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An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number

Mohamed Abdel-Basset [a,*], M. Saleh [b], Abduallah Gamal [a], Florentin Smarandache [c]

[a] Department of Decision support, Faculty of Computers and Informatics, Zagazig university, Egypt
[b] Department of Decision support, Faculty of Computers and Information, Cairo university, Egypt
[c] Math & Science Department, University of New, Mexico, Gallup, NM 87301, USA

HIGHLIGHTS

- An advanced type of neutrosophic technique, called T2NN is proposed.
- Some desirable properties and peculiar forms of operators are investigated.
- The type 2 neutrosophic numbers can accurately describe real cognitive information.
- A real case study based on the proposed T2NN-TOPSIS is introduced.

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ABSTRACT

This paper proposes an advanced type of neutrosophic technique, called type 2 neutrosophic numbers, and defines some of its operational rules. The type 2 neutrosophic number weighted averaging operator is determined in order to collective the type 2 neutrosophic number set, inferring some properties of the suggested operator. The operator is employed in a MADM problem to collect the type 2 neutrosophic numbers based classification values of each alternative over the features. The convergent classification values of every alternative are arranged with the assistance of score and accuracy values with the aim to detect the superior alternative. We introduce an illuminating example to confirm the suggested approach for multi attribute decision making issues, ordering the alternatives based on the accuracy function. Selecting an appropriate alternative among the selection options is a difficult activity for decision makers, since it is complicated to express attributes as crisp numbers. To tackle the problem, type 2 neutrosophic numbers can be efficiently used to estimate information in the decision making process. The type 2 neutrosophic numbers can accurately describe real cognitive information. We propose a novel T2NN-TOPSIS strategy combining type 2 neutrosophic numbers and TOPSIS under group decision making as application of T2NN, suggesting a type 2 neutrosophic number expression for linguistic terms. Finally, we provide a real case dealing with a decision making problem based on the proposed T2NN-TOPSIS methodology to prove the efficiency and the applicability of the type 2 neutrosophic number.

1. Introduction

Fuzzy theory was established on the notion of membership function to take linguistic variables into consideration. The theory seeks to deem uncertain data which can be related with existent fuzziness of peoples' observations and perceptions. The results indicate that they are strongly affected by self-regulation in such circumstances. Fuzzy has confirmed functionality in dealing with vagueness and ambiguity of human intellect and expression while decision making. Hence, the Neutrosophic is an extension of the fuzzy theory and intuitionistic fuzzy set (IFS). Smarandache proposed the neutrosophic sets in [1,2], attracting the attention of many scholars. The neutrosophic sets proved to be a valid workspace in describing incompatible and indefinite information. z(T, I, F) is a Type-1 Neutrosophic Number. But $z((T_T, T_I, T_F), (I_T, I_I, I_F), (F_T, F_I, F_F))$ is a Type-2 Neutrosophic Number, which means that each neutrosophic component T, I, and F is split into its truth, indeterminacy, and falsehood subparts. The procedure of splitting may be executed recurrently, as many times as needed, obtaining

Abbreviations: T2NN, Type 2 neutrosophic number; T2NNWA, Type 2 neutrosophic number weighted averaging; NPIS, Neutrosophic positive ideal solution; NNIS, Neutrosophic negative ideal solution; GDM, Group decision making; TOPSIS, Technique for order preference by similarity to ideal solution; SVN, Single valued neutrosophic; ANP, Analytical network process; MDM, Multi decision making; MADM, Multi attribute decision making; MAGDM, Multi attribute group decision making; MCDM, Multicriteria decision making

* Corresponding author.
E-mail addresses: analyst_mohamed@zu.edu.eg (M. Abdel-Basset), m.saleh@fci-cu.edu.eg (M. Saleh), abduallahgamal@gmail.com (A. Gamal), smarand@unm.edu (F. Smarandache).

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a general Type-n Neutrosophic Number, for any integer $n \geq 1$. Here, we use the type 2 neutrosophic number as advancement of neutrosophic number to solve MCDM problems.

A neutrosophic set has more possibility strength than other forming mathematical apparatus, such as fuzzy set [3], interval valued intuitionistic fuzzy set (IVIFS) [4] or IFS [5]. Smarandache combined the degree of indeterminacy as independent element in IFS and defined the neutrosophic set [6] as a generalization of IFSs. Georgiev [7] questioned that the neutrosophic logic is qualified for preserving formal operators, since there is no standardization base for the elements T, I and F. However, fuzzy sets and IFSs cannot deal with certain types of uncertain information, such as incompatible, indefinite, or incomplete information. Smarandache [8] recognized neutrosophic set as a generalization of IFS, which performs a significant role to transact unclear, unpredictable and indeterminate information in the real world. The truth, indeterminacy and falsity degrees exist in the non-standard item interval suitable for each element of the universe [8]. In these days, Neutrosophic received attentions from many researches were proceed to develop, improve and expand the neutrosophic theory [9–16]. The neutrosophic set expanded to many branches, such as topology, image conversion or social science. We used single valued neutrosophic set [17] (SVNS), a subclass of neutrosophic set, in which every component of the universe is described by the truth, indeterminacy and falsity memberships existing in the actual unit interval. Liu and Liu [18] introduced neutrosophic number generalized weighted power averaging operator (NNGWPAO) and suggested a MAGDM strategy in neutrosophic number environment. Peng et al. [19] suggested a MAGDM strategy constructed on neutrosophic number generalized hybrid weighted averaging operator. Ye [20] introduced weighted arithmetic average operator for simplified neutrosophic sets. Hence, we will refer to TOPSIS methodology that is a widespread strategy to transact MAGDM. TOPSIS [21] helps choosing the best selection, which is the nearest to the quixotic solution and the farthest from the negative quixotic solution. Information of attributes that aggregated from experts and decision maker/makers is the base of the TOPSIS strategy. In crisp setting, an extended TOPSIS strategy for MAGDM under GDM was established by Shih [22]. A TOPSIS strategy for group decision making was suggested by Hatami [23]. Ravasan et al. [24] developed a fuzzy TOPSIS strategy for an e-banking outsourcing strategy selection in fuzzy environment. Banaeian et al. [25] introduced a fuzzy TOPSIS for GDM for green supplier selection for an actual company from the agri-food sector. In intuitionistic fuzzy environment, Büyüközkan et al. [26] elaborated an MAGDM for supplier election with TOPSIS strategy. Gupta et al. [27] established a protracted TOPSIS method under interval-valued intuitionistic fuzzy environment. Wang et al. [28] suggested a TOPSIS strategy for MAGDM in single valued neutrosophic environment. Ju et al. [29] propounded a TOPSIS strategy for MAGDM established on SVN linguistic numbers. A TOPSIS strategy was presented in neutrosophic cubic set environment by Pramanik et al. [30]. A TOPSIS strategy for MADM in bipolar neutrosophic set environment was put forward by Dey et al. [31]. Abdel-Basset et al. [32] suggested an ANP-TOPSIS strategy for supplier selection problems with interval valued neutrosophic. TOPSIS strategy is yet to approach T2NN environment. To fill the research gap, we improve a MAGDM strategy built on TOPSIS in type 2 neutrosophic number environment, namely T2NN-TOPSIS strategy to solve MAGDM issues.

Contribution of this paper:

- We state a T2NN, score function and accuracy function of T2NN, and prove its basic properties.
- We define T2NNWA to aggregate T2NN decision matrices.
- We propose linguistic terms to present T2NN.
- We suggest a tangential function to locate unidentified weights of attributes in T2NN setting.
- We develop a T2NN-TOPSIS strategy to solve MAGDM problems in T2NN environment.
- The proposed T2NN-TOPSIS is comprehensive, presenting all vague and incomplete information about all elements.
- We present an illustrative model of a MADM problem.

Table 1 below provides a literature review. Section 2 introduces several basic concepts of T2NN, operations on T2NN, applications of T2NNWA operator to MADM, two properties on T2NNWA and a numerical example. Section 3 clarifies the procedure for TOPSIS-T2NN methodology for the evaluation suppliers. Section 4 provides a real example based on the proposed T2NN-TOPSIS strategy. Section 5 concludes the research.

2. Preliminaries

We introduce several basic concepts of T2NN, operations on T2NN, applications of T2NNWA operator to MADM, and two properties on T2NNWA.

Definition 1. Let Z be the limited universe of discourse and F [0, 1] be the set of all triangular neutrosophic numbers on F [0, 1]. A type 2 neutrosophic number set (T2NNS) $\tilde{U}$ in Z is represented by $\tilde{U} = \left\{\left\langle z, \tilde{T}_{\tilde{U}}(z), \tilde{I}_{\tilde{U}}(z), \tilde{F}_{\tilde{U}}(z) \mid z \in Z\right\rangle\right\}$, where $\tilde{T}_{\tilde{U}}(z): Z \rightarrow F[0,1]$, $\tilde{I}_{\tilde{U}}(z): Z \rightarrow F[0,1]$, $\tilde{F}_{\tilde{U}}(z): Z \rightarrow F[0,1]$. A T2NNS $\tilde{T}_{\tilde{A}}(z) = \left(T_{T_{\tilde{U}}}(z), T_{I_{\tilde{U}}}(z), T_{F_{\tilde{U}}}(z)\right)$, $\tilde{I}_{\tilde{U}}(z) = \left(I_{T_{\tilde{U}}}(z), I_{I_{\tilde{U}}}(z), I_{F_{\tilde{U}}}(z)\right)$, $\tilde{F}_{\tilde{U}}(z) = \left(F_{T_{\tilde{U}}}(z), F_{I_{\tilde{U}}}(z), F_{F_{\tilde{U}}}(z)\right)$, respectively, denote the truth, indeterminacy, and falsity memberships of z in $\tilde{U}$ and for every $z \in Z: 0 \leq \tilde{T}_{\tilde{U}}(z)^3 + \tilde{I}_{\tilde{U}}(z)^3 + \tilde{F}_{\tilde{U}}(z)^3 \leq 3$; for convenience, we consider that $\tilde{U} = \left\langle\left(T_{T_{\tilde{U}}}(z), T_{I_{\tilde{U}}}(z), T_{F_{\tilde{U}}}(z)\right), \left(I_{T_{\tilde{U}}}(z), I_{I_{\tilde{U}}}(z), I_{F_{\tilde{U}}}(z)\right), \left(F_{T_{\tilde{U}}}(z), F_{I_{\tilde{U}}}(z), F_{F_{\tilde{U}}}(z)\right)\right\rangle$ as a type 2 neutrosophic number.

Definition 2. Suppose $\tilde{U}_1 = \left\langle\left(T_{T_{\tilde{U}_1}}(z), T_{I_{\tilde{U}_1}}(z), T_{F_{\tilde{U}_1}}(z)\right), \left(I_{T_{\tilde{U}_1}}(z), I_{I_{\tilde{U}_1}}(z), I_{F_{\tilde{U}_1}}(z)\right), \left(F_{T_{\tilde{U}_1}}(z), F_{I_{\tilde{U}_1}}(z), F_{F_{\tilde{U}_1}}(z)\right)\right\rangle$ and $\tilde{U}_2 = \left\langle\left(T_{T_{\tilde{U}_2}}(z), T_{I_{\tilde{U}_2}}(z), T_{F_{\tilde{U}_2}}(z)\right), \left(I_{T_{\tilde{U}_2}}(z), I_{I_{\tilde{U}_2}}(z), I_{F_{\tilde{U}_2}}(z)\right), \left(F_{T_{\tilde{U}_2}}(z), F_{I_{\tilde{U}_2}}(z), F_{F_{\tilde{U}_2}}(z)\right)\right\rangle$ are two T2NNS in the set real numbers. Then the procedures are defined as Eqs. (1)–(4) in Box I.

The procedures defined in Definition 2 satisfy the following properties:

1. $\tilde{U}_1 \oplus \tilde{U}_2 = \tilde{U}_2 \oplus \tilde{U}_1$, $\tilde{U}_1 \otimes \tilde{U}_2 = \tilde{U}_2 \otimes \tilde{U}_1$;
2. $\delta(\tilde{U}_1 \oplus \tilde{U}_2) = \delta\tilde{U}_1 \oplus \delta\tilde{U}_2$, $(\tilde{U}_1 \otimes \tilde{U}_2)^{\delta} = \tilde{U}_1^{\delta} \otimes \tilde{U}_2^{\delta}$ for $\delta > 0$, and
3. $\delta_1\tilde{U}_1 \oplus \delta_2\tilde{U}_1 = (\delta_1 + \delta_2)\tilde{U}_1$, $\tilde{U}_1^{\delta_1} \oplus \tilde{U}_1^{\delta_2} = \tilde{U}_1^{(\delta_1+\delta_2)}$ for $\delta_1, \delta_2 > 0$.

Definition 3. Suppose that $\tilde{U}_1 = \left\langle\left(T_{T_{\tilde{U}_1}}(z), T_{I_{\tilde{U}_1}}(z), T_{F_{\tilde{U}_1}}(z)\right), \left(I_{T_{\tilde{U}_1}}(z), I_{I_{\tilde{U}_1}}(z), I_{F_{\tilde{U}_1}}(z)\right), \left(F_{T_{\tilde{U}_1}}(z), F_{I_{\tilde{U}_1}}(z), F_{F_{\tilde{U}_1}}(z)\right)\right\rangle$ are T2NNS in the set of real numbers, the score function $S(\tilde{U}_1)$ of $\tilde{U}_1$ is defined

Table 1
Literature review.

References	Methods	GDM	Application type	Objective of the study
Peng, X. and J. Dai [33]	SVN–TOPSIS	–	Methodology proposal	A new axiomatic definition of single-valued neutrosophic distance measure and similarity measure
Pouresmaeil, H., et al. [34]	TOPSIS and SVN	X	Methodology proposal	Multiple attribute decision making
Selvachandran, G., et al. [35]	TOPSIS-MDM–SVN Sets	–	Methodology proposal	New aggregation operator proposal
Biswas, P., et al. [36]	Neutrosophic TOPSIS	X	Methodology proposal	New aggregation operator proposal
Biswas, P., et al. [37]	TOPSIS for MAGDM under SVN	X	Methodology proposal	A new strategy for MAGDM problems
Broumi, S., et al. [38]	TOPSIS method for MADM based on interval neutrosophic	–	Methodology proposal	TOPSIS solve the MADM
Smarandache, F. and S. Pramanik [39]	Neutrosophic under bi-polar neutrosophic	–	Methodology proposal	Select the most desirable alternative

1. $\tilde{U}_1 \oplus \tilde{U}_2$

$$= \left\langle \begin{matrix} \left(\begin{matrix} \left(T_{T_{\tilde{U}_1}}(z) + T_{T_{\tilde{U}_2}}(z) - T_{T_{\tilde{U}_1}}(z).T_{T_{\tilde{U}_2}}(z)\right), \left(T_{I_{\tilde{U}_1}}(z) + T_{I_{\tilde{U}_2}}(z) - T_{I_{\tilde{U}_1}}(z).T_{I_{\tilde{U}_2}}(z)\right), \\ \left(T_{F_{\tilde{U}_1}}(z) + T_{F_{\tilde{U}_2}}(z) - T_{F_{\tilde{U}_1}}(z).T_{F_{\tilde{U}_2}}(z)\right) \end{matrix} \right), \\ \left(I_{T_{\tilde{U}_1}}(z).I_{T_{\tilde{U}_2}}(z), I_{I_{\tilde{U}_1}}(z).I_{I_{\tilde{U}_2}}(z), I_{F_{\tilde{U}_1}}(z).I_{F_{\tilde{U}_2}}(z)\right), \left(F_{T_{\tilde{U}_1}}(z).F_{T_{\tilde{U}_2}}(z), F_{I_{\tilde{U}_1}}(z).F_{I_{\tilde{U}_2}}(z), F_{F_{\tilde{U}_1}}(z).F_{F_{\tilde{U}_2}}(z)\right) \end{matrix} \right\rangle \tag{1}$$

2. $\tilde{U}_1 \otimes \tilde{U}_2$

$$= \left\langle \begin{matrix} \left(\left(T_{T_{\tilde{U}_1}}(z).T_{T_{\tilde{U}_2}}(z), T_{I_{\tilde{U}_1}}(z).T_{I_{\tilde{U}_2}}(z), T_{F_{\tilde{U}_1}}(z).T_{F_{\tilde{U}_2}}(z)\right)\right), \\ \left(\begin{matrix} \left(I_{T_{\tilde{U}_1}}(z) + I_{T_{\tilde{U}_2}}(z) - I_{T_{\tilde{U}_1}}(z).I_{T_{\tilde{U}_2}}(z)\right), \left(I_{I_{\tilde{U}_1}}(z) + I_{I_{\tilde{U}_2}}(z) - I_{I_{\tilde{U}_1}}(z).I_{I_{\tilde{U}_2}}(z)\right), \\ \left(I_{F_{\tilde{U}_1}}(z) + I_{F_{\tilde{U}_2}}(z) - I_{F_{\tilde{U}_1}}(z).I_{F_{\tilde{U}_2}}(z)\right) \end{matrix} \right) \\ \left(\begin{matrix} \left(F_{T_{\tilde{U}_1}}(z) + F_{T_{\tilde{U}_2}}(z) - F_{T_{\tilde{U}_1}}(z).F_{T_{\tilde{U}_2}}(z)\right), \left(F_{I_{\tilde{U}_1}}(z) + F_{I_{\tilde{U}_2}}(z) - F_{I_{\tilde{U}_1}}(z).F_{I_{\tilde{U}_2}}(z)\right), \\ \left(F_{F_{\tilde{U}_1}}(z) + F_{F_{\tilde{U}_2}}(z) - F_{F_{\tilde{U}_1}}(z).F_{F_{\tilde{U}_2}}(z)\right) \end{matrix} \right) \end{matrix} \right\rangle \tag{2}$$

3. $\delta\tilde{U}$

$$= \left\langle \begin{matrix} \left(1-(1-T_{T_{\tilde{U}_1}}(z))^{\delta}, 1-(1-T_{I_{\tilde{U}_1}}(z))^{\delta}, 1-(1-T_{F_{\tilde{U}_1}}(z))^{\delta}\right), \\ \left(\left(I_{T_{\tilde{U}_1}}(z)\right)^{\delta}, \left(I_{I_{\tilde{U}_1}}(z)\right)^{\delta}, \left(I_{F_{\tilde{U}_1}}(z)\right)^{\delta}\right), \\ \left(\left(F_{T_{\tilde{U}_1}}(z)\right)^{\delta}, \left(F_{I_{\tilde{U}_1}}(z)\right)^{\delta}, \left(F_{F_{\tilde{U}_1}}(z)\right)^{\delta}\right) \end{matrix} \right\rangle \text{ for } \delta > 0 \tag{3}$$

4. $\tilde{U}^{\delta}$

$$= \left\langle \begin{matrix} \left(\left(T_{T_{\tilde{U}_1}}(z)\right)^{\delta}, \left(T_{I_{\tilde{U}_1}}(z)\right)^{\delta}, \left(T_{F_{\tilde{U}_1}}(z)\right)^{\delta}\right), \\ \left(1-\left(1-I_{T_{\tilde{U}_1}}(z)\right)^{\delta}, 1-\left(1-I_{I_{\tilde{U}_1}}(z)\right)^{\delta}, 1-\left(1-I_{F_{\tilde{U}_1}}(z)\right)^{\delta}\right), \\ \left(1-\left(1-F_{T_{\tilde{U}_1}}(z)\right)^{\delta}, 1-\left(1-F_{I_{\tilde{U}_1}}(z)\right)^{\delta}, 1-\left(1-F_{F_{\tilde{U}_1}}(z)\right)^{\delta}\right) \end{matrix} \right\rangle \text{ for } \delta > 0 \tag{4}$$

Box I.

as follows:

$$S(\tilde{U}_1) = \frac{1}{12}\left\langle 8 + \left(T_{T_{\tilde{U}_1}}(z) + 2\left(T_{I_{\tilde{U}_1}}(z)\right) + T_{F_{\tilde{U}_1}}(z)\right) - \left(I_{T_{\tilde{U}_1}}(z) + 2\left(I_{I_{\tilde{U}_1}}(z)\right) + I_{F_{\tilde{U}_1}}(z)\right) - \left(F_{T_{\tilde{U}_1}}(z) + 2\left(F_{I_{\tilde{U}_1}}(z)\right) + F_{F_{\tilde{U}_1}}(z)\right)\right\rangle \tag{5}$$

$$A(\tilde{U}_1) = \frac{1}{4}\left\langle \left(T_{T_{\tilde{U}_1}}(z) + 2\left(T_{I_{\tilde{U}_1}}(z)\right) + T_{F_{\tilde{U}_1}}(z)\right) - \left(F_{T_{\tilde{U}_1}}(z) + 2\left(F_{I_{\tilde{U}_1}}(z)\right) + F_{F_{\tilde{U}_1}}(z)\right)\right\rangle \tag{6}$$

$$\mathrm{T2NNWA}_{\omega}(\tilde{U}_1, \tilde{U}_2, \ldots, \tilde{U}_n) = \omega_1\tilde{U}_1 \oplus \omega_2\tilde{U}_2 \oplus \ldots \omega_n\tilde{U}_n = \oplus_{p=1}^{n}(\omega_p\tilde{U}_p)$$
$$= \left\langle \begin{array}{c} \left(1-\prod_{p=1}^{n}\left(1-T_{T_p}(z)\right)^{\omega_p}, 1-\prod_{p=1}^{n}\left(1-T_{I_p}(z)\right)^{\omega_p}, 1-\prod_{p=1}^{n}\left(1-T_{F_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{n}\left(I_{T_p}(z)\right)^{\omega_p}, \prod_{p=1}^{n}\left(I_{I_p}(z)\right)^{\omega_p}, \prod_{p=1}^{n}\left(I_{F_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{n}\left(F_{T_p}(z)\right)^{\omega_p}, \prod_{p=1}^{n}\left(F_{I_p}(z)\right)^{\omega_p}, \prod_{p=1}^{n}\left(F_{F_p}(z)\right)^{\omega_p}\right) \end{array} \right\rangle \quad (11)$$

Box II.

Definition 4. Suppose that $\tilde{U}_1 = \left\langle\left(T_{T_{\tilde{U}_1}}(z), T_{I_{\tilde{U}_1}}(z), T_{F_{\tilde{U}_1}}(z)\right), \left(I_{T_{\tilde{U}_1}}(z), I_{I_{\tilde{U}_1}}(z), I_{F_{\tilde{U}_1}}(z)\right), \left(F_{T_{\tilde{U}_1}}(z), F_{I_{\tilde{U}_1}}(z), F_{F_{\tilde{U}_1}}(z)\right)\right\rangle$ and $\tilde{U}_2 = \left\langle\left(T_{T_{\tilde{U}_2}}(z), T_{I_{\tilde{U}_2}}(z), T_{F_{\tilde{U}_2}}(z)\right), \left(I_{T_{\tilde{U}_2}}(z), I_{I_{\tilde{U}_2}}(z), I_{F_{\tilde{U}_2}}(z)\right), \left(F_{T_{\tilde{U}_2}}(z), F_{I_{\tilde{U}_2}}(z), F_{F_{\tilde{U}_2}}(z)\right)\right\rangle$ are two T2NNS in the set of real numbers. Suppose that $S(\tilde{U}_i)$ and $A(\tilde{U}_i)$ are the score and accuracy functions of T2NNS $\tilde{U}_i (i = 1, 2)$, then the order relations are defined as follows:

1. If $\tilde{S}(\tilde{U}_1) > \tilde{S}(\tilde{U}_2)$, then $\tilde{U}_1$ is greater than $\tilde{U}_2$, that is $\tilde{U}_1$ is superior to $\tilde{U}_2$, denoted by $\tilde{U}_1 > \tilde{U}_2$;
2. If $\tilde{S}(\tilde{U}_1) = \tilde{S}(\tilde{U}_2)$, $\tilde{A}(\tilde{U}_1) > \tilde{A}(\tilde{U}_2)$)then $\tilde{U}_1$ is superior than $\tilde{U}_2$, that is $\tilde{U}_1$ is superior to $\tilde{U}_2$, denoted by $\tilde{U}_1 > \tilde{U}_2$;
3. If $\tilde{S}(\tilde{U}_1) = \tilde{S}(\tilde{U}_2)$, $\tilde{A}(\tilde{U}_1) = \tilde{A}(\tilde{U}_2)$) then $\tilde{U}_1$ is equal to $\tilde{U}_2$, that is $\tilde{U}_1$ is indifferent to $\tilde{U}_2$, denoted by $\tilde{U}_1 = \tilde{U}_2$;

Example 1. Consider two T2NNS in the group of real numbers: $\tilde{U}_1 = \left\langle\left(T_{T_{\tilde{U}_1}}(z), T_{I_{\tilde{U}_1}}(z), T_{F_{\tilde{U}_1}}(z)\right), \left(I_{T_{\tilde{U}_1}}(z), I_{I_{\tilde{U}_1}}(z), I_{F_{\tilde{U}_1}}(z)\right), \left(F_{T_{\tilde{U}_1}}(z), F_{I_{\tilde{U}_1}}(z), F_{F_{\tilde{U}_1}}(z)\right)\right\rangle$ and $\tilde{U}_2 = \left\langle\left(T_{T_{\tilde{U}_2}}(z), T_{I_{\tilde{U}_2}}(z), T_{F_{\tilde{U}_2}}(z)\right), \left(I_{T_{\tilde{U}_2}}(z), I_{I_{\tilde{U}_2}}(z), I_{F_{\tilde{U}_2}}(z)\right), \left(F_{T_{\tilde{U}_2}}(z), F_{I_{\tilde{U}_2}}(z), F_{F_{\tilde{U}_2}}(z)\right)\right\rangle$ $\tilde{U}_1 = \langle(0.65, 0.70, 0.75), (0.20, 0.15, 0.30), (0.15, 0.20, 0.10)\rangle$, $\tilde{U}_2 = \langle(0.45, 0.40, 0.55), (0.35, 0.45, 0.30), (0.25, 0.35, 0.40)\rangle$. From Eqs. (5) and (6), we get the following outcomes:

1. Score value of $\tilde{S}(\tilde{U}_1) = (8 + (2.8 - 0.8 - .065))/12 = 0.78$, and $\tilde{S}(\tilde{U}_2) = (8 + (1.8 - 1.55 - 1.35))/12 = 0.58$;
2. Accuracy value of $A(\tilde{U}_1) = (2.8 - 0.65)/4 = 0.54$, and $A(\tilde{U}_2) = (1.8 - 1.35)/4 = 0.11$; it is obvious that $A_1 > A_2$.

Example 2. Consider two T2NNS in the set of real numbers: $\tilde{U}_1 = \langle(0.50, 0.20, 0.35), (0.30, 0.45, 0.30), (0.10, 0.25, 0.35)\rangle$, $\tilde{U}_2 = \langle(0.15, 0.60, 0.20), (0.35, 0.20, 0.30), (0.45, 0.35, 0.20)\rangle$. From Eqs. (5) and (6), we obtain the following results:

1. Score value of $\tilde{S}(\tilde{U}_1) = (8 + (1.25 - 1.5 - 0.95))/12 = 0.57$, and $\tilde{S}(\tilde{U}_2) = (8 + (1.55 - 1.05 - 1.35))/12 = 0.60$;
2. Accuracy value of $A(\tilde{U}_1) = (1.25 - 0.95)/4 = 0.075$, and $A(\tilde{U}_2) = (1.55 - 1.35)/12 = 0.05$; it is obvious that $A_2 > A_1$.

2.1. Aggregation of type 2 neutrosophic number

In this part, we recall some basic descriptions of aggregation operators for real numbers.

Definition 5 ([40]). Suppose that $\omega\colon (Z)^n \to Z$, and $\alpha_p(= 1, 2, \ldots, n) = 1$ are a group of numbers. The weighted averaging operator ωA_ω is defined as:

$$\omega A_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{p=1}^{n} \omega_p\alpha_p, \quad (7)$$

where Z is the set of numbers, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\alpha_p(p = 1, 2, \ldots, n)$ such that $\omega_p \in [0, 1]$ $(p = 1, 2, \ldots, n)$ and $\sum_{p=1}^{n}\omega_p = 1$.

Definition 6 ([40]). Suppose that $\omega\colon (Z)^n \to Z$ and $\alpha_p(p = 1, 2, \ldots, n)$ are a group of numbers. The weighted averaging operator ωA_ω is defined as:

$$\omega A_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \prod_{p=1}^{n} \alpha_p^{\omega_p}, \quad (8)$$

Where Z is the set of number, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\alpha_p(p = 1, 2, \ldots, n)$ such that $\omega_p \in [0, 1]$ $(p = 1, 2, \ldots, n)$ and $\sum_{p=1}^{n}\omega_p = 1$. Based on Definitions 5 and 6, we suggest the next aggregation operator of T2NNS to be used in decision making.

Definition 7. Suppose that $\tilde{U}_p = \left\langle\left(T_{T_{\tilde{U}_p}}(z), T_{I_{\tilde{U}_p}}(z), T_{F_{\tilde{U}_p}}(z)\right), \left(I_{T_{\tilde{U}_p}}(z), I_{I_{\tilde{U}_p}}(z), I_{F_{\tilde{U}_p}}(z)\right), \left(F_{T_{\tilde{U}_p}}(z), F_{I_{\tilde{U}_p}}(z), F_{F_{\tilde{U}_p}}(z)\right)\right\rangle$ $(p = 1, 2, \ldots, n)$ is a collection T2NNS in the set of numbers and let us have T2NNWA: $\Theta^n \to \Theta$. A type 2 neutrosophic number weighted averaging (T2NNWA) operator denoted by T2NNWA $(\tilde{U}_1, \tilde{U}_2, \ldots, \tilde{U}_n)$ is defined as $\mathrm{T2NNWA}_\omega$

$$(\tilde{U}_1, \tilde{U}_2, \ldots, \tilde{U}_n) = \omega_1\tilde{U}_1 \oplus \omega_2\tilde{U}_2 \oplus \ldots \omega_n\tilde{U}_n = \oplus_{p=1}^{n}(\omega_p\tilde{U}_p), \quad (9)$$

Where $\omega_p \in [0, 1]$ is the weight vector of $U_p(p = 1, 2, \ldots, n)$ such that $\sum_{p=1}^{n}\omega_p = 1$. If $\omega = (1/n, 1/n, \ldots, 1/n)^T$, then the T2NNWA $(\tilde{U}_1, \tilde{U}_2, \ldots, \tilde{U}_n)$ operator decrease to type 2 neutrosophic number averaging (T2NNA) operator: T2NNA

$$(\tilde{U}_1, \tilde{U}_2, \ldots, \tilde{U}_n) = \frac{1}{n}(\tilde{U}_1 \oplus \tilde{U}_2 \oplus \cdots \oplus \tilde{U}_n) \quad (10)$$

Now, we can enunciate the following theorem by using the basic procedures of T2NNVs expressed in Definition 2.

Theorem 1. *Let* $\tilde{U}_p = \left\langle\left(T_{T_p}(z), T_{I_p}(z), T_{F_p}(z)\right), \left(I_{T_p}(z), I_{I_p}(z), I_{F_p}(z)\right), \left(F_{T_p}(z), F_{I_p}(z), F_{F_p}(z)\right)\right\rangle (p = 1, 2, \ldots, n)$ *be a group T2NNS in the set of numbers. Then the combined value obtained by T2NNWA is also a T2NNV, and* $\mathrm{T2NNWA}_\omega(\tilde{U}_1, \tilde{U}_2, \ldots, \tilde{U}_n)$ *is given as Eq. (11) in Box II, where* $\omega_p \in [0, 1]$ *is the weight vector of* $U_p(p = 1, 2, \ldots, n)$ *such that* $\sum_{p=1}^{n}\omega_p = 1$.

$$=\left\langle \begin{array}{c} \left(1-\left(1-\mathrm{T}_{\mathrm{T}_1}(z)\right)^{\omega_1},\ 1-\left(1-\mathrm{T}_{\mathrm{I}_1}(z)\right)^{\omega_1},1-\left(1-\mathrm{T}_{\mathrm{F}_1}(z)\right)^{\omega_1}\right), \\ \left(\left(\mathrm{I}_{\mathrm{T}_1}(z)\right)^{\omega_1},\left(\mathrm{I}_{\mathrm{I}_1}(z)\right)^{\omega_1},\left(\mathrm{I}_{\mathrm{F}_1}(z)\right)^{\omega_1}\right),\left(\left(\mathrm{F}_{\mathrm{T}_1}(z)\right)^{\omega_1},\left(\mathrm{F}_{\mathrm{I}_1}(z)\right)^{\omega_1},\left(\mathrm{F}_{\mathrm{F}_1}(z)\right)^{\omega_1}\right) \end{array} \right\rangle \tag{12}$$

$$=\left\langle \begin{array}{c} \left(1-\prod_{p=1}^{1}\left(1-\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p},1-\prod_{p=1}^{1}\left(1-\mathrm{T}_{\mathrm{I}_p}(z)\right)^{\omega_p},1-\prod_{p=1}^{1}\left(1-\mathrm{T}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{1}\left(\mathrm{I}_{\mathrm{T}_p}(z)\right)^{\omega_p},\prod_{p=1}^{1}\left(\mathrm{I}_{\mathrm{I}_p}(z)\right)^{\omega_p},\prod_{p=1}^{1}\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{1}\left(\mathrm{F}_{\mathrm{T}_p}(z)\right)^{\omega_p},\prod_{p=1}^{1}\left(\mathrm{F}_{\mathrm{I}_p}(z)\right)^{\omega_p},\prod_{p=1}^{1}\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right) \end{array} \right\rangle \tag{13}$$

Box III.

$$\oplus_{p=1}^{2}\left(\omega_p\tilde{\mathrm{U}}_p\right)=\omega_1\tilde{\mathrm{U}}_1\oplus\omega_2\tilde{\mathrm{U}}_2$$

$$=\left\langle \begin{array}{c} \left\langle \begin{array}{c} \left(1-\left(1-\mathrm{T}_{\mathrm{T}_1}(z)\right)^{\omega_1},\ 1-\left(1-\mathrm{T}_{\mathrm{I}_1}(z)\right)^{\omega_1},1-\left(1-\mathrm{T}_{\mathrm{F}_1}(z)\right)^{\omega_1}\right), \\ \left(\left(\mathrm{I}_{\mathrm{T}_1}(z)\right)^{\omega_1},\left(\mathrm{I}_{\mathrm{I}_1}(z)\right)^{\omega_1},\left(\mathrm{I}_{\mathrm{F}_1}(z)\right)^{\omega_1}\right),\left(\left(\mathrm{F}_{\mathrm{T}_1}(z)\right)^{\omega_1},\left(\mathrm{F}_{\mathrm{I}_1}(z)\right)^{\omega_1},\left(\mathrm{F}_{\mathrm{F}_1}(z)\right)^{\omega_1}\right) \end{array} \right\rangle \\ \oplus \\ \left\langle \begin{array}{c} \left(1-\left(1-\mathrm{T}_{\mathrm{T}_2}(z)\right)^{\omega_2},\ 1-\left(1-\mathrm{T}_{\mathrm{I}_2}(z)\right)^{\omega_2},1-\left(1-\mathrm{T}_{\mathrm{F}_2}(z)\right)^{\omega_2}\right), \\ \left(\left(\mathrm{I}_{\mathrm{T}_2}(z)\right)^{\omega_2},\left(\mathrm{I}_{\mathrm{I}_2}(z)\right)^{\omega_2},\left(\mathrm{I}_{\mathrm{F}_2}(z)\right)^{\omega_2}\right),\left(\left(\mathrm{F}_{\mathrm{T}_2}(z)\right)^{\omega_2},\left(\mathrm{F}_{\mathrm{I}_2}(z)\right)^{\omega_2},\left(\mathrm{F}_{\mathrm{F}_2}(z)\right)^{\omega_2}\right) \end{array} \right\rangle \end{array} \right\rangle \tag{14}$$

$$=\left\langle \begin{array}{c} \left(\begin{array}{c} \left(\begin{array}{c} \left(1-\left(1-\mathrm{T}_{\mathrm{T}_1}(z)\right)^{\omega_1}\right)+\left(1-\left(1-\mathrm{T}_{\mathrm{T}_2}(z)\right)^{\omega_2}\right) \\ -\left(1-\left(1-\mathrm{T}_{\mathrm{T}_1}(z)\right)^{\omega_1}\right).\left(1-\left(1-\mathrm{T}_{\mathrm{T}_2}(z)\right)^{\omega_2}\right) \end{array}\right), \\ \left(\begin{array}{c} \left(1-\left(1-\mathrm{T}_{\mathrm{I}_1}(z)\right)^{\omega_1}\right)+\left(1-\left(1-\mathrm{T}_{\mathrm{I}_2}(z)\right)^{\omega_2}\right) \\ -\left(1-\left(1-\mathrm{T}_{\mathrm{I}_1}(z)\right)^{\omega_1}\right).\left(1-\left(1-\mathrm{T}_{\mathrm{I}_2}(z)\right)^{\omega_2}\right) \end{array}\right), \\ \left(\begin{array}{c} \left(1-\left(1-\mathrm{T}_{\mathrm{F}_1}(z)\right)^{\omega_1}\right)+\left(1-\left(1-\mathrm{T}_{\mathrm{F}_2}(z)\right)^{\omega_2}\right) \\ -\left(1-\left(1-\mathrm{T}_{\mathrm{F}_1}(z)\right)^{\omega_1}\right).\left(1-\left(1-\mathrm{T}_{\mathrm{F}_2}(z)\right)^{\omega_2}\right) \end{array}\right) \end{array}\right), \\ \left(\left(\mathrm{I}_{\mathrm{T}_1}(z)\right)^{\omega_1}\left(\mathrm{I}_{\mathrm{T}_2}(z)\right)^{\omega_2},\left(\mathrm{I}_{\mathrm{I}_1}(z)\right)^{\omega_1}\left(\mathrm{I}_{\mathrm{I}_2}(z)\right)^{\omega_2},\left(\mathrm{I}_{\mathrm{F}_1}(z)\right)^{\omega_1}\left(\mathrm{I}_{\mathrm{F}_2}(z)\right)^{\omega_2}\right), \\ \left(\left(\mathrm{F}_{\mathrm{T}_1}(z)\right)^{\omega_1}\left(\mathrm{F}_{\mathrm{T}_2}(z)\right)^{\omega_2},\left(\mathrm{F}_{\mathrm{I}_1}(z)\right)^{\omega_1}\left(\mathrm{F}_{\mathrm{I}_2}(z)\right)^{\omega_2},\left(\mathrm{F}_{\mathrm{F}_1}(z)\right)^{\omega_1}\left(\mathrm{F}_{\mathrm{F}_2}(z)\right)^{\omega_2}\right) \end{array} \right\rangle$$

$$=\left\langle \begin{array}{c} \left(1-\prod_{p=1}^{2}\left(1-\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p},1-\prod_{p=1}^{2}\left(1-\mathrm{T}_{\mathrm{I}_p}(z)\right)^{\omega_p},1-\prod_{p=1}^{2}\left(1-\mathrm{T}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{2}\left(\mathrm{I}_{\mathrm{T}_p}(z)\right)^{\omega_p},\prod_{p=1}^{2}\left(\mathrm{I}_{\mathrm{I}_p}(z)\right)^{\omega_p},\prod_{p=1}^{2}\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{2}\left(\mathrm{F}_{\mathrm{T}_p}(z)\right)^{\omega_p},\prod_{p=1}^{2}\left(\mathrm{F}_{\mathrm{I}_p}(z)\right)^{\omega_p},\prod_{p=1}^{2}\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right) \end{array} \right\rangle \tag{15}$$

Box IV.

Proof. We verify the theorem by mathematical induction.

1. When $n = 1$, it is a normal case. We mention it here for clarification only (see Eqs. (12) and (13) in Box III).

Consequently, the theorem is true for $n = 1$.

2. When $n = 2$, we have Eqs. (14) and (15) in Box IV. Consequently, the theorem is true for $n = 2$.

3. When $n = k$, we suppose that Eq. (11) is also true.
Then, T2NNWA $(\tilde{\mathrm{U}}_1,\ \tilde{\mathrm{U}}_2,\ \ldots,\tilde{\mathrm{U}}_{\mathrm{k}})$ is given as Eq. (16) in Box V.
4. When $n = \mathrm{k}+1$, we have $T2NNWA\,(\tilde{\mathrm{U}}_1,\ \tilde{\mathrm{U}}_2,\ \ldots,\tilde{\mathrm{U}}_{\mathrm{k}+1})$ given as Eq. (17) in Box VI.

We notice that the theorem is true for $n = \mathrm{k}+1$. So, by mathematical induction, we can say that Eq. (11) holds for all values of n. As the components of all three membership functions of $\tilde{\mathrm{U}}_{\mathrm{p}}$ belong to [0, 1], the following relations are valid:

$$0\le\left(1-\prod_{p=1}^{\mathrm{k}}\left(1-\mathrm{T}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right)\le 1,\ 0\le\left(\prod_{p=1}^{\mathrm{k}}\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right)\le 1,$$
$$0\le\left(\prod_{p=1}^{\mathrm{k}}\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right)\le 1. \tag{18}$$

It follows that this relation completes the proof of Theorem 1.

$$0\le\left\langle\left(1-\prod_{p=1}^{\mathrm{k}}\left(1-\mathrm{T}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right)+\left(\prod_{p=1}^{\mathrm{k}}\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right)+\left(\prod_{p=1}^{\mathrm{k}}\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right)\right\rangle\le 3.$$

$$T2NNWA(\tilde{U}_1,\ \tilde{U}_2,\ \ldots,\tilde{U}_k) = \omega_1\tilde{U}_1 \oplus \omega_2\tilde{U} \oplus \ldots \omega_n\tilde{U}_n = \oplus_{p=1}^{k}(\omega_p\tilde{U}_p)$$

$$= \left\langle \begin{array}{c} \left(1-\prod_{p=1}^{k}\left(1-T_{T_p}(z)\right)^{\omega_p},\ 1-\prod_{p=1}^{k}\left(1-T_{I_p}(z)\right)^{\omega_p},\ 1-\prod_{p=1}^{k}\left(1-T_{F_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{k}\left(I_{T_p}(z)\right)^{\omega_p},\ \prod_{p=1}^{k}\left(I_{I_p}(z)\right)^{\omega_p},\prod_{p=1}^{k}\left(I_{F_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{k}\left(F_{T_p}(z)\right)^{\omega_p},\ \prod_{p=1}^{k}\left(F_{I_p}(z)\right)^{\omega_p},\prod_{p=1}^{k}\left(F_{F_p}(z)\right)^{\omega_p}\right) \end{array} \right\rangle \tag{16}$$

Box V.

$$T2NNWA(\tilde{U}_1,\ \tilde{U}_2,\ \ldots,\tilde{U}_{k+1}) = \oplus_{p=1}^{n}(\omega_p\tilde{U}_p) \oplus (\omega_{k+1}\tilde{U}_{k+1})$$

$$= \left\langle \begin{array}{c} \left(\begin{array}{c} \left(\begin{array}{c} 1-\prod_{p=1}^{k}\left(1-T_{T_p}(z)\right)^{\omega_p} + 1-\prod_{p=1}^{k}\left(1-T_{T_{k+1}}(z)\right)^{\omega_{k+1}} \\ -1-\prod_{p=1}^{k}\left(1-T_{T_p}(z)\right)^{\omega_p}\ 1-\prod_{p=1}^{k}\left(1-T_{T_{k+1}}(z)\right)^{\omega_{k+1}} \end{array}\right), \\ \left(\begin{array}{c} 1-\prod_{p=1}^{k}\left(1-T_{I_p}(z)\right)^{\omega_p} + 1-\prod_{p=1}^{k}\left(1-T_{I_{k+1}}(z)\right)^{\omega_{k+1}} \\ -1-\prod_{p=1}^{k}\left(1-T_{I_p}(z)\right)^{\omega_p}\ 1-\prod_{p=1}^{k}\left(1-T_{I_{k+1}}(z)\right)^{\omega_{k+1}} \end{array}\right), \\ \left(\begin{array}{c} 1-\prod_{p=1}^{k}\left(1-T_{F_p}(z)\right)^{\omega_p} + 1-\prod_{p=1}^{k}\left(1-T_{F_{k+1}}(z)\right)^{\omega_{k+1}} \\ -1-\prod_{p=1}^{k}\left(1-T_{F_p}(z)\right)^{\omega_p}\ 1-\prod_{p=1}^{k}\left(1-T_{F_{k+1}}(z)\right)^{\omega_{k+1}} \end{array}\right) \end{array}\right), \\ \left(\prod_{p=1}^{k}\left(I_{T_p}(z)\right)^{\omega_p} \cdot \overset{\omega_{p+1}}{\underset{k+1}{\left(I_{T_p}(z)\right)}},\prod_{p=1}^{k}\left(I_{I_p}(z)\right)^{\omega_p} \cdot \overset{\omega_{p+1}}{\underset{k+1}{\left(I_{I_p}(z)\right)}},\prod_{p=1}^{k}\left(I_{F_p}(z)\right)^{\omega_p} \cdot \overset{\omega_{p+1}}{\underset{k+1}{\left(I_{F_p}(z)\right)}}\right), \\ \left(\prod_{p=1}^{k}\left(F_{T_p}(z)\right)^{\omega_p} \cdot \overset{\omega_{p+1}}{\underset{k+1}{\left(F_{T_p}(z)\right)}},\prod_{p=1}^{k}\left(F_{I_p}(z)\right)^{\omega_p} \cdot \overset{\omega_{p+1}}{\underset{k+1}{\left(F_{I_p}(z)\right)}},\prod_{p=1}^{k}\left(F_{F_p}(z)\right)^{\omega_p} \cdot \overset{\omega_{p+1}}{\underset{k+1}{\left(F_{F_p}(z)\right)}}\right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(1-\prod_{p=1}^{k+1}\left(1-T_{T_p}(z)\right)^{\omega_p},\ 1-\prod_{p=1}^{k+1}\left(1-T_{I_p}(z)\right)^{\omega_p},\ 1-\prod_{p=1}^{k+1}\left(1-T_{F_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{k+1}\left(I_{T_p}(z)\right)^{\omega_p},\ \prod_{p=1}^{k+1}\left(I_{I_p}(z)\right)^{\omega_p},\prod_{p=1}^{k+1}\left(I_{F_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{k+1}\left(F_{T_p}(z)\right)^{\omega_p},\ \prod_{p=1}^{k+1}\left(F_{I_p}(z)\right)^{\omega_p},\prod_{p=1}^{k+1}\left(F_{F_p}(z)\right)^{\omega_p}\right) \end{array} \right\rangle \tag{17}$$

Box VI.

2.2. Now, we will refer to one property to confirm the T2NNWA operator

Property 1 (*Boundedness*)**.** *if all* $\tilde{U}_p(p = 1, 2, \ldots, \eta)$ *are equal* $\tilde{U}_p = \tilde{U} = \left\langle\left(T_{T_p}(z), T_{I_p}(z), T_{F_p}(z)\right), \left(I_{T_p}(z), I_{I_p}(z), I_{F_p}(z)\right), \left(F_{T_p}(z), F_{I_p}(z), F_{F_p}(z)\right)\right\rangle$, *for all p, then T2NNWA* $\tilde{U}_p(p = 1, 2, \ldots, n) = \tilde{U}$.

Suppose $\tilde{U}^{+}$

$$= \left\langle \begin{array}{c} \left(\max_p\left(T_{T_p}(z)\right),\ \max_p\left(T_{I_p}(z)\right),\ \max_p\left(T_{F_p}(z)\right)\right), \\ \left(\min_p\left(I_{T_p}(z)\right),\ \min_p\left(I_{I_p}(z)\right),\ \min_p\left(I_{F_p}(z)\right)\right), \\ \left(\min_p\left(F_{T_p}(z)\right),\ \min_p\left(F_{I_p}(z)\right),\ \min_p\left(F_{F_p}(z)\right)\right) \end{array} \right\rangle$$

$$\mathrm{S}(\tilde{\mathrm{U}}) \leq \frac{1}{12}\left\langle \begin{array}{l} 8+\left(\max_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)+2.\max_p\left(\mathrm{T}_{\mathrm{I}_p}(z)\right)+\max_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)\right) \\ -\left(\min_p\left(\mathrm{I}_{\mathrm{T}_p}(z)\right)+2.\min_p\left(\mathrm{I}_{\mathrm{I}_p}(z)\right)+\min_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)\right) \\ -\left(\min_p\left(\mathrm{F}_{\mathrm{T}_p}(z)\right)+2.\min_p\left(\mathrm{F}_{\mathrm{I}_p}(z)\right)+\min_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)\right) \end{array}\right\rangle = \mathrm{S}(\tilde{\mathrm{U}}^{+}) \tag{22}$$

Box VII.

Suppose $\tilde{\mathrm{U}}^{-}$

$$= \left\langle \begin{array}{l} \left(\min_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right),\ \min_p\left(\mathrm{T}_{\mathrm{I}_p}(z)\right),\ \min_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)\right), \\ \left(\max_p\left(\mathrm{I}_{\mathrm{T}_p}(z)\right),\ \max_p\left(\mathrm{I}_{\mathrm{I}_p}(z)\right),\ \max_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)\right), \\ \left(\max_p\left(\mathrm{F}_{\mathrm{T}_p}(z)\right),\ \max_p\left(\mathrm{F}_{\mathrm{I}_p}(z)\right),\ \max_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)\right) \end{array}\right\rangle$$

For all $p = 1, 2, \ldots, n$. *Then,* $\tilde{\mathrm{U}}^{-} \leq T2NNWA\tilde{\mathrm{U}}_p(p = 1, 2, \ldots, n) \leq \tilde{\mathrm{U}}^{+}$. *Now, we demonstrate that:*

$$\begin{array}{l} \min_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right) \leq \left(\mathrm{T}_{\mathrm{F}_p}(z)\right) \leq \max_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right), \\ \min_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right) \leq \left(\mathrm{I}_{\mathrm{F}_p}(z)\right) \leq \max_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right), \\ \min_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right) \leq \left(\mathrm{F}_{\mathrm{F}_p}(z)\right) \leq \max_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right),\ \textit{for all } p = 1, 2, \ldots, n. \end{array} \tag{19}$$

Then, $1-\prod_{p=1}^{n}\left(1-\min_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)\right)^{\omega_p} \leq 1-\prod_{p=1}^{n}\left(1-\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)\right)^{\omega_p} \leq 1-\prod_{p=1}^{n}\left(1-\max_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)\right)^{\omega_p} = 1-\left(1-\min_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)\right)^{\sum_{p=1}^{n}\omega_p} \leq 1-\prod_{p=1}^{n}\left(1-\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)\right)^{\omega_p} \leq 1-\left(1-\max_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)\right)^{\sum_{p=1}^{n}\omega_p} = \min_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right) \leq 1-\prod_{p=1}^{n}\left(1-\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)\right)^{\omega_p} \leq \max_p\left(\mathrm{T}_{\mathrm{F}_p}(z)\right)$.

Then, from Eq. (19), we have for $p = 1, 2, \ldots, n$.

$$\begin{array}{l} \prod_{p=1}^{n}\min_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p} \leq \prod_{p=1}^{n}\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p} \leq \prod_{p=1}^{n}\max_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p} \\ \quad = \prod_{p=1}^{n}\min_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p} \\ \quad \leq \prod_{p=1}^{n}\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p} \leq \prod_{p=1}^{n}\max_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p} \\ \quad = \min_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right) \leq \prod_{p=1}^{n}\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p} \leq \max_p\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)\ \textit{and} \\ \prod_{p=1}^{n}\min_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p} \leq \prod_{p=1}^{n}\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p} \leq \prod_{p=1}^{n}\max_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p} \\ \quad = \prod_{p=1}^{n}\min_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p} \leq \prod_{p=1}^{n}\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p} \\ \quad \leq \prod_{p=1}^{n}\max_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p} \\ \quad = \min_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right) \leq \prod_{p=1}^{n}\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p} \leq \max_p\left(\mathrm{F}_{\mathrm{F}_p}(z)\right). \end{array} \tag{20}$$

Then, for $\min_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right) \leq \left(\mathrm{T}_{\mathrm{T}_p}(z)\right) \leq \max_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)$, *we prove the following:*

$$\begin{array}{l} \prod_{p=1}^{n}\min_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \leq \prod_{p=1}^{n}\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \leq \prod_{p=1}^{n}\max_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \\ \quad = \prod_{p=1}^{n}\min_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p} \leq \prod_{p=1}^{n}\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \\ \quad \leq \prod_{p=1}^{n}\max_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p} \\ \quad = \min_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right) \leq \prod_{p=1}^{n}\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \leq \max_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)\ \textit{and} \\ \prod_{p=1}^{n}\min_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \leq \prod_{p=1}^{n}\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \leq \prod_{p=1}^{n}\max_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \\ \quad = \prod_{p=1}^{n}\min_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p} \leq \prod_{p=1}^{n}\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \\ \quad \leq \prod_{p=1}^{n}\max_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p} \\ \quad = \min_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right) \leq \prod_{p=1}^{n}\left(\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p} \leq \max_p\left(\mathrm{T}_{\mathrm{T}_p}(z)\right). \end{array} \tag{21}$$

Likewise, from previous Eqs. (19)–(21).

Then, for $\left\langle\left(\mathrm{T}_{\mathrm{T}_p}(z), \mathrm{T}_{\mathrm{I}_p}(z), \mathrm{T}_{\mathrm{F}_p}(z)\right), \left(\mathrm{I}_{\mathrm{T}_p}(z), \mathrm{I}_{\mathrm{I}_p}(z), \mathrm{I}_{\mathrm{F}_p}(z)\right), \left(\mathrm{F}_{\mathrm{T}_p}(z), \mathrm{F}_{\mathrm{I}_p}(z), \mathrm{F}_{\mathrm{F}_p}(z)\right)\right\rangle$ $(p = 1, 2, \ldots, n)$. *Similarly, we have:* $\min_p\left(\mathrm{T}_{\mathrm{I}_p}(z)\right) \leq \left(\mathrm{T}_{\mathrm{I}_p}(z)\right) \leq \max_p\left(\mathrm{T}_{\mathrm{I}_p}(z)\right)$, $\min_p\left(\mathrm{F}_{\mathrm{T}_p}(z)\right) \leq \left(\mathrm{F}_{\mathrm{T}_p}(z)\right) \leq \max_p\left(\mathrm{F}_{\mathrm{T}_p}(z)\right)$, $\min_p\left(\mathrm{F}_{\mathrm{I}_p}(z)\right) \leq \left(\mathrm{F}_{\mathrm{I}_p}(z)\right) \leq \max_p\left(\mathrm{F}_{\mathrm{I}_p}(z)\right)$, $\min_p\left(\mathrm{I}_{\mathrm{T}_p}(z)\right) \leq \left(\mathrm{I}_{\mathrm{T}_p}(z)\right) \leq \max_p\left(\mathrm{I}_{\mathrm{T}_p}(z)\right)$, $\min_p\left(\mathrm{I}_{\mathrm{I}_p}(z)\right) \leq \left(\mathrm{I}_{\mathrm{I}_p}(z)\right) \leq \max_p\left(\mathrm{I}_{\mathrm{I}_p}(z)\right)$, *for* $p = 1, 2, \ldots, n$.

Then, suppose that T2NNW $\mathrm{A}_{\omega}\ \tilde{\mathrm{U}}_p(p = 1, 2, \ldots, n) = \tilde{\mathrm{U}} = \left\langle\left(\mathrm{T}_{\mathrm{T}_p}(z), \mathrm{T}_{\mathrm{I}_p}(z), \mathrm{T}_{\mathrm{F}_p}(z)\right), \left(\mathrm{I}_{\mathrm{T}_p}(z), \mathrm{I}_{\mathrm{I}_p}(z), \mathrm{I}_{\mathrm{F}_p}(z)\right), \left(\mathrm{F}_{\mathrm{T}_p}(z), \mathrm{F}_{\mathrm{I}_p}(z), \mathrm{F}_{\mathrm{F}_p}(z)\right)\right\rangle$, *and the score function of* $\tilde{\mathrm{U}} = \mathrm{S}(\tilde{\mathrm{U}}) = \frac{1}{12}\left\langle 8+\left(\mathrm{T}_{\mathrm{T}_{\tilde{\mathrm{U}}_1}}(z) + 2\left(\mathrm{T}_{\mathrm{I}_{\tilde{\mathrm{U}}_1}}(z)\right)+\mathrm{T}_{\mathrm{F}_{\tilde{\mathrm{U}}_1}}(z)\right)-\left(\mathrm{I}_{\mathrm{T}_{\tilde{\mathrm{U}}_1}}(z)+2\left(\mathrm{I}_{\mathrm{I}_{\tilde{\mathrm{U}}_1}}(z)\right)+\mathrm{I}_{\mathrm{F}_{\tilde{\mathrm{U}}_1}}(z)\right)-\left(\mathrm{F}_{\mathrm{T}_{\tilde{\mathrm{U}}_1}}(z)+2\left(\mathrm{F}_{\mathrm{I}_{\tilde{\mathrm{U}}_1}}(z)\right)+\mathrm{F}_{\mathrm{F}_{\tilde{\mathrm{U}}_1}}(z)\right)\right\rangle$ *from this, we have Eq. (22) given in Box VII.*

Also, $\mathrm{S}(\tilde{\mathrm{U}}) = \frac{1}{12}\left\langle 8+\left(\mathrm{T}_{\mathrm{T}_{\tilde{\mathrm{U}}_1}}(z)+2\left(\mathrm{T}_{\mathrm{I}_{\tilde{\mathrm{U}}_1}}(z)\right)+\mathrm{T}_{\mathrm{F}_{\tilde{\mathrm{U}}_1}}(z)\right)-\left(\mathrm{I}_{\mathrm{T}_{\tilde{\mathrm{U}}_1}}(z)+2\left(\mathrm{I}_{\mathrm{I}_{\tilde{\mathrm{U}}_1}}(z)\right)+\mathrm{I}_{\mathrm{F}_{\tilde{\mathrm{U}}_1}}(z)\right)-\left(\mathrm{F}_{\mathrm{T}_{\tilde{\mathrm{U}}_1}}(z)+2\left(\mathrm{F}_{\mathrm{I}_{\tilde{\mathrm{U}}_1}}(z)\right)+\mathrm{F}_{\mathrm{F}_{\tilde{\mathrm{U}}_1}}(z)\right)\right\rangle$. *From this, we have Eq. (23) given in Box VIII. Also,* $\mathrm{S}(\tilde{\mathrm{U}}) = \frac{1}{12}\left\langle 8+\left(\mathrm{T}_{\mathrm{T}_{\tilde{\mathrm{U}}_1}}(z)+2\left(\mathrm{T}_{\mathrm{I}_{\tilde{\mathrm{U}}_1}}(z)\right)+\mathrm{T}_{\mathrm{F}_{\tilde{\mathrm{U}}_1}}(z)\right)-\left(\mathrm{I}_{\mathrm{T}_{\tilde{\mathrm{U}}_1}}(z)+2\left(\mathrm{I}_{\mathrm{I}_{\tilde{\mathrm{U}}_1}}(z)\right)+\mathrm{I}_{\mathrm{F}_{\tilde{\mathrm{U}}_1}}(z)\right)-\left(\mathrm{F}_{\mathrm{T}_{\tilde{\mathrm{U}}_1}}(z)+2\left(\mathrm{F}_{\mathrm{I}_{\tilde{\mathrm{U}}_1}}(z)\right)+\mathrm{F}_{\mathrm{F}_{\tilde{\mathrm{U}}_1}}(z)\right)\right\rangle$. *From this, we have Eq. (24) given in Box IX.*

$$S(\tilde{U}) \geq \frac{1}{12}\left\langle \begin{array}{c} 8+\left(\max_p\left(T_{T_p}(z)\right)+2.\max_p\left(T_{I_p}(z)\right)+\max_p\left(T_{F_p}(z)\right)\right) \\ -\left(\min_p\left(I_{T_p}(z)\right)+2.\min_p\left(I_{I_p}(z)\right)+\min_p\left(I_{F_p}(z)\right)\right) \\ -\left(\min_p\left(F_{T_p}(z)\right)+2.\min_p\left(F_{I_p}(z)\right)+\min_p\left(F_{F_p}(z)\right)\right) \end{array}\right\rangle = S(\tilde{U}^-) \quad (23)$$

Box VIII.

$$S(\tilde{U}) = \frac{1}{12}\left\langle \begin{array}{c} 8+\left(\max_p\left(T_{T_p}(z)\right)+2\max_p\left(T_{I_p}(z)\right)+\max_p\left(T_{F_p}(z)\right)\right) \\ -\left(\min_p\left(I_{T_p}(z)\right)+2\min_p\left(I_{I_p}(z)\right)+\min_p\left(I_{F_p}(z)\right)\right) \\ -\left(\min_p\left(F_{T_p}(z)\right)+2\min_p\left(F_{I_p}(z)\right)+\min_p\left(F_{F_p}(z)\right)\right) \end{array}\right\rangle = S(\tilde{U}^+) \quad (24)$$

Box IX.

Therefore, we found the following cases: $S(\tilde{U}) < S(\tilde{U}^+)$, $S(\tilde{U}) < S(\tilde{U}^-)$ *and* $S(\tilde{U}) = S(\tilde{U}^+)$, *hence*

$$\tilde{U}^- < T2NNWA\ \tilde{U}_p(p = 1, 2, \ldots, n) < \tilde{U}^+ \quad (25)$$

By using the previous equations and by proving the score value, we can prove in the same way the accuracy value using this equation: $A(\tilde{U}_1) = \frac{1}{4}\left\langle\left(T_{T_{\tilde{U}_1}}(z) + 2\left(T_{I_{\tilde{U}_1}}(z)\right) + T_{F_{\tilde{U}_1}}(z)\right) - \left(F_{T_{\tilde{U}_1}}(z) + 2\left(F_{I_{\tilde{U}_1}}(z)\right) + F_{F_{\tilde{U}_1}}(z)\right)\right\rangle$.

Property 2 (*Idempotency*). *if all* $\tilde{U}_p(p = 1, 2, \ldots, n)$ *are equal* $\tilde{U}_p = \tilde{U} = \left\langle\left(T_{T_p}(z), T_{I_p}(z), T_{F_p}(z)\right), \left(I_{T_p}(z), I_{I_p}(z), I_{F_p}(z)\right), \left(F_{T_p}(z), F_{I_p}(z), F_{F_p}(z)\right)\right\rangle$, *for all p, then T2NNWA* $\tilde{U}_p(p = 1, 2, \ldots, n) = \tilde{U}$. *From Eq. (11), we have T2NNWA* $\tilde{U}_p(p = 1, 2, \ldots, n)$ *given in Box X).*

Consequently,

$$\left\langle\left(T_{T_p}(z), T_{I_p}(z), T_{F_p}(z)\right), \left(I_{T_p}(z), I_{I_p}(z), I_{F_p}(z)\right), \left(F_{T_p}(z), F_{I_p}(z), F_{F_p}(z)\right)\right\rangle = \tilde{U}.$$

This proves Property 2.

Example 3. Consider the following four T2NN values. Using the T2NNWA operator defined in Eq. (11), we can aggregate ($\tilde{U}_1$, $\tilde{U}_2$, $\tilde{U}_3$, and $\tilde{U}_4$) with weight vector $\omega = (0.25, 0.20, 0.35, 0.20)$ as $\tilde{U} = T2NNWA(\tilde{U}_1, \tilde{U}_2, \tilde{U}_3, \text{and} \tilde{U}_4) = \omega_1\tilde{U}_1 \oplus \omega_2\tilde{U}_2 \oplus \omega_3\tilde{U}_3 \oplus \omega_4\tilde{U}_4$. $\tilde{U}_1, \tilde{U}_2, \tilde{U}_3$ and $\tilde{U}_4$ are given in Box XI. After aggregation, we find that $\tilde{U}_{all}$

$$= \left\langle\left(T_{T_p}(z), T_{I_p}(z), T_{F_p}(z)\right), \left(I_{T_p}(z), I_{I_p}(z), I_{F_p}(z)\right), \left(F_{T_p}(z), F_{I_p}(z), F_{F_p}(z)\right)\right\rangle$$
$$= \langle(0.881, 0.710, 0.768), (0.2851, 0.0872, 0.2093), (0.0941, 0.2163, 0.2268)\rangle$$

2.3. Application of T2NNWA operator to MADM

Consider a MADM issue in which we have the collection of $\phi_i = \{\phi_1, \phi_2, \ldots, \phi_n\}$ suitable alternatives, where i = 1, 2, …, m, assessed on n criteria $Œi_p = \{Œi_1, Œi_2, .., Œi_n\}, p = 1, 2, \ldots, n$. Assume that $\omega_p = \{\omega_1, \omega_2, \ldots, \omega_p\}$ is the weight vector of attributes, where $\omega_p > 0$ and sum $\sum_{p=1}^{n}\omega_p = 1$ for $p = 1, 2, \ldots, n$. The standing of all alternatives $\phi_i = \{\phi_1, \phi_2, \ldots, \phi_n\}$ with regard to the attributes $Œi_p = \{Œi_1, Œi_2, \ldots, Œi_n\}$, $p = 1, 2, \ldots, n$ have been supposed in T2NN values based relation matrix $R = (k_{ip})_{m \times n}$, as in Table 2. Furthermore, in the relation matrix $R = (k_{ip})_{m \times n}$, the standing $\tilde{A}_{ip} = \left\langle\left(T_{T_{ip}}(z), T_{I_{ip}}(z), T_{F_{ip}}(z)\right), \left(I_{T_{ip}}(z), I_{I_{ip}}(z), I_{F_{ip}}(z)\right), \left(F_{T_{ip}}(z), F_{I_{ip}}(z), F_{F_{ip}}(z)\right)\right\rangle$ represents a T2NN value, where the type 2 neutrosophic number $\left(T_{T_{ip}}(z), T_{I_{ip}}(z), T_{F_{ip}}(z)\right)$ signifies the degree an alternative satisfies the attribute $Œi_p = Œi_1, Œi_2, \ldots, Œi_n$, $p = 1, 2, \ldots, \eta$, with three degrees of truth (truth, indeterminacy, and falsity). Also, $\left(I_{T_{ip}}(z), I_{I_{ip}}(z), I_{F_{ip}}(z)\right)$ signifies the degree an alternative is undefined about the attribute $Œi_p = Œi_1, Œi_2, \ldots, Œi_n$, $p = 1, 2, \ldots, n$, where the uncertain degree contains three degrees of indeterminacy (truth, indeterminacy, and falsity). Also, $\left(F_{T_{ip}}(z), F_{I_{ip}}(z), F_{F_{ip}}(z)\right)$ introduces the degree an alternative does not satisfy the attribute $Œi_p = \{Œi_1, Œi_2, .., Œi_n\}$, $p = 1, 2, \ldots$, n, where the unsatisfied degree contains three degrees of dissatisfaction (truth, indeterminacy, and falsity). We improve a functional approach for solving MADM problems based on the T2NNWA, in which we rank the alternatives over the attributes. The graphical schema of the developed technique for MADM is shown in Fig. 1.

2.4. Numerical case

In this section, a mathematical example of data and methods is presented to check the competence and efficiency of submitted framework for selection the best alternative. Currently in Egypt, people seek for choosing the best bank to operate banking transactions such as deposit their money, withdraw financial loans, transfer of money, change currencies, etc. This section presents a numerical case to select the best bank for citizens and investors. There are four evaluation alternatives ϕ_1, ϕ_2, ϕ_3 and ϕ_4, five criteria are considered as selection factors $Œi_1$ (Reputation and elegance), $Œi_2$ (Customer service), $Œi_3$ (Place of the bank and its branches), $Œi_4$ (Fees), $Œi_5$ (Offers). The classification of alternatives ϕ_i (i = 1, 2, …, 4) with regard to $Œi_i$(i = 1, 2, …, 5) are expressed with T2NN values, as presented in Table 3. We suppose that $\omega = (0.20, 0.25, 0.30, 0.15, 0.10)^T$ is the proportional weight for criteria $Œi_i$ (i = 1, 2, …, 5).

$$\mathrm{T2NNWA}\tilde{\mathrm{U}}_p(p=1,2,\ldots,n) = \mathrm{T2NNWA}(\tilde{\mathrm{U}}_1,\ \tilde{\mathrm{U}}_2,\ \ldots,\tilde{\mathrm{U}}_{\mathrm{k}+1}) = \oplus_{p=1}^{n}(\omega_p \tilde{\mathrm{U}}_p)$$

$$= \left\langle \begin{array}{c} \left(1-\prod_{p=1}^{n}\left(1-\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\omega_p},\ 1-\prod_{p=1}^{n}\left(1-\mathrm{T}_{\mathrm{I}_p}(z)\right)^{\omega_p},\ 1-\prod_{p=1}^{n}\left(1-\mathrm{T}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{n}\left(\mathrm{I}_{\mathrm{T}_p}(z)\right)^{\omega_p},\ \prod_{p=1}^{n}\left(\mathrm{I}_{\mathrm{I}_p}(z)\right)^{\omega_p},\ \prod_{p=1}^{n}\left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right), \\ \left(\prod_{p=1}^{n}\left(\mathrm{F}_{\mathrm{T}_p}(z)\right)^{\omega_p},\ \prod_{p=1}^{n}\left(\mathrm{F}_{\mathrm{I}_p}(z)\right)^{\omega_p},\ \prod_{p=1}^{n}\left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\omega_p}\right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(1-\left(1-\mathrm{T}_{\mathrm{T}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p},\ 1-\left(1-\mathrm{T}_{\mathrm{I}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p},\ 1-\left(1-\mathrm{T}_{\mathrm{F}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p}\right), \\ \left(\left(\mathrm{I}_{\mathrm{T}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p},\ \left(\mathrm{I}_{\mathrm{I}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p},\ \left(\mathrm{I}_{\mathrm{F}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p}\right), \\ \left(\left(\mathrm{F}_{\mathrm{T}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p},\ \left(\mathrm{F}_{\mathrm{I}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p},\ \left(\mathrm{F}_{\mathrm{F}_p}(z)\right)^{\sum_{p=1}^{n}\omega_p}\right) \end{array} \right\rangle$$

Box X.

$$\tilde{\mathrm{U}}_1 = \langle(0.75, 0.65, 0.95), (0.30, 0.15, 0.20), (0.15, 0.25, 0.20)\rangle,$$
$$\tilde{\mathrm{U}}_2 = \langle(0.85, 0.75, 0.65), (0.20, 0.10, 0.25), (0.10, 0.30, 0.25)\rangle,$$
$$\tilde{\mathrm{U}}_3 = \langle(0.90, 0.70, 0.65), (0.30, 0.05, 0.20), (0.05, 0.25, 0.20)\rangle,$$
$$\tilde{\mathrm{U}}_4 = \langle(0.95, 0.70, 0.60), (0.35, 0.10, 0.20), (0.15, 0.10, 0.30)\rangle$$

$$= \left\langle \begin{array}{c} \left(\begin{array}{c} \left(1-(1-0.75)^{0.25}\ (1-0.85)^{0.20}\ (1-0.90)^{0.35}\ (1-0.95)^{0.20}\right), \\ \left(1-(1-0.65)^{0.25}\ (1-0.75)^{0.20}\ (1-0.70)^{0.35}\ (1-0.75)^{0.20}\right), \\ \left(1-(1-0.95)^{0.25}\ (1-0.65)^{0.20}\ (1-0.60)^{0.35}\ (1-0.60)^{0.20}\right) \end{array}\right), \\ \left(\begin{array}{c} \left((0.30)^{0.25}\ (0.20)^{0.20}\ x\ (0.30)^{0.35}\ (0.35)^{0.20}\right), \\ \left((0.15)^{0.25}\ (0.10)^{0.20}\ (0.05)^{0.35}\ (0.10)^{0.20}\right), \\ \left((0.20)^{0.25}\ (0.25)^{0.20}\ (0.20)^{0.35}\ (0.20)^{0.20}\right) \end{array}\right), \\ \left(\begin{array}{c} \left((0.15)^{0.25}\ (0.10)^{0.20}\ (0.05)^{0.35}\ (0.15)^{0.20}\right), \\ \left((0.25)^{0.25}\ (0.30)^{0.20}\ (0.25)^{0.35}\ (0.10)^{0.20}\right), \\ \left((0.20)^{0.25}\ (0.25)^{0.20}\ (0.20)^{0.35}\ (0.30)^{0.20}\right) \end{array}\right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(\begin{array}{c} (1-0.707\times0.684\times0.447\times0.549),\ (1-0.769\times0.758\times0.656\times0.758), \\ (1-0.473\times0.811\times0.726\times0.833) \end{array}\right) \\ \left(\begin{array}{c} (0.740\times0.725\times0.656\times0.811),\ (0.622\times0.631\times0.350\times0.631), \\ (0.669\times0.758\times0.569\times0.725) \end{array}\right) \\ \left(\begin{array}{c} (0.622\times0.631\times0.350\times0.684),\ (0.707\times0.786\times0.616\times0.631), \\ (0.669\times0.758\times0.569\times0.786) \end{array}\right) \end{array} \right\rangle$$

Box XI.

Table 2
Type 2 neutrosophic number value based relation matrix.

	Œi$_1$	Œi$_2$	…	Œi$_n$
ϕ_1	$\left\langle \begin{array}{l} (\mathrm{T}_{\mathrm{T}_{11}}(z), \mathrm{T}_{\mathrm{I}_{11}}(z), \mathrm{T}_{\mathrm{F}_{11}}(z)), \\ (\mathrm{I}_{\mathrm{T}_{11}}(z), \mathrm{I}_{\mathrm{I}_{11}}(z), \mathrm{I}_{\mathrm{F}_{11}}(z)), \\ (\mathrm{F}_{\mathrm{T}_{11}}(z), \mathrm{F}_{\mathrm{I}_{11}}(z), \mathrm{F}_{\mathrm{F}_{11}}(z)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (\mathrm{T}_{\mathrm{T}_{12}}(z), \mathrm{T}_{\mathrm{I}_{12}}(z), \mathrm{T}_{\mathrm{F}_{12}}(z)), \\ (\mathrm{I}_{\mathrm{T}_{12}}(z), \mathrm{I}_{\mathrm{I}_{12}}(z), \mathrm{I}_{\mathrm{F}_{12}}(z)), \\ (\mathrm{F}_{\mathrm{T}_{12}}(z), \mathrm{F}_{\mathrm{I}_{12}}(z), \mathrm{F}_{\mathrm{F}_{12}}(z)) \end{array} \right\rangle$	…	$\left\langle \begin{array}{l} (\mathrm{T}_{\mathrm{T}_{1n}}(z), \mathrm{T}_{\mathrm{I}_{1n}}(z), \mathrm{T}_{\mathrm{F}_{1n}}(z)), \\ (\mathrm{I}_{\mathrm{T}_{1n}}(z), \mathrm{I}_{\mathrm{I}_{1n}}(z), \mathrm{I}_{\mathrm{F}_{1n}}(z)), \\ (\mathrm{F}_{\mathrm{T}_{1n}}(z), \mathrm{F}_{\mathrm{I}_{1n}}(z), \mathrm{F}_{\mathrm{F}_{1n}}(z)) \end{array} \right\rangle$
ϕ_2	$\left\langle \begin{array}{l} (\mathrm{T}_{\mathrm{T}_{21}}(z), \mathrm{T}_{\mathrm{I}_{21}}(z), \mathrm{T}_{\mathrm{F}_{21}}(z)), \\ (\mathrm{I}_{\mathrm{T}_{21}}(z), \mathrm{I}_{\mathrm{I}_{21}}(z), \mathrm{I}_{\mathrm{F}_{21}}(z)), \\ (\mathrm{F}_{\mathrm{T}_{21}}(z), \mathrm{F}_{\mathrm{I}_{21}}(z), \mathrm{F}_{\mathrm{F}_{21}}(z)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (\mathrm{T}_{\mathrm{T}_{22}}(z), \mathrm{T}_{\mathrm{I}_{22}}(z), \mathrm{T}_{\mathrm{F}_{22}}(z)), \\ (\mathrm{I}_{\mathrm{T}_{22}}(z), \mathrm{I}_{\mathrm{I}_{22}}(z), \mathrm{I}_{\mathrm{F}_{22}}(z)), \\ (\mathrm{F}_{\mathrm{T}_{22}}(z), \mathrm{F}_{\mathrm{I}_{22}}(z), \mathrm{F}_{\mathrm{F}_{22}}(z)) \end{array} \right\rangle$	…	$\left\langle \begin{array}{l} (\mathrm{T}_{\mathrm{T}_{2n}}(z), \mathrm{T}_{\mathrm{I}_{2n}}(z), \mathrm{T}_{\mathrm{F}_{2n}}(z)), \\ (\mathrm{I}_{\mathrm{T}_{2n}}(z), \mathrm{I}_{\mathrm{I}_{2n}}(z), \mathrm{I}_{\mathrm{F}_{2n}}(z)), \\ (\mathrm{F}_{\mathrm{T}_{2n}}(z), \mathrm{F}_{\mathrm{I}_{2n}}(z), \mathrm{F}_{\mathrm{F}_{2n}}(z)) \end{array} \right\rangle$
…	…	…	…	…
ϕ_m	$\left\langle \begin{array}{l} (\mathrm{T}_{\mathrm{T}_{m1}}(z), \mathrm{T}_{\mathrm{I}_{m1}}(z), \mathrm{T}_{\mathrm{F}_{m1}}(z)), \\ (\mathrm{I}_{\mathrm{T}_{m1}}(z), \mathrm{I}_{\mathrm{I}_{m1}}(z), \mathrm{I}_{\mathrm{F}_{m1}}(z)), \\ (\mathrm{F}_{\mathrm{T}_{m1}}(z), \mathrm{F}_{\mathrm{I}_{m1}}(z), \mathrm{F}_{\mathrm{F}_{m1}}(z)) \end{array} \right\rangle$	$\left\langle \begin{array}{l} (\mathrm{T}_{\mathrm{T}_{m2}}(z), \mathrm{T}_{\mathrm{I}_{m2}}(z), \mathrm{T}_{\mathrm{F}_{m2}}(z)), \\ (\mathrm{I}_{\mathrm{T}_{m2}}(z), \mathrm{I}_{\mathrm{I}_{m2}}(z), \mathrm{I}_{\mathrm{F}_{m2}}(z)), \\ (\mathrm{F}_{\mathrm{T}_{m2}}(z), \mathrm{F}_{\mathrm{I}_{m2}}(z), \mathrm{F}_{\mathrm{F}_{m2}}(z)) \end{array} \right\rangle$	…	$\left\langle \begin{array}{l} (\mathrm{T}_{\mathrm{T}_{mn}}(z), \mathrm{T}_{\mathrm{I}_{mn}}(z), \mathrm{T}_{\mathrm{F}_{mn}}(z)), \\ (\mathrm{I}_{\mathrm{T}_{mn}}(z), \mathrm{I}_{\mathrm{I}_{mn}}(z), \mathrm{I}_{\mathrm{F}_{mn}}(z)), \\ (\mathrm{F}_{\mathrm{T}_{mn}}(z), \mathrm{F}_{\mathrm{I}_{mn}}(z), \mathrm{F}_{\mathrm{F}_{mn}}(z)) \end{array} \right\rangle$

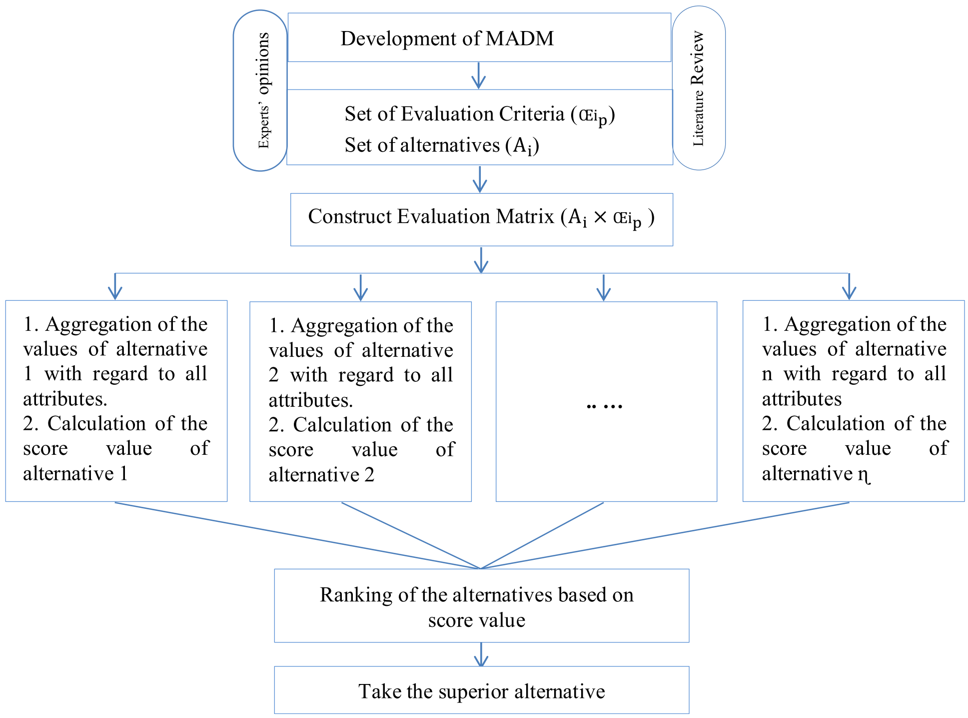

Fig. 1. The general framework of the submitted method.

Table 3
Decision matrix between alternatives and criteria using T2NN values.

	Œi$_1$	Œi$_2$	Œi$_3$	Œi$_4$	Œi$_5$
ϕ_1	$\langle(0.75, 0.80, 0.85), (0.20, 0.15, 0.30), (0.10, 0.15, 0.20)\rangle$	$\langle(0.65, 0.70, 0.75), (0.40, 0.45, 0.50), (0.35, 0.40, 0.35)\rangle$	$\langle(0.85, 0.90, 0.95), (0.30, 0.35, 0.40), (0.25, 0.40, 0.35)\rangle$	$\langle(0.50, 0.40, 0.55), (0.10, 0.15, 0.30), (0.10, 0.20, 0.20)\rangle$	$\langle(0.30, 0.45, 0.25), (0.20, 0.10, 0.30), (0.10, 0.25, 0.20)\rangle$
ϕ_2	$\langle(0.60, 0.50, 0.65), (0.30, 0.25, 0.30), (0.20, 0.30, 0.25)\rangle$	$\langle(0.65, 0.70, 0.75), (0.10, 0.15, 0.20), (0.05, 0.10, 0.15)\rangle$	$\langle(0.45, 0.35, 0.50), (0.15, 0.10, 0.10), (0.20, 0.30, 0.25)\rangle$	$\langle(0.45, 0.50, 0.60), (0.30, 0.20, 0.30), (0.25, 0.30, 0.25)\rangle$	$\langle(0.50, 0.45, 0.35), (0.30, 0.25, 0.30), (0.20, 0.30, 0.25)\rangle$
ϕ_3	$\langle(0.45, 0.50, 0.80), (0.15, 0.30, 0.55), (0.55, 0.20, 0.25)\rangle$	$\langle(0.40, 0.45, 0.50), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)\rangle$	$\langle(0.40, 0.45, 0.60), (0.05, 0.20, 0.25), (0.40, 0.20, 0.25)\rangle$	$\langle(0.45, 0.80, 0.90), (0.40, 0.70, 0.55), (0.55, 0.20, 0.40)\rangle$	$\langle(0.80, 0.50, 0.80), (0.45, 0.30, 0.55), (0.55, 0.20, 0.25)\rangle$
ϕ_4	$\langle(0.85, 0.70, 0.95), (0.60, 0.50, 0.65), (0.45, 0.15, 0.35)\rangle$	$\langle(0.60, 0.65, 0.70), (0.35, 0.40, 0.45), (0.30, 0.40, 0.45)\rangle$	$\langle(0.95, 0.70, 0.80), (0.15, 0.10, 0.30), (0.30, 0.35, 0.30)\rangle$	$\langle(0.90, 0.70, 0.95), (0.60, 0.40, 0.65), (0.45, 0.15, 0.35)\rangle$	$\langle(0.65, 0.70, 0.80), (0.40, 0.35, 0.25), (0.15, 0.15, 0.20)\rangle$

Table 4
Aggregated T2NN values based classification.

	Aggregating values
ϕ_1	$\langle(0.7131, 0.7654, 0.8302), (0.2420, 0.2444, 0.3716), (0.1801, 0.2827, 0.2721)\rangle$
ϕ_2	$\langle(0.5434, 0.5193, 0.6113), (0.1852, 0.1616, 0.1950), (0.1462, 0.2280, 0.2200)\rangle$
ϕ_3	$\langle(0.4785, 0.5408, 0.7210), (0.1395, 0.2726, 0.3565), (0.3264, 0.1861, 0.2537)\rangle$
ϕ_4	$\langle(0.8588, 0.6882, 0.8638), (0.3322, 0.2723, 0.4273), (0.3226, 0.2472, 0.3365)\rangle$

We apply the proposed aggregation operator T2NNWA to solve the best bank selection issue by using the next procedures.

Step 1. Collect the classification values of the alternatives $\phi_i(i = 1, 2, 3, 4)$ defined in the previous matrix with T2NNWA operator that is located by Eq. (11) and the values introduced in Table 4.

Step 2. Compute the score value and the accuracy value of alternatives $\phi_i(i = 1, 2, 3, 4)$ by applying Eq. (5) and Eq. (6), as shown in Table 5.

Step 3. Ranking the alternatives based on score values, we found that alternative ϕ_1 is the best alternative, and the classification of alternatives is : $\phi_1 > \phi_4 > \phi_2 > \phi_3$.

Table 5
The score and accuracy values of alternatives.

	Score values	Accuracy values
ϕ_1	0.8382	0.5141
ϕ_2	0.7809	0.3428
ϕ_3	0.7775	0.3322
ϕ_4	0.8288	0.4864

3. The proposed method procedure

We now suggest an orderly approach to TOPSIS technique to the neutrosophic environment under type 2 of neutrosophic number. We found that the GDM problem can be easily solved by this

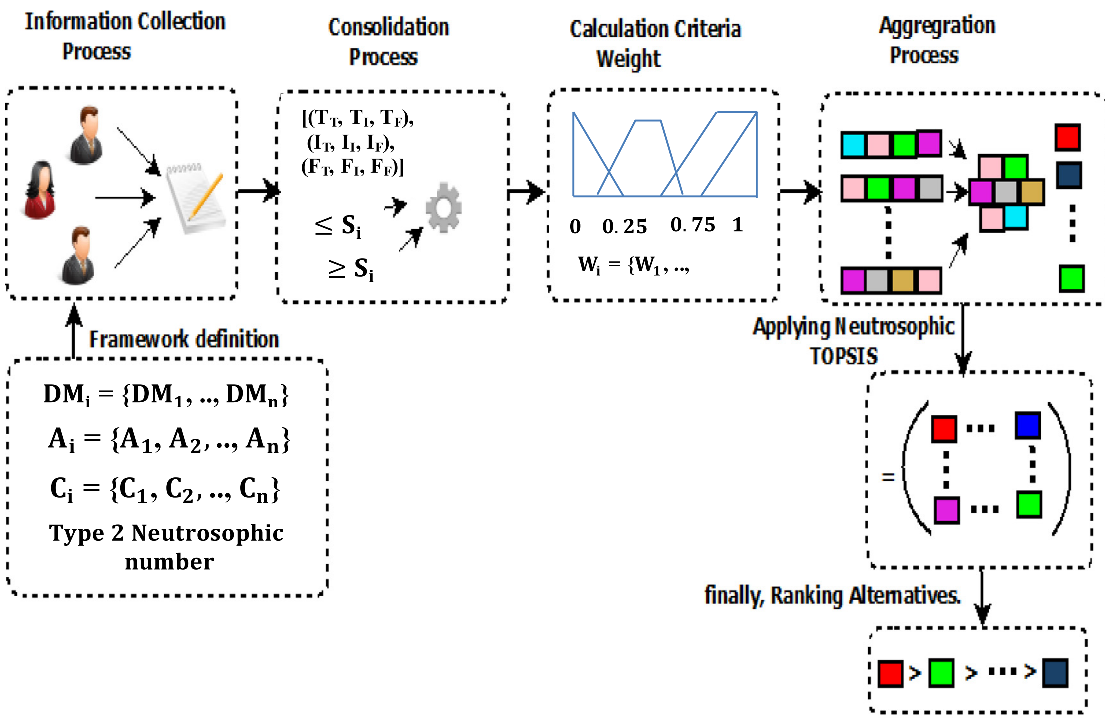

Fig. 2. The general framework for applying TOPSIS using type 2 neutrosophic number.

Table 6
Semantic terms for the significance weight of each criteria.

Linguistic variables	The type 2 neutrosophic number scale for relative importance of each criteria $[(T_T, T_I, T_F), (I_T, I_I, I_F), (F_T, F_I, F_F)]$
Weakly important (WI)	$\langle(0.20, 0.30, 0.20), (0.60, 0.70, 0.80), (0.45, 0.75, 0.75)\rangle$
Equal important (EI)	$\langle(0.40, 0.30, 0.25), (0.45, 0.55, 0.40), (0.45, 0.60, 0.55)\rangle$
Strong important (SI)	$\langle(0.65, 0.55, 0.55), (0.40, 0.45, 0.55), (0.35, 0.40, 0.35)\rangle$
Very strongly important (VSI)	$\langle(0.80, 0.75, 0.70), (0.20, 0.15, 0.30), (0.15, 0.10, 0.20)\rangle$
Absolutely important (AI)	$\langle(0.90, 0.85, 0.95), (0.10, 0.15, 0.10), (0.05, 0.05, 0.10)\rangle$

method under advanced neutrosophic environment. The general conceptualization of framework is displayed in Fig. 2.

The suggested framework consists of many phases, as presented in Fig. 2.

Phase 1. Establish a group of Exs and decide the goal, alternatives and criteria.

• Assume that EXs want to estimate the combination of n criteria and m alternatives EXs are symbolized by $\mathrm{Ex_E} = \{\mathrm{Ex_1}, \mathrm{Ex_2}, \mathrm{Ex_3}\}$, where $\mathrm{E} = 1, 2, \ldots, \mathrm{E}$, and alternatives by $\mathrm{Alt_i} = \{\mathrm{Alt_1}, \mathrm{Alt_2}, \ldots, \mathrm{Alt_m}\}$, where $\mathrm{i} = 1, 2, \ldots, \mathrm{m}$, assessed on n criteria $\text{Œi}_p = \{\text{Œi}_1, \text{Œi}_2, .., \text{Œi}_n\}$, $p = 1, 2, \ldots, n$.

Phase 2. Depict and design the linguistic scales.

- Obtain Exs' judgments on each element. Based on previous knowledge and experience on the topic, Exs are wanted to convey their judgments. Every Ex gives his/her judgment linguistically on all of these elements.
- Transform EXs' linguistic evaluations into type 2 neutrosophic numbers for every Ex providing his judgment with assistance of the linguistic terms.
- The significance weights of different criteria and the ordering of specific criteria are deemed as linguistic terms. These linguistic terms can be presented in type 2 neutrosophic number as in Tables 6 and 7. The significance weight of each criterion can be obtained either by direct allocation or indirectly by pairwise comparisons [41]. Herein, we propose that the experts and decision makers use the linguistic terms presented in Tables 6 and 7 to evaluate the weight of the criteria and the classification of alternatives with account to different criteria.
- Build the preference relation matrix to locate the weights of criteria. Exs use the linguistic terms presented in Table 6 to assess the opinions of Exs with regard to each criterion.
- A neutrosophic multicriteria GDM problem can be briefly expressed in matrix:

$$\text{Format as} \quad \tilde{A} = \begin{matrix} & \begin{matrix} \text{Œi}_p & \ldots & \text{Œi}_n \end{matrix} \\ \begin{matrix} \mathrm{Ex_1} \\ \vdots \\ \mathrm{Ex_m} \end{matrix} & \begin{bmatrix} \tilde{z}_{11} & \ldots & \tilde{z}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{z}_{\mathrm{m}1} & \ldots & \tilde{z}_{\mathrm{mn}} \end{bmatrix} \end{matrix} \tag{26}$$

$$\tilde{\omega} = [\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n] \tag{27}$$

Where $\tilde{z}_{\mathrm{ip}} = \left\langle \left(\mathrm{T_{T_{ip}}}(z), \mathrm{T_{I_{ip}}}(z), \mathrm{T_{F_{ip}}}(z)\right), \left(\mathrm{I_{T_{ip}}}(z), \mathrm{I_{I_{ip}}}(z), \mathrm{I_{F_{ip}}}(z)\right), \left(\mathrm{F_{T_{ip}}}(z), \mathrm{F_{I_{ip}}}(z), \mathrm{F_{F_{ip}}}(z)\right)\right\rangle$, $\mathrm{i} = 1, 2 \ldots, \mathrm{m}$, $p = 1, 2, \ldots, n$, where $\tilde{z}_{\mathrm{ip}}$, $\forall_{\mathrm{i},p}$ and $\tilde{\omega}_p$, $p = 1, 2 \ldots, n$ are linguistic terms. These linguistic terms can be described by type 2 neutrosophic number.

- Calculating the weights of Exs. Exs' judgments are collected by using the equation given in Box XII.
- Calculate the score value after aggregating the opinions of Exs for each criteria using Eq. (5). Then, normalize the obtained weights.

Phase 3. Construct the evaluation matrix.

$$\tilde{z}_{ip} = \frac{\left[T_{T_{ip}}(z), T_{I_{ip}}(z), T_{F_{ip}}(z),\ I_{T_{ip}}(z), I_{I_{ip}}(z), I_{F_{ip}}(z),\ F_{T_{ip}}(z), F_{I_{ip}}(z), F_{F_{ip}}(z)\right]}{n} \quad (28)$$

Box XII.

Table 7
Linguistic variables for the classification.

Linguistic variables	The type - 2 neutrosophic number scale for relative importance of comparison matrix $[(T_T, T_I, T_F), (I_T, I_I, I_F), (F_T, F_I, F_F)]$
Very Bad (VB)	⟨(0.20, 0.20, 0.10), (0.65, 0.80, 0.85), (0.45, 0.80, 0.70)⟩
Bad (B)	⟨(0.35, 0.35, 0.10), (0.50, 0.75, 0.80), (0.50, 0.75, 0.65)⟩
Medium Bad (MB)	⟨(0.50, 0.30, 0.50), (0.50, 0.35, 0.45), (0.45, 0.30, 0.60)⟩
Medium (M)	⟨(0.40, 0.45, 0.50), (0.40, 0.45, 0.50), (0.35, 0.40, 0.45)⟩
Medium Good (MG)	⟨(0.60, 0.45, 0.50), (0.20, 0.15, 0.25), (0.10, 0.25, 0.15)⟩
Good (G)	⟨(0.70, 0.75, 0.80), (0.15, 0.20, 0.25), (0.10, 0.15, 0.20)⟩
Very Good (VG)	⟨(0.95, 0.90, 0.95), (0.10, 0.10, 0.05), (0.05, 0.05, 0.05)⟩

- Build the evaluation matrix $A_i \times Œi_p$ to assess the classification of alternatives with respect to each criterion. Exs use the linguistic terms shown in Table 7.

$$\text{Format as} \quad \tilde{R} = \begin{matrix} \\ Alt_1 \\ \vdots \\ Alt_m \end{matrix} \begin{matrix} Œi_p & \cdots & Œi_n \\ \left[\begin{matrix} \tilde{z}_{11} \\ \vdots \\ \tilde{z}_{m1} \end{matrix}\right. & \begin{matrix} \cdots \\ \ddots \\ \cdots \end{matrix} & \left.\begin{matrix} \tilde{z}_{1n} \\ \vdots \\ \tilde{z}_{mn} \end{matrix}\right] \end{matrix} \quad (29)$$

- Aggregate the final evaluation matrix using Eq. (1) divided by 3.
- Use the de-neutrosophication Eq. (5) for transforming type 2 neutrosophic number to the crisp value for each factor $\tilde{z}_{ip}$.
- Then, normalize the obtained matrix by Eq. (30)

$$\tilde{y}_{ip} = \frac{\tilde{z}_{ip}}{\sqrt{\sum_{i=1}^{m} \tilde{Z}_{ip}^{2}}};\ i = 1, 2, \ldots, m;\ p = 1, 2, \ldots, n. \quad (30)$$

- Compute the weighted matrix by multiplying Eq. (27) by the normalized matrix as in Eq. (31).

$$Z_{ip} = \omega_p \times NM_{ip} \quad (31)$$

Phase 4. Rank the alternatives

- We can describe the neutrosophic positive ideal solution (NPIS, A^*) and Neutrosophic negative ideal solution (NNIS, A^-)

$$A^* = \{< \max(\delta_{ip} | i = 1, 2, \ldots, m) | p\epsilon p^+ >, < \min(\delta_{ip} | i = 1, 2, \ldots, m) | p\epsilon p^- >\} \quad (32)$$

$$A^- = \{< \min(\delta_{ip} | i = 1, 2, \ldots, m) | p\epsilon p^+ >, < \max(\delta_{ip} | i = 1, 2, \ldots, m) | p\epsilon p^- >\} \quad (33)$$

Where p^+ related with the criteria that have a profitable effect and p^- related with the criteria that have a non-beneficial effect.

- The dimension of each alternative from A^* and A^- can be currently computed as:

$$d_i^* = \sqrt{\sum_{p=1}^{n} (\tilde{A}_{ip} - A_p^*)^2},\ i = 1, 2, \ldots, m, \quad (34)$$

$$d_i^- = \sqrt{\sum_{p=1}^{n} (\tilde{A}_{ip} - A_p^-)^2},\ i = 1, 2, \ldots, m, \quad (35)$$

- A proximity factor is defined to locate the classification system of all available alternatives once the d_i^* and d_i^- of each alternative $A_i = (1, 2, \ldots, m)$ have been computed. The proximity coefficient of every available alternative is computed as:

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \qquad i = 1, 2, \ldots, m \quad (36)$$

Clearly, an alternative A_i is closer to the (NPIS, A^*) and further from (NNIS, A^-). Thus, according to the closeness coefficient, we can decide the classification order of all alternatives and select the superior one from a set of available alternatives.

4. Real case study

We introduce a numerical case which implicates methods and data analysis to test the competence and the efficiency of suggested framework for selection of the best supplier to import cars, performed on an importing company in Egypt, Ghabbour Company, founded in 1960 and based in Cairo. Egypt the Corporation seeks to increase the numbers of suppliers. For this purpose, the executive managers suggested some alternatives such as Alt_1 India, Alt_2 Japan, Alt_3 China, Alt_4 USA and Alt_5 Germany. Consequently, the organization must evaluate suppliers and their sustainability. For this study, the corporation determined the most important criteria as being $Œi_1$ competency, $Œi_2$ capacity, $Œi_3$ commitment, $Œi_4$ control, $Œi_5$ cash, $Œi_6$ cost, $Œi_7$ consistency and $Œi_8$ communication for comparing alternatives and select the best alternative. These criteria are considered by three experts. The experts are: strategic expert, marketing expert and manufacturing expert, all with more than ten years of experience in this field. The hierarchical construction of this decision problem is presented in Fig. 3. The suggested technique is employed to solve this issue and the computational steps are as follows:

Phase 1. Organize a group of Exs and determine goals, alternatives and criteria.

- A group consisting of three Exs, symbolized by $Ex_E = (Ex_1, Ex_2, Ex_3)$, is constructed to select the best supplier which the Ghabbour Company can deal with it for importing motors. Alternatives are introduced as $A_i = (Alt_1, Alt_2, Alt_3, Alt_4, Alt_5)$. These alternatives are estimated based on eight criteria $Œi_p = (Œi_1, Œi_2, Œi_3, Œi_4, Œi_5, Œi_6, Œi_7, Œi_8)$, which are collected from a comprehensive literature and EXs' opinions.

Phase 2. Depict and design the linguistic scales.

- Obtain Exs' judgments on each element. Based on the previously knowledge and experience on the topics, Exs are demanded to convey their judgments. Every Ex gives his judgment linguistically on every of these elements. Then,

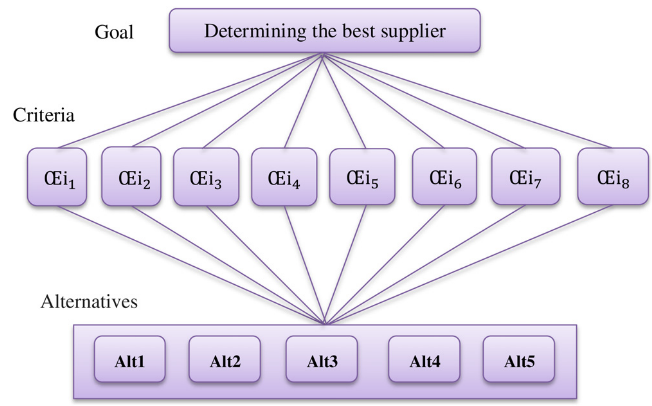

Fig. 3. The hierarchy of the problem.

Table 8
The weight of criteria by experts.

EXs	$Œi_1$	$Œi_2$	$Œi_3$	$Œi_4$	$Œi_5$	$Œi_6$	$Œi_7$	$Œi_8$
Ex_1	⟨SI⟩	⟨WI⟩	⟨SI⟩	⟨SI⟩	⟨AI⟩	⟨EI⟩	⟨EI⟩	⟨EI⟩
Ex_2	⟨VSI⟩	⟨EI⟩	⟨VSI⟩	⟨AI⟩	⟨SI⟩	⟨SI⟩	⟨WI⟩	⟨SI⟩
Ex_3	⟨AI⟩	⟨EI⟩	⟨VSI⟩	⟨AI⟩	⟨SI⟩	⟨EI⟩	⟨EI⟩	⟨VSI⟩

transform EXs' linguistic evaluations into type 2 neutrosophic numbers as in Tables 6 and 7.

- Build the preference relation matrix to locate the weights of criteria using Eq. (26) as presented in Table 8. EXs employ the semantic terms displayed in Table 6 to assess the opinions of EXs with consideration to every criterion.
- Calculate the weights of Exs; Exs' judgments are collected by using Eq. (28). Then, calculate the score value after aggregating the opinions of Exs for each criteria using Eq. (5). Then, normalize the obtaining weights as presented in Table 9.

Phase 3. Create the valuation matrix.

- Form the valuation matrix $A_i \times Œi_j$ using Eq. (29) to assess the ratings of alternatives with esteem to every criterion, as in Table 10. Exs use the linguistic terms presented in Table 7.
- Aggregate the final evaluation matrix using Eq. (1) as in Table 11.

• Use the de-neutrosophication Eq. (5) for transforming type 2 neutrosophic numbers to the crisp values, as shown in Table 12.

• Then, construct the normalized decision matrix by Eq. (30), as presented in Table 13.

• Compute the weighted matrix by multiplying Eq. (27) by the normalized matrix as in Eq. (31), as shown in Table 14.

Phase 4. Rank the alternatives

- We can define the neutrosophic positive ideal solution (NPIS, A^*) and the Neutrosophic negative ideal solution (NNIS, A^-) by Eqs. (32) and (33).

Table 10
Classification of alternatives and criteria by EXs.

Exs	Alt_n	$Œi_1$	$Œi_2$	$Œi_3$	$Œi_4$	$Œi_5$	$Œi_6$	$Œi_7$	$Œi_8$
	Alt_1	⟨MG⟩	⟨G⟩	⟨VG⟩	⟨MG⟩	⟨B⟩	⟨VG⟩	⟨VB⟩	⟨VG⟩
	Alt_2	⟨VB⟩	⟨VG⟩	⟨G⟩	⟨B⟩	⟨MG⟩	⟨G⟩	⟨G⟩	⟨G⟩
Ex_1	Alt_3	⟨MG⟩	⟨MG⟩	⟨MG⟩	⟨M⟩	⟨B⟩	⟨MG⟩	⟨G⟩	⟨MG⟩
	Alt_4	⟨G⟩	⟨MB⟩	⟨VG⟩	⟨MG⟩	⟨VG⟩	⟨VG⟩	⟨MG⟩	⟨VG⟩
	Alt_5	⟨VB⟩	⟨B⟩	⟨B⟩	⟨VG⟩	⟨VB⟩	⟨MG⟩	⟨MG⟩	⟨M⟩
	Alt_1	⟨G⟩	⟨MB⟩	⟨MB⟩	⟨VG⟩	⟨VG⟩	⟨VG⟩	⟨MG⟩	⟨G⟩
	Alt_2	⟨MB⟩	⟨VB⟩	⟨G⟩	⟨M⟩	⟨M⟩	⟨G⟩	⟨VB⟩	⟨MG⟩
Ex_2	Alt_3	⟨VG⟩	⟨MG⟩	⟨VG⟩	⟨MG⟩	⟨VB⟩	⟨MG⟩	⟨G⟩	⟨VG⟩
	Alt_4	⟨VG⟩	⟨VB⟩	⟨VG⟩	⟨VG⟩	⟨MB⟩	⟨VB⟩	⟨MB⟩	⟨VG⟩
	Alt_5	⟨MB⟩	⟨B⟩	⟨VG⟩	⟨VG⟩	⟨VB⟩	⟨VG⟩	⟨MB⟩	⟨M⟩
	Alt_1	⟨VG⟩	⟨B⟩	⟨VG⟩	⟨VG⟩	⟨G⟩	⟨VG⟩	⟨VG⟩	⟨VG⟩
	Alt_2	⟨M⟩	⟨VG⟩	⟨MB⟩	⟨MB⟩	⟨MG⟩	⟨M⟩	⟨M⟩	⟨M⟩
Ex_3	Alt_3	⟨G⟩	⟨B⟩	⟨MG⟩	⟨MG⟩	⟨VB⟩	⟨B⟩	⟨MG⟩	⟨G⟩
	Alt_4	⟨B⟩	⟨MB⟩	⟨VG⟩	⟨MB⟩	⟨MG⟩	⟨M⟩	⟨G⟩	⟨VG⟩
	Alt_5	⟨MB⟩	⟨VG⟩	⟨M⟩	⟨MB⟩	⟨MG⟩	⟨VG⟩	⟨MB⟩	⟨MB⟩

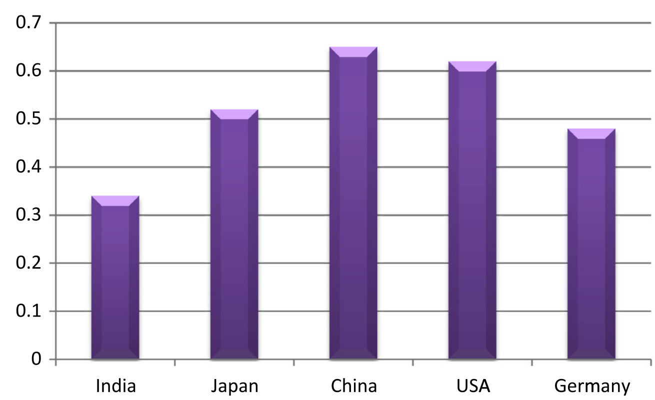

Fig. 4. Ranking the alternatives according to the best supplier.

- The distance of each alternative from A^* and A^- can be currently calculated by Eqs. (34) and (35) as: $d^* = (0.021, 0.016, 0.012, 0.012, \text{and } 0.018)$, $d^- = \{0.011, 0.017, 0.019, 0.022, \text{and } 0.017)$.
- The proximity coefficient of each available alternative is computed by Eq. (36) as in Table 15.

• The ordering for the optimal alternatives of selecting the best supplier is: Alt_3, Alt_4, Alt_2, Alt_5, and Alt_1, as presented in Fig. 4.

5. Concluding remarks

MADM issues generally occur in difficult environments related to uncertainty and imprecise data. The type 2 neutrosophic number is an efficient tool to deal with expert's impreciseness or incompleteness, and the decision maker's appreciations and assessments over alternative with esteem to attribute. In the first part of the article, we present the proposed method, introducing the type 2 neutrosophic number and defining its operations, properties and

Table 9
The final results of normalized criteria weights.

Weight $\tilde{\omega}_n$	Aggregation weight by T2NN	Crisp	Normalized weight
$Œi_1$	$\langle(0.78, 0.72, 0.73), (0.23, 0.25, 0.32), (0.18, 0.18, 0.22)\rangle$	0.7617	0.16
$Œi_2$	$\langle(0.33, 0.30, 0.23), (0.50, 0.60, 0.53), (0.45, 0.65, 0.62)\rangle$	0.3800	0.08
$Œi_3$	$\langle(0.75, 0.68, 0.65), (0.27, 0.25, 0.38), (0.22, 0.20, 0.25)\rangle$	0.7283	0.15
$Œi_4$	$\langle(0.82, 0.75, 0.82), (0.20, 0.25, 0.25), (0.15, 0.17, 0.18)\rangle$	0.7933	0.17
$Œi_5$	$\langle(0.73, 0.65, 0.68), (0.30, 0.35, 0.40), (0.25, 0.28, 0.27)\rangle$	0.6858	0.14
$Œi_6$	$\langle(0.48, 0.38, 0.35), (0.43, 0.52, 0.45), (0.42, 0.53, 0.48)\rangle$	0.4758	0.10
$Œi_7$	$\langle(0.33, 0.30, 0.23), (0.50, 0.60, 0.53), (0.45, 0.65, 0.62)\rangle$	0.3800	0.08
$Œi_8$	$\langle(0.62, 0.53, 0.50), (0.35, 0.38, 0.42), (0.32, 0.37, 0.37)\rangle$	0.6017	0.12

Table 11
The consolidated decision matrix.

EXs	Œi$_1$	Œi$_2$	Œi$_3$	Œi$_4$
Alt$_1$	⟨(0.617, 0.599, 0.623), (0.013, 0.001, 0.014), (0.003, 0.005, 0.005)⟩	⟨(0.476, 0.440, 0.453), (0.013, 0.018, 0.030), (0.008, 0.011, 0.026)⟩	⟨(0.650, 0.619, 0.650), (0.003, 0.004, 0.004), (0.001, 0.008, 0.005)⟩	⟨(0.653, 0.629, 0.650), (0.006, 0.005, 0.006), (0.008, 0.006, 0.001)⟩
Alt$_2$	⟨(0.353, 0.308, 0.358), (0.043, 0.042, 0.064), (0.024, 0.032, 0.063)⟩	⟨(0.640, 0.613, 0.637), (0.002, 0.003, 0.007), (0.004, 0.007, 0.006)⟩	⟨(0.552, 0.544, 0.593), (0.004, 0.005, 0.009), (0.002, 0.002, 0.008)⟩	⟨(0.393, 0.351, 0.358), (0.033, 0.039, 0.060), (0.026, 0.030, 0.059)⟩
Alt$_3$	⟨(0.617, 0.599, 0.623), (0.013, 0.001, 0.014), (0.003, 0.005, 0.005)⟩	⟨(0.475, 0.393, 0.358), (0.006, 0.006, 0.017), (0.002, 0.016, 0.005)⟩	⟨(0.603, 0.539, 0.571), (0.001, 0.008, 0.001), (0.001, 0.001, 0.004)⟩	⟨(0.485, 0.420, 0.458), (0.005, 0.003, 0.011), (0.001, 0.008, 0.003)⟩
Alt$_4$	⟨(0.589, 0.588, 0.591), (0.006, 0.005, 0.003), (0.008, 0.005, 0.002)⟩	⟨(0.383, 0.261, 0.358), (0.054, 0.003, 0.057), (0.030, 0.024, 0.084)⟩	⟨(0.664, 0.657, 0.664), (0.003, 0.003, 0.004), (0.004, 0.004, 0.004)⟩	⟨(0.588, 0.510, 0.571), (0.003, 0.001, 0.002), (0.008, 0.001, 0.002)⟩
Alt$_5$	⟨(0.383, 0.261, 0.358), (0.054, 0.003, 0.057), (0.030, 0.024, 0.084)⟩	⟨(0.511, 0.497, 0.380), (0.008, 0.019, 0.011), (0.004, 0.009, 0.007)⟩	⟨(0.522, 0.519, 0.501), (0.007, 0.011, 0.007), (0.003, 0.005, 0.005)⟩	⟨(0.650, 0.619, 0.650), (0.002, 0.001, 0.004), (0.004, 0.003, 0.005)⟩
EXs	Œi$_5$	Œi$_6$	Œi$_7$	Œi$_8$
Alt$_1$	⟨(0.589, 0.588, 0.591), (0.006, 0.005, 0.003), (0.008, 0.005, 0.002)⟩	⟨(0.664, 0.657, 0.664), (0.003, 0.003, 0.004), (0.004, 0.004, 0.004)⟩	⟨(0.545, 0.490, 0.501), (0.004, 0.004, 0.004), (0.003, 0.001, 0.002)⟩	⟨(0.656, 0.648, 0.659), (0.005, 0.007, 0.003), (0.008, 0.001, 0.002)⟩
Alt$_2$	⟨(0.485, 0.420, 0.458), (0.005, 0.003, 0.011), (0.001, 0.008, 0.003)⟩	⟨(0.535, 0.566, 0.593), (0.003, 0.006, 0.021), (0.001, 0.003, 0.006)⟩	⟨(0.415, 0.444, 0.453), (0.013, 0.024, 0.035), (0.005, 0.016, 0.021)⟩	⟨(0.511, 0.499, 0.533), (0.004, 0.005, 0.010), (0.001, 0.005, 0.005)⟩
Alt$_3$	⟨(0.245, 0.245, 0.099), (0.070, 0.160, 0.193), (0.034, 0.160, 0.106)⟩	⟨(0.408, 0.365, 0.232), (0.017, 0.028, 0.053), (0.008, 0.047, 0.021)⟩	⟨(0.535, 0.566, 0.593), (0.003, 0.006, 0.021), (0.001, 0.003, 0.006)⟩	⟨(0.617, 0.599, 0.623), (0.013, 0.001, 0.014), (0.003, 0.005, 0.005)⟩
Alt$_4$	⟨(0.588, 0.510, 0.571), (0.003, 0.001, 0.002), (0.008, 0.001, 0.002)⟩	⟨(0.491, 0.490, 0.501), (0.008, 0.012, 0.007), (0.003, 0.005, 0.005)⟩	⟨(0.530, 0.466, 0.533), (0.005, 0.004, 0.009), (0.002, 0.004, 0.006)⟩	⟨(0.664, 0.657, 0.664), (0.003, 0.003, 0.004), (0.004, 0.004, 0.004)⟩
Alt$_5$	⟨(0.325, 0.277, 0.232), (0.028, 0.032, 0.060), (0.007, 0.053, 0.025)⟩	⟨(0.653, 0.629, 0.650), (0.006, 0.005, 0.006), (0.008, 0.006, 0.001)⟩	⟨(0.483, 0.337, 0.458), (0.017, 0.006, 0.017), (0.007, 0.008, 0.018)⟩	⟨(0.407, 0.380, 0.458), (0.027, 0.024, 0.038), (0.018, 0.016, 0.041)⟩

Table 12
The final aggregated matrix.

Œi$_n$/Alt$_n$	Œi$_1$	Œi$_2$	Œi$_3$	Œi$_4$	Œi$_5$	Œi$_6$	Œi$_7$	Œi$_8$
Alt$_1$	0.8659	0.8061	0.8751	0.8765	0.8598	0.8844	0.8336	0.8814
Alt$_2$	0.7488	0.8720	0.8497	0.7614	0.8118	0.8509	0.8002	0.8385
Alt$_3$	0.8659	0.7954	0.8523	0.8118	0.6493	0.7600	0.8509	0.8759
Alt$_4$	0.8597	0.7487	0.8844	0.8467	0.8467	0.8263	0.8298	0.8844
Alt$_5$	0.7487	0.8166	0.8339	0.8763	0.7351	0.8765	0.7940	0.7851

Table 13
The normalized decision matrix.

Œi$_n$/Alt$_n$	Œi$_1$	Œi$_2$	Œi$_3$	Œi$_4$	Œi$_5$	Œi$_6$	Œi$_7$	Œi$_8$
Alt$_1$	0.34	0.31	0.33	0.34	0.33	0.34	0.32	0.35
Alt$_2$	0.32	0.38	0.37	0.32	0.35	0.36	0.34	0.36
Alt$_3$	0.38	0.35	0.37	0.36	0.28	0.33	0.37	0.39
Alt$_4$	0.36	0.31	0.37	0.35	0.35	0.35	0.35	0.37
Alt$_5$	0.32	0.35	0.36	0.38	0.32	0.38	0.34	0.33

functioning rules. Then, we suggest an aggregation operator, called T2NNWA operator, the score function and the accuracy function, and apply them to solve a MADM problem under neutrosophic environment using type 2 neutrosophic numbers. We discuss two properties of the T2NNWA operator. Finally, the competence, the performance and the applicability of the suggested technique is illustrated with the best bank selection problem to do some banking transactions. In the second part, we present a powerful application of the proposed method under GDM in neutrosophic environment and employ the TOPSIS method in the neutrosophic environment by the type 2 neutrosophic numbers. We apply the proposed method in a problem of selection of the best supplier for importing cars. The method can be easily used to compute and rank the alternatives under group decision making process. The suggested technique can be as well employed in other decision making issues, such as pattern recognition, medical diagnosis, personnel selection, information project selection, material selection and other management decision problems.

Table 14
The weighted matrix.

Œi$_n$/Alt$_n$	Œi$_1$	Œi$_2$	Œi$_3$	Œi$_4$	Œi$_5$	Œi$_6$	Œi$_7$	Œi$_8$
Alt$_1$	0.0544	0.0248	0.0495	0.0578	0.0462	0.0340	0.0256	0.0420
Alt$_2$	0.0512	0.0304	0.0555	0.0544	0.0490	0.0360	0.0272	0.0432
Alt$_3$	0.0608	0.0280	0.0555	0.0612	0.0392	0.0330	0.0296	0.0468
Alt$_4$	0.0576	0.0248	0.0555	0.0595	0.0490	0.0350	0.0280	0.0444
Alt$_5$	0.0512	0.0280	0.0540	0.0664	0.0448	0.0380	0.0272	0.0396

Table 15
The final result of ranking.

$Œi_n/Alt_n$	D_i^+	D_i^-	$Œi_i$	Arranging
Alt_1	0.021	0.011	0.34	5
Alt_2	0.016	0.017	0.52	3
Alt_3	0.012	0.022	0.65	1
Alt_4	0.012	0.019	0.62	2
Alt_5	0.018	0.017	0.48	4

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