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An Extended Single-Valued Neutrosophic Projection-Based Qualitative Flexible Multi-Criteria Decision-Making Method

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Abstract: With respect to multi-criteria decision-making (MCDM) problems in which the criteria denote the form of single-valued neutrosophic sets (SVNSs), and the weight information is also fully unknown, a novel MCDM method based on qualitative flexible multiple criteria (QUALIFLEX) is developed. Firstly, the improved cosine measure of the included angle between two SVNSs is defined. Then, the improved single-valued neutrosophic projection is developed, and the corresponding improved single-valued neutrosophic bidirectional projection and single-valued neutrosophic bidirectional projection difference are investigated. Moreover, the partial ordering relation of SVNSs is developed. Secondly, an extended QUALIFLEX method based on an improved single-valued neutrosophic projection measure is proposed to handle MCDM problems in which the weights of criteria are completely unknown. Finally, an example for selection of a green supplier, as well as a performance comparison analysis, are provided to demonstrate the effectiveness of the proposed method.

1. Introduction

It is well-known that successful decision-making often requires consideration of several factors or criteria. This kind of decision-making problem, simultaneously involving different criteria, is called a multi-criteria decision making (MCDM) problem. In many cases, it is in fact difficult for decision-makers to definitely express their preference in solving MCDM problems with inaccurate, uncertain, or incomplete information. Under these circumstances, Zadeh's fuzzy sets (FSs) [1], where the membership degree is represented by a real number between zero and one, are regarded as an important tool to solve the problems associated with MCDM [2,3], fuzzy logic and optimization [4], approximate reasoning [5], and pattern recognition [6].

However, FSs cannot handle some cases in which the membership degree is difficult to define by only a particular value. In order to overcome the lack of knowledge on non-membership degrees, Atanassov [7] presented intuitionistic fuzzy sets (IFSs), as the extension of Zadeh's FSs. IFSs simultaneously take the membership degree, the non-membership degree, and the degree of hesitation into account. Therefore, they are more appropriate and practical in addressing fuzziness and uncertainty than the FSs.

Although the FS theory has been developed and generalized, it still cannot deal with all sorts of uncertainties in different real problems. Some types of uncertainties, such as indeterminate

information and inconsistent information, cannot be properly handled. To better elucidate the scope of FSs and IFSs, Smarandache et al. [8–10] proposed neutrosophic logic and neutrosophic sets (NSs). Rivieccio [11] pointed out that an NS is a set in which each element of the universe has a degree of truth, indeterminacy, and falsity, respectively, and it lies in $[0^-, 1^+]$, the non-standard unit interval. Obviously, it is the extension of the standard interval $[0, 1]$ of IFSs. In addition, the uncertainty presented here, that is, the indeterminacy factor, is dependent on truth and falsity values, while the incorporated uncertainty is dependent on the degree of belongingness, as well as the degree of non-belongingness of IFSs [12]. It is noteworthy that because of the uncertain, incomplete, and inaccurate information, NSs are difficult to apply to real-life situations. Hence, a single-valued neutrosophic set (SVNS) was proposed, which is an instance of NS [8,12]. Wang et al. [13] and Lupiáñez [14] proposed the concept of interval neutrosophic sets (INSs), and presented the set-theoretic operators of INSs. Hu et al. [15] developed an interval neutrosophic projection-based the VlseKriterijumskaOptimizacija I KompromisnoResenje (VIKOR) method; Wang et al. [16] studied the fuzzy stochastic MCDM method associated with interval neutrosophic probability. Wang et al. [17] investigated a cloud service reliability method based on multi-valued neutrosophic measures. Besides, other extensions of NSs, including neutrosophic cubic sets [18,19], single-valued neutrosophic hesitant fuzzy sets (SVNHFSs) [20,21], rough neutrosophic sets (RNSs) [22,23], interval rough neutrosophic sets (IRNSs) [24], single-valued neutrosophic graphic [25,26], bipolar single-valued neutrosophic graphic [27], and other extensions [28], were proposed and applied to solve various problems.

In particular, SVNS, as an important tool to express ambiguous information, has always received great attention from researchers and has been studied from different aspects, including outranking relations, aggregation operators, and information measures. For a single-valued neutrosophic MCDM method based on outranking relations, Peng et al. [29] proposed a single-valued neutrosophic outranking method by combing elimination and choice expressing reality (ELECTRE) III. For a single-valued neutrosophic MCDM method based on aggregation operators, Wu et al. [30] defined a simplified neutrosophic prioritized aggregation operator, Ye [31] also developed a simplified neutrosophic aggregation operator, Garg [32] presented a single-valued neutrosophic operator based on Frank's norm, and Liu and Wang [33] proposed a single-valued neutrosophic normalized weighted Bonferroni mean (SVNNWBM) operator. For a single-valued neutrosophic MCDM method based on information measures, Ye [34,35] studied similarity of SVNSs, and applied them to cluster analysis, Ye [36] defined a single-valued neutrosophic cross-entropy, and Ye [37] investigated projection and bidirectional projection measures of SVNSs. To our knowledge, entropy, cross entropy, similarity, and distance and projection of SVNSs are significant measures to deal with ambiguous information. In particular, projection based on the included angle between objects plays a key role in solving MCDM problems.

Moreover, the qualitative flexible multiple criteria (QUALIFLEX) method was developed by Paelinck [38–40], who utilized outranking relations or priority functions for ranking the alternatives in terms of priority among the criteria, which was based on the pair-wise comparisons of alternatives with respect to each criterion under all possible permutations of the alternatives, and identified the optimal permutation, maximizing the value of concordance/discordance index [41]. Recently, several extensions have been developed to enhance the QUALIFLEX method [42,43]. For instance, Ji et al. [44] defined a triangular neutrosophic QUALIFLEX and an acronym in Portuguese of interactive and multicriteria decision making (TODIM) method for treatment selection, and Li and Wang [45] developed a probability hesitant fuzzy QUALIFLEX approach to select green suppliers.

After reviewing the existing references presented above, we can conclude that there are some limitations about existing single-valued neutrosophic MCDM methods. Firstly, those aforementioned methods based on aggregation operators always involve operations, impacting the final results to some extent. In other words, different aggregation operators are always involved in different operations that can lead to different rankings. Secondly, the outranking method based on elimination and choice expressing reality (ELECTRE) is only appropriate for MCDM problems in which the

number of alternatives is much bigger than the number of criteria. Thirdly, a single-valued neutrophic projection defined by Ye is unreasonable, as previously discussed in Ye [37] in some cases. Furthermore, the principal advantage of the QUALIFLEX method is that it can effectively handle the decision-making problems in which the number of criteria is clearly greater than the number of alternatives, and which are not extended to a single-valued neutrosophic environment. Therefore, the main objectives of this research are concluded as follows: (1) provide the improved projection measure of SVNSs based on the improved included angle of SVNSs, in addition to defining the corresponding single-valued neutrosophic bidirectional projection and the corresponding bidirectional projection difference; (2) extend the QUALIFLEX method to a single-valued neutrosophic environment based on the proposed projection. Thus, for a single-valued neutrosophic MCDM problem, where the weight information is fully unknown, the methodology can be concluded in the following steps. Firstly, the weight of criteria can be calculated by utilizing the maximizing deviation method. As a result, the possible permutations can be determined. Secondly, the concordance index of each permutation is calculated based on the proposed projection. Eventually, according to the value of corresponding concordance index, the maximum of all permutations is selected. Consequently, the corresponding permutation is the optimal ranking of alternatives. Moreover, the proposed method is more appropriate for handling single-valued neutrosophic information, where the weight of criteria is completely unknown, and the number of alternatives is smaller than the number of criteria.

The present study is summarized as follows. In Section 2, some definitions and operations of SVNSs are introduced. The projection and bidirectional projection of SVNSs are reviewed as well. Then, an improved single-valued neutrosophic projection, a single-valued neutrosophic bidirectional projection, and the corresponding bidirectional projection difference are developed in Section 3. Next, the single-valued projection-based QUALIFLEX method accompanied with unknown weight information is developed in Section 4. In Section 5, the selection of a green supplier is presented to assess the efficacy of the proposed approach. Finally, we summarize the paper with a further discussion in Section 6.

2. Preliminaries

In this section, some definitions, operations, and projection measures of SVNSs are reviewed, which may be utilized in the latter analysis.

Definition 1 [8]. *An SVNS M on the universe X can be expressed as $M = \{\langle x, T(x), I(x), F(x)\rangle | x \in X\}$, where, $T(x)$, $I(x)$, and $F(x)$ are numerical numbers in [0,1], that is, $T(x) : X \to [0, 1]$, $I(x) : X \to [0,1]$, and $F(x) : X \to [0,1]$, denoting the truth-membership degree, indeterminacy-membership degree, and falsity-membership degree, respectively.*

If there exists only one element in X, then M is called a single-valued neutrosophic number (SVNN), denoted by $M = (T, I, F)$ for convenience.

Definition 2 [46]. *Let $M = (M(x_1), M(x_2), \ldots, M(x_n))$, $M_1 = (M_1(x_1), M_1(x_2), \ldots, M_1(x_n))$, and $M_2 = (M_2(x_1), M_2(x_2), \ldots, M_2(x_n))$ be three SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$, then the following operations of SVNSs can be achieved:*

(1) $M_1 \subseteq M_2$, if and only if $T_1(x) \leq T_2(x)$, $I_1(x) \geq I_2(x)$, and $F_1(x) \geq F_2(x)$ for any $x \in X$;
(2) If $M_1 \subseteq M_2$ and $M_1 \supseteq M_2$, then $M_1 = M_2$;
(3) $M^C = \{\langle x, F(x), 1 - I(x), T(x)\rangle | x \in X\}$.

Definition 3 [37]. *Let $M_1 = (M_1(x_1), M_1(x_2), \ldots, M_1(x_n))$ and $M_2 = (M_2(x_1), M_2(x_2), \ldots, M_2(x_n))$ be two SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$. Then, the cosine measure of the included angle between two SVNSs can be defined as follows:*

$$\cos(M_1, M_2) = \frac{M_1 \cdot M_2}{\|M_1\|\|M_2\|} \tag{1}$$

Here, $M_1 \cdot M_2 = \sum\limits_{i=1}^{n} (T_{1i}(x_i)T_{2i}(x_i) + I_{1i}(x_i)I_{2i}(x_i) + F_{1i}(x_i)F_{2i}(x_i))$ denotes the inner product between M_1 and M_2. $\|M_1\| = \sqrt{\sum\limits_{i=1}^{n} \left(T_{1i}(x_i)^2 + I_{1i}(x_i)^2 + F_{1i}(x_i)^2\right)}$ and $\|M_2\| = \sqrt{\sum\limits_{i=1}^{n} \left(T_{2i}(x_i)^2 + I_{2i}(x_i)^2 + F_{2i}(x_i)^2\right)}$ represent the modules of M_1 and M_2, respectively.

Definition 4 [37]. *Let $M_1 = (M_1(x_1), M_1(x_2), \ldots, M_1(x_n))$ and $M_2 = (M_2(x_1), M_2(x_2), \ldots, M_2(x_n))$ to be two SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$. The projection of M_1 on M_2 can be denoted as follows:*

$$\mathrm{Pro}_{M_2}(M_1) = \|M_2\|\cos(M_1, M_2) = \frac{\sum\limits_{i=1}^{n} (T_{1i}(x_i)T_{2i}(x_i) + I_{1i}(x_i)I_{2i}(x_i) + F_{1i}(x_i)F_{2i}(x_i))}{\sqrt{\sum\limits_{i=1}^{n} \left(T_{2i}(x_i)^2 + I_{2i}(x_i)^2 + F_{2i}(x_i)^2\right)}} \tag{2}$$

The projection $\mathrm{Pro}_{M_2}(M_1)$ defined in Definition 4 reflects the degree to which an object is similar to another. Generally speaking, the greater the value of $\mathrm{Pro}_{M_2}(M_1)$, the more M_1 is similar to M_2. However, as discussed in Ye [37], the projection measure is unreasonable in some cases.

Example 1. *Let $M_1 = \{x, \langle 1, 1, 1\rangle\}$ and $M_2 = \{x, \langle 1, 0, 0\rangle\}$ be two SVNSs. From Equation (2), we can achieve $\mathrm{Pro}_{M_2}(M_1) = \frac{1\times1+1\times0+1\times0}{\sqrt{1^2}} = 1$ and $\mathrm{Pro}_{M_2}(M_2) = \frac{1\times1+0\times0+0\times0}{\sqrt{1^2}} = 1$. Apparently, $\mathrm{Pro}_{M_2}(M_1) = \mathrm{Pro}_{M_2}(M_2)$, that is, M_1 is close to M_2 as M_2 to M_2. However, it can be seen that M_1 and M_2 are different. Thus, cosine measure of the included angle between two SVNSs and the projection measure defined in Definitions 3 and 4, respectively, are unreasonable.*

Definition 5 [37]. *Let $M_1 = (M_1(x_1), M_1(x_2), \ldots, M_1(x_n))$ and $M_2 = (M_2(x_1), M_2(x_2), \ldots, M_2(x_n))$ to be two SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$. Therefore, the single-valued neutrosophic bidirectional projection measure of M_1 on M_2 is developed as follows:*

$$BPro_{SVNS}(M_1, M_2) = \frac{\|M_1\|\|M_2\|}{\|M_1\|\|M_2\| + |\|M_1\| - \|M_2\|| M_1 \cdot M_2} \tag{3}$$

where $M_1 \cdot M_2 = \sum\limits_{i=1}^{n} (T_1(x_i)T_2(x_i) + I_1(x_i)I_2(x_i) + F_1(x_i)F_2(x_i))$ denotes the inner product between M_1 and M_2. $\|M_1\| = \sqrt{\sum\limits_{i=1}^{n} \left(T_1(x_i)^2 + I_1(x_i)^2 + F_1(x_i)^2\right)}$ and $\|M_2\| = \sqrt{\sum\limits_{i=1}^{n} \left(T_2(x_i)^2 + I_2(x_i)^2 + F_2(x_i)^2\right)}$ represent the modules of M_1 and M_2, respectively.

Example 2. *Based on Example 1 and Definition 5, we have $BPro_{SVNS}(M_1, M_2) = \frac{\|M_1\|\|M_2\|}{\|M_1\|\|M_2\| + |\|M_1\| - \|M_2\||M_1\cdot M_2} = \frac{\sqrt{3}}{\sqrt{3}+|\sqrt{3}-1|} = \frac{1}{2-\frac{1}{\sqrt{3}}} < 1$ and $BPro_{SVNS}(M_2, M_2) = \frac{\|M_2\|\|M_2\|}{\|M_2\|\|M_2\| + |\|M_2\| - \|M_2\||M_2\cdot M_2} = \frac{1}{1+0} = 1$. Also, $BPro_{SVNS}(M_2, M_2) \geq BPro_{SVNS}(M_1, M_2)$. Thus, M_2 is closer to M_2 than M_1 to M_2, which is consistent with our intuition.*

Apparently, the single-valued neutrosophic bidirectional projection measure defined by Ye [37] is more reasonable than the projection measure.

3. Improved Projection Measures of SVNSs

In this section, the improved single-valued neutrosophic projection, single-valued bidirectional projection, and single-valued bidirectional projection difference measures are defined, and the related properties are discussed.

3.1. The Improved Single-Valued Neutrosophic Projection Measure

On the basis of the existing projection measure of SVNSs presented in Ye [37], the improved single-valued neutrosophic cosine measure and the corresponding projection measure are defined in the following.

Definition 6. *Let $\overline{M}_1 = (\overline{M}_1(x_1), \overline{M}_1(x_2), \ldots, \overline{M}_1(x_n))$ and $\overline{M}_2 = (\overline{M}_2(x_1), \overline{M}_2(x_2), \ldots, \overline{M}_2(x_n))$ be two SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$. Thus, the improved cosine measure of the included angle between $\overline{M}_1$ and $\overline{M}_2$ is proposed as follows:*

$$
\begin{aligned}
&\mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_2) \\
&= \frac{\sum\limits_{i=1}^{n}\left(\overline{T}_1(x_i)\overline{T}_2(x_i) + \left(1 - \overline{I}_1(x_i)\right)\left(1 - \overline{I}_2(x_i)\right) + \left(1 - \overline{F}_1(x_i)\right)\left(1 - \overline{F}_2(x_i)\right)\right)}{\sqrt{\sum\limits_{i=1}^{n}\left(\overline{T}_1(x_i)^2 + \left(1 - \overline{I}_1(x_i)\right)^2 + \left(1 - \overline{F}_1(x_i)\right)^2\right)}\sqrt{\sum\limits_{i=1}^{n}\left(\overline{T}_2(x_i)^2 + \left(1 - \overline{I}_2(x_i)\right)^2 + \left(1 - \overline{F}_2(x_i)\right)^2\right)}}
\end{aligned} \tag{4}
$$

Here, $\overline{M}_1 \cdot \overline{M}_2 = \sum\limits_{i=1}^{n}\left(\overline{T}_1(x_i)\overline{T}_2(x_i) + \left(1 - \overline{I}_1(x_i)\right)\left(1 - \overline{I}_2(x_i)\right) + \left(1 - \overline{F}_1(x_i)\right)\left(1 - \overline{F}_2(x_i)\right)\right)$ denotes the

inner product between $\overline{M}_1$ and $\overline{M}_2$. Besides, $\|\overline{M}_1\| = \sqrt{\sum\limits_{i=1}^{n}\left(\overline{T}_1(x_i)^2 + \left(1 - \overline{I}_1(x_i)\right)^2 + \left(1 - \overline{F}_1(x_i)\right)^2\right)}$

and $\|\overline{M}_2\| = \sqrt{\sum\limits_{i=1}^{n}\left(\overline{T}_2(x_i)^2 + \left(1 - \overline{I}_2(x_i)\right)^2 + \left(1 - \overline{F}_2(x_i)\right)^2\right)}$ represent the modules of $\overline{M}_1$ and $\overline{M}_2$, respectively.

If $n = 1$, then $\overline{M}_1$ and $\overline{M}_2$ become two SVNNs. In this case, Equation (4) is reduced to the cosine measure of the included angle between two SVNNs, that is,

$$
\mathrm{Cos}_{SVNN}(\overline{M}_1, \overline{M}_2) = \frac{\overline{T}_1\overline{T}_2 + \left(1 - \overline{I}_1\right)\left(1 - \overline{I}_2\right) + \left(1 - \overline{F}_1\right)\left(1 - \overline{F}_2\right)}{\sqrt{\left(\overline{T}_1^2 + \left(1 - \overline{I}_1\right)^2 + \left(1 - \overline{F}_1\right)^2\right)}\sqrt{\left(\overline{T}_2^2 + \left(1 - \overline{I}_2\right)^2 + \left(1 - \overline{F}_2\right)^2\right)}} \tag{5}
$$

Theorem 1. *The improved cosine measure of the included angle between $\overline{M}_1$ and $\overline{M}_2$ satisfies the following properties:*
  P1. $0 \leq \mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_2) \leq 1$;
  P2. $\mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_2) = \mathrm{Cos}_{SVNS}(\overline{M}_2, \overline{M}_1)$;
  P3. If $\overline{M}_1 = \overline{M}_2$, then $\mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_2) = 1$.

Proof. P1. Because $\overline{M}_1 \cdot \overline{M}_2 \geq 0$, $\|\overline{M}_1\| \geq 0$, and $\|\overline{M}_2\| \geq 0$, $\mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_2) = \frac{\overline{M}_1 \cdot \overline{M}_2}{\|\overline{M}_1\| \cdot \|\overline{M}_2\|} \geq 0$. According to the Cauchy–Schwartz inequality, we achieve $(x_1 y_1 + x_2 y_2 + x_3 y_3) \leq \sqrt{\left(x_1^2 + x_2^2 + x_3^2\right)\left(y_1^2 + y_2^2 + y_3^2\right)}$, that is, $\overline{M}_1 \cdot \overline{M}_2 \leq \|\overline{M}_1\| \cdot \|\overline{M}_2\|$. Therefore, $\mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_2) = \frac{\overline{M}_1 \cdot \overline{M}_2}{\|\overline{M}_1\| \cdot \|\overline{M}_2\|} \leq 1$ can be obtained accordingly. As a result, $0 \leq \mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_2) \leq 1$.

  P2. Apparently, $\mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_2) = \frac{\overline{T}_1\overline{T}_2 + \left(1 - \overline{I}_1\right)\left(1 - \overline{I}_2\right) + \left(1 - \overline{F}_1\right)\left(1 - \overline{F}_2\right)}{\sqrt{\left(\overline{T}_1^2 + \left(1 - \overline{I}_1\right)^2 + \left(1 - \overline{F}_1\right)^2\right)}\sqrt{\left(\overline{T}_2^2 + \left(1 - \overline{I}_2\right)^2 + \left(1 - \overline{F}_2\right)^2\right)}} = \frac{\overline{T}_2\overline{T}_1 + \left(1 - \overline{I}_2\right)\left(1 - \overline{I}_1\right) + \left(1 - \overline{F}_2\right)\left(1 - \overline{F}_1\right)}{\sqrt{\left(\overline{T}_2^2 + \left(1 - \overline{I}_2\right)^2 + \left(1 - \overline{F}_2\right)^2\right)}\sqrt{\left(\overline{T}_1^2 + \left(1 - \overline{I}_1\right)^2 + \left(1 - \overline{F}_1\right)^2\right)}} = \mathrm{Cos}_{SVNS}(\overline{M}_2, \overline{M}_1)$.

  P3. If $\overline{M}_1 = \overline{M}_2$, then we have $\mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_2) = \mathrm{Cos}_{SVNS}(\overline{M}_1, \overline{M}_1) = \frac{\overline{M}_1 \cdot \overline{M}_1}{\|\overline{M}_1\| \cdot \|\overline{M}_1\|} = 1$.

  Moreover, the cosine measure of the included angle between $\overline{M}_1$ and $\overline{M}_2$, that is, $\mathrm{Cos}_{SVNN}(\overline{M}_1, \overline{M}_2)$, always satisfies the properties presented in Theorem 1. □

Definition 7. *Let $\overline{M}_1 = \left(\overline{M}_1(x_1), \overline{M}_1(x_2), \ldots, \overline{M}_1(x_n)\right)$ and $\overline{M}_2 = \left(\overline{M}_2(x_1), \overline{M}_2(x_2), \ldots, \overline{M}_2(x_n)\right)$ be two SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$. Then, the improved single-valued neutrosophic projection measure of $\overline{M}_1$ on $\overline{M}_2$ can be developed as follows:*

$$
\begin{aligned}
IPro_{\overline{M}_2}\left(\overline{M}_1\right) &= \|\overline{M}_1\| Cos_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) \\
&= \frac{\overline{M}_1 \cdot \overline{M}_2}{\|\overline{M}_2\|} \\
&= \frac{\sum\limits_{i=1}^{n}\left(\overline{T}_1(x_i)\overline{T}_2(x_i) + \left(1 - \overline{I}_1(x_i)\right)\left(1 - \overline{I}_2(x_i)\right) + \left(1 - \overline{F}_1(x_i)\right)\left(1 - \overline{F}_2(x_i)\right)\right)}{\sqrt{\sum\limits_{i=1}^{n}\left(\overline{T}_2(x_i)^2 + \left(1 - \overline{I}_2(x_i)\right)^2 + \left(1 - \overline{F}_2(x_i)\right)^2\right)}}.
\end{aligned}
\tag{6}
$$

Example 3. *Based on Example 1, $IPro_{\overline{M}_2}\left(\overline{M}_1\right) = \frac{1 \times 1 + (1-1) \times (1-0) + (1-1) \times (1-0)}{\sqrt{1^2 + (1-0)^2 + (1-0)^2}} = \frac{1}{\sqrt{3}}$ and $IPro_{\overline{M}_2}\left(\overline{M}_2\right) = \frac{1 \times 1 + (1-0) \times (1-0) + (1-0) \times (1-0)}{\sqrt{1^2 + (1-0)^2 + (1-0)^2}} = \sqrt{3}$ can be obtained. Because $IPro_{\overline{M}_2}\left(\overline{M}_2\right) > IPro_{\overline{M}_2}\left(\overline{M}_1\right)$, $\overline{M}_2$ is much closer to $\overline{M}_2$ than $\overline{M}_1$, the results are consistent with our intuition. Therefore, the improved single-valued neutrosophic projection measure is more reasonable and effective than the existing projection defined by Ye [37].*

Theorem 2. *Let $\overline{M}_1 = \left(\overline{M}_1(x_1), \overline{M}_1(x_2), \ldots, \overline{M}_1(x_n)\right)$, $\overline{M}_2 = \left(\overline{M}_2(x_1), \overline{M}_2(x_2), \ldots, \overline{M}_2(x_n)\right)$, and $\overline{M}_3 = \left(\overline{M}_3(x_1), \overline{M}_3(x_2), \ldots, \overline{M}_3(x_n)\right)$ be three SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$, then the improved single-valued neutrosophic projection measure presented in Definition 7, that is, Equation (6), satisfies the following properties:*

P1. $0 \leq IPro_{\overline{M}_2}\left(\overline{M}_1\right) \leq \sqrt{3n}$;

P2. If $\overline{M}_1 = \overline{M}_2$, then $IPro_{\overline{M}_2}\left(\overline{M}_1\right) = IPro_{\overline{M}_1}\left(\overline{M}_2\right)$;

P3. If $\overline{M}_1 \subseteq \overline{M}_2 \subseteq \overline{M}_3$, then $IPro_{\overline{M}_3}\left(\overline{M}_1\right) \leq IPro_{\overline{M}_3}\left(\overline{M}_2\right)$.

Proof. P1. As $0 \leq Cos_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) \leq 1$ and $0 \leq \|\overline{M}_1\| \leq \sqrt{3n}$, $0 \leq IPro_{\overline{M}_2}\left(\overline{M}_1\right) = \|\overline{M}_1\| Cos_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) \leq \sqrt{3n}$ can be obtained.

P2. If $\overline{M}_1 = \overline{M}_2$, then we have $IPro_{\overline{M}_2}\left(\overline{M}_1\right) = \|\overline{M}_1\| Cos_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) = \|\overline{M}_2\| Cos_{SVNS}\left(\overline{M}_2, \overline{M}_1\right) = IPro_{\overline{M}_1}\left(\overline{M}_2\right)$.

P3. If $\overline{M}_1 \subseteq \overline{M}_2 \subseteq \overline{M}_3$, then we have $\overline{T}_1(x_i) \leq \overline{T}_2(x_i) \leq \overline{T}_3(x_i)$, $\overline{I}_1(x_i) \geq \overline{I}_2(x_i) \geq \overline{I}_3(x_i)$ and $\overline{F}_1(x_i) \geq \overline{F}_2(x_i) \geq \overline{F}_3(x_i)$ for any $x_i \in X$. Thus, $\overline{T}_1(x_i) \cdot \overline{T}_3(x_i) \leq \overline{T}_2(x_i)\overline{T}_3(x_i)$, $\left(1 - \overline{I}_1(x_i)\right) \cdot \left(1 - \overline{I}_3(x_i)\right) \leq \left(1 - \overline{I}_2(x_i)\right) \cdot \left(1 - \overline{I}_3(x_i)\right)$, and $\left(1 - \overline{F}_1(x_i)\right) \cdot \left(1 - \overline{F}_3(x_i)\right) \leq \left(1 - \overline{F}_2(x_i)\right) \cdot \left(1 - \overline{F}_3(x_i)\right)$. Thus, $\overline{T}_1(x_i) \cdot \overline{T}_3(x_i) + \left(1 - \overline{I}_1(x_i)\right) \cdot \left(1 - \overline{I}_3(x_i)\right) + \left(1 - \overline{F}_1(x_i)\right) \cdot \left(1 - \overline{F}_3(x_i)\right) \leq \overline{T}_2(x_i) \cdot \overline{T}_3(x_i) + \left(1 - \overline{I}_2(x_i)\right) \cdot \left(1 - \overline{I}_3(x_i)\right) + \left(1 - \overline{F}_2(x_i)\right) \cdot \left(1 - \overline{F}_3(x_i)\right)$, that is, $\overline{M}_1 \cdot \overline{M}_3 \leq \overline{M}_2 \cdot \overline{M}_3$. According to Definition 7, we can achieve $\frac{\overline{M}_1 \cdot \overline{M}_3}{\|\overline{M}_3\|} \leq \frac{\overline{M}_2 \cdot \overline{M}_3}{\|\overline{M}_3\|}$, that is, $IPro_{\overline{M}_3}\left(\overline{M}_1\right) \leq IPro_{\overline{M}_3}\left(\overline{M}_2\right)$. □

3.2. The Single-Valued Neutrosophic Bidirectional Projection Measure

On the basis of the improved single-valued neutrosophic inner product and single-valued neutrosophic projection measure, the corresponding improved single-valued neutrosophic bidirectional projection measure is developed in this section.

Definition 8. *Let $\overline{M}_1 = \left(\overline{M}_1(x_1), \overline{M}_1(x_2), \ldots, \overline{M}_1(x_n)\right)$ and $\overline{M}_2 = \left(\overline{M}_2(x_1), \overline{M}_2(x_2), \ldots, \overline{M}_2(x_n)\right)$ be two SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$. Then, the corresponding improved single-valued neutrosophic bidirectional projection measure of $\overline{M}_1$ on $\overline{M}_2$ is developed as follows:*

$$
IBPro_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) = \frac{1}{1 + \left|\|\overline{M}_1\| - \|\overline{M}_2\|\right| \overline{M}_1 \cdot \overline{M}_2}
\tag{7}
$$

where $\overline{M}_1 \cdot \overline{M}_2 = \sum\limits_{i=1}^{n} \left(\overline{T}_1(x_i)\overline{T}_2(x_i) + \left(1 - \overline{I}_1(x_i)\right)\left(1 - \overline{I}_2(x_i)\right) + \left(1 - \overline{F}_1(x_i)\right)\left(1 - \overline{F}_2(x_i)\right) \right)$ denotes the

inner product between $\overline{M}_1$ and $\overline{M}_2$. In addition, $\|\overline{M}_1\| = \sqrt{\sum\limits_{i=1}^{n} \left(\overline{T}_1(x_i)^2 + \left(1 - \overline{I}_1(x_i)\right)^2 + \left(1 - \overline{F}_1(x_i)\right)^2 \right)}$

and $\|\overline{M}_2\| = \sqrt{\sum\limits_{i=1}^{n} \left(\overline{T}_2(x_i)^2 + \left(1 - \overline{I}_2(x_i)\right)^2 + \left(1 - \overline{F}_2(x_i)\right)^2 \right)}$ represent the modules of $\overline{M}_1$ and $\overline{M}_2$, respectively.

Example 4. *According to Example 1 and Definition 8, we have $IBPro_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) = \frac{1}{1+\left|\|\overline{M}_1\|-\|\overline{M}_2\|\right|\overline{M}_1\cdot\overline{M}_2} = \frac{1}{1+\left|1-\sqrt{3}\right|\times 1} = \frac{\sqrt{3}}{3}$ and $IBPro_{SVNS}\left(\overline{M}_2, \overline{M}_2\right) = \frac{1}{1+\left|\|\overline{M}_2\|-\|\overline{M}_2\|\right|\overline{M}_2\cdot\overline{M}_2} = \frac{1}{1+0} = 1$. Then, $IBPro_{SVNS}\left(\overline{M}_2, \overline{M}_2\right) > IBPro_{SVNS}\left(\overline{M}_1, \overline{M}_2\right)$. Apparently, the results achieved using the corresponding single-valued neutrosophic bidirectional projection measure are the same as the results of the bidirectional projection measure in Ye [37]. In other words, the corresponding single-valued neutrosophic bidirectional projection measure based on improved single-valued neutrosophic projection is as effective as the bidirectional projection measure presented by Ye [34].*

Theorem 3. *Let $\overline{M}_1 = \left(\overline{M}_1(x_1), \overline{M}_1(x_2), \ldots, \overline{M}_1(x_n)\right)$ and $\overline{M}_2 = \left(\overline{M}_2(x_1), \overline{M}_2(x_2), \ldots, \overline{M}_2(x_n)\right)$ be two SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$, accordingly, the improved single-valued neutrosophic bidirectional projection measure presented in Definition 8, that is, Equation (7), satisfies the following properties:*
    P1. $0 \le IBPro_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) \le 1$;
    P2. $IBPro_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) = IBPro_{SVNS}\left(\overline{M}_2, \overline{M}_1\right)$;
    P3. If $\overline{M}_1 = \overline{M}_2$, then $IBPro_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) = 1$.

Proof. P1. Because $0 \le \frac{1}{1+\left|\|\overline{M}_1\|-\|\overline{M}_2\|\right|\overline{M}_1\cdot\overline{M}_2} \le 1, 0 \le IBPro_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) \le 1$.
    P2. $IBPro_{SVNS}\left(\overline{M}_1, \overline{M}_2\right) = \frac{1}{1+\left|\|\overline{M}_1\|-\|\overline{M}_2\|\right|\overline{M}_1\cdot\overline{M}_2} = \frac{1}{1+\left|\|\overline{M}_2\|-\|\overline{M}_1\|\right|\overline{M}_2\cdot\overline{M}_1} = IBPro_{SVNS}\left(\overline{M}_2, \overline{M}_1\right)$.
    P3. If $\overline{M}_1 = \overline{M}_2$, then $IBPro_{SVNS}\left(\overline{M}_1, \overline{M}_1\right) = \frac{1}{1+\left|\|\overline{M}_1\|-\|\overline{M}_1\|\right|\overline{M}_1\cdot\overline{M}_1} = \frac{1}{1+0} = 1$. $\square$

Theorem 4. *Let $\overline{M} = \left(\overline{M}(x_1), \overline{M}(x_2), \ldots, \overline{M}(x_n)\right)$, $\overline{M}_1 = \left(\overline{M}_1(x_1), \overline{M}_1(x_2), \ldots, \overline{M}_1(x_n)\right)$, and $\overline{M}_2 = \left(\overline{M}_2(x_1), \overline{M}_2(x_2), \ldots, \overline{M}_2(x_n)\right)$ be three SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$, and M^* be an ideal SVNS, consequently, the following properties can be achieved:*
    P1. $0 \le IBPro_{SVNS}\left(\overline{M}, M^*\right) \le 1$;
    P2. $IBPro_{SVNS}\left(\overline{M}, M^*\right) = IBPro_{SVNS}\left(M^*, \overline{M}\right)$;
    P3. $IBPro_{SVNS}\left(M^*, M^*\right) = 1$.

Proof. According to the proof of Theorem 3, Theorem 4 is true as well. $\square$

3.3. The Single-Valued Neutrosophic Bidirectional Projection Difference Measure

    According to single-valued neutrosophic bidirectional projection, the corresponding difference measure and the partial ordering relation of SVNSs are discussed in the following.

Definition 9. *Let $\overline{M}_1 = \left(\overline{M}_1(x_1), \overline{M}_1(x_2), \ldots, \overline{M}_1(x_n)\right)$ and $\overline{M}_2 = \left(\overline{M}_2(x_1), \overline{M}_2(x_2), \ldots, \overline{M}_2(x_n)\right)$ be two SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$, and M^* be the ideal SVNS, then the single-valued neutrosophic bidirectional projection difference measure between $\overline{M}_1$ and $\overline{M}_2$ can be defined as follows:*

$$Diff_{IBPro}\left(\overline{M}_1, \overline{M}_2\right) = IBPro_{SVNS}\left(\overline{M}_1, M^*\right) - IBPro_{SVNS}\left(\overline{M}_2, M^*\right) \tag{8}$$

Theorem 5. *The bidirectional projection difference measure defined in Definition 9, that is, Equation (8), satisfies the following properties:*
    P1. $-1 \le Diff_{IBPro}\left(\overline{M}_1, \overline{M}_2\right) \le 1$;

P2. If $\overline{M}_1 = \overline{M}_2$, then $Diff_{IBPro}(\overline{M}_1, \overline{M}_2) = 0$;

P3. If $Diff_{IBPro}(\overline{M}_1, \overline{M}_2) \geq 0$ and $Diff_{IBPro}(\overline{M}_2, \overline{M}_3) \geq 0$, thus $Diff_{IBPro}(\overline{M}_1, \overline{M}_3) \geq 0$.

Proof. P1. On the basis of Theorem 4, we have $0 \leq IBPro_{SVNS}(\overline{M}_1, M^*) \leq 1$ and $0 \leq IBPro_{SVNS}(\overline{M}_2, M^*) \leq 1$, that is, $-1 \leq -IBPro_{SVNS}(\overline{M}_2, M^*) \leq 0$. Thus, $-1 \leq IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_2, M^*) \leq 1$, that is, $-1 \leq Diff_{IBPro}(\overline{M}_1, \overline{M}_2) \leq 1$.

P2. If $\overline{M}_1 = \overline{M}_2$, then $Diff_{IBPro}(\overline{M}_1, \overline{M}_2) = IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_1, M^*) = 0$.

P3. As $Diff_{IBPro}(\overline{M}_1, \overline{M}_2) = IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_1, M^*) \geq 0$ and $Diff_{IBPro}(\overline{M}_2, \overline{M}_3) = IBPro_{SVNS}(\overline{M}_2, M^*) - IBPro_{SVNS}(\overline{M}_3, M^*) \geq 0$, $Diff_{IBPro}(\overline{M}_1, \overline{M}_3) = IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_3, M^*) = (IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_2, M^*)) + (IBPro_{SVNS}(\overline{M}_2, M^*) - IBPro_{SVNS}(\overline{M}_3, M^*)) \geq 0$. □

Definition 10. *Let $\overline{M}_1 = (\overline{M}_1(x_1), \overline{M}_1(x_2), \ldots, \overline{M}_1(x_n))$ and $\overline{M}_2 = (\overline{M}_2(x_1), \overline{M}_2(x_2), \ldots, \overline{M}_2(x_n))$ be two SVNSs on the universe $X = \{x_1, x_2, \ldots, x_n\}$, and M^* be an ideal SVNS. Then, the partial ordering relation of SVNSs can be formulated as follows:*

(1) If $Diff_{IBPro}(\overline{M}_1, \overline{M}_2) > 0$, that is, $IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_2, M^*) > 0$, that is, $IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_2, M^*) > 0$, then $\overline{M}_1$ is preferred to $\overline{M}_2$, denoted by $\overline{M}_1 \succ \overline{M}_2$;

(2) If $Diff_{IBPro}(\overline{M}_1, \overline{M}_2) = 0$, that is, $IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_2, M^*) = 0$, then $\overline{M}_1$ is indifferent to $\overline{M}_2$, denoted by $\overline{M}_1 \sim \overline{M}_2$;

(3) If $Diff_{IBPro}(\overline{M}_1, \overline{M}_2) < 0$, that is, $IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_2, M^*) < 0$, then $\overline{M}_1$ is inferior to $\overline{M}_2$, denoted by $\overline{M}_1 \prec \overline{M}_2$.

Example 5. *Let $\overline{M}_1 = \{x, \langle 0.6, 0.2, 0.2 \rangle\}$ and $\overline{M}_2 = \{x, \langle 0.6, 0.1, 0.1 \rangle\}$ be two SVNSs. Based on Equations (7) and (8), we $IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_2, M^*) < 0$ can obtain $Diff_{IBPro}(\overline{M}_1, \overline{M}_2) = IBPro_{SVNS}(\overline{M}_1, M^*) - IBPro_{SVNS}(\overline{M}_2, M^*) = \frac{1}{1 + \left| \sqrt{0.6^2 + 0.8^2 + 0.8^2} - \sqrt{1^2 + 1^2 + 1^2} \right| \times (0.6 \times 1 + 0.8 \times 1 + 0.8 \times 1)} - \frac{1}{1 + \left| \sqrt{0.6^2 + 0.9^2 + 0.9^2} - \sqrt{1^2 + 1^2 + 1^2} \right| \times (0.6 \times 1 + 0.9 \times 1 + 0.9 \times 1)} = -0.0101 < 0$. Thus, $\overline{M}_1$ is inferior to $\overline{M}_2$, that is, $\overline{M}_1 \prec \overline{M}_2$, which is consistent with our intuition.*

4. The Single-Valued Neutrosophic Projection-Based QUALIFLEX Method

The QUALIFLEX method is based on the pair-wise comparisons of alternatives with respect to each criterion under all possible permutations of the alternatives. It is a useful outranking method because of its flexibility with respect to cardinal and ordinal information [47] and has been extensively extended to different environments. Although the method of Ye [37] based on single-valued neutrosophic bidirectional projection is simple and effective, it cannot consider the relations between criteria. Moreover, because of decision-maker's knowledge, the weight information of criteria is always partly known or incompletely known in some real decision-making situations, so the single-valued neutrosophic projection-based QUALIFLEX method with incomplete weight information is proposed in this section.

In an actual decision-making process, because of the intangibility of the decision environment and the decision-maker's subjectivity, it is difficult to express their preferences. SVNS can overcome these shortcomings by consideration of three different aspects of decision-makers. Moreover, management-based decision-making problems always involve several criteria. Thus, the MCDM method with single-valued neutrosophic information plays a major role in management. Assume a group of alternatives denoted as $\overline{M} = \{\overline{M}_1, \overline{M}_2, \ldots, \overline{M}_n\}$ and corresponding criteria denoted by $C = \{c_1, c_2, \ldots, c_m\}$, in which the weight of criterion w_j is fully unknown. $\overline{M}_{ij} = \left\langle T_{\overline{M}_{ij}}, I_{\overline{M}_{ij}}, F_{\overline{M}_{ij}} \right\rangle$ represents the evaluation value of $\overline{M}_i$ with respect to criterion c_j. $T_{\overline{M}_{ij}}$, $I_{\overline{M}_{ij}}$, and $F_{\overline{M}_{ij}}$ indicate the truth-membership, the indeterminacy-membership, and the falsity-membership, respectively. The proposed method consists of the following steps.

Step 1. Normalization of the decision matrix.

Each criterion can be divided into two types—benefit-based criteria, meaning the lager the better; and cost-based criteria, meaning the smaller the better. For the benefit-based criteria, nothing needs to be conducted; for the cost-based criteria, the criterion values can be transformed as $\overline{M}_{ij} = (\overline{M}_{ij})^c$ ($i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$). Here, $(\overline{M}_{ij})^c$ is the complement of $\overline{M}_{ij}$ as presented in Definition 2.

Step 2. Calculation of the weight of criteria.

On the basis of the maximizing deviation method of SVNSs defined by Sahin and Liu [48], the weight of criteria $\varpi_j (j = 1, 2, \ldots, m)$ can be achieved as follows:

$$\varpi_j^* = \frac{\sum\limits_{i=1}^{n} \sum\limits_{t=1}^{n} \left(\left| T_{\overline{M}_{ij}} - T_{\overline{M}_{tj}} \right| + \left| I_{\overline{M}_{ij}} - I_{\overline{M}_{tj}} \right| + \left| F_{\overline{M}_{ij}} - F_{\overline{M}_{tj}} \right| \right)}{\sum\limits_{j=1}^{m} \sum\limits_{i=1}^{n} \sum\limits_{t=1}^{n} \left(\left| T_{\overline{M}_{ij}} - T_{\overline{M}_{tj}} \right| + \left| I_{\overline{M}_{ij}} - I_{\overline{M}_{tj}} \right| + \left| F_{\overline{M}_{ij}} - F_{\overline{M}_{tj}} \right| \right)} (i, t = 1, 2, \ldots, n; j = 1, 2, \ldots, m) \quad (9)$$

Step 3. Determination of the possible permutations.

For a group of alternative $\overline{M}_i (i = 1, 2, \ldots, n)$, there exist $n!$ permutations of different ranks of alternatives. Assume P^δ represents the δ-th permutation as follows:

$$P^\delta = (\ldots, \overline{M}_\xi, \ldots, \overline{M}_\varsigma, \ldots) (\delta = 1, 2, \ldots, n!; \xi, \varsigma = 1, 2, \ldots, n) \quad (10)$$

Here, $\overline{M}_\xi, \overline{M}_\varsigma \in \overline{M}$ and $\overline{M}_\xi$ is superior than or equal to $\overline{M}_\varsigma$.

Step 4. Calculation of the concordance index.

For each pair of alternatives $(\overline{M}_\xi, \overline{M}_\varsigma)$ $(\overline{M}_\xi, \overline{M}_\varsigma \in \overline{M})$ with respect to the j-th criterion, the corresponding concordance index $\varphi_j^\delta (\overline{M}_\xi, \overline{M}_\varsigma)$, can be defined as follows:

$$\varphi_j^\delta (\overline{M}_\xi, \overline{M}_\varsigma) = Diff_{IBPro} (\overline{M}_\xi, \overline{M}_\varsigma) \quad (11)$$

According to the partial ordering relation of SVNSs, the following assumptions can be true:

(1) If $\varphi_j^\delta (\overline{M}_\xi, \overline{M}_\varsigma) > 0$, that is, $Diff_{IBPro} (\overline{M}_{\xi j}, \overline{M}_{\varsigma j}) > 0$, then $\overline{M}_\xi$ ranks over $\overline{M}_\varsigma$ with respect to the j-th criterion under the δ-th permutation;

(2) If $\varphi_j^\delta (\overline{M}_\xi, \overline{M}_\varsigma) = 0$, that is, $Diff_{IBPro} (\overline{M}_{\xi j}, \overline{M}_{\varsigma j}) = 0$, then both $\overline{M}_\xi$ and $\overline{M}_\varsigma$ have the same rank with respect to the j-th criterion under the δ-th permutation;

(3) If $\varphi_j^\delta (\overline{M}_\xi, \overline{M}_\varsigma) < 0$, that is, $Diff_{IBPro} (\overline{M}_{\xi j}, \overline{M}_{\varsigma j}) < 0$, then $\overline{M}_\varsigma$ ranks over $\overline{M}_\xi$ with respect to the j-th criterion under the δ-th permutation.

Step 5. Determination of the weighted concordance index.

Considering the importance weight ϖ_j of each criterion that $c_j \in C$ is expressed by SVNNs, the weighted concordance/discordance index $\varphi_j^\delta (\overline{M}_\xi, \overline{M}_\varsigma)$ for each pair of alternatives $(\overline{M}_\xi, \overline{M}_\varsigma)$ $(\overline{M}_\xi, \overline{M}_\varsigma \in \overline{M})$ can be denoted as follows:

$$\varphi^\delta (\overline{M}_\xi, \overline{M}_\varsigma) = \sum_{j=1}^{m} \varphi_j^\delta (\overline{M}_\xi, \overline{M}_\varsigma) \cdot \varpi_j (j = 1, 2, \ldots, m) \quad (12)$$

Here, $\varpi_j (j = 1, 2, \ldots, m)$ is the weight of criteria $c_j (j = 1, 2, \ldots, m)$.

Step 6. Calculation of the comprehensive concordance index.

For the δ-th permutation, the corresponding comprehensive concordance index φ^δ can be calculated as follows:

$$\varphi^\delta = \sum_{\overline{M}_\xi, \overline{M}_\varsigma \in \overline{M}} \varphi^\delta (\overline{M}_\xi, \overline{M}_\varsigma) \quad (13)$$

Step 7. Ranking the alternatives.

According to the partial ordering relation of SVNNs, it can be observed that the greater the comprehensive concordance index value, the more optimal the final ranking. Thus, the optimal rank can be obtained with the help of the maximal comprehensive concordance/discordance index φ^δ, that is,

$$P^* = \max_{\delta=1}^{n!}\left\{\varphi^\delta\right\} \tag{14}$$

5. An Illustrative Example

In this section, an example of an MCDM problem (adapted from that provided in the work of [49]) is used to demonstrate the application and effectiveness of the proposed decision-making approach.

In order to attenuate environmental impacts and increase ecological efficiency, ABC automobile manufacturing company attempts to implement green practices at all stages of the manufacturing process to achieve profit and market share objectives. Thus, how to choose a proper green supplier from several potential suppliers is an MCDM problem. Assume that there are three possible green suppliers, $\overline{M}_i(i = 1, 2, 3)$, that can be selected. Each supplier is assessed based on nine criteria, which are denoted by $c_j(j = 1, 2, \ldots, 9)$; where c_1 is the pollution produced, such as average volume of air emission pollutant, waster, solid wastes, and harmful materials, which can be released per day during measurement period; c_2 is the resource consumption, such as resource consumption in terms of raw materials, energy, and water during the measurement period; c_3 is the eco-design, including design of products for reduced consumption of material/energy, design of products for reuse, recycle, and recovery of materials; c_4 is a green image, such as the ratio of green customers to total customers; c_5 is the environmental management system, including environmental certifications, such as ISO-1400, environmental policies, planning of environmental objectives, as well as checking and control of environmental activities; c_6 represents the commitment to green supply chain management on behalf of managers, including senior and mid-level managers' commitment and support to improve green supply chain management practices and environmental performance; c_7 denotes the use of an environmentally-friendly technology, such as the application of the environmental science to conserve the natural environment and resources, and to curb the negative impacts of human involvement; c_8 represents the use of environmentally-friendly materials, including the level of green recyclable materials used in packaging and manufacturing of goods; and c_9 is the staff environmental training, involving training targets. Moreover, c_1 is a minimizing type, and other criteria are of the maximizing type. A decision-maker was invited to assess the performance of these three potential suppliers using each criterion in form of SVNNs. The evaluation results are presented in Table 1.

Table 1. Evaluation of the green suppliers by decision-makers in presence of different criteria.

Criteria	Suppliers		
	$\overline{M}_1$	$\overline{M}_2$	$\overline{M}_3$
c_1	(0.5,0.6,0.7)	(0.6,0.5,0.5)	(0.5,0.6,0.6)
c_2	(0.5,0.3,0.4)	(0.6,0.5,0.2)	(0.6,0.3,0.2)
c_3	(0.5,0.4,0.4)	(0.7,0.3,0.3)	(0.5,0.4,0.3)
c_4	(0.7,0.3,0.2)	(0.5,0.2,0.4)	(0.6,0.1,0.3)
c_5	(0.6,0.2,0.3)	(0.5,0.4,0.2)	(0.7,0.2,0.5)
c_6	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.6,0.2,0.6)
c_7	(0.8,0.6,0.4)	(0.7,0.4,0.6)	(0.6,0.2,0.4)
c_8	(0.7,0.3,0.5)	(0.8,0.2,0.4)	(0.7,0.2,0.3)
c_9	(0.7,0.3,0.4)	(0.8,0.2,0.4)	(0.7,0.3,0.4)

5.1. An Illustration of the Proposed Method

The procedures for obtaining the optimal alternative, using the developed method, are as follows.
Step 1. Normalization of the decision-making matrix.

As c_1 is a minimizing type, and other criteria are of the maximizing type, the normalized SVNN decision matrix is calculated and shown in Table 2.

Table 2. Normalized decision matrix.

Criteria	Suppliers		
	$\overline{M}_1$	$\overline{M}_2$	$\overline{M}_3$
c_1	(0.7,0.4,0.5)	(0.5,0.5,0.6)	(0.6,0.4,0.5)
c_2	(0.5,0.3,0.4)	(0.6,0.5,0.2)	(0.6,0.3,0.2)
c_3	(0.5,0.4,0.4)	(0.7,0.3,0.3)	(0.5,0.4,0.3)
c_4	(0.7,0.3,0.2)	(0.5,0.2,0.4)	(0.6,0.1,0.3)
c_5	(0.6,0.2,0.3)	(0.5,0.4,0.2)	(0.7,0.2,0.5)
c_6	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.6,0.2,0.6)
c_7	(0.8,0.6,0.4)	(0.7,0.4,0.6)	(0.6,0.2,0.4)
c_8	(0.7,0.3,0.5)	(0.8,0.2,0.4)	(0.7,0.2,0.3)
c_9	(0.7,0.3,0.4)	(0.8,0.2,0.4)	(0.7,0.3,0.4)

Step 2. Calculation of the weight of criteria.

From Equation (9), the weight of criteria can be obtained as $\varpi = (0.0851, 0.1064, 0.0851, 0.1277, 0.1489, 0.1489, 0.1702, 0.0851, 0.0426)^T$.

Step 3. Determination of all the possible permutations.

As $n = 3$, we have 6(3! = 6) permutations of alternative rankings, that is, $P^1 = (\overline{M}_1, \overline{M}_2, \overline{M}_3)$, $P^2 = (\overline{M}_1, \overline{M}_3, \overline{M}_2)$, $P^3 = (\overline{M}_2, \overline{M}_1, \overline{M}_3)$, $P^4 = (\overline{M}_2, \overline{M}_3, \overline{M}_1)$, $P^5 = (\overline{M}_3, \overline{M}_1, \overline{M}_2)$, and $P^6 = (\overline{M}_3, \overline{M}_2, \overline{M}_1)$.

Step 4. Calculation of the concordance index.

According to the single-valued neutrosophic bidirectional projection and corresponding difference measure, that is, Equations (7) and (10), the concordance index $\varphi_j^\delta(\overline{M}_\xi, \overline{M}_\varsigma)$ for each pair of alternatives $(\overline{M}_\xi, \overline{M}_\varsigma)$ $(\overline{M}_\xi, \overline{M}_\varsigma \in \overline{M})$ in the permutation P^δ under criterion c_j can be obtained (see Table 3).

Table 3. The concordance index.

P^1	$\varphi_j^1(\overline{M}_1, \overline{M}_2)$	$\varphi_j^1(\overline{M}_1, \overline{M}_3)$	$\varphi_j^1(\overline{M}_2, \overline{M}_3)$	P^2	$\varphi_j^2(\overline{M}_1, \overline{M}_3)$	$\varphi_j^2(\overline{M}_1, \overline{M}_2)$	$\varphi_j^2(\overline{M}_3, \overline{M}_2)$
c_1	-0.0505	-0.0476	0.0029	c_1	-0.0476	-0.0505	-0.0029
c_2	-0.0658	-0.0698	-0.0040	c_2	-0.0698	-0.0658	0.0039
c_3	-0.0078	-0.0427	-0.0349	c_3	-0.0427	-0.0078	0.0349
c_4	0.0774	0.0596	-0.0178	c_4	0.0596	0.0774	0.0178
c_5	-0.0786	0.0900	0.1686	c_5	0.0900	-0.0786	-0.1686
c_6	0.0265	0.1290	0.1025	c_6	0.1290	0.0265	-0.1025
c_7	0.0243	-0.0843	-0.1086	c_7	-0.0843	0.0243	0.1086
c_8	-0.0045	-0.0461	-0.0416	c_8	-0.0461	-0.0045	0.0416
c_9	0.0188	0	-0.0188	c_9	0	0.0188	0.0188
P^3	$\varphi_j^3(\overline{M}_2, \overline{M}_1)$	$\varphi_j^3(\overline{M}_2, \overline{M}_3)$	$\varphi_j^3(\overline{M}_1, \overline{M}_3)$	P^4	$\varphi_j^4(\overline{M}_2, \overline{M}_3)$	$\varphi_j^4(\overline{M}_2, \overline{M}_1)$	$\varphi_j^4(\overline{M}_3, \overline{M}_1)$
c_1	0.0505	0.0029	-0.0476	c_1	0.0029	0.0505	0.0476
c_2	0.0658	-0.0040	-0.0698	c_2	-0.0040	0.0658	0.0698
c_3	0.0078	-0.0349	-0.0427	c_3	-0.0349	0.0078	0.0427
c_4	-0.0774	-0.0178	0.0596	c_4	-0.0178	-0.0774	-0.0596
c_5	0.0786	0.1686	0.0900	c_5	0.1686	0.0786	-0.0900
c_6	-0.0265	0.1025	0.1290	c_6	0.1025	-0.0265	-0.1290
c_7	-0.0243	-0.1086	-0.0843	c_7	-0.1086	-0.0243	0.0843
c_8	0.0045	-0.0416	-0.0461	c_8	-0.0416	0.0045	0.0461
c_9	-0.0188	-0.0188	0	c_9	-0.0188	-0.0188	0
P^5	$\varphi_j^5(\overline{M}_3, \overline{M}_1)$	$\varphi_j^5(\overline{M}_3, \overline{M}_2)$	$\varphi_j^5(\overline{M}_1, \overline{M}_2)$	P^6	$\varphi_j^6(\overline{M}_3, \overline{M}_2)$	$\varphi_j^6(\overline{M}_3, \overline{M}_1)$	$\varphi_j^6(\overline{M}_2, \overline{M}_1)$
c_1	0.0476	-0.0029	-0.0505	c_1	-0.0029	0.0476	0.0505
c_2	0.0698	0.0040	-0.0658	c_2	0.0040	0.0698	0.0658
c_3	0.0427	0.0349	-0.0078	c_3	0.0349	0.0427	0.0078
c_4	-0.0596	0.0178	0.0774	c_4	0.0178	-0.0596	-0.0774
c_5	-0.0900	-0.1686	-0.0786	c_5	-0.1686	-0.0900	0.0786
c_6	-0.1290	-0.1025	-0.0265	c_6	-0.1025	-0.1290	-0.0265
c_7	0.0843	0.1086	0.0243	c_7	0.1086	0.0843	-0.0243
c_8	0.0461	0.0416	-0.0045	c_8	0.0416	0.0461	0.0045
c_9	0	0.0188	0.0188	c_9	0.0188	0	-0.0188

Step 5. Calculation of the weighted concordance index.

The weighted concordance index $\varphi^\delta\left(\overline{M}_\xi, \overline{M}_\varsigma\right)$ can be calculated as presented in Table 4.

Table 4. The weighted concordance index.

P^1	$\varphi^1\left(\overline{M}_1,\overline{M}_2\right)$	$\varphi^1\left(\overline{M}_1,\overline{M}_3\right)$	$\varphi^1\left(\overline{M}_2,\overline{M}_3\right)$	P^2	$\varphi^2\left(\overline{M}_1,\overline{M}_3\right)$	$\varphi^2\left(\overline{M}_1,\overline{M}_2\right)$	$\varphi^2\left(\overline{M}_3,\overline{M}_2\right)$
	-0.0053	0.0068	0.0121		0.0068	-0.0053	-0.0121
P^3	$\varphi^3\left(\overline{M}_2,\overline{M}_1\right)$	$\varphi^3\left(\overline{M}_2,\overline{M}_3\right)$	$\varphi^3\left(\overline{M}_1,\overline{M}_3\right)$	P^4	$\varphi^4\left(\overline{M}_2,\overline{M}_3\right)$	$\varphi^4\left(\overline{M}_2,\overline{M}_1\right)$	$\varphi^4\left(\overline{M}_3,\overline{M}_1\right)$
	0.0053	0.0121	0.0068		0.0121	0.0053	-0.0069
P^5	$\varphi^5\left(\overline{M}_3,\overline{M}_1\right)$	$\varphi^5\left(\overline{M}_3,\overline{M}_2\right)$	$\varphi^5\left(\overline{M}_1,\overline{M}_2\right)$	P^6	$\varphi^6\left(\overline{M}_3,\overline{M}_2\right)$	$\varphi^6\left(\overline{M}_3,\overline{M}_1\right)$	$\varphi^6\left(\overline{M}_2,\overline{M}_1\right)$
	-0.0069	-0.0121	-0.0053		-0.0121	-0.0069	0.0053

Step 6. Calculation of the comprehensive concordance index.

From Equation (13), the comprehensive concordance index φ^δ can be calculated as shown in Table 5.

Table 5. The comprehensive concordance index.

φ^1	φ^2	φ^3	φ^4	φ^5	φ^6
0.0137	-0.0106	0.0243	0.0106	-0.0243	-0.0137

Step 7. Ranking the alternatives.

On the basis of the results presented in Table 5, $\varphi^3 > \varphi^1 > \varphi^4 > \varphi^2 > \varphi^6 > \varphi^5$ and $P^* = \max_{\kappa=1}^{n!}\left\{\varphi^\delta\right\} = P^3$ can be obtained. Thus, the final order of the three plans is as follows: $\overline{M}_2 \succ \overline{M}_1 \succ \overline{M}_3$. The best plan is $\overline{M}_2$, while $\overline{M}_3$ is the worst.

5.2. Comparison Analysis

To further validate the practicability of the proposed method, a comparison analysis was undertaken by utilizing some existing methods with the help of single-valued neutrosophic information.

The first comparative method is the projection measure method defined in Ye [37]. Because the compared method cannot appropriately handle single-valued neutrosophic information where the weight is fully unknown, the weights of criteria can be determined as $\omega = (0.0851, 0.1064, 0.0851, 0.1277, 0.1489, 0.1489, 0.1702, 0.0851, 0.0426)^T$. Then, from $\overline{M}_j^* = \left(T_j^*, I_j^*, F_j^*\right) = \left(\max_i\left(T_j^*\right), \min_i\left(I_j^*\right), \min_i\left(F_j^*\right)\right)$ $(j = 1, 2, \ldots, 9)$, the corresponding ideal solution can be obtained as follows:

$\overline{M}^* = \{(0.7, 0.4, 0.5), (0.6, 0.3, 0.2), (0.7, 0.3, 0.3), (0.7, 0.1, 0.2), (0.7, 0.2, 0.2), (0.7, 0.2, 0.2)$ $\langle 0.8, 0.2, 0.4\rangle, \langle 0.8, 0.2, 0.3\rangle, \langle 0.8, 0.2, 0.4\rangle\}$.

Thus, the following results is determined as follows:

$\overline{M}^* = 0.2871$; $\left|\overline{M}\right|_1 = 0.3033$; $\left|\overline{M}\right|_2 = 0.2935$; $\left|\overline{M}\right|_3 = 0.2801$. Thus, according to the bidirectional projection measure defined in Ye [37], that is, Equation (3), the weighted bidirectional projection measure between an alternative $\overline{M}_i(i = 1, 2, 3)$ and the ideal solution $\overline{M}^*$ can be calculated as follows:

$BProj_w\left(\overline{M}_1, \overline{M}^*\right) = 0.9847$; $BProj_w\left(\overline{M}_2, \overline{M}^*\right) = 0.9939$; $BProj_w\left(\overline{M}_3, \overline{M}^*\right) = 0.9934$.

Then, the final ranking is $\overline{M}_2 \succ \overline{M}_3 \succ \overline{M}_1$.

Similarly, the projection measure between an alternative $\overline{M}_i(i = 1, 2, 3)$ and the ideal solution $\overline{M}^*$ can be calculated as follows:

$Proj_{w_{\overline{M}^*}}\left(\overline{M}_1\right) = 0.2908$; $Proj_{w_{\overline{M}^*}}\left(\overline{M}_2\right) = 0.2794$; $Proj_{w_{\overline{M}^*}}\left(\overline{M}_3\right) = 0.2666$.

The final ranking is $\overline{M}_1 \succ \overline{M}_2 \succ \overline{M}_3$. Apparently, the result using the projection measure is different from those using the bidirectional projection measure. In other words, we cannot utilize the proposed method in Ye [37] to choose the optimal ones of three potential suppliers.

In addition, some other existing methods with single-valued neutrosophic information are compared with the proposed approach. Those methods mainly focus on the outranking methods [29], aggregation operators [31,32,50,51], and measures [34,36,37]. Because the compared methods cannot deal with single-valued neutrosophic information where the weight information is completely unknown, the weight is determined as $\varpi = (0.0851, 0.1064, 0.0851,$ $0.1277, 0.1489, 0.1489, 0.1702, 0.0851, 0.0426)^T$. Then, the compared results are eventually achieved and presented in Table 6.

Table 6. The comparison results.

References	Aggregation Operator	Rankings
Ye [31]	Algebraic norm	$\overline{M}_1 \succ \overline{M}_2 \succ \overline{M}_3$
Garg [32]	Frank norm ($\lambda = 1, 3$)	$\overline{M}_2 \succ \overline{M}_1 \succ \overline{M}_3$
Liu et al. [50]	Hamacher operator ($\lambda = 1, 3$)	$\overline{M}_2 \succ \overline{M}_1 \succ \overline{M}_3$ or $\overline{M}_1 \succ \overline{M}_2 \succ \overline{M}_3$
Peng et al. [51]	Einstein norm	$\overline{M}_2 \succ \overline{M}_3 \succ \overline{M}_1$

References	Measure	Rankings
Ye [34]	Cosine similarity	$\overline{M}_2 \succ \overline{M}_3 \succ \overline{M}_1$
Ye [36]	Cross entropy	$\overline{M}_2 \succ \overline{M}_1 \succ \overline{M}_3$
Ye [37]	Bidirectional projection	$\overline{M}_2 \succ \overline{M}_1 \succ \overline{M}_3$
	Projection	$\overline{M}_1 \succ \overline{M}_2 \succ \overline{M}_3$

References	Outranking	Rankings
Peng et al. [29]	ELECTRE III	$\overline{M}_2 \succ \overline{M}_1 \succ \overline{M}_3$
The proposed method	Projection	$\overline{M}_2 \succ \overline{M}_1 \succ \overline{M}_3$

According to the results presented in Table 6, it can be seen that the results obtained from the proposed approach are consistent with the compared methods presented by Garg [32], Ye [36], and Peng et al. [29]; the optimal green supplier is $\overline{M}_2$, whereas the worst green supplier is $\overline{M}_3$. For the other compared methods, there always exists a slight difference in the final rankings to some extent. Especially, although the preference relations between $\overline{M}_1$ and $\overline{M}_3$ cannot be discerned, the optimal alternative using the bidirectional projection of Ye [37] is the same as the one of the proposed approach.

According to the results presented above, we can conclude the following results. Firstly, the methods based on aggregation operators [31,32,50,51] involve different norms and parameters, which may produce different rankings. Moreover, it is difficult for decision-makers to choose an optimal operator, as well as corresponding parameters in a real decision-making process. Secondly, the results obtained using the method presented by Ye [37] are different form the proposed method's findings. The reason is that the projection was defined based on the inner product. However, the inner product is unreasonable as discussed in Section 2. Thirdly, Peng et al.'s outranking method [29] is only appropriate for solving the MCDM problems in which the number of alternatives is more than the number of criteria. Finally, all the compared methods cannot deal with some special cases, in which the weight information is fully unknown. However, the proposed approach can avoid these shortcomings. Thus, the proposed method based on the projection is reasonable and effective, and it can further broaden the application of decision-making methods.

6. Conclusions

In this study, an improved cosine measure of the included angle between two SVNSs was initially defined. Then, an improved projection, bidirectional projection, and corresponding bidirectional projection difference of SVNSs were investigated. On the basis of the developed measures, a single-valued neutrosophic bidirectional projection-based QUALIFLEX approach was proposed to

deal with MCDM problems in which the weights of criteria were fully unknown. A green supplier example demonstrated the effectiveness of the proposed method, and showed that the results are reasonable and credible. The main contribution of this research is that the developed improved projection can overcome the limitations, as we discussed previously. Furthermore, the proposed method based on the projection would be appropriate to handle the MCDM problems in which the number of alternatives is less than the number of criteria and the weight information is completely unknown, which can be used to obtain credible and realistic results. However, the main limitation of the proposed method is that it cannot properly deal with some problems in which the number of alternatives is greater than the number of criteria. In the future, the related distance measures of SVNSs will be further investigated.

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