_3), (u_3, u_4), (u_2, u_4)\}$, $E(H_3) = \{(u_1, u_4), (u_1, u_6)\}$ and $V(H_1) = (u_1, u_2, u_6)$, $V(H_2) = (u_2, u_3, u_4)$, $V(H_3) = (u_4, u_5, u_6)$.

Biraud and Blum and Ribes [5] were most likely the initial ones to watch a connection between graceful labeling of graphs and PSDS. The ordinary PSDS (1,n,1) is a PSDS with one segment beginning with 1. There exists just two normal PSDS (1,n,1) [5]. They are



The mirror images of the above PSDS are also PSDS.

LABELING, COVERING AND DECOMPOSING OF GRAPHS

Definition 5 A standards in a numerical framework $(\Sigma; R)$ is said to be Smarandachely denied on the off chance that it carries on in at any rate two unique ways inside a similar set Σ , i.e., approved and invalidated, or just invalidated however in different unmistakable manners. A Smarandache framework $(\Sigma; R)$ is a scientific framework which has at any rate one Smarandachely denied rule in R .

Definition 6 For an integer $m \geq 2$, let $(\Sigma_1; R_1), (\Sigma_2; R_2), \dots, (\Sigma_m; R_m)$ be m mathematical systems different two by two. A Smarandache multi-space is a pair $(\Sigma; e \text{ Re})$ with

$$\tilde{\Sigma} = \bigcup_{i=1}^m \Sigma_i, \quad \text{and} \quad \tilde{R} = \bigcup_{i=1}^m R_i.$$

Definition 7 A maxims is said to be Smarandachely denied if the saying carries on in at any rate two unique ways inside a similar space, i.e., approved and invalidated, or just invalidated yet in numerous particular manners.

Example 1 Let us consider an Euclidean plane R^2 and three non-collinear points A, B and C . Characterize s -points as all standard Euclidean points on R^2 and s -lines any Euclidean line that goes through one and only one of points A, B and C , for example, those appeared in Fig.6.

(i) The adage (A5) that through a point outside to a given line there is just one equal going through it is presently supplanted by two articulations: one equal, and no equal. Leave L alone a s -line goes through C and is equal in the Euclidean sense to AB . Notice that through any s -point

not lying on AB there is one s-line corresponding to L and through some other s-point lying on AB there is no s-lines corresponding to L, for example, those appeared in Fig.6(a).

(ii) The maxim that through any two particular points there exist one line going through them is presently supplanted by; one s-line, and no s-line. Notice that through any two unmistakable spoints D, E collinear with one of A, B and C, there is one s-line going through them and through any two particular s-points F, G lying on AB or non-collinear with one of A, B and C, there is no s-line going through them, for example, those appeared in Fig.6(b).

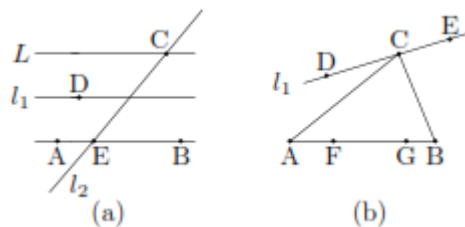


Fig.6

Definition 8 A combinatorial system CG is a union of mathematical systems $(\Sigma_1; R_1), (\Sigma_2; R_2), \dots, (\Sigma_m; R_m)$ for an integer m, i.e.,

$$\mathcal{C}_G = \left(\bigcup_{i=1}^m \Sigma_i; \bigcup_{i=1}^m R_i \right)$$

with an underlying connected graph structure G, where

$$V(G) = \{\Sigma_1, \Sigma_2, \dots, \Sigma_m\},$$

$$E(G) = \{(\Sigma_i, \Sigma_j) \mid \Sigma_i \cap \Sigma_j \neq \emptyset, 1 \leq i, j \leq m\}.$$

Vertex-Edge Labeled Graphs with Applications

1 Application to Principal Fiber Bundles

Definition 9 A labeling on a graph $G = (V, E)$ is a mapping $\theta_L : V \cup E \rightarrow L$ for a name set L, meant by GL.

In the event that $\theta L: E \rightarrow \emptyset$ or $\theta L: V \rightarrow \emptyset$, at that point GL is known as a vertex labeled graph or an edge labeled graph, meant by GV or GE, separately. Else, it is known as a vertex-edge labeled graph.

Example:

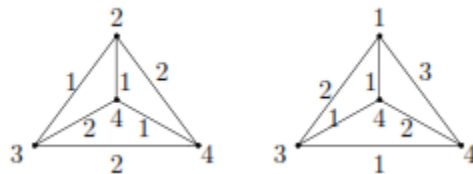


Fig.7

CONCLUSION

A vertex v of a graph G is known as a cut-vertex of G if the evacuation of v expands the quantity of parts. An edge e of a graph G is known as a cut edge or extension if the evacuation of e expands the quantity of parts. A lot of edges S is called an edge cut of G if the quantity of segments of $G - S$ is more prominent than that of G . A square of a graph is a maximal associated, non-unimportant subgraph without cut-vertices. A graph is non-cyclic in the event that it has no cycles. A tree is an associated non-cyclic graph. A tree with precisely one vertex of degree > 3 is known as a creepy crawly tree and an established tree comprising of k - branches where i th branch is a path of length I , is called an olive tree. Let G be a graph with vertex set $\{ v, v_2, \dots, v \}$. At that point the graph obtained by presenting n new vertices u_1, u_2, \dots, u and edges $u_1 v_i$ is indicated by G . The separation between two vertices u and v in an associated graph G is the length of the most limited $u - v$ path in G and is signified by $d(u, v)$. The level of a vertex v in a graph G , indicated by $d(v)$, is the quantity of edges episode with v . The base degree among the vertices of G is indicated by $\delta(G)$, while the greatest degree among the vertices of G is signified by $\Delta(G)$. In the event that $\delta(G) = \Delta(G) = r$, at that point all vertices have a similar degree and G is known as a customary graph of degree r . In the event that $d(v) = 0$, v is called a disconnected vertex.

REFERENCES

- L. Pandiselvi, S. NavaneethaKrishnan and A. NellaiMurugan, Path Related V4 Cordial Graphs. *International Journal Of Recent Advances in Multidisciplinary Research*, Vol. 03, Issue 02, pp.1285-1294, February 2016.
- A. Rosa, On certain valuations of the vertices of a graph, *Theory of graphs (International Symposium, Rome)*, July (1966).
- C. Sekar, *Studies in Graph Theory*, Ph.D.Thesis, Madurai Kamaraj University, 2002.
- R. Sridevi, S. Navaneethakrishnan and K. Nagarajan, Odd-Even graceful graphs, *J.Appl.Math.& Informatics* Vol.30 (2012), No. 5-6, pp. 913-923.
- M. Sundaram, R. Ponraj and S. Somasundaram Prime Cordial Labeling of graphs, *Journal of Indian Academy of Mathematics*, 27 (2005) 373-390.
- R. Sridevi, A. NellaiMurugan, A. Nagarajan, and S. Navaneethakrishnan Fibonacci Divisor Cordial Graphs, *International Journal of Mathematics and Soft Computing*.
- R. Sridevi, S. Navaneethakrishnan Some New Graph Labeling Problems and Related Topics(Thesis). ManonmaniamSundaranar University, January 2013.
- Solomon W. Golombo, How to number a graph, *Graph Theory and Computing*, Academic Press, New York (1972), 23-37.
- R. Tao, On k-cordiality of cycles, crowns and wheels, *Systems Sci.*, 11 (1998), 227-229.
- R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan, Special classes of Divisor Cordial graphs, *International Mathematical Forum*, vol.7, 2012, no.35, 1737-1749.
- R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan, Divisor cordial graph, *International Journal of Mathematical Comb.*, Vol .4(2011),15-25.
- S.K. Vaidya, U.M. Prajapati Fibonacci and Super Fibonacci Graceful Labelings of some cycle related graphs, *International J.Math.Combi.* Vol.4, (2011), 59-69.

R. Vasuki, A.Nagarajan *Studies in Graph Theory – Labeling Problems in Graphs*(Thesis), Manonmaniam Sundaranar University, December 2010.

M.Z. Youssef, On k-cordial labeling, *Australas .J. Combin.*, Vol 43(2009), 31- 37.

G.J. Gallian, A Dynamic survey of graph labeling, *The electronic journal of combinotorics*, 16 (2009), #DS6.

Z. Gao, Odd graceful labelings of some union of graphs, *J. Nat. Sci. Heilongjiang Univ*, 24(2007) 35-39.

R.B. Gnanajothi, *Topics in Graph Theory*, Ph.D. Thesis, Madurai Kamaraj University, 1991.

S.W. Golombo, *How to number a graph in graph Theory and Computing*, R.C.Read, ed., Academic Press, New York(1972) 23-37.