Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 13734 - 13741
Received 05 March 2021; Accepted 01 April 2021.

Algebraic Structures of Semigroup by Rough Neutrosophic Ideals

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Abstract: In this article, we will look at the definition of rough neutrosophic set in semigroups and investigate their properties. We also introduce the concept of rough Neutrosophic (right,left,two-sided,bi-) ideals in a semigroup and study some properties of the ideals of semigroup.

Keywords: Rough set, Neutrosophic set, Rough neutrosopic set, Rough neutrosopic left(right) ideal, Rough neutrosophic bi-ideal.

1. Introduction

The idea of fuzzy sets was first proposed by (Zadeh.1965). As an extension of it, (Atanassov.1986)introduced intuitionistic fuzzy set, where a degree of non-membership was considered besides the degree of membership of each element with (membership value + non-membership value) ≤ 1. After that several generalizations such as, rough sets, vague sets, interval-valued sets etc. are considered as mathematical tools for dealing with uncertainties. (Smarandache.2005) introduced Neutrosophic set in which he introduced the indeterminacy to intuitionistic fuzzy sets. There are also several authors who have enriched the theory of neutrosophic sets.

(Elavarasan, Balasubramanian; FlorentinSmarandache; and Young Bae Jun.1990) introduced the Neutrosophic ideals in semigroups and investigated several properties. Based on an equivalence relation, (Dubois and Prade.1990) introduced the lower and upper approximations of fuzzy sets in a Pawlak approximation space to obtain an extended notion called rough fuzzy sets. The notions of rough prime ideals and rough fuzzy prime ideals in semigroup are introduced by (Xiao.Q. M, Zhang.Z.L,(2006)).(Jayanta Ghosh, Samanta.2012) introduces the concept of Rough intuitionistic fuzzy ideals in semigroups. It motivates us to define the notion of rough neutrosophic in semigroup.

In this paper, the notion of rough neutrosophic ideals in semigroups is introduced and several properties are investigated.

2. Preliminaries

This paper recalls the concepts of neutrosophic ideals in a semigroup and in previous versions of this paper (Elavarasan, Balasubramanian; FlorentinSmarandache; and Young Bae Jun. (2019),JayantaGhosh, T. K. Samanta. (2012Kuroki.N(1997)) some simple definitions are presented.

3. Rough neutrosophic subsets in a semigroup

In this section we establish some results dealing with rough neutrosophic subsets in a semigroup.It should be noted that throughout this section ζ denote the congruence relation on S_G .

Definition 3.1:

Let ζ be a congruence relation on S_G , such that $(m,n)\in\zeta \Rightarrow (mx,nx)\in\zeta,(xm,xn)\in\zeta,\forall x\in S_G.$

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Consider $[m]_\zeta$ as a ζ congruence class containing the element $m \in S_G$. For ζ on S_G , we have $[m]_\zeta [n]_\zeta \subseteq [mn]_\zeta \forall m,n \in S_G$. ζ on S_G is complete if $[m]_\zeta [n]_\zeta = [mn]_\zeta \forall m,n \in S_G$.

Let κ be a *NS*of S_G . Then the *NS* $\underline{\zeta}(\kappa) = \{\underline{\zeta}(t), \underline{\zeta}(i), \underline{\zeta}(f)\}$ and $\overline{\zeta}(\kappa) = \{\overline{\zeta}(t), \overline{\zeta}(i), \overline{\zeta}(f)\}$ are respectively called ζ lower and ζ upper approximations of NS.

Where,

$$\underline{\zeta}(\kappa)(t)(\alpha) = \wedge_{p\in[\alpha]_\zeta} (t(p))$$

$$\underline{\zeta}(\kappa)(i)(\alpha) = \vee_{p\in[\alpha]_\zeta} (i(p))$$

$$\underline{\zeta}(\kappa)(f)(\alpha) = \vee_{p\in[\alpha]_\zeta} (f(p))$$

$$\overline{\zeta}(\kappa)(t)(\alpha) = \vee_{p\in[\alpha]_\zeta} (t(p))$$

$$\overline{\zeta}(\kappa)(i)(\alpha) = \wedge_{p\in[\alpha]_\zeta} (i(p))$$

$$\overline{\zeta}(\kappa)(f)(\alpha) = \wedge_{p\in[\alpha]_\zeta} (f(p))$$

$$\forall \alpha \in S_G$$

For a *NS*of S_G , $\zeta(\kappa) = (\underline{\zeta}(\kappa), \overline{\zeta}(\kappa))$ is called RNS with respect to ζ if $\underline{\zeta}(\kappa) \neq \overline{\zeta}(\kappa)$.

Theorem 3.2 :

Let ζ and ζ' be any two congruence relations on S_G . If M and N are any two NS of S_G , then thefollowing holds:

$(i)\underline{\zeta}(\mathrm{M}) \subseteq \mathrm{M} \subseteq \overline{\zeta}(\mathrm{M})$

$(ii)\underline{\zeta}(\underline{\zeta}(\mathrm{M})) = \underline{\zeta}(\mathrm{M})$

$(iii)\overline{\zeta}(\overline{\zeta}(\mathrm{M})) = \overline{\zeta}(\mathrm{M})$

$(iv)\overline{\zeta}(\underline{\zeta}(\mathrm{M})) = \underline{\zeta}(\mathrm{M})$

$(v)\underline{\zeta}(\overline{\zeta}(\mathrm{M})) = \overline{\zeta}(\mathrm{M})$

$(vi)(\overline{\zeta}(\mathrm{M}^c))^c = \underline{\zeta}(\mathrm{M})$

$(vii)(\underline{\zeta}(\mathrm{M}^c))^c = \overline{\zeta}(\mathrm{M})$

$(viii)\underline{\zeta}(\mathrm{M} \cap \mathrm{N}) = \underline{\zeta}(\mathrm{M}) \cap \underline{\zeta}(\mathrm{N})$

$(ix)\overline{\zeta}(\mathrm{M} \cap \mathrm{N}) \subseteq \overline{\zeta}(\mathrm{M}) \cap \overline{\zeta}(\mathrm{N})$

$(x)\overline{\zeta}(\mathrm{M} \cup \mathrm{N}) = \overline{\zeta}(\mathrm{M}) \cup \overline{\zeta}(\mathrm{N})$

$(xi)\underline{\zeta}(\mathrm{M} \cup \mathrm{N}) \supseteq \underline{\zeta}(\mathrm{M}) \cup \underline{\zeta}(\mathrm{N})$

$(xii)\mathrm{M} \subseteq \mathrm{N} \Rightarrow \overline{\zeta}(\mathrm{M}) \subseteq \overline{\zeta}(\mathrm{N})$

$(xiii)\mathrm{M} \subseteq \mathrm{N} \Rightarrow \underline{\zeta}(\mathrm{M}) \subseteq \underline{\zeta}(\mathrm{N})$

$(xiv)\zeta \subseteq \zeta' \Rightarrow \underline{\zeta}(\mathrm{M}) \supseteq \underline{\zeta}'(\mathrm{M})$

$(xv)\zeta \subseteq \zeta' \Rightarrow \overline{\zeta}(\mathrm{M}) \subseteq \overline{\zeta}'(\mathrm{M})$

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Proof:

The Proof is obvious.

Theorem 3.3:

If M and N be any two NS of S_G, then $\overline{\zeta}(\mathrm{M}) \circ \overline{\zeta}(\mathrm{N}) \subseteq \overline{\zeta}(\mathrm{M} \circ \mathrm{N})$.

Proof:

Let M and N be any two NS of S_G.Then,

$$\overline{\zeta}(\mathrm{M}) \circ \overline{\zeta}(\mathrm{N}) = \{[\overline{\zeta}(\mathrm{M}(t)) \circ \overline{\zeta}(\mathrm{N}(t))], [\overline{\zeta}(\mathrm{M}(i)) \circ \overline{\zeta}(\mathrm{N}(i))], [\overline{\zeta}(\mathrm{M}(f)) \circ \overline{\zeta}(\mathrm{N}(f))]\}$$

and

$$\overline{\zeta}(\mathrm{M} \circ \mathrm{N}) = \{[\overline{\zeta}(\mathrm{M}(t) \circ \mathrm{N}(t))], [\overline{\zeta}(\mathrm{M}(i) \circ \mathrm{N}(i))], [\overline{\zeta}(\mathrm{M}(f) \circ \mathrm{N}(f))]\}$$

To prove,

$$\overline{\zeta}(\mathrm{M}) \circ \overline{\zeta}(\mathrm{N}) \subseteq \overline{\zeta}(\mathrm{M} \circ \mathrm{N})$$

For this we need to prove,

$$\forall \alpha \in S_G$$

$$(\overline{\zeta}(\mathrm{M}(t)) \circ \overline{\zeta}(\mathrm{N}(t)))(\alpha) \le \overline{\zeta}(\mathrm{M}(t) \circ \mathrm{N}(t))(\alpha)$$
$$(\overline{\zeta}(\mathrm{M}(i)) \circ \zeta(\mathrm{N}(i)))(\alpha) \ge \overline{\zeta}(\mathrm{M}(i) \circ \mathrm{N}(i))(\alpha)$$
$$(\overline{\zeta}(\mathrm{M}(f)) \circ \overline{\zeta}(\mathrm{N}(f)))(\alpha) \ge \overline{\zeta}(\mathrm{M}(f) \circ \mathrm{N}(f))(\alpha)$$

Consider

$$(\overline{\zeta}(\mathrm{M}(t)) \circ (\mathrm{N}(t)))(\alpha) = \vee_{\alpha=\beta\gamma} [\overline{\zeta}(\mathrm{M}(t))(\beta) \wedge \overline{\zeta}(\mathrm{N}(t))(\gamma)]$$

$$= \vee_{\alpha=\beta\gamma} [(\vee_{x\in[\beta]_\zeta} (\mathrm{M}(t)(x)) \wedge (\vee_{y\in[\gamma]_\zeta} (\mathrm{N}(t)(y))]$$

$$= \vee_{\alpha=\beta\gamma} [(\vee_{\substack{x\in[\beta]_\zeta \\ y\in[\gamma]_\zeta}} (\mathrm{M}(t)(x) \wedge \mathrm{N}(t)(y))]$$

$$\le \vee_{\alpha=\beta\gamma} [(\vee_{x+y\in[\beta\gamma]_\zeta} (\mathrm{M}(t)(x) \wedge \mathrm{N}(t)(y))]$$

$$= \vee_{xy\in[\alpha]_\zeta} (\mathrm{M}(t)(x) \wedge \mathrm{N}(t)(y))$$

$$= \vee_{\substack{z\in[\alpha]_\zeta \\ z=xy}} (\mathrm{M}(t)(x) \wedge \mathrm{N}(t)(y))$$

$$= \vee_{z\in[\alpha]_\zeta} \vee_{z=x+y} (\mathrm{M}(t)(x) \wedge \mathrm{N}(t)(y))$$

$$= \vee_{z\in[\alpha]_\zeta} [\mathrm{M}(t) \circ \mathrm{N}(t)](z)$$

$$= \overline{\zeta}(\mathrm{M}(t) \circ \mathrm{N}(t))(z)$$

And

$$(\overline{\zeta}(\mathrm{M}(i)) \circ (\mathrm{N}(i)))(\alpha) = \wedge_{\alpha=\beta\gamma} [\overline{\zeta}(\mathrm{M}(i))(\beta) \wedge \overline{\zeta}(\mathrm{N}(i))(\gamma)]$$

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$$= \wedge_{\alpha=\beta\gamma} [(\wedge_{x\in[\beta]_\zeta} (\mathrm{M}(i)(x)) \vee (\wedge_{y\in[\gamma]_\zeta} (\mathrm{N}(i)(y))]$$
$$= \wedge_{\alpha=\beta\gamma} [(\wedge_{\substack{x\in[\beta]_\zeta \\ y\in[\gamma]_\zeta}} (\mathrm{M}(i)(x) \vee \mathrm{N}(i)(y))]$$
$$\geq \wedge_{\alpha=\beta\gamma} [(\wedge_{x+y\in[\beta\gamma]_\zeta} (\mathrm{M}(i)(x) \vee \mathrm{N}(i)(y))]$$
$$= \wedge_{xy\in[\alpha]_\zeta} (\mathrm{M}(i)(x) \vee \mathrm{N}(i)(y))$$
$$= \wedge_{\substack{z\in[\alpha]_\zeta \\ z=xy}} (\mathrm{M}(i)(x) \vee \mathrm{N}(i)(y))$$
$$= \wedge_{z\in[\alpha]_\zeta} \wedge_{z=x+y} (\mathrm{M}(i)(x) \vee \mathrm{N}(i)(y))$$
$$= \wedge_{z\in[\alpha]_\zeta} [\mathrm{M}(i) \circ \mathrm{N}(i)](z)$$
$$= \overline{\zeta}(\mathrm{M}(i) \circ \mathrm{N}(i))(z)$$

Also

$$(\overline{\zeta}(\mathrm{M}(f)) \circ (\mathrm{N}(f)))(\alpha) = \wedge_{\alpha=\beta\gamma} [\overline{\zeta}(\mathrm{M}(f))(\beta) \wedge \overline{\zeta}(\mathrm{N}(f))(\gamma)]$$
$$= \wedge_{\alpha=\beta\gamma} [(\wedge_{x\in[\beta]_\zeta} (\mathrm{M}(f)(x)) \vee (\wedge_{y\in[\gamma]_\zeta} (\mathrm{N}(f)(y))]$$
$$= \wedge_{\alpha=\beta\gamma} [(\wedge_{\substack{x\in[\beta]_\zeta \\ y\in[\gamma]_\zeta}} (\mathrm{M}(f)(x) \vee \mathrm{N}(f)(y))]$$
$$\geq \wedge_{\alpha=\beta\gamma} [(\wedge_{x+y\in[\beta\gamma]_\zeta} (\mathrm{M}(f)(x) \vee \mathrm{N}(f)(y))]$$
$$= \wedge_{xy\in[\alpha]_\zeta} (\mathrm{M}(f)(x) \vee \mathrm{N}(f)(y))$$
$$= \wedge_{\substack{z\in[\alpha]_\zeta \\ z=xy}} (\mathrm{M}(f)(x) \vee \mathrm{N}(f)(y))$$
$$= \wedge_{z\in[\alpha]_\zeta} \wedge_{z=x+y} (\mathrm{M}(f)(x) \vee \mathrm{N}(f)(y))$$
$$= \wedge_{z\in[\alpha]_\zeta} [\mathrm{M}(f) \circ \mathrm{N}(f)](z)$$
$$= \overline{\zeta}(\mathrm{M}(f) \circ \mathrm{N}(f))(z)$$

Hence Proved.

By the same arguments we prove the following theorem.

Theorem 3.4:

If M and N be any two NS of S_G, then $\underline{\zeta}(\mathrm{M}) \circ \underline{\zeta}(\mathrm{N}) \subseteq \underline{\zeta}(\mathrm{M} \circ \mathrm{N})$.

4. Rough Neutrosophic ideals in a Semigroup

In this section rough neutrosophic subsemigroup (RNSSG)andrough neutrosophic ideals(RNI) in S_G are introduced and related theorems are proved.

Definition 4.1:

A NS κ of S_G is said to be an upper (lower) (RNSSG)of S_G, if $\overline{\zeta}(\kappa)(\underline{\zeta}(\kappa))$ is a (NSSG) of S_G .A NS κ is called a RNSSG of S_G if it is both upper and lower RNSSG of S_G .

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 13734 - 13741
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Theorem 4.2:

If κ is a NSSG of S_G, then κ is an upper RNSSG of S_G.

Proof:

Let κ be a NSSG of S_G, $a,b \in S_G$. Then

$$\overline{\zeta}\kappa(t)(ab) = \vee_{c\in[xy]_\zeta} \kappa(t)(c)$$

$$\geq \vee_{c\in[x]_\zeta[y]_\zeta} \mathrm{K}(t)(c) = \vee_{xy\in[x]_\zeta[y]_\zeta} \mathrm{K}(t)(xy) \geq \vee_{c\in[x]_\zeta[y]_\zeta} [\mathrm{K}(t)(x) \wedge \mathrm{K}(t)(y)]$$

$$= \vee_{c\in[x]_\zeta} \mathrm{K}(t)(x) \wedge \vee_{c\in[y]_\zeta} \mathrm{K}(t)(y)$$

$$\overline{\zeta}\kappa(t)(ab) = \overline{\zeta}\kappa(t)(a) \wedge \overline{\zeta}\kappa(t)(b)$$

And

$$\overline{\zeta}\kappa(i)(ab) = \wedge_{c\in[xy]_\zeta} \kappa(i)(c)$$

$$\leq \wedge_{c\in[x]_\zeta[y]_\zeta} \mathrm{K}(i)(c) = \wedge_{xy\in[x]_\zeta[y]_\zeta} \mathrm{K}(i)(xy) \leq \wedge_{c\in[x]_\zeta[y]_\zeta} [\mathrm{K}(i)(x) \vee \mathrm{K}(i)(y)]$$

$$= \wedge_{c\in[x]_\zeta} \mathrm{K}(i)(x) \vee \wedge_{c\in[y]_\zeta} \mathrm{K}(i)(y)$$

$$\overline{\zeta}\kappa(i)(ab) = \overline{\zeta}\kappa(i)(a) \vee \overline{\zeta}\kappa(i)(b)$$

Also

$$\overline{\zeta}\kappa(f)(ab) = \wedge_{c\in[xy]_\zeta} \kappa(f)(c)$$

$$\leq \wedge_{c\in[x]_\zeta[y]_\zeta} \mathrm{K}(f)(c) = \wedge_{xy\in[x]_\zeta[y]_\zeta} \mathrm{K}(f)(xy) \leq \wedge_{c\in[x]_\zeta[y]_\zeta} [\mathrm{K}(f)(x) \vee \mathrm{K}(f)(y)]$$

$$= \wedge_{c\in[x]_\zeta} \mathrm{K}(f)(x) \vee \wedge_{c\in[y]_\zeta} \mathrm{K}(f)(y)$$

$$\overline{\zeta}\kappa(f)(ab) = \overline{\zeta}\kappa(f)(a) \vee \overline{\zeta}\kappa(f)(b)$$

This implies $\overline{\zeta}(\kappa)$ is an NSSG of S_G and hence, κ is an upper RNSSG of S_G.

By the same arguments we prove the following theorem.

Theorem 4.3:

If κ is a NSSG of S_G, then κ is an lower RNSSG of S_G.

Definition 4.4:

A NS κ of S_G is said to be an upper (lower) rough Neutrosophic left (right, two sided) ideal RNLI(RNRI,RNTI) of S_G, if $\overline{\zeta}(\kappa)(\underline{\zeta}(\kappa))$ is a Neutrosophic left(right,two sided) ideal NLI(NRI,NTI) of S_G respectively.

Theorem 4.5:

If κ is a NLI(NRI,NTI)of S_G, then κ is an upper RNLI(RNRI,RNTI) of S_G.

Proof:

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 13734 - 13741
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Let κ be a neutrosophic left ideal of S_G and $a,b \in S_G$. Then

$$\overline{\zeta}\kappa(t)(ab) = \vee_{c\in[xy]_\zeta} \kappa(t)(c)$$

$$\geq \vee_{c\in[x]_\zeta[y]_\zeta} \kappa(t)(c) = \vee_{xy\in[x]_\zeta[y]_\zeta} \kappa(t)(xy) \geq \vee_{c\in[x]_\zeta[y]_\zeta} \kappa(t)(y)$$

$$= \vee_{y\in[y]_\zeta} \kappa(t)(y)$$

$$= \overline{\zeta}\kappa(t)(b)$$

And

$$\overline{\zeta}\kappa(i)(ab) = \wedge_{c\in[xy]_\zeta} \kappa(i)(c)$$

$$\leq \wedge_{c\in[x]_\zeta[y]_\zeta} \kappa(i)(c) = \wedge_{xy\in[x]_\zeta[y]_\zeta} \kappa(i)(xy) \leq \wedge_{c\in[x]_\zeta[y]_\zeta} \kappa(i)(y)$$

$$= \wedge_{y\in[y]_\zeta} \kappa(i)(y)$$

$$= \overline{\zeta}\kappa(i)(b)$$

Also

$$\overline{\zeta}\kappa(f)(ab) = \wedge_{c\in[xy]_\zeta} \kappa(f)(c)$$

$$\leq \wedge_{c\in[x]_\zeta[y]_\zeta} \kappa(i)(c) = \wedge_{xy\in[x]_\zeta[y]_\zeta} \kappa(f)(xy) \leq \wedge_{c\in[x]_\zeta[y]_\zeta} \kappa(f)(y)$$

$$= \wedge_{y\in[y]_\zeta} \kappa(f)(y)$$

$$= \overline{\zeta}\kappa(f)(b)$$

This implies $\overline{\zeta}(\kappa)$ is an NLI(NRI,NTI)of S_G and hence, κ is an upper RNLI(RNRI,RNTI) of S_G.

By the same arguments we prove the following theorem.

Theorem 4.6:

If κ is a NLI(NRI,NTI)of S_G, then κ is an RNLI(RNRI,RNTI) of S_G.

Definition 4.7:

A NSSG κ of S_G is said to be an upper(lower) rough neutrosophic bi-ideal (RNBI) of S_G, if $\overline{\zeta}(\kappa)(\underline{\zeta}(\kappa))$ is a neutrosophic bi-ideal (NBI) of S_G.

Theorem 4.8:

If κ is a NBIof S_G, then κ is an upper RNBI of S_G.

Proof:

Let κ be the NBI of S. Then

$$\forall \alpha, \beta, x \in S_G$$

$$\overline{\zeta}\kappa(t)(\alpha x\beta) = \vee_{\gamma\in[\alpha x\beta]_\zeta} \kappa(t)(\gamma) \geq \vee_{\gamma\in[\alpha]_\zeta[x]_\zeta[\beta]_\zeta} \kappa(t)(\gamma)$$

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$$= \bigvee_{asb\in[\alpha]_\zeta[x]_\zeta[\beta]_\zeta} \kappa(t)(asb) \geq \bigvee_{a\in[\alpha]_\zeta, s\in[x]_\zeta, b\in[\beta]_\zeta} \kappa(t)(a)\wedge\kappa(t)(b)$$
$$= \bigvee_{a\in[\alpha]_\zeta, b\in[\beta]_\zeta} \kappa(t)(a)\wedge\kappa(t)(b) = \bigvee_{a\in[\alpha]_\zeta} \kappa(t)(a)\wedge \bigvee_{b\in[\beta]_\zeta} \kappa(t)(b)$$
$$= \overline{\zeta}\kappa(t)(\alpha)\wedge\overline{\zeta}\kappa(t)(\beta)$$

Similarly we can prove

$$\forall \alpha,\beta,x\in S_G$$
$$\overline{\zeta}\kappa(i)(\alpha x\beta) \leq \overline{\zeta}\kappa(i)(\alpha)\vee\overline{\zeta}\kappa(i)(\beta)$$
$$\overline{\zeta}\kappa(f)(\alpha x\beta) \leq \overline{\zeta}\kappa(f)(\alpha)\vee\overline{\zeta}\kappa(f)(\beta)$$

Therefore, κ is an upper RNBIof S_G .

By the same arguments we prove the following theorem.

Theorem 4.9:

If κ is a NBIof S_G , then κ is an lower RNBI of S_G .

Theorem 4.10:

If M and N are any two NRI and NLI of S_G respectively, then $\overline{\zeta}(\mathrm{M}\circ\mathrm{N}) \subseteq \overline{\zeta}(\mathrm{M})\cap\overline{\zeta}(\mathrm{N})$

Proof:

Let M and N are any two NRI and NLI of S_G respectively

Then

$$\overline{\zeta}(\mathrm{M}\circ\mathrm{N}) = \{[\overline{\zeta}(\mathrm{M}(t)\circ\mathrm{N}(t))],[\overline{\zeta}(\mathrm{M}(i)\circ\mathrm{N}(i))],[\overline{\zeta}(\mathrm{M}(f)\circ\mathrm{N}(f))]\}$$

$$\overline{\zeta}(\mathrm{M})\bigcap\overline{\zeta}(\mathrm{N}) = \{[\overline{\zeta}(\mathrm{M}(t))\bigcap\overline{\zeta}(\mathrm{N}(t))],[\overline{\zeta}(\mathrm{M}(i))\bigcap\overline{\zeta}(\mathrm{N}(i))],[\overline{\zeta}(\mathrm{M}(f))\bigcap\overline{\zeta}(\mathrm{N}(f))]\}$$

To Prove, $\overline{\zeta}(\mathrm{M}\circ\mathrm{N}) \subseteq \overline{\zeta}(\mathrm{M})\cap\overline{\zeta}(\mathrm{N})$.

For this we need to prove,

$$\forall \alpha\in S_G$$

$$(\overline{\zeta}(\mathrm{M}(t))\circ\overline{\zeta}(\mathrm{N}(t)))(\alpha) \leq \overline{\zeta}(\mathrm{M}(t)\bigcap\mathrm{N}(t))(\alpha)$$
$$(\overline{\zeta}(\mathrm{M}(i))\circ\overline{\zeta}(\mathrm{N}(i)))(\alpha) \geq \overline{\zeta}(\mathrm{M}(i)\bigcap\mathrm{N}(i))(\alpha)$$
$$(\overline{\zeta}(\mathrm{M}(f))\circ\overline{\zeta}(\mathrm{N}(f)))(\alpha) \geq \overline{\zeta}(\mathrm{M}(f)\bigcap\mathrm{N}(f))(\alpha)$$

$$(\overline{\zeta}(\mathrm{M}(t))\circ(\mathrm{N}(t)))(\alpha) = \bigvee_{x\in[\alpha]_\zeta} [(\mathrm{M}(t))\circ\mathrm{N}(t)))(x)]$$

$= \vee_{x\in[\alpha]_\zeta} \vee_{x=\beta\gamma} [(\mathrm{M}(t)(\beta) \wedge (\mathrm{N}(t)))(\gamma)]$

$\leq \vee_{x\in[\alpha]_\zeta} \vee_{x=\beta\gamma} [(\mathrm{M}(t)(\beta\gamma) \wedge (\mathrm{N}(t)))(\beta\gamma)]$

$= \vee_{x\in[\alpha]_\zeta} [(\mathrm{M}(t)(x)) \wedge (\mathrm{N}(t)(x))]$

$\leq \vee_{x\in[\alpha]_\zeta, y\in[\alpha]_\zeta} [(\mathrm{M}(t)(x)) \wedge (\mathrm{N}(t)(y))]$

$= (\vee_{x\in[\alpha]_\zeta} (\mathrm{M}(t)(x)) \wedge (\vee_{y\in[\alpha]_\zeta} (\mathrm{N}(t)(y))]$

$= \overline{\zeta}\mathrm{M}(t)(x) \wedge \overline{\zeta}\mathrm{N}(t)(x)$

$= [\overline{\zeta}\mathrm{M}(t) \cap \overline{\zeta}\mathrm{N}(t)](x)$

Similarly we can prove

$(\overline{\zeta}(\mathrm{M}(i)) \circ \overline{\zeta}(\mathrm{N}(i)))(\alpha) \geq \overline{\zeta}(\mathrm{M}(i) \cap \mathrm{N}(i))(\alpha)$

$(\overline{\zeta}(\mathrm{M}(f)) \circ \overline{\zeta}(\mathrm{N}(f)))(\alpha) \geq \overline{\zeta}(\mathrm{M}(f) \cap \mathrm{N}(f))(\alpha)$

Hence proved.

By the same arguments we prove the following theorem.

Theorem 4.11:

If M and N are any two NRI and NLIof S_G respectively, then $\underline{\zeta}(\mathrm{M} \circ \mathrm{N}) \subseteq \underline{\zeta}(\mathrm{M}) \cap \underline{\zeta}(\mathrm{N})$.

5.Conclusion

In this paper, we have introduced rough Neutrosophic left(right, two sided,bi-)ideals and some properties of these ideals. This concept can be extended in other algebraic structures such as Gamma-semigroups, Ordered semigroups, This study can be used in some real life applications namely decision making, medical diagnosis, multi attribute decision making etc.,

References (APA)

1. Atanassov.K.T.(1986) “Intuitionsitic fuzzy sets”, Fuzzy Sets and Systems, 20, 87 – 96.
2. Broumi.S, F. Smarandache and P. K. Maji. (2014), Intuitionistic neutrosphic soft set over Rings , Mathematics and Statistics, 2(3), 120 - 126.
3. Broumi.S, I. Deli and F. Smarandache. (2014), Relations on interval valued neutrosophic soft sets, Journal of New Results in Science, 5, 01 - 20.
4. Dubios, D., Prade, H.,(1990) Rough fuzzy sets and fuzzy rough sets, Int. J.Gen. Syst., 17, 191-208.
5. Elavarasan, Balasubramanian; FlorentinSmarandache; and Young Bae Jun. (2019). Neutrosophic ideals in semigroups. Neutrosophic Sets and Systems 28, 1.
6. Jayanta Ghosh, T. K. Samanta. (2012), Rough intuitionistic fuzzy ideals in semigroups, 4(1) 155- 168,ISSN 2093–9310
7. Kuroki.N(1997), Rough ideals in semigroups, Inform. Sci. 100, 139–163
8. Pawlak, Z.(1982),Rough Sets, Int. J. Comput. Inform. Sci., 11, 341-356.
9. Smarandache, F.(2005). Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, Int. J. Pure Appl. Math. ,24,pp.287-297.
10. Xiao.Q. M, Zhang.Z.L,(2006) Rough prime ideals and rough fuzzy prime ideals in semigroups,Inform. Sci. 176, 725–733.
11. Zadeh, L. A. (1965) Fuzzy sets, Information and Control, 8, 338 - 353.