IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received October 2, 2018, accepted November 1, 2018, date of publication November 5, 2018, date of current version December 3, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2879741

Aggregation Functions Considering Criteria Interrelationships in Fuzzy Multi-Criteria Decision Making: State-of-the-Art

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This work was supported in part by the National Natural Science Foundation of China under Grant 61702274, in part by the Natural Science Foundation of Jiangsu Province under Grant BK20170958, and in part by the China-USA Computer Science Researcher Center.

ABSTRACT Aggregation function is an important component in an information aggregation or information fusion system. Interrelationships usually exist between the input arguments (e.g., the criteria in the multi-criteria decision making) of an aggregation function. In this paper, we make a comprehensive survey on the aggregation operators (AOs) that consider the argument interrelationships in crisp and fuzzy settings. In particular, we discuss the mechanisms of modeling the argument interrelationships of the Choquet integral (CI), the power average (PA), the Bonferroni mean (BM), the Heronian mean (HM), and the Maclaurin symmetric mean (MSM) operators, and introduce their extended (e.g., generalized or weighted) forms and their applications in different fuzzy sets. In addition, we compare these five types of operators and summarize their advantages and disadvantages. Furthermore, we discuss the applications of these operators. Finally, we identify some future research directions in the AOs considering the argument interrelationships. The reviewed papers are mainly about the development of the CI, the PA, the BM, the HM, and the MSM in (fuzzy) MCDMs, most of which fall in the period of 2009–2018.

INDEX TERMS Aggregation function, criteria interrelationship, Bonferroni mean, Choquet integral, Heronian mean, power average, Maclaurin symmetric mean.

I. INTRODUCTION

Information aggregation is a basic concern in an information processing system like pattern recognition, decision making, and image processing [38]. A mathematical aggregation operator (AO) is generally used to simultaneously estimate several information pieces (e.g. numerical values) from different sources to make a decision, answer questions or prove hypotheses [65]. In this paper, we focus on the discussion of the mathematical AOs in a context of multi-criteria decision making (MCDM). However, the properties and the functions of the discussed operators can be applied to the other contexts of the information aggregation. In this paper, we use the word 'variable' to represent the value of a criterion (or an attribute, an input argument). In addition, all the values we discuss in this paper are nonnegative real values.

The MCDM devotes to the development of the decision support tools to solve the complex problems that involve multiple conflicting objectives or goals [61]. Aggregation functions for reducing the dimensions of the criteria play a fundamental role in MCDMs [38], e.g. t-norm [51], t-conorm [51], arithmetic mean (AM) [111], and geometric mean (GeoM) [111]. Traditional aggregation functions assume the criteria are independent, and the effects of the criteria are additive [189], e.g. the weighted averaging [242]. However, practical applications always contain different types of interrelationships between the decision criteria or the input attributes, so the independent assumption usually cannot be satisfied [189]. Hence, a group of aggregation functions modelling the criteria interrelationships appear, e.g. the Choquet integral (CI) [152].

The imprecision and uncertainty are another two critical problems in the MCDM [123], which usually emerge along with the incomplete, ambiguous, subjective, or conflicting information [190]. To model such imprecision and uncertainties, Zadeh [263] established the fuzzy set theory, which has been generalized and applied in different decision making scenarios. Since then, a series of fuzzy sets (FSs) have been proposed for different purposes. For example, Zadeh [264] introduced the type-2 fuzzy set to model the 'attribution of membership degrees to the elements'. The intuitionistic fuzzy set (IFS) [6] is characterized by the membership and non-membership values which are assigned to the factors in a universe of discourse. The Pythagorean fuzzy set [271] (PyFS) improves the IFS by extending the range of the situations to be modelled. As another extension of the IFS, the hesitant fuzzy set [202] (HFS) allows the membership degree of an element to be a set of values rather than a specific value or a value interval.

The fuzzy MCDM (FMCDM) is a decision making technique having the capability of using the fuzzy number (FN) to measure the imprecision and the uncertainties [123]. The AOs having the capability of aggregating different types of the FNs play a key role in the FMCDM. Researchers have extended the classical aggregation functions to adapt to different types of FSs, and proposed a series of fuzzy aggregation functions for each FS [2], [47], [233]. In particular, several fuzzy multi-criteria AOs were proposed to measure and integrate the effects of the criteria interrelationships on the results of the FMCDM, among which the most basic and popular AOs are: *the fuzzy integral (Choquet and Sugeno integral)* [152], *the power average (PA)* [243], *the Bonferroni mean (BM)* [246], *the Heronian mean (HM)* [259] and the *Maclaurin symmetric mean (MSM)* [135]. In particular, the Sugeno integral is an ordinal version of the CI. However, its properties associated with the MCDM have not been well studied [60]. Therefore, in the category of the fuzzy integral, we will focus on reviewing the work related to the CI.

Marichal [134] identified three types of criteria interrelationships: the criteria correlation, the substitutivity of the criteria, and the preferential dependence of the criteria, which can be modelled using the five operators mentioned above by properly defining the interaction types and setting the interaction values. In addition, Chen *et al.* [28] introduced two forms of interrelationships: the homogeneous (*homo*) and the heterogeneous (*hete*). The *hete* interrelationship considers both the dependent and the independent criteria, whereas, the *homo* interrelationship assumes that all the criteria considered are dependent to each other.

The CI [146] is an efficient method to support the decision making problems having the interactive criteria. In addition, Yager [243] introduced the PA that enables the criteria to reinforce each other based on the degree of *Support* between two criteria. In 2009, Yager [246] further introduced the BM to model the *homo* interrelationships. Chen *et al.* [28] improved the BM and proposed an extended BM (EBM) to deal with the *hete* interrelationships. Furthermore, Yu *et al.* [259] proposed using the HM to model the interrelationships, which improves the BM by additionally considering the interrelationships between a criterion and itself, and by reducing the redundant consideration of the interrelationships between two criteria by the BM. Compared to the BM, one of the main advantages of the MSM is to allow the modelling of the relationships among a set of more than two criteria.

In this paper, we make a comprehensive survey on the AOs that consider the criteria interrelationships in the MCDM and the FMCDM. In particular, we briefly introduce the definitions of some important FSs and their development. We also introduce the types of the criteria interrelationships. In the main section, we discuss the mechanisms of the CI, the PA, the BM, the HM, and the MSM for modelling the criteria interrelationships, and introduce their basic extensions in terms of their concepts, where the basic extension is co-occurred with the fuzzy extension. For example, we state that the geometric BM (GeoBM) [229] and the extended BM (EBM) [28] are the basic extensions of the BM; on the other hand, the extensions of the BM, the GeoBM and the EBM to different FSs are the fuzzy extensions. For each operator and its basic extensions, we summarize their mechanisms of modelling the interrelationships, their input parameters, the interrelationships they can model, and their fuzzy extensions. Furthermore, we review the application areas of these AOs. As the PA-, BM-, HM- and MSM-based AOs are mostly applied to the MCDMs, we only focus on the applications of the CI-based operators, which have been applied to improve the performance of some traditional aritificial intelligence technologies, such as the fuzzy rule-based classification system (FRBCS) [9], the classification [97], the clustering [13], the evolutionary algorithms [19], and the TOPSIS-based MCDMs [252]. We also summarize and categorize the applications of the CI-based operators in terms of seven practical application scenarios. Finally, we identify six future research directions of these AOs.

The motivation of this work is to find methods to identify and model the interrelationships that may exist among the criteria in the MCDMs, as Marichal [134] pointed out: the decision criteria have some interactions in many practical situations. However it is still very difficult to model such interaction. In addition to reviewing the popular AOs that model the interrelationships, our work also aims to show the gaps of these AOs in solving the MCDM problems, which can be a reference for the future research in the areas of the AOs and the MCDMs. The main contributions of this paper are as follows:

- we summarize the types of the interrelationships that may exist among the criteria;
- we review the popular AOs that model the criteria interaction, analyze their properties and behavioral patterns, and summarize their basic and fuzzy extensions;
- we review the technical and practical applications of the CI-based operators, and summarize the main purposes and functions of applying the CI in the area;
- we summarize the possible future research directions based on this literature review.

The structure of this paper is as follows: Section II introduces the related work; Section III introduces the types of the interrelationships; Section IV reviews the five AOs modelling the criteria interrelationships; Section V presents the applications of these operators; Section VI summarizes the identified research gaps and the future research directions; and Section VII concludes this paper.

II. RELATED WORK

A. MULTI-CRITERIA DECISION MAKING AND AGGREGATION FUNCTIONS

A multi-criteria decision making (MCDM) problem [246] is to select one alternative from a set of alternatives $O = \{o_1, \ldots, o_m\}$ based on the satisfaction degree of a decision maker to an alternative in terms of a set of criteria $X = \{x_1, \ldots, x_n\}$. We use $x_j(i) \in [0, 1]$ to represent the satisfaction degree to the alternative o_i in terms of the criterion x_j. An MCDM problem can then be represented by a decision matrix M:

$$M = \begin{bmatrix} x_1(1) & \cdots & x_n(1) \\ \cdots & \cdots & \cdots \\ x_1(m) & \cdots & x_n(m) \end{bmatrix}$$

An MCDM method uses a pointwise valuation function to valuate o_i with respect to (w.r.t.) the n criteria, which is defined as $D(o_i) = f(x_1, \ldots, x_j, \ldots, x_m)$. The selection result is o_i having the largest valuation D. Function f is an aggregation function that has three significant properties: (1) indifference: $D(o_i)$ only depends on the satisfaction degree of the decision maker to o_i in terms of $x_j, \forall j \in [1, n]$, but does not depend on the satisfaction degrees of the decision maker to the other alternatives; (2) monotonicity: if o_i and o_k are two alternatives and $x_j(i) \geq x_j(k)$ for all $x_j \in X$, then $D(o_i) \geq D(o_k)$; (3) grounding: if $x_j(i) = 0, \forall x_j \in X$, then $D(o_i) = 0$; and if $x_j(i) = 1, \forall x_j \in X$, then $D(o_i) = 1$. An aggregation function is formally defined in Def. 1 [12].

Definition 1: Let $I = [0, 1]$, then an aggregation function f is a mapping $f : I^n \rightarrow I$, where $f(0, \ldots, 0) = 0$, $f(1, \ldots, 1) = 1$, and $f(g_1, \ldots, g_n) \geq f(h_1, \ldots, h_n)$ if $g_i \geq h_i$, for $\forall i$.

Aggregation functions are divided into three categories by the boundedness of their outputs: if $min(x) \leq f(x) \leq max(x)$, f is averaging or idempotent, i.e. $f(x, x, \cdots, x) = x$; if $f(x) \leq min(x)$, f is conjunctive; and if $f(x) \geq max(x)$, f is disjunctive. One necessary condition of an aggregation function is that the criteria considered should be mutually independent.

B. OVERVIEW OF FUZZY SETS

In this section, we summarize the concepts and relations of the popular fuzzy sets to which the AOs of modelling the criteria interrelationships have been applied, which is based on the work of Bustince *et al.* [20]. We also reference other important survey work as the complementary material [123], [193], [258].

Based on our literature review, the AOs of modelling the criteria interrelationships have been applied to the following fuzzy sets and their extensions: type-1 FS (T1FS), type-2 FS (T2FS), interval-valued type-2 FS (IVT2FS), intuitionistic FS (IFS), hesitant FS (HFS), Pythagorean FS (PyFS), qth rung orthopair FS (qROF), fuzzy multiset (FM), neutrosophic set (NS), and linguistic term set (LTS).

The concept of the fuzzy set (FS) was first proposed by Zadeh [263] to process the imperfect information which is *imprecise, uncertain, vague, incomplete, partially true or partially possible* in certain aspects [20]. A fuzzy set maps a crisp element in a universe to a membership degree of a fuzzy element through a membership function, which is now normally called the T1FS. One of the most important problems of T1FS is how to define the membership function so as to determine the membership degree of each element. However, it is usually very non-evident and difficult to define the membership functions.

To better capture the fuzzy information, a series of generalizations of the FS were proposed. A T2FS has the membership functions that induce the membership degrees as the FSs in [0, 1]. By extending this concept, type-n FS can be defined. For example, a type-3 FS is a FS whose membership degrees are T2FSs. The T2FS has been utilized by many applications [137]. However, compared to the T1FS, T2FS has much higher computional complexity.

Grattan-Guinness [62] defined the set valued FS (SVFS) whose membership degrees are the subsets of [0, 1]. Torra [202] defined the HFS that has similar definition as the SVFS. Then Torra [202] further defined the union and the intersection operators for the HFS to extend those operations proposed by Zadeh. The typical HFS (THFS) [10] is a HFS whose membership degree of each element is represented by a finite and non-empty subset of [0, 1]. Most of the applications employed the THFS [172].

The concept of the IVFS was proposed in 1975 [177] to define the membership degree of an element as an interval rather than a precise value or a set of values. In 1976, Grattan-Guinness [62] defined that an IVFS is a FS where the membership degree of an element is a closed subintervals of [0, 1]. The IVFS is a particular case of the L-FS, the SVFS, the HFS, and the T2FS. The grey set (GS) and the shadowed set (SS) are a particular case of the IVFS. A variety of applications have adopted the IVFS to improve the performance of processing the imperfect information by using the T1FS. In addition, compared to the T2FS, working with the IVFS has lower computaional complexity, which is just slightly higher than that of working with the T1FS. The interval-valued type-2 FS (IVT2FS) is defined in 2006 [82]. Given an IVFS, if all the membership degrees of the FS always equal to 1, this IVFS is an IVT2FS. The concept of the IVT2FS is not the same as that of the IVFS, but it is a generalization of the IVFS [184].

Atanassov [7] proposed the concept of the IFS in 1983. An IFS is defined by two functions: the membership and nonmembership, which respectively represent the degrees of an element belonging to and not belonging to an IFS. In addition, the IFS uses a definition of the intuitionistic or hesitant index to measure the hesitation of the experts to determine the membership and nonmembership values. The FS is a particular case of the IFS.

To relax the restriction given to the IFS, Smarandache [183] proposed the Neutrosophic set (NS) in 2002, and Yager proposed the Pythagorean fuzzy set (PyFS) in 2013 [247]. An IFS is a particular case of a PyFS, a bipolar-valued fuzzy set [85], an HFS and a SVFS. The IFS and the IVFS are conceptually different as the IVFS does not consider the nonmembership degree of an element, and the hesitance of an expert to assign the membership and nonmembership values. Ye [251] used an example to analyze the efficiency of the IFS and the IVFS, which shows that the results based on the IFS are closer to the true values than those based on Zadeh's FS [263], and the interval representation does not make much sense in practical applications. The IFS has been proved very useful in the area of decision making [258], psychology [64], medicine [198], and image processing [275].

Anther two major extensions of the FS are the interval-valued IFS (IVIFS) [5] and the fuzzy multiset (FMS) [241]. The IVIFS extends the IFS by assigning the interval values to both of the membership and nonmembership degrees. The FMS assigns several membership degrees to an element. Based on their definitions, the IVIFS and the FMS are special cases of the HFS and the SVFS respectively. Although a number of scholars have applied the IVIFS to a series of applications, none of them proved a better performance of the IVIFS than that of the FS and the other extensions of the FS.

We have analyzed the main extensions of the FS. There have been a variety of the FS extensions based on these existing extensions. For example, Yager [248] further generalized the PyFS to the qth rung orthopair FS (qROF). To allow the membership and nonmembership degrees of a Pythagorean FS (PyFS) to have the form of interval values, Peng and Yang [160] proposed the interval-valued PyFS (IVPyFS). To consider the hesitant situation in the PyFS, Liang and Xu [93] combined the HFS and the PyFS to define the hesitant PyFS (HPyFS), in which the membership and nonmembership degrees of a PyFS are evaluated by the HFSs. Zhu *et al.* [282] proposed the intuitionistic hesitant fuzzy set (IHFS) that integrates the concepts of the membership and nonmembership into the HFS, which enables the HFS to manifest a precise gradual composite entity or to process an epistemic construction of an ill-known object by using a set of possible values. The probabilistic hesitant fuzzy set (PHFS) [269] improves the HFS by solving three main problems to avoid the information loss: (1) the sum of the occurrence probability of different memberships of an element may be less than 1; (2) the occurrence probability of an element membership may be an irrational number; and (3) it is difficult to use the complex expression of the fuzzy multiset.

Based on our review of literature, the AOs with critera interrelationships are also used to operate the linguistic variable (LV). We introduce the definition of the LV that is used in this paper as follows.

A linguistic variable (LV) is defined in Def. 2 [136].

Definition 2: A linguistic variable (LV) is characterized by a quintuple (x, T, X, L, M), where x is the variable, T is the set of linguistic terms of x, X is an universe of discourse, L is a syntactic rule for generating the terms, and M is a semantic rule to associate each term with its meaning.

The $T = \{t_i | i = -k, \cdots, -1, 0, 1, \cdots, k\} (k \in N^+)$ of an LV should satisfy the following conditions: (1) $t_i \prec t_j$ iff $i < j$; (2) $neg(t_i) = t_{-i}$ and $neg(t_0) = t_0$; (3) if $i \geq j$, $\max(t_i, t_j) = t_i$; and (4) if $i \geq j$, $\min(t_i, t_j) = t_j$.

Scholars have integrated different concepts of the FS into the LV. For example, Liu and Jin [101] proposed the trapezoidal fuzzy LV (TraFLV) which represents the value of the linguistic term as the trapezoidal FNs. Herrera and Herrera-Viedma [70] defined the 2-tuple linguistic term (2TLT) to represent the linguistic information using a 2-tuple (t_i, a_i), where t_i is a linguistic label in an LTS T and $a_i \in [-0.5, 0.5)$ represents the symbolic translation of t_i. Ye [252] defined the single valued neutrosophic linguistic set (SVNLS), and then Liu [115] proposed the single valued neutrosophic uncertain linguistic set (SVNULS) by considering the uncertain LV in the SVNLS. In addition, researchers have defined the intuitionistic uncertain linguistic set (IULS) [106], the Pythagorean uncertain linguistic set (PyULS) [128], the hesitant fuzzy linguistic set (HFLS) [262] and the hesitant linguistic intuitionistic fuzzy set [217] based on the concept of the LV.

III. TYPES OF INTERRELATIONSHIPS BETWEEN CRITERIA

We interpret an interrelationship between criteria from two perspectives: their meanings [134] and their interaction forms [41].

A. TYPES OF INTERRELATIONSHIPS IN TERMS OF MEANINGS

There are three types of interrelationships distinguished by their definitions [134]: the correlation between criteria, the substitutivity/complementary of two criteria, and the preferential dependence between criteria.

The *correlation* between two criteria usually reflects a linear relationship between them, which has the positive, the negative, and the independent forms. Given two criteria x_i and x_j, suppose $f_i \in [0, 1]$ and $f_j \in [0, 1]$ are their partial utilities (or satisfaction degrees) respectively, and f_{ij} is their combined utility (or satisfaction degree). If $f_{ij} < f_i + f_j$, then x_i and x_j are positively correlated or negatively synergistic (negatively correlated); if $f_{ij} > f_i + f_j$, then they are negatively correlated or positively synergistic (positively interacted); and if $f_{ij} = f_i + f_j$, then they are independent. For example, if we evaluate the academic ability of a group of students on three subjects: statistics, probability, and literature. The criteria *statistics* and *probability* are redundant (i.e. positively correlated or negatively synergistic) in some degree, as we believe a student who is good at statistics is usually good at probability [134]. The correlation is usually determined based on the sample data or the objective knowledge.

If we say two criteria are *substitutive* with each other, we mean each of them has similar contribution with the contribution of their combination, i.e. $f_{ij} \approx f_i \approx f_j$. In this case, we only need to consider one criterion x_i or x_j rather than considering both of them to evaluate a decision. If we say two criteria are complementary with each other, we mean one of them alone cannot make an evident contribution to the utility of an alternative. However, considering both of them simultaneously will have a significant contribution, i.e. $f_T \approx f_{T\bigcup i} \approx f_{T\bigcup j} < f_{T\bigcup ij}$, where T represents a subset of criteria $T \subseteq X/ij$. In this case, we would like to evaluate an alternative by taking into account x_i and x_j simultaneously.

The preferential independence is defined as: let $S \subset X$ be a subset of criteria, X/S be the subset of criteria excluding S, and a and b be two alternatives, if a is prefered to b w.r.t. the values of S, represented by $a(S_a) \succ b(S_b)$, we have $a(S_a \bigcup T_a) \succ b(S_b \bigcup T_b)$ for all $T \subseteq X/S$ and $T_a = T_b$, then we say S is preferentially independent with T.

B. TYPES OF INTERRELATIONSHIPS IN TERMS OF INTERACTION FORMS

The interrelationships between criteria are categorized into two groups in terms of their interaction forms: the *homo* and *hete*. We explain these two types of interrelationships based on Fig. 1 and Fig. 2 respectively [41]. Given a set of criteria X, if each $X_i \in X$ has relations with $\forall X_j \subseteq X/X_i$, where X/X_i represents the criteria in X except x_i, then the criteria in X have *homo* interrelationships (Fig. 1). If X is separated into two groups: B and D (Fig. 2), where B contains a subset of criteria $x_{b1}, \ldots, x_{br}$ in which each criterion $x_{bi}, \forall i \in [1, \ldots, r]$ has interrelationships with the criteria in B_i ($B_i \subseteq B/x_{bi}$); and D contains a subset of criteria where for all $x_{di} \in D$, x_{di} is independent with x_j for $\forall x_j \in X/x_{di}$, the criteria in X have *hete* interrelationships.

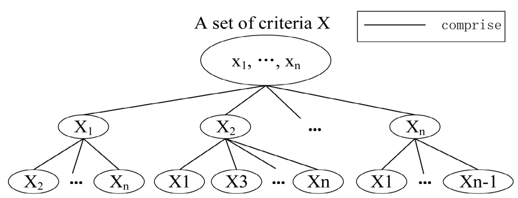

FIGURE 1. ***Homo*** **interrelationships among criteria, where a solid line from the node in an upper level to the node in a lower level represents an inclusion relation [41].**

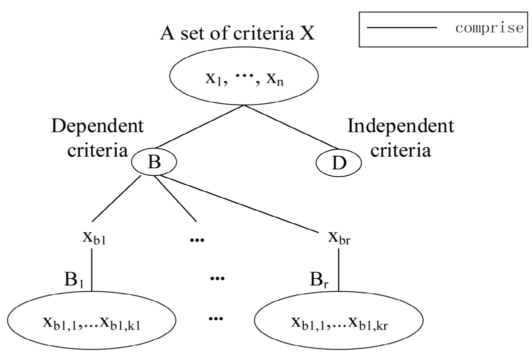

FIGURE 2. ***Hete*** **interrelationships among criteria, where a solid line from the node in an upper level to the node in a lower level represents an inclusion relation [41].**

IV. AGGREGATION OPERATORS PROCESSING CRITERIA INTERRELATIONSHIPS

The AOs are an important tool in the information fusion. Many researchers explored ways to process the interaction between attributes (e.g. criteria in MCDM or the fuzzy rules in fuzzy reasoning) using AOs. Typical examples include the CI [152], the PA [243], the BM [11], the HM [259] and the MSM [135]. In this section, we investigate their definitions, mechanisms of processing the criteria interactions, and applications.

A. CHOQUET INTEGRAL

A CI is a kind of utility AO that is capable of measuring the influence of the importance of the individual criteria and the importance of the interrelationships among criteria [204]. A set of importance values needs to be determined by users or learning algorithms before using the CI, which is called a fuzzy measure (FM). One of the critical steps of using the CI is to precisely define a FM. In this section, we introduce the CI-related work of the criteria aggregation with the consideration of their interrelationships.

1) BASIC DEFINITIONS REGARDING THE CI

Let $X = x_1, \cdots, x_n$ be a set of variables ($\forall i \in [1, n], x_i \in [0, 1]$), and $\Gamma = pow(X)$ be the set of all subsets of X, the FM is defined in Def. 3.

Definition 3: A discrete FM on X is defined as a set function $\phi : \Gamma \to [0, 1]$ which satisfies the following conditions: (1) $\phi(\emptyset) = 0$ and $\phi(X) = 1$, and (2) if $S \subseteq T \subseteq X$, then $\phi(S) \leq \phi(T)$.

The CI is defined in Def. 4.

Definition 4: Let ϕ be a FM on X, the discrete CI of X w.r.t. ϕ is

$$C_\phi(x_1, \cdots, x_n) = \sum_{i=1}^{n} x_{(i)}[\phi(x_{(i)}, \cdots, x_{(n)}) - \phi(x_{(i+1)}, \cdots, x_{(n)})] \quad (1)$$

where (.) represents a permutation of X: $x_{(1)} \leq \cdots \leq x_{(n)}$.

Though the fuzzy integral shows more rationality and richness by comparing with the linear additive measures (e.g., weighted averaging (WA) [219]), it has not been widely applied due to the complexity of determining the FM [59]. For example, suppose X is a set of n elements ($|X| = n$), the number of subsets of X is $|pow(X)| = 2^n$. That is, defining a FM on X requires identifying 2^n coefficients for $pow(X)$, which would become very complex when n is large. However, WA only requires n coefficients. Grabisch [59] introduced the k-additive FM to reduce the complexity. In practical applications, it is very difficult for users to determine the importance degree of three or more interrelated criteria, and knowing the weight of a single criterion and the weight of a pair of interrelated criteria is usually enough to derive an efficient fuzzy integral. A 2-additive FM can model this situation, which only requires the determination of $n(n + 1)/2$ parameters [59]. Therefore, most applications assume their problems can be modelled by the 2-additive FM. The k-additive FM (see Def. 5 [59]) is defined based on the Möbius representation of a FM.

Definition 5: A Möbius representation of a FM ϕ is defined as a function m: $\Gamma \rightarrow R$, for each $C \subseteq X$, $m(C) = \sum_{T \subseteq C} (-1)^{|C-T|}\phi(T)$. If $\exists k \in R$, $m(T) = 0$ for all $|T| > k$, then the FM is called k-additive.

Based on the definition of Möbius representation, for a single attribute, e.g., $\{x_i\}$, $\phi(\{x_i\}) = m(\{x_i\})$. In this paper, we use $\phi(x_i)$ to represent $\phi(\{x_i\})$. For a couple of criteria (non-ordered), e.g., $\{x_i, x_j\}$, then $\phi(x_i, x_j) = \phi(x_i) + \phi(x_j) + m(x_i, x_j)$. For a subset $C \subseteq X$ with any number of elements, its 2-additive FM is $\phi(T) = \sum_{x_i \in T} m(x_i) + \sum_{\{x_i,x_j\} \subset T} m(x_i, x_j)$, $T \subset C$.

The Möbius representation of the CI of $\{x_1, \cdots, x_n\}$ w.r.t. a 2-additive FM ϕ' is $C_{\phi'}(x_1, \cdots, x_n) = \sum_{x_i \in T} a(x_i)x_i + \sum_{\{x_i,x_j\} \subset T} a(x_i, x_j) * min\{x_i, x_j\}$.

Shapley [181] proposed the definition of an importance coefficient, namely Shapley index, which has been extended to the CI to evaluate the overall importance of an attribute. Given a FM ϕ of X, the Shapley importance of an attribute x_j w.r.t. ϕ is defined by Formula 2.

$$I_\phi(x_j) = \sum_{T \subseteq X/x_j} \frac{(n - |T| - 1)!|T|!}{n!}(\phi(T \cup x_j) - \phi(T)) \qquad (2)$$

where n and $|T|$ are the number of criteria in X and T respectively.

The Shapley interaction index between criteria measures the interaction degree between two criteria in any subset of criteria [134]. Given a FM ϕ of X, the interaction index between x_j and x_l w.r.t. ϕ is defined by Formula 3.

$$\begin{aligned} I_\phi(x_j, x_l) = &\sum_{T \subset X/\{x_j,x_l\}} \frac{(n - |T| - 2)!|T|!}{(n-1)!} \\ &\times \big(\phi(T \cup \{x_j, x_l\}) - \phi(T \cup x_j) \\ &- \phi(T \cup x_l) + \phi(T)\big) \end{aligned} \qquad (3)$$

Assume ϕ' is a 2-additive FM, the Shapley index of a subset $S \subseteq X$ ($|S| \geq 2$) w.r.t. ϕ' is defined by Formula 4 [146].

$$\begin{aligned} I_{\phi'}(S) = &\sum_{\forall \{x_i,x_j\} \subseteq S} \phi'(x_i, x_j) \\ &+ \frac{1}{2} \sum_{\forall x_i \in S, \forall x_k \in X/S} (\phi'(x_i, x_k) - |S|\phi'(x_k)) \\ &- \frac{|X| + |S| - 4}{2} \sum_{i \in S} \phi'(x_i) \end{aligned} \qquad (4)$$

The Shapley importance of a variable $x_i \in X$ w.r.t. ϕ' is defined by Formula 5 [146].

$$I_{\phi'}(x_i) = \frac{3 - |X|}{2}\phi'(x_i) + \frac{1}{2} \sum_{x_j \in X/x_i} (\phi(x_i, x_j) - \phi(x_i)) \qquad (5)$$

The Shapley importance of a criterion $a_i \in X$ can also be derived by Formula 6 [146].

$$I(x_i) = I(S \bigcup x_i) - I(S) \qquad (6)$$

where $\forall S \subseteq X$, $|S| \geq 2$, and $x_i \notin S$.

Given the Shapley importance (I_i) and the interaction index (I_{ij}) for all $x_i, x_j \in X$, the 2-additive FM ϕ' of a subset $S \subseteq X$ is determined by Formula 7.

$$\phi'(S) = \sum_{x_i \in S} (I_i - \frac{1}{2} \sum_{x_j \in S/x_i} I_{ij}) + \sum_{x_i,x_j \in S} I_{ij} \qquad (7)$$

Then the CI of X w.r.t. ϕ' can be calculated using Formula 1.

An important extension of the discrete CI is the Choquet ordered aggregation (COA) [244], in which the aggregation weights of the variables are formed based on the order of the variables. Yager [244] then proposed a more general form of the COA: the induced Choquet ordered averaging (ICOA). The ICOA assumes that each of the aggregated variables is represented by a tuple $x_i = \langle v_i, a_i \rangle$, where v_i is the order inducing variable (OIV) and a_i is the argument variable. The ICOA is defined in Def. 6.

Definition 6: Given a set of induced variables $X = \{\langle v_1, a_1 \rangle, \ldots, \langle v_n, a_n \rangle\}$, let $vidx(i)$ be the i^{th} largest of the v_i, $b_i = a_{vidx(i)}$ be the $vidx(i)^{th}$ argument variable of a_i, $H_i = \{X_{vidx(k)} | k = 1, \ldots, k\}$, and $m_i = \mu(H_i) - \mu(H_{i-1})$, then

$$ICOA(\langle v_1, a_1 \rangle, \ldots, \langle v_n, a_n \rangle) = M_v^T B_v = \sum_{i=1}^{n} m_i b_i \qquad (8)$$

From the definition, the OIVs are used to induce the order of the argument variables, and the induced order is used to generate the weights of the argument variables. If the OIVs are the same as the argument variables, i.e. $x_i = \langle a_i, a_i \rangle$, the ICOA becomes the COA.

The ICOA is monotonically increasing w.r.t. the argument variables, and has the properties of boundedness, idempotency, permutative, and ratio-scale invariant. In addition, its behavioral pattern depends on the pattern of the FM of the argument variables.

If ϕ is an additive FM, the ICOA becomes the induced OWA. If $\phi_i = 1/n$, the ICOA becomes the arithmetic average operator (see Table 2).

The generalized CI (GCI) was proposed by Yager [245] which uses a parameter to control the raising power of the arguments, which is defined in Def.7.

Definition 7: Given a set of variables $X = \{x_1, \ldots, x_n\}$, let $idx(i)$ be the i^{th} largest of the x_i, $H_i = \{x_{idx(k)} | k = 1, \ldots, k\}$, $w_i = \mu(H_i) - \mu(H_{i-1})$, and $\beta \in [-\infty, +\infty]$ then the GCI of X is defined as:

$$GCI(x_1, \ldots, x_n) = \left(\sum_{i=1}^{n} w_i x_{idx(i)}^{\beta} \right)^{\frac{1}{\beta}} \qquad (9)$$

The FM ϕ and the value of the β control the behavioral pattern of the GCI. Especially, when $\beta = 1$, the GCI becomes the CI.

The induced generalized Choquet ordered averaging (IGCOA) [188] combines the definitions of the ICOA and the GCI, which applies the parameter β to the ICOA. Different values of the β can determine different patterns of the IGCOA. For example, if $\beta = 1$, the IGCOA becomes the

ICOA; if $\beta = 2$, the IGCOA behaves like the order weighted square mean; and if $\beta = -1$, it behaves like the ordered Harmonic average (see Table 2). Based on the IGCOA, Tan and Chen [188] further developed two operators to process the importance of the experts and the fuzzy preference relations of the criteria in MCGDMs, which are the importance IGCOA (IIGCOA) and the preference IGCOA (PIGCOA) respectively. The IIGCOA uses the importance values associated with each expert as the OIV; and the PIGCOA uses the relative preference values as the OIV.

Xu [234] combined the concepts of the CI and the GeoM to define the geometric CI (GeoCI) for the IFS and the IVIFS, and discussed the special cases of the GeoCI. If all the variables are independent, the GeoCI becomes the weighted GeoM (WGeoM) having variable weights $\phi(x_i)$ for $\forall i \in \{1, \ldots, n\}$; and if $\phi(x_i) = \frac{1}{n}$, this WGeoM becomes the GeoM. If $w_i = \phi(A_{(i)}) - \phi(A_{(i-1)})$ satisfies $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$, then the GeoCI becomes the ordered weighted GeoM (OWGeoM); and if $\phi(A) = \frac{|A|}{n}$ for $\forall A \subseteq X$, this OWGeoM becomes the GeoM. If $w_i = Q(\sum_{j \leq i} \phi(x_{(i)})) - Q(\sum_{j<i} \phi(x_{(i-1)}))$ where Q is a basic unit-interval monotonic function (BUM), then the GeoBI becomes the weighted OWGeoM (WOWGeoM); and if $\phi(x_i) = \frac{1}{n}$, then the WOWGeoM becomes the OWGeoM (see Table 2).

The λ-FM [220] and the Shapley index are two efficient tools to reduce the calculation complexity of the usual CI. Meng *et al.* [148] defined the generalized Shapley index based on the λ-FM. By using the generalized Shapley index as the FM, they defined the arithmetic generalized λ-Shapley CI (GλSCI) and the geometric GλSCI (GeoGλSCI). They then extended the GeoGλSCI to the IVIFS.

2) EXTENSIONS OF CI-BASED OPERATORS TO DIFFERENT FSS

The basic extensions of the CI have been extended to different fuzzy sets. We call these fuzzy CI-based operators as their fuzzy extensions.

The usual CI has been extended to different fuzzy sets for fuzzy information aggregation, including the interval 2-tuple linguistic set (I2TLS) [212], the dual hesitant fuzzy set (DHFS) [80], the interval neutrosophic uncertain linguistic set (INULS) [116], the PyFS [161], the HFS [223], the multiset hesitant fuzzy element (MHFE) [159], the interval neutrosophic uncertain linguistic variable (INULV) [116], the interval grey number (IVGN) [216]. Yu [256] proposed the CI operators of the IFS based on the Einstein operation laws.

Much effort has been made for the application of the CI and the COA in the IFS. Tan and Chen [189] proposed an intuitionistic fuzzy CI (IFCI) to aggregate the intuitionistic fuzzy numbers (IFNs) to consider the interrelations among decision criteria. Xu [234] generalized the IFCI and proposed the correlated averaging and geometric operators for the IFS and the IVIFS. Tan and Chen [190] then extended the IFCI based on the induced ordered weighted averaging (IOWA) [249], and proposed an induced intuitionistic fuzzy CI (IIFCI). The IIFCI allows the weights and the order of the criteria to be induced by multiple features of alternatives, and enables the measure of criteria interactions based on the CI. Similar to the ICOA, in IIFCI, the reordering of $a_1, \cdots, a_n$ is induced by the reordering of $v_1, \cdots, v_n$. Wu *et al.* [225] further discussed the properties of the IFCI and the intuitionistic fuzzy conjugate CI. Wei [221] extended the IFCI to the interval-valued intuitionistic hesitant fuzzy set (IVIHFS).

There are some fuzzy extensions of the COA, the ICOA and the GCI. Bustince *et al.* [21] discussed the comparison procedure between two interval values and extended the COA to the IVFS. Lin *et al.* [95] extended the COA to the triangular fuzzy linguistic variable (TriFLV). Qu *et al.* [168] developed the COA for the dual hesitant fuzzy set (DHFS). Li and Zhang [87] defined the COA for the single-valued neutrosophic hesitant fuzzy set (SVNHFS). Xu and Xia [238] combined the definitions of the ICOA and the GCI, and proposed the induced generalized CI operators for the IFS. Wei *et al.* [222] proposed a number of the ICOA operators for the IFS. Ning *et al.* [155] extended the IGCOA to the 2-tuple linguistic variable (2TLV). Tan *et al.* [191] extended the GCI to the HFS.

The extensions of the GeoCI are as follows. Zhang and Yu [270] generalized the GeoCI based on the Einstein operation rules and extended the Einstein CGeo to the IVIFS. Tan *et al.* [192] combined the concepts of the Quasi OWA and the CI to define the Quasi Choquet Geometric (QCGeo) operator, and extended the QCGeo to the IFS. Sajjad *et al.* [176] proposed the GeoCI for the interval-valued Pythagorean fuzzy set (IVPyFS).

Researchers has extended the GλSCI to various fuzzy sets, including the 2-additive IVIFS (2AIVIFS) [149], the uncertain environment [143], the HFS [140], the IVHFS [145], the interval-valued intuitionistic uncertain linguistic variable (IVIULV) [141], the interval-valued hesitant fuzzy linguistic variable (IVHFLV) [139], the interval-valued intuitionistic uncertain linguistic variable (IVIULV) [141], the DHFS [168]–[170], the triangular intuitionistic fuzzy set (TriIFS) [125], [163], the interval-valued dual hesitant fuzzy set (IVDHFS) [171], the IVIFS [144], [149], the uncertain linguistic variable (ULV) [145], the 2TLV [79]. Furthermore, Meng *et al.* proposed the generalized Shapley hybrid (SH) aggregation for the IFS [147], and the induced generalized SHs for the IFS [138] and the HFS [142]. Cheng and Tang [29] proposed the Shapley geometric CI for the IVIFS.

3) SUMMARY OF CI-BASED OPERATORS

We introduced the basic extensions of the CI in this section, which include the COA, the ICOA, the GCI, the IGCOA, the GeoCI, and the GλSCI. The COA formalizes the FM of the aggregated variables based on the ordered argument values. The ICOA generalizes the COA by introducing an OIV. The main difference between the COA and the ICOA is that the reordering step in COA is only based on the

TABLE 1. Summary of CI-based operators.

Operators	Inputs	Relations	Environment
CI	ϕ, X	homo & hete	crisp [154], I2TLS [214], DHFS [80], INULS [118], PyFS [163], HFS [225], MHFE [161], INULV [118], IVGN [218], IFS [258] [191] [236] [192] [227]
IGCOA	ϕ, X, OIV, β	homo & hete	crisp [246], IFS [240] [224], 2TLV [157], IVFS [21], TriFLV [96], DHFS [170], SVNHFS [88], HFS [193]
GeoCI	ϕ, X	homo & hete	IFS [236] [194], IVIFS [236] [272], IVPyFS [178]
GλSCI	λ-FM, X	homo & hete	IFS [149] [140], IVIFS [150] [144], 2AIVIFS [151], uncertain [145], HFS [142] [144], IVHFS [147], IVIULV [143], IVHFLV [141], IVIULV [143], DHFS [172] [170] [171], TriIFS [127], IVDHFS [173], IVIFS [151] [146], ULV [147], 2TLV [79]

TABLE 2. Behavioral patterns of CI-based operators w.r.t. different parameter values.

Parameters	ICOA	IGCOA	GeoCI
additive ϕ	COA	IOWA	WGeoM
additive ϕ and $\phi_i = \frac{1}{n}$	AM	AM	GeoM
$w_i = \phi(A_{(i)}) - \phi(A_{(i-1)})$ and $\sum_i w_i = 1$			OWGeoM
$w_i = \phi(A_{(i)}) - \phi(A_{(i-1)})$, $\sum_i w_i = 1$ and $\phi(A) = \frac{\lvert A \rvert}{n}$			GeoM
$\beta = 1$		ICOA	
$\beta = 2$		OWSM	
$\beta = -1$		OHA	

argument variables, but the ICOA generalizes the COA by inducing an order based on the OIV. When an OIV equals to its corresponding argument variable, the ICOA becomes the COA. The GCI adopts an additional parameter β to control the raising power of the arguments. When $\beta = 1$, the GCI becomes the CI. The IGCOA generalizes both the GCI and the ICOA. The GeoCI introduces the function of the GeoM to the CI. The GλSCI adopts the Shapley index to represent the FM in a simpler way compared to the original CI.

We summarize some basic CIs in Table 1. The inputs of the usual CI and the GeoCI are the FM ϕ and the set of aggregated variables X. The IGCOA requires the OIVs and the power controlling parameter β. The GλSCI requires a λ-FM to generate the Shaley index. The CI-based operators can model both the *homo* and *hete* interrelationships among variables. Because by using the FM, the CI can simultaneously consider the weight of a single variable and the weight of any subset of X. These operators have been extended to different fuzzy sets.

The CI is one of the most representative nonadditive aggregation operators. It is a generalized form of the weighted AM, the OWA, and the weighted minimum and maxmum operators [58]. We have mentioned that when the value of an OIV of a variable equals to the value of the argument variable, the COA becomes the CI. Table 2 shows the special behavioral patterns of the ICOA, the IGCOA, and the GeoCI when their parameters take special values. We can see that the COA is a special case of the ICOA. The IGCOA is a general form of the CI, the COA, and the GCI. When $\beta = 1$ and the FM ϕ is additive, the IGCOA becomes the COA. The GeoCI is the generalization of the GeoM operators. The fuzzy extensions of a CI-based operator can be applied to the special cases of this operator.

B. PA-BASED AGGREGATION OPERATORS

To take into account the interrelationship between the input arguments in an information fusion, Yager [243] introduced the concept of the PA (see Def. 9) to enable the arguments to support and reinforce each other. In the PA, the contribution of a variable to the aggregating result depends on the values of the other variables.

1) BASIC EXTENSIONS OF PA-BASED OPERATORS

To define the PA, we first give the definition of *support*. The *support* between two criteria is defined in Def. 8 [243].

Definition 8: Let $X = \{x_1, \cdots, x_n\}$ be a set of n criteria, where for all $i \in [1, n]$, $x_i \in [0, 1]$, $Supp(x_i, x_j)$ is the support for x_i from x_j, which satisfies the following properties: (1)

1) $Supp(x_i, x_j) \in [0, 1]$.
2) $Supp(x_i, x_j) = Supp(x_j, x_i)$.
3) If $d(x_i, x_j) < d(x_k, x_r)$, then $Supp(x_i, x_j) \geq Supp(x_k, x_r)$, where $d(x_i, x_j)$ is the distance between x_i and x_j such that $0 \leq d(x_i, x_j) \leq 1$, $d(x_i, x_j) = 0$ iff $x_i = x_j$, and $d(x_i, x_j) = d(x_j, x_i)$.

The *support* is like a similarity index such that the more similar x_i and x_j are, the more they support each other.

Definition 9: Given a set of variables $X = \{x_1, \cdots, x_n\}$, the PA of X is defined as:

$$PA(x_1, \cdots, x_n) = \frac{\sum_{i=1}^{n}(1 + T(x_i))x_i}{\sum_{i=1}^{n}(1 + T(x_i))} \tag{10}$$

where $T(x_i) = \sum_{j=1;j\neq i}^{n} Supp(x_i, x_j)$.

Let $V_i = 1 + T(x_i)$ and $\widetilde{w}_i = V_i / \sum_{i=1}^{n} V_i$, then $\sum_{i=1}^{n} \widetilde{w}_i = 1$ and $PA(x_1, \cdots, x_n) = \sum_{i=1}^{n} \widetilde{w}_i x_i$. We can see that the PA is a nonlinear weighted average of X, where the $\widetilde{w}_i$ represents the contribution of the ith variable to the aggregating result, and the value of $\widetilde{w}_i$ depends on the interrelationships of the variables.

The interrelationship between a variable and the other variables is demonstrated by the support value between this variable and the others, that is, the more similar two variables are, the more they support each other. For example, if $Supp(a_i, a_j) = k$ for $\forall i, j \in \{1, \ldots, n\}$ and $k \geq 0$, then $T(a_i) = k(n-1)$ and so $PA(a_1, \ldots, a_n) = \frac{1}{n}\sum_i a_i$, which means if there is no support between variables or all the supports are the same, the PA becomes the arithmetic average.

The determination of the *Supp* function is usually context dependent. Yager [243] discussd a useful form of the *Supp* function: $Supp(a, b) = Ke^{-\alpha(a-b)^2}$, which can provide a continuous transition between different attribute clusters. A support mountain-based algorithm is proposed to learn the parameters (e.g. V_i and w_i) by using this *Supp*.

Furthermore, Yager [243] combined the ordered weighted average (OWA) operator and the PA, and proposed a power ordered weighted average operator (POWA), where the OWA supports the determination of the argument weights based on an order of the arguments.

PA is an arithmetic AO. One disadvantage of PA is that it cannot deal with the unduly high or low values in aggregating multiple variables. On the other hand, the geometric AO, e.g. the geometric mean (GeoM), can balance and coordinate multiple values effectively [72], so Xu and Yager [239] proposed the power geometric (PGeo, see Def. 10) to aggregate the input arguments. The PGeo is a nonlinear weighted-geometric AO. Similar to the PA, the contribution of x_i (represented by $\frac{1+T(x_i)}{\sum_{j=1}^{n}(1+T(x_j))}$) is determined by all the attributes in X.

Definition 10: Given a set of variables $X = \{x_1, \cdots, x_n\}$, the power geometric (PGeo) operator is defined as [239]:

$$PGeo(x_1, \cdots, x_n) = \prod_{i=1}^{n} x_i^{\frac{1+T(x_i)}{\sum_{j=1}^{n}(1+T(x_j))}} \tag{11}$$

where $T(x_i) = \sum_{j=1; j\neq i}^{n} Supp(x_i, x_j)$.

Compared to the PA, the PGeo reduces the influence of the unduly high and low attribute values on the aggregating results, so it satisfies $PGeo(x_1, \ldots, x_n) \leq PA(x_1, \ldots, x_n)$, where the equation is hold if and only if $x_1 = \ldots. = x_n$. Similar to the PA, the form of the *Supp* function depends on the application contexts and can determine the aggregating pattern of the PGeo. If the supports between any two variables are the same, the PGeo becomes the GeoM. The PGeo has the properties of commutativity, idempotency and boundedness.

Xu *et al.* [239] further considered the preference weight of each variable in the PGeo by proposing the weighted PGeo (WPGeo) where the weights are determined by the decision maker, and proposed the power ordered weighted geometric mean (POWGeo) where the weights are determined by the orders and values of the aggregated variables. The WPGeo has the properties of idempotency and boundedness, but not the commutativity. In addition, it cannot become the GeoM when we set the same support value between variables. The POWGeo can be applied when the weights of the variables are unavailable. In addition, Xu and Yager [239] developed a decision making approach by taking into account the multiplicative preference relations. Examples show that using the proposed PGeo operators is more efficient than using the arithmetic multiplicative preference-based approach proposed by Saaty [175].

To further generalize the PA operators, Zhou *et al.* [279] proposed a generalized PA (GPA, see Def. 11), which inherits the generalization capability of the generalized mean [45] so that the GPA can be used to generalize the PA, the PGeo, the power harmonic average (PHA) [91], and the power quadratic average (PQA) [26]. They further defined the weighted GPA and the generalized power ordered weighted average (GPOWA).

Definition 11: A generalized PA (GPA) of n real variables $X = \{x_1, \cdots, x_n\}$ is a mapping function $GPA : R^n \to R$ such that:

$$GPA(x_1, \cdots, x_n) = \left(\frac{\sum_{i=1}^{n}(1+T(x_i))x_i^{\lambda}}{\sum_{i=1}^{n}(1+T(x_i))}\right)^{\frac{1}{\lambda}} \tag{12}$$

where $T(x_i) = \sum_{j=1; j\neq i}^{n} Supp(x_i, x_j)$, and λ is a real-value parameter satisfying $\lambda \neq 0$.

The value of λ can influence the behavioral pattern of the GPA (see Table 4). When $\lambda = 1$, the GPA collapses to the PA; when $\lambda = -1$, the GPA becomes the PHA; when $\lambda = 2$, the GPA becomes the PQA; and when $\lambda \to 0$, the GPA behaves like the PGeo. Similar to the GPA, the behavioral patterns of the GPOWA are also determined by the λ (see Table 4). In particular, when $\lambda = -1$, GPOWA becomes the power ordered weighted harmonic averaging (POWHA), and when $\lambda = 2$, GPOWA becomes the power ordered weighted quadratic averaging (POWQA) [279].

2) EXTENSIONS OF PA-BASED OPERATORS TO DIFFERENT FSS

Researchers have proposed a series of PA-related AOs in different fuzzy environments (see Table 3).

There are various extensions of the PA-based operators in the IFS. Xu [236] extended the PA to the IFS, and developed a number of PA-based IFS operators. Xu and Wang [232] extended the PA, the weighted PA (WPA) and the POWA to process the 2-tuple linguistic terms (2TLTs). Zhang [273] extended the GPA and the GPOWA to the IFS, and proposed some generalized PGeo operators for the IFS. Wan [207] extended the PA to operate the trapezoidal intuitionistic fuzzy numbers (TraIFNs), and proposed the PA, the WPA, the POWA, and the power hybrid average (PHA) operators of TraIFNs. Wan and Dong [208] then further extended the PGeo-related operators to the TraIFNs. Wan and Yi [209] extended four PA operators to the normalized TraIFN (NorTraIFN) based on the strict t-norms and t-conorms, which include the PA, the WPA, the POWA and the PHA. These normalized PA operators can be generalized by using different types of operational laws. Zhao *et al.* [277] extended the GPA, the WGPA, and the GPOWA to the intuitionistic trapezoidal fuzzy set (ITraFS). He *et al.* [69] proposed some interactive

TABLE 3. Summary of PA-based operators.

Operators	Inputs	Relations	Types of arguments
PA	X, Supp	homo	crisp [245], 2TLT [234], IVFS and IFS [281], TraIFS [209], PyFS [176], NLS [114], LV [280], ILTs [120], 2DULT [121], ULTS [76], IVIFS [68], INS [119], HFS [84], DHFS [217], LHFS [286], LI2T [175], LIFS [116], NorTraIFN [211], IULV [103], HMFS [216], IVHULV [129], IFS [238]
PGeo	X, Supp	homo	crisp [241], IVFS [241], PyFS [176], NS [161], IFS [275], IVIFS [238], TraIFN [210], IFS [275], DHFS [217], LHFS [286], LIFS [116], IVFHS [232], IULV [103], HMFS [216], IVHULV [129]
GPA	X, Supp, λ	homo	crisp [281], LV [280], ILTs [120], 2DULT [121], IVDHFLS [164], TraIFS [209], INS [119], DHFS [217], LHFS [286], LIFS [116], TraF2DLV [90], 2TLV [228], ITraFS [279], IFS [69] [275]
WGPA	X, Supp, λ, W	homo	crisp [281], 2DULT [121], INS [119], DHFS [217], LIFS [116], TraF2DLV [90], ITraFS [279], IFS [69]
GPOWA	X, Supp, λ, g	homo	crisp [281], IVDHFLS [164], ILTs [120], TraIFS [209], INS [119], DHFS [217], LHFS [286], 2TLV [228], ITraFS [279], IFS [69] [275]

PA operators for the IFS, including the GPA, the WGPA, and the GPOWA of the IFNs.

The main extensions of the PA-based operators in the HFS are as follows. Wang *et al.* [214] investigated the application of the PA to the hesitant multiplicative fuzzy set (HMFS), and proposed the PA, PGeo, POWA, and POWGeo operators for the HMFS. Xiong *et al.* [230] applied the PGeo to the interval-valued hesitant fuzzy set (IVHFS). Wang *et al.* [215] proposed a variety of PA operators for the dual HFS (DHFS) based on the Archimedean t-norm and t-conorm. Keikha *et al.* [83] proposed the PA, the WPA, the ordered WPA (OWPA), and the hybrid WPA (HWPA) for the HFS.

The PA-based operators have also been extended to operate the interval values in the uncertain environment. Xu and Yager [239] applied the PGeo and the POWGeo to the MCGDMs in a crisp environment and to the uncertain MCGDMs having the interval values. They proposed an uncertain PGeo operator and an uncertain POWA operator to operate the interval values. Zhou *et al.* [279] developed the uncertain GPA (UGPA), the uncertain generalized WPA (GWPA), and the uncertain GPOWA (UGPOWA). They also developed the generalized intuitionistic fuzzy PA (GIFPA) and the generalized intuitionistic fuzzy POWA (GIFPOWA) to operate the IFNs.

Researchers also extended the PA-based operators to the linguistic environment. Zhou *et al.* [278] applied the GPA and the GPOWA to the LTS. Ruan *et al.* [173] extended the PA, the WPA and the POWA to the linguistic interval 2-tuple (LI2T) environment. Liu and Wang [118] applied the PA to deal with the intuitionistic linguistic terms (ILTs), and proposed the intuitionistic linguistic GWPA and the intuitionistic linguistic GPOWA. Liu and Yu [119] proposed two PA-based AOs to operate the 2-dimension uncertain linguistic terms (2DULTs), which are the 2-dimension uncertain linguistic GPA and the 2-dimension uncertain linguistic GWPA. In addition, Peng *et al.* [159] proposed a multi-valued neutrosophic WPG. Jiang *et al.* [76] extended the PA to the unbalanced linguistic term set (ULTS). Wu *et al.* [226] extended the GPA and the GPOWA to the 2TLV and the interval 2TLV. Liu and Shi [102] explored the combination of the PA and the Einstein operators. They extended such a combination to the intuitionistic uncertain linguistic variable (IULV), and proposed the Einstein PA and the Einstein PGeo for the IULV. Qi *et al.* [162] applied the GPA, the GPOWA and the generalized power ordered weighted geometric mean (GPOWGeo) to the interval-valued dual hesitant fuzzy linguistic set (IVDHFLS). Zhu *et al.* [284] extended a series of PA operators (i.e. PA, PGeo, POWA, and POWGeo) to the linguistic hesitant fuzzy set (LHFS). Li *et al.* [89] developed the GPA and the weighted GPA (WGPA) operators for the trapezoidal fuzzy two-dimension linguistic variable (TraF2DLV). Liu and Qin [114] extended the PA, the PGeo, the GPA, and the GWPA to the LIFS. Liu *et al.* [127] proposed the PA and the PGeo operators for the interval-valued hesitant uncertain linguistic variable (IVHULV). In addition, Liu and Tang [117] extended the PA, the GPA, the WGPA, and the GPOWA to the interval neutrosophic set (INS).

3) SUMMARY OF PA-BASED OPERATORS

The PA enables the support and the reinforcement between the attributes based on a mechanism of the *support*, where the contribution of a variable to the aggregation result depends on the values of the other variables. Researchers developed a variety of the PA-based AOs. We summarize some typical examples in Table 3. The PGeo is proposed to incorporate the GeoM to the PA to process the unduly high and low criteria values. The value of the $PGeo(x_1, \ldots, x_n)$ is not greater than that of the $PA(x_1, \ldots, x_n)$. The PA and the PGeo become the GeoM if the supports between any variables are the same. The GPA deals with the more general forms based on the power $\lambda \in R$ and $\lambda \neq 0$. It can become the PA, the PGeo, the PHA, and the PQA. The POWA and the POWGeo combine the ordered weighted average with the PA and the PGeo, which allows a decision maker to determine the criteria weights based on a predefined priority order of the criteria. The GPOWA generalizes the POWA and the POWGeo. In addition, the PA-based operators can only deal with the *homo* interrelationships among the variables.

The common inputs of the operators in Table 3 are the variables X and the support function *Supp*. The GPA requires a new parameter $\lambda \neq 0$ to control its behavioral pattern (see Table 4). The WGPA requires a set of preference weights W of the variables X; on the other hand, the GPOWA does not require these weights, but it requires a basic unit-interval monotonic (BUM) function $g : [0, 1] \to [0, 1]$ to help to form the ordered weights of the variables. The relations between the PA, the PGeo, the POWA, the GPA, and the GPOWA are summarized in Table 4, in which we can see that the PA and the PG are the special cases of the GPA, and the POWA and the POWG are the special cases of the GPOWA.

TABLE 4. Behavioral patterns of PA-based operators w.r.t. different parameter values.

Parameters	GPA	GPOWA
$\lambda = -1$	PHA	POWHA
$\lambda \to 0$	PGeo	POWGeo
$\lambda = 1$	PA	POWA
$\lambda = 2$	PQA	POWQA

The PA-based operators have been extended to different FSs, which is summarized in the column 'Types of arguments' of Table 3, where the extensions of the GPA and the GPOWA can also be applied to their special cases (e.g. PA, PG, POWA and POWG).

C. BM-BASED AGGREGATION OPERATORS

1) BASIC EXTENSIONS OF BM-BASED OPERATORS

The BM (see Def. 12 [17]) was first proposed by Bonferroni in 1950. It is an averaging aggregation function that is capable of modelling the *homo* interrelationships among the criteria in a decision-making problem, which is based on an assumption that each criterion is related to all the other criteria [240].

Definition 12: Let $X = \{x_1, \ldots, x_n\}$ be a collection of values of criteria, where $x_i \in [0, 1]$ for $\forall i \in [1, n]$, and $p, q \geq 0$. Then the BM of X w.r.t. p, q is represented by Formula 13:

$$B^{p,q}(x_1, \ldots, x_n) = (\frac{1}{n}\frac{1}{n-1}\sum_{i,j=1;i\neq j}^{n} x_i^p x_j^q)^{\frac{1}{p+q}} \quad (13)$$

The meanings of the BM depend on the values of p, q and n. For example, given specific values to p and q, the BM can be seen as a general extension of the PA and the GeoM. In addition, if $n > 2$ and $p = q$, the BM is soft partial conjunctive [28].

In 2016, Chen *et al.* [28] interpreted the BM by transforming Formula 13 to Formula 14, which indicates that the BM is like the PA of the satisfaction degree of an intersection $\{x_i \bigcup \textit{the power mean of } \forall x_{j,i\neq j} | \forall i \in [1, n]\}$.

$$B^{p,q}(x_1, \cdots, x_n) = \left(\underbrace{\frac{1}{n}\sum_{i=1}^{n} x_i^p \left(\overbrace{\left(\frac{1}{n-1}\sum_{j=1,j\neq i}^{n} x_j^q \right)^{\frac{1}{q}}}^{\text{power mean with power } q} \right)^q}_{x^p y^q} \right)^{\frac{1}{p+q}} \quad (14)$$

The BM satisfies the properties of the boundedness and the monotonicity [40]: $B^{p,q}(0, \ldots, 0) = 0$, $B^{p,q}(1, \ldots, 1) = 1$ and $\forall i \in [1, n]$, if $g_i \geq h_i$, $B^{p,q}(g_1, \ldots, g_n) \geq B^{p,q}(h_1, \ldots, h_n)$. It is a mean type AO [246].

Yager [246] proposed the concept of the generalized BM (GBM) and based on which he discussed two extensions of the BM: the Bonferroni ordered weighted averaging (BON-OWA) and the Bonferroni CI (BON-CHO), by combining the BM with another two popular AOs: the OWA and the CI. Furthermore, Yager [246] assigned the individual importance to each criterion in BON-OWA and proposed the weighted BON-OWA (WBON-OWA), which associates two types of weight to each x_i: the t_i and the w_i. The t_i refers to the individual importance of x_i. The w_i are the ordering weights used in the OWA, which represents the social relationships between x_i and the other criteria and indicates the way of aggregating all of the criteria in X except x_i. On the other side, the BON-CHO uses the CI to aggregate the vector v_i.

Beliakov *et al.* [14] then defined the GBM (see Def. 13) in formal.

Definition 13: Let $M = \langle M_1, M_2, C \rangle$, where $M_1 : [0, 1]^n \to [0, 1]$, $M_2 : [0, 1]^{n-1} \to [0, 1]$ and $C : [0, 1]^2 \to [0, 1]$ are aggregation functions, the GBM is defined as:

$$GBM(X) = d_C^{-1}(M_1(C(x_1, M_2(X_{j\neq 1})), \cdots, C(x_n, M_2(X_{j\neq n})))) \quad (15)$$

where d_C^{-1} is the inverse of the diagonal $d_C(t) = C(t, t)$.

The BM is a special case of the GBM: let $M_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$ be an AM, $M_2 = \frac{1}{n-1}\sum_{j=1,j\neq i}^{n} x_j^q$ be a PA, and $C = x^p y^q$ so that $d_C(x) = x^{p+q}$ and $d_C^{-1}(x) = x^{\frac{1}{p+q}}$.

Different M_1, M_2 and C assign different properties and meanings to the GBM. If M_1 and M_2 are averaging functions or are symmetric, then no matter what C is, the GBM is an averaging function or is symmetric. The symmetric property means that if an aggregation function f is symmetric, then for any permutation X_α of $X = \{x_1, \cdots, x_n\}$, $f(X) = f(X_\alpha)$.

Beliakov *et al.* [14] summarized the primary properties of the GBM: (1) the GBM satisfies the property of the partial conjunction; (2) it can model the k-tolerance and k-intolerance, and model the averages in cases having the nonzero inputs; and (3) the GBM becomes a quasi-arithmetic mean if M_1, M_2 and C are all generated from the same generator. Furthermore, based on the definition of GBM, Beliakov *et al.* [14] extended the BM to the $BM^{\vec{k}}$ ($|k| \geq 2$), e.g. the BM of triples represented by $BM^{p,q,r}$ [228]. Some scholars used the BM of triples to represent the GBM [28], [228], [280].

Formula 13 demonstrates that the BM captures the *homo* interrelationships among the criteria but not the *hete* interrelationships. However, two criteria may not interrelate with each other all the time, so it is necessary to consider the *hete* interrelationships. Dutta *et al.* [44] proposed a partitioned

BM (ParBM) to aggregate the criteria having *hete* interrelationships. Given a set of variables $X = \{x_1, \cdots, x_n\}$, they partitioned the variable set X into d disjoint classes $P = \{P_1, \cdots, P_d\}$ based on the *hete* interrelationships among the variables, where for $\forall x_i, x_j \in P_k$, x_i and x_j are interrelated with each other; and for $\forall x_i \in P_k, \forall x_j \in P_r, \forall P_k, P_r \in P$ and $P_k \neq P_r$, x_i and x_j are independent with each other.

To further formalize the ParBM to process both the *homo* and *hete* interrelationships, Dutta *et al.* [41] defined an extended BM (EBM, see Def. 14), which is a combination of the averaging and the conjunctive functions.

Definition 14: Let $D \subset X$ be a subset of variables such that each variable in D is independent with the other variables in X, $B = X/D$, and for all $x_i \in B$, B_i represents the set of variables in B except x_i: $B_i = B/x_i$. The extended BM (EBM) is defined as:

$$EBM^{p,q}(x_1, \ldots, x_n) = \left(\frac{|B|}{n} \left(\frac{1}{|B|} \sum_{x_i \in B} x_i^p \left(\frac{1}{|B_i|} \sum_{x_j \in B_i} x_j^q \right) \right)^{\frac{p}{p+q}} + \frac{|D|}{n} \left(\frac{1}{|D|} \sum_{x_k \in D} x_k^p \right) \right)^{\frac{1}{p}} \quad (16)$$

where by convention the empty sum is 0 and $\frac{0}{0} = 0$.

The EBM can be decomposed into a set of components:

$$EBM^{p,q} = WPA^p(d_C^{-1}(AM(C^{|B|}(x_i \in B, PA^q(\overrightarrow{x_i}|x_{i_m} \in B_i)))), PA^p(\overrightarrow{x_D}|x_{D_k} \in D)) \quad (17)$$

where WPA^p represents the weighted power mean with power p, AM is the arithmetic mean, $C(x, y) = x^p y^q$, PA^q and PA^p refer to the power mean with power q and p respectively.

In 2016, Chen *et al.* [28] proposed a generalized form of the EBM, called the generalized EBM (GEBM, see Def. 15), which uses the general conjunctive and disjunctive operators to replace the average and power operations in the original EBM.

Definition 15: Given a set of variables $X = \{x_1, \cdots, x_n\}$, let B and D are two disjoint subsets of X such that $B \bigcup D = X$, where $\forall x_i (\in B)$ is dependent to a nonempty subset $B_i (\subseteq B/x_i)$, and $\forall x_j (\in D)$ is independent with any variables in X/x_j. Let M be a quintuple of the aggregation functions: $M = \langle M_1, M_2, M_3, M_4, C \rangle$, where $M_1 : [0, 1]^{|B|} \to [0, 1]$, $M_2 : [0, 1]^{|B|-1} \to [0, 1]$, $M_3 : [0, 1]^{|D|} \to [0, 1]$, $M_4 : [0, 1]^2 \to [0, 1]$, $C : [0, 1]^2 \to [0, 1]$, and the diagonal of C is defined as $d_C(t) = C(t, t)$ and $d_C^{-1}(t)$ is the inverse diagonal of C, the generalized extended BM (GEBM) is defined as:

$$GEBM(x_1, \cdots, x_n) = M_4 \left(d_C^{-1} \left(M_1 \left(C^{|B|} (x_i \in B, M_2(B_i)) \right) \right), M_3(D) \right) \quad (18)$$

where $C^{|B|}$ represents a $|B|$-ary tuple: $\langle C(x_i \in B, M_2(B_i)) \rangle$

The M_1, M_2, M_3, M_4 and C of GEBM generalize the AM, PA^q, PA^p, WPA^p, and C in Formula 17 respectively.

The interrelationships among the inputs determine the aggregation forms of the GEBM. Table 6 summarizes its behavioral forms, where the M_3 is an AO replacement defined in Def. 15. When all variables are dependent, and each variable depends on some of the other variable but not always all of the others, the GEBM becomes the $DGEBM = d_C^{-1} (M_1 (C (x_1, M_2 (x \in B_1)), \ldots, C(x_n, M_2 (x \in B_n)))))$, where DGEBM refers to the dependent GEBM.

Xia *et al.* [229] proposed the geometric BM (GeoBM) (see Def. 16). Furthermore, they extended the GeoBM to the IFS environment, and proposed the weighted IFS geometric BM (WIFGeoBM) and applied it to the MCDM problems. They also proved that the GeoBM has four main properties: idempotency, monotonicity, boundedness, and commutativity. The commutativity means the GeoBM of a set of variables X equals to the GeoBM of any permutation of the variables in X.

Definition 16: Given $p, q > 0$, and a set of variables $X = \{x_1, \ldots, x_n\}$, $\forall x_i (\in X) \geq 0$, the GeoBM of X w.r.t. p, q is defined by formula 19.

$$GeoBM^{p,q}(x_1, \ldots, x_n) = \frac{1}{p+q} \prod_{i,j=1; i \neq j}^{n} (px_i + qx_j)^{\frac{1}{n(n-1)}} \quad (19)$$

Based on the work of Xia *et al.* [229], Dutta *et al.* [42] proposed an extended GeoBM (EGeoBM, see Def. 17) to enable the GeoBM to model the *hete* interrelationship among criteria.

Definition 17: Let $D \subseteq X$ be a subset of variables such that each variable in D is independent with the other variables in B, where B is the set of variables in X but not in D: $B = X/D$, and for all $x_i \in B$, $B_i = B/x_i$. The extended geometric BM (EGeoBM) is defined as:

$$EGeoBM^{p,q}(x_1, \cdots, x_n) = \frac{|B|}{n} \left(\frac{1}{p+q} \prod_{i \in B} \left(\prod_{j \in B_i} (px_i + qx_j)^{\frac{1}{|B_i|}} \right)^{\frac{1}{|B|}} \right) + \frac{|D|}{n} \left(\prod_{i \in D} x_i^{\frac{1}{|D|}} \right) \quad (20)$$

Dutta *et al.* [42] interpreted the EGeoBM from the perspective of aggregating the opinions (or the satisfaction degrees) of a group of users. They interpreted the component $\left(\frac{1}{p+q} \prod_{i \in B} \left(\prod_{j \in B_i} (px_i + qx_j)^{\frac{1}{|B_i|}} \right)^{\frac{1}{|B|}} \right)$ as an averaging satisfaction degree of a subgroup of interrelated individuals, the component $\left(\prod_{i \in D} x_i^{\frac{1}{|D|}} \right)$ as an averaging opinion of a subgroup of independent individuals, and the EGeoBM is the weighted average of the averaging opinions of the interrelated individuals and the independent individuals.

Comparing to the BM and the EBM, the efficiency of the EGeoBM is proved by a location selection example with mandatory requirements [42]. The example shows that the EGeoBM can provide a more compound decision as it considers both the *hete* and *homo* attribute relations, and also processes the mandatory requirements.

The EGeoBM has the properties of idempotency, commutativity, nondecreasing, boundedness, and ratio-scale invariant [42], where the nondecreasing is a more specific property of the monotonicity and means that the EGeoBM of a set $A = \{a_1, \cdots, a_n\}$ is equal to or greater than the EGeoBM of a set $C = \{c_1, \cdots, c_n\}$ if $a_i \geq c_i$ for $\forall i \in \{1, \cdots, n\}$ and variables in A and C have the same *hete* interrelationships; and the ratio-scale invariant means that for $\forall r > 0$, $EGeoBM(ra_1, \cdots, ra_n) = rEGeoBM(a_1, \cdots, a_n)$.

When the interrelationships among the inputs are fixed, the aggregation forms of the EGeoBM depend on the values of p and q [42]: when $p = q$, the result of EGeoBM is independent with p and q; when $q = 0$, the result is neither relevant to q nor to the interrelationships among the variables, but it can be seen as the weighted average of the GeoM of the dependent variables and the GeoM of the independent variables; when $p = 0$, the result is irrelevant to the value of q; and the results under the conditions of $p \to \infty$ and $q \to \infty$ equal to the results under the conditions of $q = 0$ and $p = 0$ respectively.

In addition, Dutta *et al.* [42] pointed out that most of the work focusing on the BM-based decision making problem assumes that the *hete* interrelationship between the criteria are known in advance. However, this is not always true in practical applications, so it is necessary to design a mechanism to establish an accurate *hete* interrelationship. They proposed an algorithm based on the similarity feature of the criteria to learn their interrelationships.

He *et al.* [67] pointed out that when decision makers fuse the interrelationships between the criteria, the unduly high or low attribute values may badly affect the aggregation results. Therefore, He *et al.* [67] proposed the power BM (PBM, see Def. 18 [67]) and extended the PBM to the hesitant fuzzy environment (i.e. HFPBM). The HFPBM operator cannot only deal with the fusion of the attribute interrelationships provided by different decision makers, but also can deal with the interrelaionships between the criteria provided by one decision maker. Furthermore, they defined the hesitant fuzzy power geometric BM (HFPGeoBM). By assigning different values to p, q and $S = n(T(x_j) + 1)/(\sum_{t=1}^{n}(T(x_t) + 1))$ for $\forall j \in \{1, 2, \ldots, n\}$, the HFPBM can be converted to different operators. For example, if $q \to 0$ and $S = 1/n$, then the HFPBM becomes the generalized hesitant fuzzy mean.

Definition 18: Let $X = \{x_1, \cdots, x_n\}$ be a set of hesitant fuzzy numbers, and $p, q \geq 0$, then the power BM of X w.r.t. p, q is defined as:

$$PBM^{p,q}(X) = \left(\frac{1}{n(n-1)} \sum_{i=1,j=1,i\neq j}^{n} \left(\left(\frac{n(T(x_i)+1)}{\sum_{t=1}^{n}(T(x_t)+1)} x_i\right)^p \times \left(\frac{n(T(x_j)+1)}{\sum_{t=1}^{n}(T(x_t)+1)} x_j\right)^q\right)\right)^{\frac{1}{p+q}} \quad (21)$$

where $T(x_i) = \sum_{j=1,j\neq i}^{n} Supp(x_i, x_j)$.

2) EXTENSIONS OF BM-BASED OPERATORS TO DIFFERENT FSS

Xu and Yager [240] extended the BM to the IFS environment and proposed the intuitionistic fuzzy BM (IFBM) and the intuitionistic fuzzy weighted BM (IFWBM). However, Zhou and He [281] pointed out that the IFWBM does not has the reducibility, that is, the IFWBM cannot become the IFBM when $w_i = 1/n$ for all $i \in [1, n]$. Therefore, Zhou and He [281] proposed a normalized weighted BM (NWBM), which is represented by Formula 22 [281].

$$NWBM^{p,q}(x_1, \cdots, x_n) = \left(\sum_{i=1}^{n} w_i x_i^p \sum_{j=1,j\neq i}^{n} \frac{w_j}{1-w_i} x_j^q\right)^{\frac{1}{p+q}} \quad (22)$$

We can see from Formula 22 that $\sum_{j=1,j\neq i}^{n} \frac{w_j}{1-w_i} x_j^q$ represents the WPA satisfaction degree of the X except x_i. Therefore, this NWBM can overall reflect the interrelationships between the individual variable and the other variables. Zhou and He [281] proved that the NWBM has the property of reducibility, idempotency, monotonicity, and boundedness, where the reducibility means if the weight vector of X is $W = (1/n, \cdots, 1/n)$, then $NWBM^{p,q}(x_1, \cdots, x_n) = BM^{p,q}(x_1, \cdots, x_n)$. They then applied the NWBM to the IFS.

To consider the interrelationship between the membership function and the nonmembership function of an IFN, He *et al.* [66] proposed the intuitionistic fuzzy interaction BM (IFIBM). He *et al.* [66] proved that the IFIBM has properties of idempotency and commutativity; and presented the efficiencies of the IFIBM by comparing it to the other important AOs of the IFS, e.g. the intuitionistic fuzzy average (IFA), the intuitionistic fuzzy geometric (IFG), and the generalized intuitionistic fuzzy geometric interaction average (GIFGIA).

In 2017, Liu *et al.* [104] proposed the intuitionistic fuzzy interaction partitioned BM (IFIParBM), which combines the ParBM and the IFIBM. They [104] then defined the intuitionistic fuzzy interaction partitioned geometric BM (IFIParGeoBM). In addition, Liu *et al.* [104] discussed the properties of the IFIParBM and defined the weighted and geometric extensions of the IFIParBM. In 2018, Liu *et al.* [109] extended the BM to the qROF, and extended the qROFBM to the q-rung orthopair fuzzy weighted BM (qROFWBM), the q-rung orthopair fuzzy geometric BM (qROFGBM) and the q-rung orthopair fuzzy weighted geometric BM (qROFWGBM). In 2018, Ji *et al.* [75] extended the PA and the NWBM to the SVNS. The proposed BM operator obeys the Frank operational law [48] that provides more flexibility and robustness than the algebraic operational law. Ji *et al.* [75] also extended the frank operations to operate the SVNNs. They then proposed the single-valued neutrosophic Frank BM (SVNFBM), and extended the SVNFBM to the SVNFN PBM based on the definitions of PA and NWBM.

The work of extending the EBM, GBM, GeoBM, PBM and NWBM is as follows. Sun and Sun [187] combined the BM with the HM, and proposed the fuzzy Bonferroni harmonic mean (FBHM). Su *et al.* [185] extended the FBHM to the trapezoidal intuitionistic fuzzy set (TraIFS), and proposed a trapezoidal intuitionistic fuzzy BHM (TraIFBHM). Dutta and Guha [43] proposed the trapezoidal intuitionistic fuzzy BM (TraIFBM). Liu *et al.* [124] generalized the BM to the interval-valued 2-tuple linguistic terms (IV2TLTs). Yu [257] proposed the BM-based AOs for the triangular IFNs (TriIFNs). Gou *et al.* [56] introduced two BM-based AOs for the hesitant fuzzy linguistic term (HFLT). Garg and Arora [52] extended the BM and the weighted BM to the intuitionistic fuzzy soft set (IFSS). Liang *et al.* [92] extended the EBM to the interval-valued Pythagorean FS (IVPyFS). Yu *et al.* [260] extended the GBM to the HFS, and proposed the generalized hesitant fuzzy BM (GHFBM). Beliakov and James [11] extended the GBM to operate the lattices. Zhang *et al.* [268] extended the GBM to the PyFS. Jiang and Wei [77] extended the GeoBM to the 2-tuple linguistic set (2TLS). Gong *et al.* [55] proposed some AOs based on the GeoBM to operate the trapezoidal interval type-2 fuzzy set (TraIT2FS). Li *et al.* [86] introduced the concept of the generalized GeoBM based on the definitions of the GBM and the GeoBM. Zhang [274] extended the GeoBM to the IVIFS, and proposed the interval-valued intuitionistic fuzzy GeoBM (IVIFGeoBM) and the weighted IVIFGeoBM (WIVIFGeoBM). Liu *et al.* [107] extended the PBM to operate the IVIFS. They defined four AOs for the IVIFS: the power BM (IVIFPBM), the weighted PBM (IVIFWPBM), the power GeoBM (IVIFPGeoBM), and the weighted PGeoBM (IVIFWPGeoBM). Zhou [280] extended the RWBM to the HFS, and proposed the hesitant fuzzy reducible weighted BM (HFRWBM) and the generalized HFRWBM (GHFRWBM). Xia *et al.* [228] improved the BM of triples and the NWBM, and proposed a reducible generalized weighted BM (RGWBM). However, Zhou [280] stated that one problem of the RGWBM is that it does not evitably indicate the interrelationship between an individual variable and the other variables. Zhou then [280] proposed a reducible weighted BM (RWBM).

Some researchers extended the BM-based operators to the neutrosophic environment. Liu *et al.* [121] extended the BM to the multi-valued neutrosophic set (MVNS). Liu and Liu [126] introduced the BM-based operators for the normal intuitionistic fuzzy set (NIFS). Liu and Li [108] defined the normal neutrosophic number (NorNN) and proposed four BM-based operators for NorNNs.

In addition, the BM-based operators were applied to different linguistic variables (LVs). Tian *et al.* [199] extended the BM to the grey LV (GLV). Liu and Jin [101] proposed four BM-based operators to operate the trapezoidal fuzzy LVs (TraFLVs). Zhu *et al.* [286] developed the triangular fuzzy BM and the triangular fuzzy WBM for operating the TriFNs. Tian *et al.* [200] proposed a method to apply the NWBM to the neutrosophic linguistic set (NLS). Liu *et al.* [128] extended the BM to the PULS. As the classical BM only deals with the *homo* interrelationships among criteria, Liu *et al.* [128] proposed a partitioned BM to process the *hete* interrelationships. They then applied the EBM to the linguistic 2-tuple fuzzy set (L2TFS) [41]. Xu [235] extended the BON-OWA and the BON-CHO to the uncertain environment. Liu *et al.* [106] applied the BON-OWA to IULS.

3) SUMMARY OF BM-BASED OPERATORS

Based on the above discussion, there are a number of basic extensions of the BM, among which the most popular ones are the GBM, the EBM, the GEBM, the GeoBM, and the PBM (see Table 5). We can see that the BM, the GBM, the GeoBM and the PBM only consider the *homo* interrelationship, and the EBM extends the BM by integrating the capability of modelling the *hete* interrelationship, so the inputs of the EBM include the common parameters p, q, X and the interrelationships among the input variables (Rl). The GBM generalizes the BM by allowing the other operation rules to replace the arithmatic summation, multiplication and exponential laws in the BM. The operators (M_1, M_2, C) combining with the values of p, q determine the meanings and properties of the GBM. The GeoBM extends the BM by using the GeoM to replace the AM, where the AM emphasizes the overall impact and the complement of the aggregated data. On the other side, the GeoM stresses the balance and the coordination between the data [111]. The PBM integrates the capability of the PA to the BM to reduce the negative effects of the unduly high or low values. The WBM models the weighted relations among criteria by assigning a preference weight to each criterion. In particular, this weighted form can be extended to all the other extensions of the BM, e.g. the weighted GeoBM [109] and the weighted power GeoBM [107].

The BM-based operators have been applied to various fuzzy sets (see column 'Types of Variables' in Table 5). We only summarized the work that proposes the direct applications of one BM-based operator. However, if an operator is applied to a FS, then all of its special behavioral patterns (see Table 6) can also be extended to this FS. Table 6 summarizes the behavioral patterns of the EBM, GEBM, and EGeoBM in terms of different types of interrelationships among variables, where $|D| = n$ indicates each of the variables is independent with the others, and $|D| = 0$ means there is no independent variable in X. From this Table, we can see that the PA and the BM are special patterns of the EBM; the GBM is a special pattern of the GEBM; the GeoM and the GeoBM are special patterns of the EGeoBM; and when $|D| = n$, the value of the GEBM is only determined by the operator M_3.

We summarize some typical examples of the aggregation patterns and the capability of satisfying mandatory requirements of the BM, the GBM, the EBM, and the GEBM after setting the specific inputs (e.g. p, q, Rl, and (M_1, M_2, M_3, M_4, C)). For the BM, assigning different values to p and q allows it to model different degrees of conjunction and disjunction [14]. Therefore, the BM can be seen as a generalization of some other AOs. For example, when $p = q$ and $|X| = 2$, the BM behaves like the GeoBM; when $p \neq q, q = 0$, the BM behaves like the PA; and when $\lim \frac{p}{q} \rightarrow \infty$ or 0, the BM behaves like the MAX operator. The BM is a special case of the GBM

TABLE 5. Summary of BM-based operators.

Operators	Inputs	Relations	Types of Variables
BM	p, q, X	homo	crisp [17], IFS [242] [66], qROF [111], HFS [282], SVNS [75], GLV [201], TraIFS [187], IV2TLT [126], HFLT [56], IFSS [52], MVNS [123], NIFS [128], NorNN [110], TraFLV [102], PyULS [130]
GBM	p, q, X, (M_1, M_2, C)	homo	crisp [14], HFS [262], Lattice [11], PyFS [270],
EBM	p, q, X, *Rl*	homo, hete	crisp [41], 2TLT [41], IFS [36], IVPyFS [93], PyULS [130], L2TFS [41]
GEBM	p, q, X, (M_1, M_2, M_3, M_4, C), *Rl*	homo, hete	crisp [28]
GeoBM	p, q, X	homo	crisp [231], IFS [87], HFS [285], 2TLV [77], TraIT2FS [55], IVT2FS [126], IVIFS [276], NIFS [128], NorNN [110]
EGeoBM	p, q, X, *Rl*	homo, hete	crisp [42]
PBM	p, q, X, Supp	homo	HFS [67], IVIFS [109],

TABLE 6. Behavioral patterns of the EBM given different types of variable interrelationships.

Interrelationship	EBM	GEBM	EGeoBM
$\|D\| = n$	$PA(X)$	$M_3(X)$	$GeoM(X)$
$\|D\| = 0$ and each variable is dependent to all the other variables	$BM^{p,q}(X)$	$GBM(X)$	$GeoBM^{p,q}(X)$
$\|D\| = 0$ and each variable depends on some of the other variables but not always all the others	$\left(\frac{1}{n}\sum_{i=1}^{n} x_i^p \left(\frac{1}{\|B_i\|}\sum_{j\in B_i} x_j\right)\right)^{\frac{1}{p+q}}$	$DGEBM(X)$	$\frac{1}{p+q}\prod_{i\notin D}\left(\prod_{j\in B_i}(pa_i+qa_j)^{\frac{1}{\|B_i\|}}\right)^{\frac{1}{n-\|B_i\|}}$

when M_1 and M_2 are the AMs, and C is the production operator.

For the GBM, M_1, M_2 and C (see Def. 13) influence its aggregation behavior. The M_1 and M_2 can be both symmetric and weighted functions. The weights defined in the M_1 and M_2 can be used to indicate the satisfaction degrees of the decision makers for the criteria. For example, the satisfaction of the final decision of the $GBM^{p,q}$ requires the satisfaction of at least two mandatory requirements. When we increase the dimension of the parameters, e.g. the $BM^{p,q}$ becomes the $BM^{p,q,r}$, the BM can model the cases that need to satisfy at least three mandatory requirements. Correspondingly, the iterative GBM (IGBM) is defined to aggregate the increasing number of the hard partial conjunctions. In addition, there are a number of important properties of the $GBM^{p,q}$, which can be extended to its high dimensional extensions $GBM^{\overrightarrow{k}} = GBM^{p_1,p_2,\ldots,p_k}$, $|\overrightarrow{k}| > 2$ [14].

For the EBM and the GEBM, the interrelationships among the input variables can determine their aggregation behavior. Table 6 summarizes the behavioral patterns of the EBM given different types of variable interrelationships. The EBM generalizes the BM by modelling different types of criteria interrelationships. When there is no independent variables and each variable is dependent on all the other variables, the EBM becomes the BM. The GEBM generalizes the EBM by generalizing its AM, PA, WPA, and production operations. The capability of the GEBM to model the mandatory requirements depends on the variable relationships and the aggregation parameters (e.g. M_1, M_2, M_3, M_4, C). If there are at least two nonzero dependent variables, the positive value of the GEBM is determined by M_4 (see Def. 15); and if there is at least one nonzero independent variables, the positive value of the GEBM is determined by M_3 and M_4. The DGEBM (see Table 6) behaves like the GBM in terms of modelling the mandatory requirements. However, it is different from the GBM in terms of the aggregation behavior of the interrelationships among dependent variables.

Overall speaking, the $GBM^{\overrightarrow{k}}$ can model the mandatory requirements (through their hard partial conjunction) w.r.t. the $|k|$ nonzero variables and model the average contributions of all the other variables. The GEBM has similar capabilities of modelling the mandatory requirements to the EBM. Furthermore, it can also model the *hete* interrelationships among variables.

D. HM-BASED AGGREGATION OPERATORS

1) BASIC EXTENSIONS OF HM-BASED OPERATORS

The BM-based operators consider the interrelationships between different variables in MCDM problems. Yu *et al.* [259] identified two drawbacks of the BM-based operators: it cannot deal with the interrelationship between a variable and itself; and it does not distinguish the interrelationship between variables x_i and x_j from the interrelationship between x_j and x_i. Yu *et al.* [259] believed that these two relations are similar, so BMs redundantly consider the interrelationship between x_i and x_j. They proposed to replace the BM using the HM [15] to solve these problems. HM (see Def. 19 [259]) is a mean type AO.

Definition 19: The HM of a set of nonnegative values $X = \{x_1, \cdots, x_n\}$ is:

$$HM(x_1, \cdots, x_n) = \frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \sqrt{x_i x_j} \tag{23}$$

Yu *et al.* [259] defined the generalized HM (GHM, see Def. 20). When $p = q = \frac{1}{2}$, the GHM becomes the HM.

Definition 20: Given $p, q \geq 0$ and a set of nonnegative values $X = \{x_1, \cdots, x_n\}$, the generalized HM of X

w.r.t. p and q is:

$$GHM^{p,q}(x_1, \cdots, x_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} x_i^p x_j^q \right)^{\frac{1}{p+q}} \quad (24)$$

Chu and Liu [32] proposed a reducible weighted GHM (WGHM, see Def. 21) and a reducible weighted generalized geometric HM (WGGeoHM, see Def. 22).

Definition 21: Given $p, q \geq 0$, a set of nonnegative values $X = \{x_1, \cdots, x_n\}$, and a set of weights $W = \{w_1, \ldots, w_n\}$ such that $w_i > 0$ for $\forall i \in \{1, \ldots, n\}$ and $\sum_{i=1}^{n} w_i = 1$, then the WGHM is defined as:

$$WGHM^{p,q}(x_1, \ldots, x_n) = \frac{\left(\sum_{i=1}^{n} \sum_{j=i}^{n} (w_i x_i)^p (w_j x_j)^q \right)^{\frac{1}{p+q}}}{\left(\sum_{i=1}^{n} \sum_{j=i}^{n} w_i^p w_j^q \right)} \quad (25)$$

Definition 22: Given $p, q \geq 0$, a set of nonnegative values $X = \{x_1, \cdots, x_n\}$, and a set of weights $W = \{w_1, \ldots, w_n\}$ such that $w_i > 0$ for $\forall i \in \{1, \ldots, n\}$ and $\sum_{i=1}^{n} w_i = 1$, then the WGGeoHM is defined as:

$$WGGeoHM^{p,q}(x_1, \ldots, x_n) = \frac{1}{p+q} \prod_{i=1}^{n} \prod_{j=i}^{n} (px_i + qx_j)^{\frac{2(n+1-i)w_j}{n(n+1)\sum_{k=1}^{n} w_k}} \quad (26)$$

The WGHM and WGGeoHM has the properties of reducibility, idempotency, monotonicity and boundedness.

The Archimedean t-norm and t-conorm can generalize most of the exsiting aggregation functions and provide the general operational rules for the IFNs. The general t-norm and t-conorm are defined in Def. 23 and Def. 24 respectively [100]. If the general t-norm and t-conorm satisfy the following two conditions, they are Archimedean t-norm and t-conorm respectively [100]: (1) S and T are continous; (2) $S(x, x) > x$ and $T(x, x) < x$.

Definition 23: A t-norm is a binary function T: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies the following axioms: (1) $T(x, 0) = 0$ and $T(x, 1) = x$; (2) $T(x, y) = T(y, x)$; (3) $T(x, T(y, z)) = T(T(x, y), z)$; and (4) if $x_1 \leq x_2$ and $y_1 \leq y_2$, then $T(x_1, y_1) \leq T(x_2, y_2)$.

Definition 24: A t-conorm is a binary function S: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies the following axioms: (1) $S(x, 0) = x$ and $S(x, 1) = 1$; (2) $S(x, y) = S(y, x)$; (3) $S(x, S(y, z)) = S(S(x, y), z)$; and (4) if $x_1 \leq x_2$ and $y_1 \leq y_2$, then $S(x_1, y_1) \leq S(x_2, y_2)$.

Liu *et al.* [100] generalized the HM based on the general Archimedean t-norm and t-conorm, and proposed the intuitionistic fuzzy Archimedean HM aggregation (IFAHA, see Def. 25). Furthermore, they defined the intuitionistic fuzzy weighted AHA (IFWAHA) to consider the variable weights. Liu *et al.* [100] emphasized the significance of their work: (1) the AOs are very suitable for solving MAGDMs; (2) the Archimedean t-norm and t-conorm can generalize the operational rules of the IFNs and the AOs of the IFNs. However, most of the AOs cannot deal with the variable interrelationships; and (3) the HM is capable of modelling the variable interrelationships. However, the existing HM-based AOs are the Algebraic operations, so they do not have generalities.

Definition 25: Given a set of intuitionistic fuzzy variables $X = \{x_1, \cdots, x_n\}$, and $p, q \geq 0$, the IFAHA operator is defined as:

$$IFAHA^{p,q}(x_1, \cdots, x_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} x_i^p \otimes x_j^q \right)^{\frac{1}{p+q}} \quad (27)$$

where $\otimes$ represents the multiplication operations of the IFS.

As the PA can relieve the influence of the biased values (e.g. the unduly low or high) given by different decision makers and the HM is capable of modelling the variable interrelationships, Liu [103] combined the PA and the HM for the IVIFN, and proposed the interval-valued intuitionistic fuzzy power HM aggregation (IVIFPHA, see Def. 26) and the weighted IVIFPHA. Chen and Liu [27] combined the HM with the OWA, and proposed the Heronian OWA (H-OWA).

Definition 26: Given a set of IVIFNs $X = x_1, \cdots, x_n$ where $X_i = ([a_i, b_i], [c_i, d_i])$, and $p, q \geq 0$, IVIFPHA of X is a mapping $\Omega^n \rightarrow \Omega$ such that

$$IVIFPHA^{p,q}(x_1, \cdots, x_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(n \frac{1 + T(x_i)}{\sum_{k=1}^{n}(1 + T(x_k))} x_i \right)^p \otimes \left(n \frac{1 + T(x_j)}{\sum_{k=1}^{n}(1 + T(x_k))} x_j \right)^q \right)^{\frac{1}{p+q}} \quad (28)$$

where Ω is the space of IVIFN, and $\otimes$ is the multiplication law of IVIFNs. Let $t_i = n\frac{1+T(x_i)}{\sum_{k=1}^{n}(1+T(x_k))} x_i$, then $\sum_{i=1}^{n} t_i = 1$.

2) EXTENSIONS OF HM-BASED OPERATORS TO THE OTHER FSS

Some work extend the HM to the TraFS. Chen and Liu [27] extended the H-OWA to the TraIFS. Das and Guha [35] extended the power HM (PHM) to the TraIFS. Das and Guha [34] extended the HM to the TraIFS.

The applications of the HM-based AOs to the neutrosophic set are as follows. Li *et al.* [88] introduced the improved HM operators to improve the traditional HM by enabling the idempotency of the HM. They proposed the improved generalized weighted HM and the improved generalized weighted geometric HM. They then extended these operators to the SVNS. Liu *et al.* [112] extended the PA and the HM to the linguistic neutrosophic set (LNS). Liu and Shi [115] extended the HM to the neutrosophic uncertain linguistic set (NULS). Liu and Zhang [120] extended the HM to the neutrosophic HFS (NHFS).

In addition, Chu and Liu [32] applied the WGHM and the WGGeoHM to operate the two dimensional uncertain linguistic variables (2DULV), and proposed the two dimensional

TABLE 7. Summary of HM-based operators.

Operators	Inputs	Relations	Application environment
HM	X	homo	crisp [261], IVIFS [261], IFS [101], LV [114], TraIFS [27], SVNS [89], NULS [117]
GHM	p, q, X	homo	crisp [32], IVIFS [261], SVNS [89], NULS [117], 2DULV [32], NHFS [122], 2DULV [32],
GGeoHM	p, q, X	homo	crisp [32], 2DULV [32], SVNS [89], NULS [117], NLS [114], 2DULV [32], NHFS [122]
AHM	p, q, X, generators	homo	IFS [101]
PHM	p, q, X, Supp	homo	IVIFS [104], LV [114], TraIFS [35], LNS [114]

uncertain linguistic weighted GHM and the two dimensional uncertain linguistic weighted GGeoHM.

3) SUMMARY OF HM-BASED OPERATORS

The HM is different to the BM by considering the interrelationship between a variable and itself, and by making the interrelationship from i to j to be same as the interrelationship from j to i. A number of the extensions of the HM have been proposed (see Table 7), among which the most basic ones are the GHM, the GGeoHM, the AHM, and the PHM. The GHM extends the HM by using the general exponents p and q ($p, q \geq 0$) rather than setting $p = q = 1$. The GGeoHM takes advantage of the GeoM in the GHM to valuate the balance and the coordination of the aggregated criteria. The AHM extends the Archimedean t-norm and t-conorm to the HM to make the operational laws in the HM of the fuzzy numbers easy to be generalized. The PHM integrates the capability of the power mean to the HM to deal with the unduly high and low values of the criteria. In addition, the H-OWA takes advantage of the OWA to determine the criteria weights based on a predefined priority order of the criteria.

The HM is a special case of the GHM and the AHM. If $p = q = \frac{1}{2}$, the GHM collapses to the HM. If the additive generator of the AHM is the algebraic operations, the AHM collapses to the HM. In addition, assigning different p, q values of the GHM can result in different aggregation patterns of the GHM; assigning different p, q values of the AHM can transform the AHM to different heavy weighted averaging patterns; and setting different additive generators can convert the AHM to the corresponding types of HM operators.

Table 7 (column 'Application environment') shows the FSs where these HM-based operators have been applied to. We only summarized the work that proposes the direct applications of one HM-based operator. However, if an operator is applied to a FS, then all of its special behavioral patterns can also be extended to this FS.

E. MSM-BASED AGGREGATION OPERATORS

The MSM [135] is another mean type operator that has been applied to model the interrelationships among multiple criteria. Compared to the BM and the HM, the MSM has two advantages [262]: (1) the BM and the HM only consider the interrelationships between two variables. However, the MSM can model the interrelationships of more than two; and (2) the BM and the HM require the determination of at least two parameters (p and q) from an infinite set. However, the MSM only requires one parameter from a finite integer set. Thus, the MSM is more flexible and robust than the BM and the HM [166].

1) BASIC EXTENSIONS OF MSM-BASED OPERATORS

The MSM is defined in Def. 27 [135]. It has the properties of idempotency, monotonicity, and boundedness, and strictly satisfies the schur convexity [166]. In particular, assigning different values to m leads to different forms of MSM [167] (see Table 9).

Definition 27: Let $X = \{x_1, \cdots, x_n\}$ be a set of n nonnegative real numbers and $m \in \{1, 2, \cdots, n\}$, the m^{th} MSM of X is a mapping $MSM : (R^+)^n \to R^+$ such that:

$$MSM^m(x_1, \cdots, x_n) = \left(\frac{\sum_{1 \leq i_1 \leq \cdots \leq i_m \leq n} \prod_{j=1}^m x_{i_j}}{C_n^m} \right)^{\frac{1}{m}} \tag{29}$$

where $(i_1, \cdots, i_m)$ represents all the m-tuple combination of $(1, 2, \cdots, n)$ and C_n^m is the binomial coefficient.

Qin *et al.* [166] defined the dual MSM (DMSM, see Def. 28) that satisfies the idempotency, monotonicity, and boundedness; and strictly satisfies the schur convexity. Identical to the MSM, different values of m convert the DMSM to different AOs (see Table 9).

Definition 28: Let $X = \{x_1, \cdots, x_n\}$ be a set of nonnegative real numbers, and $m \in \{1, 2, \cdots, n\}$, the m^{th} DMSM of X is defined as:

$$DMSM^m(x_1, \cdots, x_n) = \frac{1}{m} \left(\prod_{i_1 \leq \cdots \leq i_m} \left(\sum_{j=1}^m x_{i_j} \right)^{\frac{1}{C_n^m}} \right) \tag{30}$$

Wang *et al.* [213] extended the MSM to the generalized arithmetic MSM (GMSM, see Def. 29) and the generalized geometric MSM (GGeoMSM, see Def. 30).

Definition 29: Let $X = \{x_1, \cdots, x_n\}$ be a set of nonnegative real numbers, $m \in \{1, 2, \cdots, n\}$, $p_1, \cdots, p_n \geq 0$, the m^{th} GMSM of X is defined as a mapping $GMSM^{m, p_1, \cdots, p_m} : (R^+)^n \to R^+$ such that:

$$GMSM^{m,p_1,\cdots,p_m}(x_1, \cdots, x_n) = \left(\frac{\sum_{1 \leq i_1 \leq \cdots \leq i_m \leq n} \prod_{j=1}^m x_{i_j}^{p_j}}{C_n^m} \right)^{\frac{1}{p_1 + \cdots + p_m}} \tag{31}$$

Definition 30: Let $X = \{x_1, \cdots, x_n\}$ be a set of nonnegative real numbers, $m \in \{1, 2, \cdots, n\}$, $p_1, \cdots, p_n \geq 0$, the m^{th} GGeoMSM of X is defined as a mapping

$GGeoMSM^{m,p_1,\cdots,p_m} : (R^+)^n \rightarrow R^+$ such that:

$$GGeoMSM^{m,p_1,\cdots,p_m}(x_1,\cdots,x_n) = \frac{1}{p_1+\cdots+p_m} \times \left(\prod_{1\leq i_1\leq\cdots\leq i_n\leq n} (p_1 x_{i_1} + \cdots + p_m x_{i_m}) \right)^{\frac{1}{C_n^m}} \tag{32}$$

Liu *et al.* [105] combined the PA and the MSM to reduce the influence of the extreme values on the information fusion results of the MSM. They defined the power MSM (PMSM) for the q-ROFS, which is shown in Def. 31.

Definition 31: Let $A = \{a_1, \cdots, a_n\}$ be a q-ROF, for $\forall i \in \{1, \ldots, n\}$, $a_i = (u_i, v_i)$ is a q-ROF number (qROFN), where u_i and v_i are the membership and nonmembership functions, $0 \leq u_i^q + v_i^q \leq 1$, and $q \geq 1$. The m^{th} qROF power MSM (qROFPMSM) is defined as:

$$PMSM^{m,q}(a_1,\cdots,a_n) = \left(\frac{\sum_{1\leq i_1\leq\cdots\leq i_n\leq n} \prod_{j=1}^{m} \frac{m(1+T(a_{i_j}))}{\sum_{i=1}^{n}(1+T(a_{i_j}))} a_{i_j}}{C_n^m} \right)^{\frac{1}{m}} \tag{33}$$

where $T(a_i) = \sum_{j=1, j\neq i}^{n} Sup(a_i, a_j)$.

2) EXTENSIONS OF MSM-BASED OPERATORS TO DIFFERENT FSS

The MSM has been extended to the IFS [165], IVFS [186], PyFS [250], HFS [167], qROF [105] and LVs [98]. In particular, Qin and Liu [165] extended the MSM to the IFS, and proposed the MSM for the IFS (IFSMSM). Liu and Liu [110] further improved the work of [165] to explore the influence of the interactions between the membership function and the non-membership function on the IFSMSM. Sun and Xia [186] then extended the MSM to the IVIFS. Yang and Pang [250] proposed some MSM-based operators for the PyFS, which integrates the interactions between the membership and nonmembership functions into the original MSM. Wang *et al.* [210] extended the MSM to the trapezoidal interval type-2 fuzzy set (TraIT2FS). Wang [218] extended the MSM to process the 2-tuple linguistic variables (2TLVs), and proposed the dependent 2-tuple linguistic MSM (D2TLMSM). Liu *et al.* [98] proposed the MSM operators to process the uncertain or unknown information represented by the Pythagorean fuzzy uncertain LVs (PyFULVs). Geng *et al.* [54] applied the MSM to the interval neutrosophic linguistic variables (INLVs). Ju *et al.* [78] proposed the concept of the single-valued neutrosophic interval 2-tuple linguistic set (SVN-ITLS), and then applied the MSM to process the SVN-ITL numbers (SVN-ITLNs). Liu [122] extended the MSM to the single-valued trapezoidal neutrosophic set (SVTNS). Wang *et al.* [211] proposed a number of MSM operators for the simplified neutrosophic linguistic variables (NLVs). Yu *et al.* [262] extended the MSM to operate the hesitant fuzzy linguistic variables (HFLVs).

Some researchers explored the applications of the DMSM [196], GMSM [164], GGeoMSM [213], and PMSM [197]. In particular, Qin *et al.* [166] applied the DMSM to operate the uncertain linguistic variables (ULVs). Liu and Qin [113] applied the MSM and DMSM to the linguistic intuitionistic fuzzy set (LIFS). Teng *et al.* [196] extended the MSM and DMSM to the unbalanced linguistic variables and proposed a series of unbalanced linguistic MSM operators. Wang *et al.* [213] applied the MSM, GMSM and GGeoMSM to the single-valued neutrosophic linguistic set (SVNLS). In addition, they proved that the weighted MSM operators for the SVNLS are all reducible. Qin [164] extended the GMSM to the PyFS. They also applied the proposed operators to solve the classical superiority and inferiority ranking group decision problems [231]. Liu *et al.* [129] applied the MSM and the PA to the IVIFS, and proposed the interval-valued intuitionistic fuzzy PMSM (IVIFPMSM). Teng *et al.* [197] extended the PMSM to process the Pythagorean fuzzy linguistic variables (PyFLVs).

3) SUMMARY OF MSM-BASED OPERATORS

We summarize the inputs, variable relations and application environments of the MSM-based operators in Table 8. The common inputs of MSM operators are the m and X, where X is a set of variables that need to be aggregated, and m is the parameter that controls the behaviors of the MSM operator. The GMSM and the GGeoMSM additionally require the m inputs $\{p_1, \cdots, p_m\}$, where $p_j > 0 (j \in \{1, \ldots, m\})$ generalizes the power 1 of $x_{i_j} (i \in \{1, \ldots, n\})$ in the original MSM. The relation that can be modelled by the MSM-based operators is the *homo* interrelationship between variables. The MSMs have been extended to various fuzzy sets. However, the PMSM has not been defined for the crisp values. The operators MSM, DMSM, GMSM, GGeoMSM and PMSM all have the properties of idempotency, monotonicity, and boundedness [105], [167], [213].

When assigning different values to the parameters, these MSM-based operators have different behavioral patterns (see Table 9). In this table, *N/A* represents *not available*, and each operator is operated on n values: $X = \{x_1, \ldots, x_n\}$. We note that based on the definitions of MSM (Def. 27), DMSM (Def. 28), GMSM (Def. 29) and GGeoMSM (Def. 30), it is definitely that the GMSM and the GGeoMSM are the generalized forms of the MSM and the DMSM respectively:

Proposition 1: $GMSM^{m,1,\ldots,1} = GMSM^m$ and $GGeoMSM^{m,1,\ldots,1} = DMSM^m$.

Proposition 2: When $m = 1$, GMSM is the PA.

$$GMSM^{m=1,p_1,\ldots,p_m}(x_1,\ldots,x_n) = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^p\right)^{\frac{1}{p}} \tag{34}$$

The work of [105], [167], [213] have proved all the other behavioral patterns in Table 9 given different values of $m, p_1, \ldots, p_n$.

TABLE 8. Summary of MSM-based operators.

Operators	Inputs	Relations	Application environment
MSM	m, X	homo	crisp [137], IFS [167], IVIFS [188], PyFS [252], HFS [169], qROF [107] and LVs [99], SVNLS [215], INLV [54], SVN-ITLS [78], TraIT2FS [212], 2TLV [220], HFLV [264], PyFULV [99], PyFS [252], NLV [213]
DMSM	m, X	homo	crisp [168], LIFS [115], UbLV [198]
GMSM	m, X, $p_1, \cdots, p_m$	homo	crisp [215], SVNLS [215], PyFS [166]
GGeoMSM	m, X, $p_1, \cdots, p_m$	homo	crisp [215], SVNLS [215]
PMSM	m, X, Supp	homo	qROF [107], IVIFS [131], PyFLV [199]

Proposition 3: The *BM* and *GeoBM* are special cases of the *GMSM* and *GGeoMSM* respectively.

F. COMPARISON OF CI, PA, BM, HM AND MSM

We have reviewed five AOs (CI, PA, BM, HM, and MSM) and their extensions which consider the variable interrelationships. We compare them from the following perspectives: (1) their basic extensions in terms of their concepts; (2) can the operator model the *hete* or the *homo* interrelationship; (3) What the inputs an operator requires and how the input parameters of the operator influence the decision making results; and (4) can the operators model the self-relation, the symmetric relation and the relation among more than two variables.

1) EXTENSIONS OF CI, PA, BM, HM, AND MSM IN TERMS OF BASIC CONCEPTS

In Section II-A, we have summarized some of the basic extensions (see Tables 1, 3, 5, 7 and 8) and their relations (see Tables 2, 4, 6, and 9) of the CI, the PA, the BM, the HM, and the MSM . We can see that the extensions of these operators mainly include their generalized forms, their combinations with the GeoM and the PA, and the consideration of the weights or the induced ordered weights of the variables. The development of the CI and the BM are relatively mature compared to the other three operators, where the CI is one of the most representative nonadditive aggregation operators; and the BM has been extended to the EBM to model both the *homo* and *hete* interrelationships.

2) MODELLING *HOMO* AND *HETE* INTERRELATIONSHIPS

The CI-based operators can process both *homo* and *hete* interrelationships, because the CI-based aggregation can simutanously consider any subset of X by using a FM, including the importance of a single criterion (ϕ_{x_i}), the interactive importance of two criteria ($\phi(\{x_i, x_j\}), i \neq j$), and the coalition importance of a subset of the criteria ($\phi(\{x_i, x_{i+1}, \cdots, x_j\})$). The EBM and its extensions can process both the *homo* and *hete* interrelationships. The existing research work have not introduced the mechanism of modelling the *hete* interrelationship into the PA-, HM-, and MSM-based operators.

3) PARAMETERS OF OPERATORS

The types of the input parameters of these operators depend on their definitions. One common input is the aggregated variables X. In addition, the CI operators require the FM. The PA operators need the determination of a support function in advance. If an operator has the function of inducing the variable weights, it requires the input of an order inducing procedure. For example, the IGCOA adopts the OIV, and the GPOWA uses a BUM function g to generate the weights of the variables. If an extension of the CI, the BM, the HM, or the MSM has the capability of power averaging, this extension requires the input of a support function. The CI can use the pre-defined FM values to determine the types of the interrelationships. Compared to the CI, the EBM takes less time complexity to aggregate different types of the interrelationships, because the determination of a FM in the CI requires the determination of 2^n values for a set of n variables. However, the EBM only requires the pre-determination of the interrelationships between variables. Especially, Chen *et al.* [28] used an example to compare the performance of the CI and the GEBM for solving an MCDM problem. The result demonstrates that the GEBM performs as well as the CI. On the other hand, the GEBM is superior to the CI because the CI requires that a decision maker knows the weights of the interrelationships among the criteria in advance, while the GEBM only requires that a decision maker knows which criteria are independent or dependent to the others in advance.

The BM and the HM operators require the inputs of the p, q values. A series of examples [100], [103] have shown that the p, q values of the BM- and the HM-based operators influence the interaction degrees of the variables so as to influence the ranking results of the alternatives, where the increase of the values of p and q enhances the interaction degrees between the variables. The values of p, q reflect the risk attitude of the decision maker. If the values of p, q are relatively bigger, then the decision maker is risk averse; otherwise, the decision maker is risk seeking [44]. If the p, q are not given in advance, setting $p = q = 1$ or $p = q = 2$ can effectively support the MCDM with criteria interactions [199]. When the values of p and q are less than 1, the ranking is similar to the ranking of the case without considering the criteria interrelationships [100].

One import parameter of the MSM operators is the m, which determines the number of a set of interactive variables

TABLE 9. Behavioral patterns of MSMs w.r.t. different parameter values.

Parameters	MSM^m	$DMSM^m$	$PMSM^m$	$GMSM^{m,p_1,\cdots,p_m}$	$GGeoMSM^{m,p_1,\cdots,p_m}$
$m=1$	AM	$GeoM$	PA	PA	$\left(p^{n-1}\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}}$
$m=2$	$BM^{1,1}$	$GeoBM^{1,1}$	$BM^{1,1}$	BM^{p_1,p_2}	$GeoBM^{p_1,p_2}$
$m=n$	$GeoM$	AM	$GeoM$	$\left(\prod_{i=1}^{n} x_i^{p_i}\right)^{\frac{1}{p_1+\ldots+p_n}}$	$\frac{1}{p_1+\ldots+p_n}\left(\sum_{i=1}^{n} p_i x_i\right)$
$p_1=\cdots=p_m=1$	N/A	N/A	N/A	MSM^m	$DMSM^m$

considered simutaneously, and influences the evaluation scores of the alternatives in the MCDMs. We summarize the influence of the m as follows: (1) the value of m can represent the risk attibude of the decision maker in the MCDMs [105]. When m increases, the evaluation score of an alternative decreases. That is, if m becomes larger (e.g. $m > \lfloor \frac{n}{2} \rfloor$), the decision maker becomes more risk-preferred; and if $m = \lfloor \frac{n}{2} \rfloor$, the decision maker has the neutral risk attitude. (2) When m increases, the alternative that has the smaller difference between different evaluation scores is better than the other alternatives. And (3) the default m is usually set as 2 if m is not given by the decision maker [262].

4) SELF-RELATION, SYMMETRIC RELATION AND RELATION AMONG MORE THAN TWO VARIABLES

The CI-based operator does not require the FM of the self-relation of a variable (e.g. $\phi(\{x_i, x_i\})$). Similarly, the PA-, BM- and MSM-based operators do not consider the interactive value between a criterion and itself. The HM-based operators are the only type of operators to process the self-relation.

A symmetric relation between i and j means that the relation from i to j is the same as the relation from j to i. Both CI- and HM-based operators treat the relation between two criteria as a symmetric relation, and do not repeatedly aggregate the relations from i to j and from j to i simultaneously. In contrast, the PA- and the BM-based operators redundantly calculate the interrelationship values from i to j and from j to i, which requires more computational complexity. Based on the definition of the MSM, when $m = 2$, the MSM considers the interactions between any two variables once and only once. Therefore, the MSM-based operators do not repeatedly consider the symmetric relations.

The PA, the BM, and the HM focus on investigating the interrelationships between two variables. However, the CI and the MSM take into accout the relations among more than two variables. The CI uses the FM to represent the contribution of a variable coalition to the final aggregating result. The special cases of the CI include the weighted means, the OWA, the minimum and maximum and the order statistics [15]. The MSM is a generalization of the BM (see Table 9, row '$m = 2$'). It extends the BM by allowing the consideration of the relation among the subsets of X, where one subset includes more than two variables.

V. APPLICATIONS OF CI-, PA, BM-, HM- AND MSM-BASED OPERATORS

Based on a comprehensive survey, we find that most of the applications of the PA-, BM-, HM- and MSM-based operators are in the area of MCDMs [14], [237], [259]. However, the CI has been widely applied to different area, among which the most popular one is the fuzzy rule-based classification system (FRBCS). In addition, the CI has been combined with the techniques of the classification, the clustering, the evolutionary algorithm, and the TOPSIS-based MCDMs to improve the performance of these traditional techniques.

A. APPLICATIONS OF THE CI IN THE FRBCS

A fuzzy rule-based model [71] is defined as: if $\overrightarrow{x}$ is A_i then $f_i(\overrightarrow{x})$, $i \in [1, c]$, where c is the number of rules, $\overrightarrow{x}$ is an n-ary variable, A_i is an n-ary information granules (formed by the fuzzy sets) in the n-dimensional input space, and $f_i(\overrightarrow{x})$ is a member of the output space. The reasoning scheme of a model [71] is to determine the activation levels of the rules and aggregate their outputs, which is represented by $y = \oplus_{i=1}^{c} A_i(\overrightarrow{x})f_i(\overrightarrow{x})$, where $\oplus$ represents an aggregation law, such as the summation.

A FRBCS can deal with the classification problems effectively [131] because of its two features: (1) the nonlinearity of a fuzzy classifier helps to reduce the classification error, and (2) the interpretability of a fuzzy classifier makes the classification models understandable to the users. The fuzzy reasoning method (FRM) in an FRBCS is the main component to form the fuzzy reasoning rules of the object classification, which is based on the aggregation of different information sources using the aggregation [63] or the preaggregation [133] operators. A classical FRBCS is shown in Fig. 3 [72], which is a mapping from an n dimensional vector $\overrightarrow{x} = \{x_1, \cdots, x_n\}$ to a classification space $\{0, 1\}$, where 0 and 1 refer to two classes of negative and positive patterns respectively. We interpret a classification fuzzy rule in terms of a TP-topology based on Fig. 3 [72]: if the value of $\overrightarrow{x}$ is A_i then the probability of the object $\overrightarrow{x}$ belonging to class 1 is $f_i(\overrightarrow{x}) \in [0, 1]$. A fuzzy rule contains two parts: the condition part and the conclusion part [72]. $A_i (i \in [1, c])$ are the condition part and are identified by clustering algorithms (e.g. fuzzy c means [16]) based on the n-ary feature space. The number of clusters determines the number of rules. Each rule (e.g. rule i) determines the membership degree of

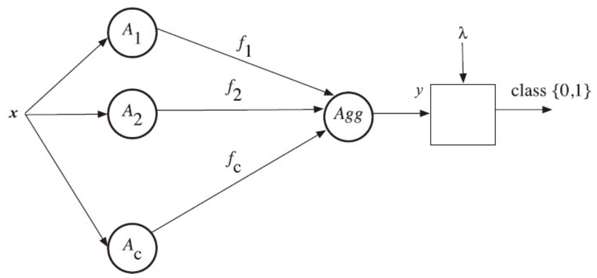

FIGURE 3. Classical fuzzy rule-based classification system [72].

$x_k (k \in [1, n])$ belonging to the ith cluster. The result of the clustering is a mapping $\overrightarrow{x} \rightarrow [A_1, \cdots, A_c] \in \{0, 1\}^c$, which reduces the dimensionality of $\overrightarrow{x}$ from n to c. The consequence of a fuzzy rule f_i is determined by $f_i = n_1/(n_0 + n_1)$, where n_0 and n_1 counts the number of the objects in the ith cluster that are classfied as class 0 and 1 respectively. The y in Fig. 3 is an aggregation of the conditions and the consequences of c fuzzy rules: $y = \sum_{i=1}^{c} A_i(\overrightarrow{x}) f_i(\overrightarrow{x})$. λ is a threshold that determines the class of $\overrightarrow{x}$ based on the value y.

Traditional fuzzy rule-based models treat each rule as an individual entity. However, Hu *et al.* [71] pointed out that considering the interaction between rules yields to a more efficient structure of the model, which is a promising research area. They referred the realization of the rule interaction to a mechanism of measuring and incorporating the rule interaction to the model to improve the quality of the reasoning results. The fuzzy rule-based model with the rule interaction is defined as: $B_k = g(WA)$, $k \in [1, p]$, where p represents the number of outputs (fuzzy sets), g is a non-linear mapping, $A = (A_1, \cdots, A_c)$ is a vector of the activation levels inferenced by the input $\overrightarrow{x}$, and $W = (w_{ij})$ is a $c \times c$ matrix where each value w_{ij} represents the interaction between rules i and j for all $i, j \in [1, c]$.

Barrenechea *et al.* [9] proposed to use the CI to aggregate the information associated with each fuzzy rule by considering the interaction between rules. Lucca *et al.* [133] introduced the pre-aggregation function which replaces the product operator used in the classical CI by the t-norms. In [132], they further generalized the pre-aggregation operator using the copula, and proposed a CC-integral operator. Based on the CC-integral, Lucca *et al.* [131] proposed a CF-integral operator to investigate the classification performance of an FRBCS based on the non-averaging characteristics. We introduce the concepts of the pre-aggregation, the CC-integral, and the CF-integral operators as follows.

Lucca *et al.* [133] discussed the restriction of the monotonicity in an aggregation function, and proposed the concept of a pre-aggregation function that requires directional monotonicity, i.e. the monotonicity along some certain directions but not all directions, which relaxes the monotonicity of the aggregation function. They defined a CI-based pre-aggregation function, which uses the minimum or the Hamacher product t-norm operation to replace the product operation in the classical CI. Furthermore, they applied the proposed CI-based pre-aggregation function in a FRBCS. The experiment results show a better performance of using the pre-aggregation function in an FRM compared to the use of the classical CI and the winning rule.

On the basis of the work of [133], Lucca *et al.* [132] proposed a Choquet-like copula-based aggregation function, called the CC-integral, which replaces the product operator in the classical CI by a copula, and produces an aggregation function rather than a pre-aggregation function. Lucca *et al.* [132] applied the CC-integral to the FRBCS, which outperforms the performance of the best Choquet-like based pre-aggregation function.

Lucca *et al.* [131] proposed a CF-integral operator that improves the CC-integral by generalizing the copula in CC-integral to a left 0-absorbent bivariate function F. This F satisfies a minimal set of properties of guaranteeing the CF-integral be a pre-aggregation function. They then applied the CF-integral to the FRM of a FRBCS. They also proved that the CF-integral has the non-averaging characteristic, which can yield to a better classification result comparing with the averaging AOs.

Hu *et al.* [72] proposed an enhanced generic topology to introduce the interaction between the fuzzy rules and the membership functions into the FRBCS. They generalized the rule aggregation in the classical FRBCS by replacing the sum and the product operators using the t-norm and the t-conorm, e.g. $y = T_{i=1}^{c}(A_i T' f_i)$ and $y = T_{i=1}^{c}((1 - A_i) T' f_i)$, where T and T' are a certain t-norm or t-conorm operator, and $T \neq T'$. To incorporate the interactions into the FRBCS, Hu *et al.* [72] enhanced the aggregation of the information granule by introducing an interaction matrix $V = [v_{ij}] (\forall i, j \in [1, c])$ of the fuzzy rules, which is shown in Fig. 4. The new information granule is defined as: $A_i = T_{j=1}^{c}(A_j T' v_{ij})$, $v_{ij} \in [0, 1]$.

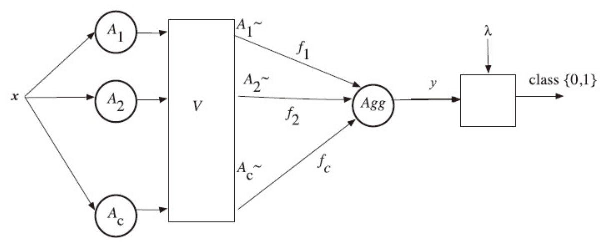

FIGURE 4. Fuzzy rule-based classification system with the consideration of information interaction [72].

Summary of the applications of the CI in the FRBCS: The CI has been applied to the FRBCS, which integrates the interaction of the fuzzy inference rules to yield a more efficient classification model by comparing with the traditional FRBCS. The CI requires that each variable in a rule should have the property of the monotonicity. To relax this restriction, a CI preaggregation function is proposed which requires the monotonicity of a subset of the variables. To futher improve the performance of the preaggregation function in the CI, a Choquet-like copula-based aggregation function (CC-integral) is developed which replaces the product operator using a copula. Furthermore, a more general CI operator, called the CF-integral, is proposed which uses a left 0-absorbent bivariate function to replace the copula in the CC-integral. Overall, the existing literature has proved the better performance of these CI-based FRBCSs compared to the traditional FRBCSs.

B. APPLICATIONS OF THE CI IN CLASSIFICATION

Tehrani *et al.* [195] applied the discrete CI to the multipartite ranking, which depends on the identification of an appropriate FM ϕ to calculate the CI of an alternative. The learning of the multipartite ranking is to learn a ranking model which is used to determine the order of a subset of the alternatives. This work formalizes the problem of identifying the ϕ as a margin maximization problem and solves it using a cutting plane algorithm. Let $O = \{o_1, \cdots, o_r\}$ be a set of objects (or alternatives) , $X = \{x_1, \cdots, x_n\}$ be a set of criteria that is used to describe these objects, and ϕ be a FM on X, then each object $o \in O$ can be represented by a feature vector: $f_o = \{f_o(x_1), \cdots, f_o(x_r)\}$, and the CI utility of o is represented by $U(o) = C_\phi(f_o)$.

Torra and Narukawa [203] discussed the problem of integrating the CI to the distance calculation for forming the probability density distribution of a sample set. They reviewed the Mahalanobis distance in the Gaussian distribution, which calculates the distance between the samples by taking into account their correlations. In addition, they defined a CI-based distance and discussed the probability density distributions based on this distance calculation. Finally, they combined the Mahalanobis distance with the CI, and proposed a Choquet Mahalanobis integral operator, a Choquet Mahalanobis distance, and a generalized probability density function. The combination of the Mahalanobis distance and the CI takes advantage of both the covariance matrix and the FM for modelling the attribute interactions, and enables us to learn classification models based on more general density distribution functions comparing with the Gaussian-based models.

Tehrani *et al.* [194] proposed a generalized logistic regression, called the choquistic regression, which uses the CI to represent the predictor variables. The choquistic regression is capable of capturing the non-linear dependencies and the interactions among variables, and keeps the comprehensibility and the monotonicity of the individual predictors.

Pacheco and Krohling [158] proposed a CI-based method for the aggregation of neural classifiers. They focused on solving the core issue of deriving the FM of the set of the neural classifier ensemble based on the calculation of the Shannon's entropy.

C. APPLICATIONS OF THE CI IN CLUSTERING

Tseng *et al.* [205] applied the CI to the metric learning in the semisupervised clustering. The discrete CI is capable of modelling the importance of the single attribute, the coalitions of the criteria, and the interactions between criteria, so a CI-based metric provides a great flexibility to model the attribute-level constraints in a clustering process. The proposed CI-based semisupervised learning method takes into account new forms of partial information: the interaction-order preference, the attribute-order preference, and the unlabeled data with the instance-level constraints, where the interaction-order preferences indicate the correlations between criteria and are measured by an interaction index. Ng *et al.* [154] introduced a subspace clustering technique that considers the feature interactions based on the CI to improve the clustering-based pattern recognition without considering the feature interactions.

D. APPLICATIONS OF THE CI IN EVOLUTIONARY ALGORITHMS

Branke *et al.* [19] introduced an interactive multiobjective evolutionary algorithm to find the most preferred objects in a Pareto-optimal set, which uses the CI to model users' preferences by considering the attribute interactions. To achieve a trade-off between the flexibility of representing a user's preferences and the complexity of learning the model, this work designs a dynamic procedure to switch between a simple linear model and the CI model according to the complexity of users' preferences.

E. APPLICATIONS OF THE CI IN TOPSIS-BASED DECISION MAKING

Lourenzutti *et al.* [130] proposed the CI based fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) and the TODIM (an acronym in Portuguese for Interactive and Multi-Criteria Decision Making) methods that take into account the criteria interrelations through the FM and the CI. The authors identified one drawback of using the FM, which is the difficulty and the complexity of determining a FM, either based on the expert opinions or a large amount of data. They focused on investigating the ways of determining the FM of an MCDM problem based on the expert opinions when only a small supporting dataset available. In a group decision making environment, different decision makers may provide different FMs for the criteria.

Lourenzutti *et al.* [130] allowed each decision maker giving different FMs in terms of different states. They then defined an objective function to select an optimal FM from the FM set given by decision makers. The selected FM is capable of descriminating the alternatives that have controversial ranking orders. Their method adopts a module separation procedure, where each module includes a set of variables in a similar type (e.g. crisp numbers, T1FNs, T2FNs, and random variables), to aggregate different types of variables.

Based on the above discussion, we can see that the CI plays an important role in clustering, classification, evolutionary learning, and TOPSIS. The capacity of the CI to model the weight of a single criterion and the weight of a subset of the criteria improves the performance of the other artificial intelligence technologies.

F. APPLICATIONS OF THE CI IN PRACTICAL SCENARIOS

In the previous section, we introduced the capabilities of the CI for improving the other artificial intelligence techniques. In this section, we summarize the applications of the CI to a number of practical scenarios, including the brain computer interface [24], [227], [254], [265], image processing [30], [53], [179], [227], encryption and security [57], [99], [272], sustainable city developement [22], [157], [266], [276], supply chain [4], [261], [285], risk assessment [1], [49], [90], [151] and other problems in economics [50], [73], and [180].

1) BRAIN COMPUTER INTERFACE

Some researchers applied the CI to the area of the brain computer interface (BCI) mainly to aggregate multiple classifiers or criteria for the pattern recognition of the electroencephalography (EEG) signals, or for the control of robot behaviors. In particular, based on the FM and the CI, Yoo and Kim conducted a series of researches about the gaze control to improve the performance of the BCI [253]–[255]. Yoo and Kim [255] developed a gaze control architecture based on the CI to implement the field-based navigation for the humanoid robots. This work [255] defines four criteria (i.e. local map confidence, self-localization, obstacles and waypoint) and their partial utility functions to calculate the score of each direction and determine the optimal gaze direction. The CI-based gaze control architecture is shown in Fig. 5, where the partial values of the four criteria are assigned to a gaze direction (represented by X_{gd} in Fig. 5). The 'visibility check' component checks whether a gaze direction is located in the gaze area or not. Then the global evaluation is taken for the gaze directions located in the gaze area. The user-defined preference is defined by a FM and the hierarchy diagram is used to transform the FM to a hierarchy-based FM. Finally, the CI calculates the global evaluation of a gaze direction based on the partial evaluation criteria scores of the direction and the hierarchy-based FM. Yoo and Kim [253] extended the work of [255] by defining seven criteria to evaluate a gaze direction. Then Yoo and Kim [254] further extended the work of [253] by developing an evolutionary fuzzy integral-based gaze control algorithm, which derives the individual preference and controls the human-like gaze based on the individual preference.

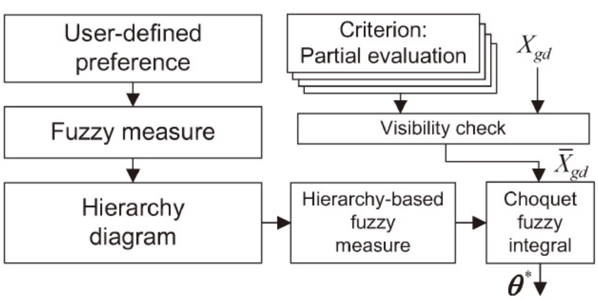

FIGURE 5. The fuzzy-integral-based gaze control architecture [255].

In addition, Cavrini et al. [24] proposed a CI-based method to combine the outputs of a set of classifiers for the pattern recognition of the EEG in the BCI. Wu et al. [227] applied the fuzzy fusion approch to improve the performance of the BCI by monitoring and analysing the EEG signals. Zhang et al. [265] used the fuzzy integral to analyze the EEG signals to implement the human intention recognition. They proposed a deep learning framework based on the 3D convolutional neural network (3D-CNN) and the recurrent neural network (RNN) to extract the local spatio-temporal features and the global temporal features, and then used the fuzzy integral to integrate these two types of information based on the optimized FM, where the optimized FM is derived based on the deep Q-network (DQN). Wu et al. [227] explored the applications of the CI to the fusion of the Motor imagery signals. They used an ensemble of LDA classifiers to classify a user's mental signals, and used the fuzzy integrals to integrate the information in this classifier ensemble process.

2) IMAGE PROCESSING

The CI operator was applied in the area of image processing typically for the edge detection and the object extraction. Sesma-Sara et al. [179] proposed an image edge detection algorithm based on the ordered directionally monotone functions to consider the direction of the edges at each pixel in the edge detection. The ordered directionally monotone is defined by Def. 32. A CI is an ordered-directionally $\overrightarrow{r}$-increasing function if and only if it satisfies the following condition: let $\overrightarrow{r} = (r_1, \cdots, r_n)$ be a nonnull real vector, μ be a FM and for $\forall\sigma \in S_n$, if $\sum_{i=1}^{n} r_i\mu_\sigma(i) \geq 0$, where $\mu_\sigma(1) = \mu(\{\sigma(n)\})$ and $\mu_\sigma(i) = \mu(\{\sigma(n-i+1), \cdots, \sigma(n)\})$ for $\forall i \in \{2, \cdots, n\}$. Based on the ordered-directionally monotone CI function, Sesma-Sara et al. [179] assigned a magnitude value of the gradient vector to each pixel of an image to extract features of the image. The FM of the ordered-directionally monotone CI is constructed based on the overlap indices.

Definition 32: Assume a function $F : [0, 1]^n \to [0, 1]$, $\overrightarrow{r} \in R^n$ with $\overrightarrow{r} \neq \overrightarrow{0}$, S_n is the set of all permutation operators of $\{1, \cdots, n\}$, if F satisfies the following conditions: for $\forall\overrightarrow{x} \in [0, 1]^n$, $\forall\sigma \in S_n$ that $x_{\sigma(1)} \geq \cdots \geq x_{\sigma(n)}$, and any constant value $c > 0$ such that $1 \geq x_{\sigma(1)} + cr_1 \geq \cdots \geq x_{\sigma(n)} + cr_n \geq 0$, if $F(\overrightarrow{x} + c\overrightarrow{r}_{\sigma^{-1}}) \geq F(\overrightarrow{x})$, where $\overrightarrow{r}_{\sigma^{-1}} = (r_{\sigma^{-1}(1)}, \cdots, r_{\sigma^{-1}(n)})$, then F is ordered directionally $\overrightarrow{r}$-increasing; if $F(\overrightarrow{x} + c\overrightarrow{r}_{\sigma^{-1}}) \leq F(\overrightarrow{x})$, then F is ordered-directionally $\overrightarrow{r}$-decreasing.

Chiranjeevi and Sengupta [31] investigated the problem of object detection in a video that has heavy dynamic background. They used the fuzzy integral to compute the fuzzy similarities between the feature vectors of images, and update and classify the image models. Then, Chiranjeevi and Sengupta [30] further extended the CI of the real numbers to the interval-valued environment, which is capable of modelling the 'adaptive uncertain values at the pixel level', and is used to calculate the interval-valued similarity among the feature models of each pixel, where a feature model is a vector containing four feature elements.

Furthermore, Du [39] investigated the application of the CI in the imagery fusion to improve the locating accuracy in the images with different resolution levels. The author pointed out that it is difficult to precisely label the image training data pixel by pixel, and the locating of an image cannot be accurate at the level of single pixel. In addition, different types of images with different resolutions may be fused and processed in an application. Therefore, the author proposed a Multiple Instance CI framework (MICI) to fuse the images of the multi-resolutions from multiple sensors. Karczmarek *et al.* [81] applied the CI-based preaggregation function to solve the recognition problems by aggregating the classifiers using the t-norm operators. Martínez *et al.* [153] made a comparison of the applications of the Choquet and Sugeno integrals in the area of pattern recognition. The Choquet and Sugeno integrals were used to integrate different sources of information in an uncertain environment. The authors then used the proposed methods to aggregate the outputs of a neural network for face recognition. Karczmarek *et al.* [81] applied the pre-aggregation functions of CI to aggregate classifiers for face recognition. Taştimur *et al.* [201] used the CI to aggregate the outputs of multiple traffic sign recognition systems, and the output of the CI is the final sign recognition results. Wei *et al.* [224] proposed an image fusion approach for object detection based on a FM agreement analysis and the CI-based fusion method. They extended the CI to aggregate the two-dimensional interval-valued information, based on which an Axis-Aligned Bounding Box Fuzzy Integral (AABBFI)-based fusion method was developed to improve the accuracy of the object detection.

3) ENCRYPTION AND SECURITY

The CI was also applied to improve the image encryption [99], [272] and the system security [57]. Seyedzadeh *et al.* [180] proposed a CI-based keystream generator for the encryption of the RGB color image, where the generation of the pseudo-random keystreams is based on the generation of the λ-FM [150]. The output of the CI shifts the bits of the three gray level images randomly, and then the generated keystreams combined with the RGB color values are used to encrypt the shifted gray level images.

Furthermore, Zhang *et al.* [272] cryptanalyzed the CI-based color image cryptosystem proposed by Seyedzadeh *et al.* [180]. Liu *et al.* [99] proposed a color image encryption scheme based on the chaos theory and the CI. Goztepe [57] developed a decision model based on the analitic network process and the CI integration for selecting the operating systems to avoid the cyber threats.

4) SUSTAINABLE CITY DEVELOPMENT

Büyüközkan *et al.* [22] applied the IFCI to aggregate the interactive sustainability related criteria in public transportation systems. In addition, the authors proposed a pair-wise comparison method to identify the FM of the IFCI. Bottero *et al.* [18] presented another work of using the CI to solve a sustainability problem: selecting the optimal location for a waste incinerator plant. Zhao *et al.* [276] used the CI operator and the Shapely entropy to evaluate the sustainable development level of cities. Zhang *et al.* [266] formulated the problem of the city sustainability evaluation as a MCDM problem and developed an optimization approach to determine the FM of the interactive criteria. The CI was then used to aggregate the scores of the criteria based on the derived FM. Zhang *et al.* [267] used the λ-FM and the CI to process the mutual interaction among the criteria of selecting the sustainable energy plan of Nanjing City. Ozdemir and Ozdemir [157] explored the application of the CI in solving the energy saving issues. They used the GCI to aggregate the evaluation criteria with nonlinear relationships to rank the residential heating systems.

5) SUPPLY CHAIN

The applications of the CI in the supply chain mainly include the supplier selection [4], [261], [285] and the warehouse selection [37]. The supplier selection is a complex decision making problem and the CI operator has been proved to be a useful tool for processing the complexities [4]. Ashayeri *et al.* [4] introduced an IFCI to select the partners and the configurations of the supply chains. Hwang and Shen [74] employed the fuzzy integral approach to model the criteria dependence, the information vagueness and the fuzziness of the human expression in the supplier selection. Tuzkaya [206] proposed a decision making method based on the intuitionistic fuzzy CI for supplier selection. Zhu and Li [285] proposed an integrated framework in the hesitant fuzzy environment for green supplier selection, which uses the CI operator to rank the green suppliers. Yu *et al.* [261] developed a fuzzy CI model to measure the correlations among the criteria of selecting the supply chain partners. Demirel *et al.* [37] applied the multi-criteria CI to select the warehouse locations for a Turkish logistic firm.

6) RISK ASSESSMENT

Many researchers studied the applications of the CI to the risk assessment. Smith *et al.* [182] investigated the composition of multiple CIs. They associated the CI with a genetic program, proposed a genetic program CI (named GpCI) and used a genetic optimization algorithm to learn the parameters of the GpCI. The GpCI is applied to fuse the values from multi-sensors of the electromagnetic induction (EMI) and the ground penetrating radar (GPR) to detect the explosive hazards. Li *et al.* [90] studied the application of the CI in the area of risk assessment. They proposed a CI-based method to calculate the risk value by considering the correlation of multiple risks. Namvar and Naderpour [151] used the CI to fuse the base classifiers to predict the credit risk in the peer-to-peer (P2P) lending system. The CI fuses the prediction results of a set of base classifiers to enhance the credit worthiness, which effectively reduces the risk of financial losses in a P2P

TABLE 10. Summary of the applications of the CI in practical scenarios.

Application area	Functions	Purposes
brain computer interface	integrate classifiers or criteria	pattern recognition; decision making for robot moving
image processing	integrate image feature vectors or classifiers	edge detection; object extraction
encryption and security	integrate encryption keystreams or criteria	image encryption; system security
sustainable city developement	integrate criteria	energy saving element selection
supply chain	integrate criteria	supplier selection; warehouse location selection
risk assessment	fuse sensor data; integrate risks or classifiers	decision making; risk prediction
other problems in economics	integrate agent preference or criteria	economic equilibria; investment decision making

lending transaction. Furman *et al.* [49] used the signed CI to represent the Gini functional for the risk management in finance.

7) OTHER PROBLEMS IN ECONOMICS

We have introduced the applications of the CI in some cross-cutting area of economics, e.g. the sustainable city development, the supply chain and the risk assessment. We then review some economic problems that did not appear in the above cross-cutting area, for example, the searching of the equilibria of an economic model [178], and the making of the investment decisions in cloud computing [84].

In the exchange economies, the space of agents is assumed to be non-atomic measure space [3]. Sambucini [178] explored the way of searching the equilibria of the exchange economies in the finite dimensional commodity space with a more general structure of the agent set S. In Sambucini's work [178], S is modelled by a FM μ to represent the weight of the agent coalitions in the economic market. S can be decomposed into a number of coalitions, where the agents in one coalition have the same initial endowment and criteria preference. Candeloro *et al.* [23] studied one central problem of the Mathematical Economics which searches the equilibria of an Economic model. They decomposed the agent sapce into several sections which correspond to a set of autonomous economic models. In addition, coalitions are defined to represent the interactions among sections.

The CI has also been applied to help a decision maker make the investment decision. For example, Sun *et al.* [84] applied the CI operator in the cloud service selection to rank cloud services by considering the service criteria interactions. Ozdemir and Basligil [156] explored the application of the CI to the investment decision in the airway transportation by modelling the nonlinear relationships among the main criteria and the subcriteria. Ferreira *et al.* [46] used the CI to evaluate the ethical determinants in the banking activities. Cebi [25] developed a quality evalution method of websites based on two multi-criteria decision making methods: the decision-making trial and the generalized CI, where the CI is used to aggregate the degrees of the importance of the website quality evaluation criteria. Coletti *et al.* [33] represented the partial preference relation of a set of generalized lotteries as a strictly increasing Choquet expected utility function. Lin and Jerusalem [96] used the diamond pairwise comparison method and the CI to induce the criteria weights and the evaluation utilities to select the optimal fashion design.

8) SUMMARY OF APPLICATIONS OF THE CI IN PRACTICAL SCENARIOS

Based on our literature review, we summarize the main functions and purposes of the CI in the seven practical scenarios in Table 10. We can see that in the area of BCI, the CI is mainly used in the classifier integration to improve the classification accuracy for EEG pattern recognition, and in the criteria integration to make informed decisions for controlling the robot behaviors. In image processing, the CI is mainly used to integrate the feature vectors of the images or integrate the classifiers for image classification, and the main purposes are to detect edges of objects and to extract objects from the dynamic environment. In the area of encryption and security, the CI is mainly used to integrate the randomly generated keystreams for image encryption or integrate the system evaluation criteria for secure system selection. In the area of sustainable city development and supply chain, the CI is mainly used to integrate the decision making criteria to select the energy saving elements, the suppliers or the warehouse locations. As to the risk assessment, the CI fuses the data from different sensors, or integrates the correlated risks or classifiers to reduce or predict the risks. Furthermore, the CI is used to integrate the preference of agents in the exchange economics to achieve the economic equilibria, or integrate the decision criteria to make investment decisions.

VI. FUTURE RESEARCH DIRECTIONS FOR THE AGGREGATION OPERATORS CONSIDERING CRITERIA INTERRELATIONSHIPS

We summarize the future research directions of the AOs considering the criteria interrelationships.

- *Design mechanisms to establish the interrelationships among criteria.* Most of the work investigating the criteria interrelationships assumes that the interrelationships among criteria are known in advance [42]. However, this is not always true in practical applications, so it is necessary to design a mechanism to establish accurate interrelationships.
- *Enable the wide application of the AOs*. Based on our survey, the CI-based operators have been applied to different areas. However, the PA-, BM-, HM-, and MSM-based operators are only developed for solving the MCDMs. In particular, many research works developed the extended and the generalized forms of the BM. Based on our analysis, the BM-based operators are superior

to the CI-based operators in terms of certain aspects (e.g. the BM-based operators do not require the decision makers determining the FM in advance). Therefore, exploring the application or the integration of the BM-based operators (and the PA-, HM- and MSM-based operators) to the other area (e.g. pattern recognition, deep learning, and crowdsourcing) may enhance the capabilities of both the AOs and the other technologies.

- *Develop the basic extended forms of the PA-, HM- and MSM-based operators to enable them to process the hete interrelationships.* From Tables 3, 7 and 8, there has not been any basic extension of the PA, HM and MSM, which is capable of processing the *hete* interrelationship. Though they have certain advantages compared to the BM-based operators, the capability of modelling the *hete* interrelationship can make their aggregating results have more comprehensive meaning.
- *Reduce the computational complexity of the AOs.* None of the work of the PA-, BM-, HM- and MSM-based operators discusses the computational complexity of the information aggregation by using their proposed operators. Assume a problem has n input arguments, the computational complexity of the aggregation using the basic BM (see Def. 12) or the 2-additive CI (see Def. 4 and Def. 7) is $\mathcal{O}(n^2)$, which is a very high in the applications requiring real time responses. The computational complexity would be higher than $\mathcal{O}(n^2)$ if the AOs are applied to a more complex fuzzy environment (e.g. the IVIFS, the qROF, or the INULS). Therefore, it may have a wider application if we develop a CI-, PA-, BM-, HM- or MSM-based AO by taking into account its computational complexity.
- *Design measurement indices to validate the performance of the AOs considering criteria interrelationships.* It is not intuitive to understand the evaluation results of the alternatives using the AO considering criteria interrelationships. The evaluation results require an in-depth analysis [60] for the decision makers. On the other hand, the measurement indices that can accurately and dynamically measure the performance of the AOs would dramatically improve the application and reduce the analysis complexity of using these operators.
- *Explore more fuzzy extensions of the CI-, PA-, BM-, HM- or MSM-based operators.* We summarized the fuzzy extensions of these operators in Tables 1, 3, 5, 7 and 8. We can see that each operator has been widely extended to different FSs. However, there are still some gaps. For example, the GEBM and EGeoBM can only be applied to crisp dataset. Their extensions to the fuzzy environment would greatly improve the efficiency of the fuzzy MCDM. In addition, the CI- and HM-based operators have not been extended to the hesitant fuzzy linguistic term set [94]; and all of the CI-, PA-, BM-, HM- or MSM-based operators have not been extended to the probability fuzzy linguistic term set [8].

VII. CONCLUSION

In this paper, we explored the definitions, properties and development of five aggregation operators (i.e. CI, PA, BM, HM and MSM) which consider different types of interrelationships among criteria. We first reviewed the concepts of different fuzzy sets, and introduced the definitions of the *homo* and *hete* interrelationships. Then we made a comprehensive survey on the basic extensions and the fuzzy extensions of the CI, PA, BM, HM and MSM. We summarized these operators in terms of their parameters, the types of the interrelationship they can model, the capability of dealing with the self-relation of a criterion and the symmetric relation between criteria, and the fuzzy sets they have been extended to. We discussed the special behavioral patterns of the five operators and their basic extensions given special values of their parameters, based on which we analyzed the relations between an operator and its basic extensions, and between different types of operators. As the PA-, BM-, HM- and MSM- based operators are mostly applied to the MCDM, we only reviewed the applications of the CI-based operators, which have been applied to improve the performance of the other traditional artificial intelligence technologies, e.g. the FRBCS, classification, clustering, evolutionary algorithm and TOPSIS-based MCDM. We further analyzed their applications in seven practical scenarios: the brain computer interface, image processing, encryption and security, sustainable city development, supply chain, risk assessment and other problems in economics. Finally, we pointed out six future research directions of the AOs considering the criteria interaction.

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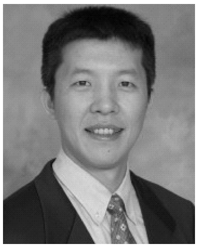

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