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An interval-valued hesitant fuzzy multigranulation rough set over two universes model for steam turbine fault diagnosis

Chao Zhang[a,b], Deyu Li[a,b,*], Yimin Mu[c], Dong Song[d]

[a] School of Computer and Information Technology, Shanxi University, Taiyuan 030006, Shanxi, China
[b] Key Laboratory of Computational Intelligence and Chinese Information Processing of Ministry of Education, Taiyuan 030006, Shanxi, China
[c] College of Mechanical and Electrical Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, Jiangsu, China
[d] Taiyuan Institute of China Coal Technology and Engineering Group, Taiyuan 030006, Shanxi, China

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ABSTRACT

In the field of mechanical engineering, steam turbine fault diagnosis is a difficult task for mechanical engineers who are confronted with challenges in dealing with copious amounts of uncertain information. Different mechanical engineers may have their own opinions about the system fault knowledge base that differs slightly from other mechanical engineers. Thus, to solve the problems presented by uncertain data analysis and group decision-making in steam turbine fault diagnosis, we propose a new rough set model that combines interval-valued hesitant fuzzy sets with multigranulation rough sets over two universes, called an interval-valued hesitant fuzzy multigranulation rough set over two universes. In the multigranulation framework, both basic definitions and some important properties of the proposed model are presented. Then, we develop a general approach to steam turbine fault diagnosis by using the proposed model. Lastly, an illustrative example is provided to verify the established approach and demonstrate its validity and applicability.

1. Introduction

Fault diagnosis technology has grown in importance in multiple areas of engineering, and issues with fault diagnosis technology have attracted much attention from researchers and practitioners. Until now, there are various data-based fault diagnosis techniques that have been developed, including expert systems [1], neural networks [2], and fuzzy approaches [3–5]. Taking advantages of human expertise and experiential knowledge, the expert system has been successfully utilized in steam turbine fault diagnosis. However, the limitations of acquiring expert knowledge and maintaining the rules database for steam turbines are revealed in expert system-based fault diagnosis. An additional potential weakness of this approach is that the rules database may include conflicting expert knowledge that may vary from case to case. This variation may preclude a general mathematical formulation that can be utilized for fault diagnosis in different fields. Another typical fault diagnosis technique, neural networks, addresses the shortcomings of expert systems by direct acquisition of fault knowledge from a set of training samples and exhibition of highly nonlinear input-output relationships. However, neural network-based fault diagnosis requires sufficient and compatible training samples to ensure proper training of the model.

* Corresponding author at: School of Computer and Information Technology, Shanxi University, No.92 Wucheng Road, Taiyuan 030006, China.
E-mail addresses: zhch3276152@163.com (C. Zhang), lidysxu@163.com (D. Li), minminwell@163.com (Y. Mu), tymkysd@126.com (D. Song).

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Free from the limitations of these models, the fuzzy approach can be used to sufficiently model the complicated relationship between faults and their features in steam turbine fault diagnosis. Additionally, many kinds of inherent uncertainties, such as inaccuracy, incompleteness, and inconsistency, are rampant in expressions of fault information available to engineers. Based on fuzzy set theory [6] and fuzzy logic [7], fuzzy approaches have been widely applied to fault diagnosis processes. The classical fuzzy set is limited by imperfect and uncertain information induced by several sources of vagueness, so several modifications and extensions have been introduced, including interval-valued fuzzy sets [8], type-2 fuzzy sets [9], intuitionistic fuzzy sets [10], and Pythagorean fuzzy sets [11]. These extensions overcome the limitations of classical fuzzy sets in different ways but are based on similar rationales.

Recently, considering that decision-makers are usually hesitant and irresolute for one thing or another when establishing a common membership degree, Torra and Narukawa [12] and Torra [13] introduced the concept of hesitant fuzzy sets, opening a new door for research on decision-making under hesitant environments. However, for real-life decision-making problems, the available information may not be sufficient for experts to provide their preferences with crisp values. Thus, a possible solution is to represent such preferences by interval values. As an extension form of hesitant fuzzy sets, Chen et al. [14] presented the concept of interval-valued hesitant fuzzy sets, which permits the membership degrees of an element to a given set are defined by several possible interval values. It is evident that interval-valued hesitant fuzzy sets can reflect individual's hesitancy more objectively than other widely developed fuzzy set approaches. Thereafter, many scholars have studied interval-valued hesitant fuzzy sets from different points of view and obtained many meaningful results [15–19]. Compared to classical fuzzy sets, since interval-valued hesitant fuzzy sets can better model insufficiency and hesitancy in available information, this approach can better handle the more uncertain information in fault diagnosis well, and provide mechanical engineers with an improved way to include their understanding about the system fault knowledge base.

Considering the above facts, as one of the commonly used multi-attribute decision-making methods, rough set theory [20] is an effective tool to acquire knowledge with its core concepts of lower and upper approximations. Moreover, rough set theory has demonstrated advantages in selecting significant attributes and explaining decisions according to attribute reductions and distinguishable relations. Over the past few years, two kinds of important generalizations of rough set theory, fuzzy rough sets [21] and multigranulation rough sets over two universes [22], have been developed in different application contexts. To effectively solve the problems of interval-valued hesitant fuzzy data analysis in fault diagnosis technology, we intend to integrate rough set theory with interval-valued hesitant fuzzy data analysis. We next itemize the necessities of combining those two theories with interval-valued hesitant fuzzy sets in detail.

(1) Rough sets and fuzzy sets, as two primary theories utilized for handling uncertain information in various information systems, are commonly viewed as correlative, but distinct and complementary. Determination of the best approach to generalize a rough set model to the fuzzy case is required for the development of rough set theory. In order to deal with various fuzzy information systems by utilizing rough set theory, Dubois and Prade [21] first constructed the concept of fuzzy rough sets. This work has been expanded by many recent studies of the fusion of hesitant fuzzy set theory with rough sets [23–26].

(2) In realistic rough set-based decision-making problems, when considering concept approximations and rules acquisitions in the background of multi-source information systems, in order to acquire knowledge efficiently, a reasonable way is to analyze these multi-source information systems directly rather than gathering each information systems as an entire information system. In this situation, the single-granulation rough set model exposes limitations that the computational times of knowledge discovery are long. Therefore, it is useful to describe a target concept through multiple binary relations according to a user's different requirements. From the perspective of granular computing [27], Qian et al. [28,29] introduced multigranulation rough sets in optimistic and pessimistic styles. Based on multigranulation rough sets model, Sun and Ma [22] further pointed out many kinds of decision-making information, such as the relationship between faults and their features in steam turbine fault diagnosis, involves two different types of objects, each of which belongs to a different universe of discourse, and the utilization of two universes and their interrelations may invalidate multigranulation rough sets model. Thus, by utilizing rough sets over two universes model [30–34], they proposed multigranulation rough sets over two universes. In light of the above, multigranulation rough sets over two universes could not only describe the decision-making information in different universes, but also synthesize multi-group expert preferences to form a final decision result by aggregating multiple binary relations. Thus, multigranulation rough sets over two universes can be seen as a relatively superior information fusion strategy under conditions of several different granulation levels.

On the basis of the above analysis, it is meaningful to utilize multigranulation rough sets over two universes model for multi-source information systems analysis. However, multigranulation rough sets over two universes can only handle crisp information systems and have limitations in processing various fuzzy information systems. Considering that interval-valued hesitant fuzzy sets can model insufficiency and hesitancy in decision-making information effectively, we aim to develop a new data analysis model in fault diagnosis techniques under the environment of interval-valued hesitant fuzzy information. Thus, to expand the application domain of multigranulation rough sets over two universes, through combining interval-valued hesitant fuzzy sets with multigranulation rough sets over two universes, it is necessary to construct an interval-valued hesitant fuzzy multigranulation rough set over two universes model and further explore its application in steam turbine fault diagnosis.

The paper is organized as follows. In Section 2, we briefly introduce interval-valued hesitant fuzzy sets, interval-valued hesitant fuzzy rough sets over two universes, and multigranulation rough sets over two universes. In Section 3, we present interval-valued hesitant fuzzy multigranulation rough sets over two universes and discuss relevant properties. Section 4 presents an analysis of the decision-making problems in steam turbine fault diagnosis. In Section 5, we give the application of the developed decision-making approach to a steam turbine fault diagnosis case and make some comparison analysis. In Section 6, the paper is concluded and some directions for future work are suggested.

2. Preliminaries

Some preliminary concepts are given in this section to explain our proposal, including interval-valued hesitant fuzzy sets, interval-valued hesitant fuzzy rough sets over two universes, and multigranulation rough sets over two universes. These will be utilized in the subsequent analysis.

2.1. Interval-valued hesitant fuzzy sets

Interval-valued hesitant fuzzy sets (IVHFSs) were originally introduced by Chen et al. [14]. By replacing crisp numbers with interval numbers in the environment of hesitant fuzzy sets, interval-valued hesitant fuzzy sets can be regarded as a more powerful structure in reflecting an expert's hesitance in expressing preferences over objects. Prior to the introduction of interval-valued hesitant fuzzy sets, we first present the concept of hesitant fuzzy sets.

Hesitant fuzzy sets (HFSs) were introduced by Torra and Narukawa [12] and Torra [13]. HFSs permit the membership degree of an element to a reference set expressed by several possible values between 0 and 1.

Definition 2.1 [13]**.** Let U be the universe of discourse, and a hesitant fuzzy set F on U is defined as a function $h_F(x)$ that returns a subset of [0, 1], which can be expressed as the following mathematical symbol:

$$F = \{\langle x, h_F(x)\rangle | x \in U\},$$

where $h_F(x)$ is a set of some different finite values in [0, 1], describing the possible membership degrees of the element $x \in U$ to the set F. For convenience, $h_F(x)$ is called a hesitant fuzzy element. The set of all hesitant fuzzy elements is called HFEs.

In what follows, we present the definition of interval-valued hesitant fuzzy sets introduced by Chen et al. [14].

Definition 2.2 [14]**.** Let U be the universe of discourse, and $D[0, 1]$ be the set of all closed subintervals of [0, 1], an interval-valued hesitant fuzzy set A on U can be expressed as the following mathematical symbol:

$$A = \{\langle x, h_A(x)\rangle | x \in U\},$$

where $h_A(x)$: $U \to D[0, 1]$ denotes all possible interval-valued membership degrees of the element $x \in U$ to the set A. For convenience, $h_A(x)$ is called an interval-valued hesitant fuzzy element, which reads $h_A(x) = \{\gamma | \gamma \in h_A(x)\}$, where $\gamma = [\gamma^L, \gamma^U]$ is an interval number, and $\gamma^L = \inf \gamma$ and $\gamma^U = \sup \gamma$ denote the lower and upper limits of γ, respectively. An interval-valued hesitant fuzzy element is the basic unit of an interval-valued hesitant fuzzy set. The set of all interval-valued hesitant fuzzy elements is called IVHFEs.

Suppose that U is the universe of discourse, then we denote the set of all interval-valued hesitant fuzzy sets on U as $IVHF(U)$. Therefore, $\forall A \in IVHF(U)$.

In what follows, we introduce some special interval-valued hesitant fuzzy sets.

(1) A is referred to as an empty interval-valued hesitant fuzzy set [24] if and only if $h_A(x) = \{[0, 0]\}$ for all x in U, the empty interval-valued hesitant fuzzy set is denoted by $\emptyset$ in this paper.
(2) A is referred to as a full interval-valued hesitant fuzzy set [24] if and only if $h_A(x) = \{[1, 1]\}$ for all x in U, the full interval-valued hesitant fuzzy set is denoted by U in this paper.

Example 2.1. Let $U = \{x_1, x_2, x_3\}$ be a universe set, $h_A(x_1) = \{[0.1, 0.2], [0.3, 0.4]\}$, $h_A(x_2) = \{[0.4, 0.5], [0.5, 0.6]\}$ and $h_A(x_3) = \{[0.3, 0.5], [0.6, 0.7]\}$ are three IVHFEs to a set A. Then, the interval-valued hesitant fuzzy set A can be expressed as follows:

$$A = \{\langle x_1, \{[0.1, 0.2], [0.3, 0.4]\}\rangle, \langle x_2, \{[0.4, 0.5], [0.5, 0.6]\}\rangle, \langle x_3, \{[0.3, 0.5], [0.6, 0.7]\}\rangle\}.$$

To compare the magnitude of different interval numbers and IVHFEs, we first present the methods of comparing two interval numbers introduced by Xu and Da [35].

Definition 2.3 [35]**.** Let $a = [a^L, a^U]$ and $b = [b^L, b^U]$ be two interval numbers with the following: (1) $a = b$ if $a^L = b^L$ and $a^U = b^U$; (2) $a + b = [a^L + b^L, a^U + b^U]$. Let $l_a = a^U - a^L$ and $l_b = b^U - b^L$, then the degree of possibility of $a \geq b$ is defined as

$$p(a \geq b) = \max\left\{1 - \max\left(\frac{b^U - a^L}{l_a + l_b}, 0\right), 0\right\}.$$

Similarly, the degree of possibility of $a \leq b$ is defined as

$$p(a \leq b) = \max\left\{1 - \max\left(\frac{a^U - b^L}{l_a + l_b}, 0\right), 0\right\}.$$

The above definition is proposed to compare and rank different interval numbers. In what follows, we further present the score function for IVHFEs.

Definition 2.4 [14]**.** For an IVHFE $h_A(x)$, $s(h_A(x)) = \frac{1}{l(h_A(x))}\sum_{\gamma\in h_A(x)}\gamma$, where $l(h_A(x))$ is the number of interval values in an IVHFE $h_A(x)$ and $s(h_A(x))$ is an interval value belonging to [0, 1]. For two IVHFEs, $h_A(x)$ and $h_B(x)$, if $s(h_A(x)) \geq s(h_B(x))$, then $h_A(x) \geq h_B(x)$.

It is clear that different IVHFEs have different number of interval values, and the interval values might be out of order. Thus, Chen et al. [14] introduced the following assumptions.

(1) All elements in each IVHFE $h_A(x)$ are arranged in increasing order by Definition 2.3. Suppose that $h_A^{\sigma(k)}(x)$ is the *kth* largest interval number in the IVHFE $h_A(x)$. Then, $h_A^{\sigma(k)}(x)$ is expressed as $h_A^{\sigma(k)}(x) = [h_A^{\sigma(k)L}(x), h_A^{\sigma(k)U}(x)]$, where $h_A^{\sigma(k)L}(x) = \inf h_A^{\sigma(k)}(x)$ and $h_A^{\sigma(k)U}(x) = \sup h_A^{\sigma(k)}(x)$ denote the lower and upper limits of $h_A^{\sigma(k)}(x)$, respectively.
(2) If for two IVHFEs $h_A(x)$ and $h_B(x)$, $l(h_A(x)) \neq l(h_B(x))$, then the two IVHFEs $h_A(x)$ and $h_B(x)$ should be of the same length $l = \max\{l(h_A(x)), l(h_B(x))\}$. If the number of elements in $h_A(x)$ is less than in $h_B(x)$, we should extend $h_A(x)$ by adding the maximum element until $h_A(x)$ and $h_B(x)$ are the same length.

Based on the above assumptions, Zhang et al. [24] defined some operational laws for interval-valued hesitant fuzzy sets.

Definition 2.5 [24]**.** Let U be the universe of discourse, $\forall A, B \in IVHF(U)$, then

(1) The complement of A is denoted by A^c such that $\forall x \in U$, $h_{A^c}(x) = \sim h_A(x) = \{[1 - h_A^{\sigma(k)U}(x), 1 - h_A^{\sigma(k)L}(x)] | k = 1, 2, \ldots l\}$.
(2) The intersection of A and B is denoted by $A \cap B$ such that $\forall x \in U$, $h_{A\cap B}(x) = h_A(x) \wedge h_B(x) = \{[h_A^{\sigma(k)L}(x) \wedge h_B^{\sigma(k)L}(x), h_A^{\sigma(k)U}(x) \wedge h_B^{\sigma(k)U}(x)] | k = 1, 2, \ldots l\}$.
(3) The union of A and B is denoted by $A \cup B$ such that $\forall x \in U$, $h_{A\cup B}(x) = h_A(x) \vee h_B(x) = \{[h_A^{\sigma(k)L}(x) \vee h_B^{\sigma(k)L}(x), h_A^{\sigma(k)U}(x) \vee h_B^{\sigma(k)U}(x)] | k = 1, 2, \ldots l\}$.

In the above definition, the operations ^{c}, $\cap$ and $\cup$ are defined on interval-valued hesitant fuzzy sets, respectively. Conversely, the operations $\sim$, $\wedge$ and $\vee$ are defined on corresponding interval-valued hesitant fuzzy elements, respectively. We next present the properties of the above operations.

Theorem 2.1 [24]**.** *Let U be the universe of discourse, and suppose that A, B and C are three interval-valued hesitant fuzzy sets. Then, the following properties are true:*

(1) *Double negation law:* $\sim(\sim h_A(x)) = h_A(x)$;
(2) *De Morgan's laws:* $\sim(h_A(x) \vee h_B(x)) = (\sim h_A(x)) \wedge (\sim h_B(x))$ *and* $\sim(h_A(x) \wedge h_B(x)) = (\sim h_A(x)) \vee (\sim h_B(x))$;
(3) *Commutativity:* $h_A(x) \vee h_B(x) = h_B(x) \vee h_A(x)$ *and* $h_A(x) \wedge h_B(x) = h_B(x) \wedge h_A(x)$;
(4) *Associativity:* $h_A(x) \vee (h_B(x) \vee h_C(x)) = (h_A(x) \vee h_B(x)) \vee h_C(x)$ *and* $h_A(x) \wedge (h_B(x) \wedge h_C(x)) = (h_A(x) \wedge h_B(x)) \wedge h_C(x)$;
(5) *Distributivity:* $h_A(x) \wedge (h_B(x) \vee h_C(x)) = (h_A(x) \wedge h_B(x)) \vee (h_A(x) \wedge h_C(x))$ *and* $h_A(x) \vee (h_B(x) \wedge h_C(x)) = (h_A(x) \vee h_B(x)) \wedge (h_A(x) \vee h_C(x))$.

Example 2.2. Suppose that A and B are two interval-valued hesitant fuzzy sets. Let $h_A(x) = \{[0.1, 0.2], [0.3, 0.4]\}$ and $h_B(x) = \{[0.4, 0.5], [0.5, 0.6]\}$ be two IVHFEs. Then, we obtain the complement, intersection and union as follows:

(1) $\sim h_A(x) = \{[1 - 0.2, 1 - 0.1], [1 - 0.4, 1 - 0.3]\} = \{[0.6, 0.7], [0.8, 0.9]\}$; similarly, $\sim h_B(x) = \{[0.4, 0.5], [0.5, 0.6]\}$;
(2) $h_A(x) \wedge h_B(x) = \{[0.1 \wedge 0.4, 0.2 \wedge 0.5], [0.3 \wedge 0.5, 0.4 \wedge 0.6]\} = \{[0.1, 0.2], [0.3, 0.4]\}$;
(3) $h_A(x) \vee h_B(x) = \{[0.1 \vee 0.4, 0.2 \vee 0.5], [0.3 \vee 0.5, 0.4 \vee 0.6]\} = \{[0.4, 0.5], [0.5, 0.6]\}$.

Definition 2.6 [24]**.** Let U be the universe of discourse, $\forall A, B \in IVHF(U)$, if $h_A(x) \preceq h_B(x)$ holds for each $x \in U$ such that $h_A(x) \preceq h_B(x) \Leftrightarrow h_A^{\sigma(k)L}(x) \leq h_B^{\sigma(k)L}(x), h_A^{\sigma(k)U}(x) \leq h_B^{\sigma(k)U}(x)$, $k = 1, 2, \ldots l$, then A is referred to as an interval-valued hesitant fuzzy subset of B, and is denoted by $A \subseteq B$. Note that $\subseteq$ is reflexive, transitive and antisymmetric on $IVHF(U)$.

2.2. Interval-valued hesitant fuzzy rough sets over two universes

In this subsection, we first present interval-valued hesitant fuzzy (IVHF) relations over two universes.

Definition 2.7 [24]**.** Let U, V be two universes of discourse. An interval-valued hesitant fuzzy relation R from U to V is defined by:

$$R = \{\langle (x, y), h_R(x, y)\rangle | (x, y) \in U \times V\},$$

where $h_R(x, y)$: $U \times V \to D[0, 1]$ is a set of interval values in $D[0, 1]$. The family of all IVHF relations on $U \times V$ is denoted by $IVHFR(U \times V)$.

According to the IVHF relation over two universes, we present the definition of interval-valued hesitant fuzzy rough sets over two universes as follows.

Definition 2.8 [24]**.** Let U, V be two universes of discourse, and $R \in IVHFR(U \times V)$, the pair (U, V, R) is called an interval-valued hesitant fuzzy approximation space over two universes. For any $A \in IVHF(V)$, the lower and upper approximations of A are defined by:

$$\underline{R}(A) = \left\{\left\langle x, h_{\underline{R}(A)}(x)\right\rangle | x \in U\right\};$$
$$\overline{R}(A) = \left\{\left\langle x, h_{\overline{R}(A)}(x)\right\rangle | x \in U\right\},$$

where $h_{\underline{R}(A)}(x) = \wedge_{y\in V}\{h_{R^c}(x, y) \vee h_A(y)\}$; $h_{\overline{R}(A)}(x) = \vee_{y\in V}\{h_R(x, y) \wedge h_A(y)\}$. The pair $(\underline{R}(A), \overline{R}(A))$ is referred to as an interval-valued hesitant fuzzy rough set over two universes of A with respect to (U, V, R), both $\underline{R}$ (A) and $\overline{R}(A)$ are interval-valued hesitant fuzzy sets, $\underline{R}$ and $\overline{R}$ are lower and upper IVHF rough approximation operators, respectively.

2.3. Multigranulation rough sets over two universes

According to two different approximation strategies, Sun and Ma [22] established two different multigranulation rough sets over two universes including optimistic and pessimistic ones.

Definition 2.9 [30]**.** Let U, V be two universes of discourse. R is a family of binary compatibility relations from U to V in terms of a binary mapping family F_i, where F_i: $U \to 2^V$, $u \mapsto \{v \in V | (u, v) \in R_i.\}$, $R_i \in R$, $i = 1, 2, \ldots, m$. Then, we call the ordered triple set (U, V, R) the multigranulation approximation space over two universes.

By Definition 2.9, the following definition gives the formal representation of optimistic and pessimistic multigranulation rough sets over two universes.

Definition 2.10 [22]**.** Let F and G be two binary mappings from universe U to V. For any $X \subseteq V$, the optimistic and pessimistic lower and upper multigranulation approximations with respect to (U, V, R) are defined by:

$$\underline{apr}^{O}_{F+G}(X) = \{x \in U | F(x) \subseteq X \vee G(x) \subseteq X\};$$
$$\overline{apr}^{O}_{F+G}(X) = \underline{apr}^{O}_{F+G}(X^c)^c;$$
$$\underline{apr}^{P}_{F+G}(X) = \{x \in U | F(x) \subseteq X \wedge G(x) \subseteq X\};$$
$$\overline{apr}^{P}_{F+G}(X) = \underline{apr}^{P}_{F+G}(X^c)^c.$$

The pair $(\underline{apr}^{O}_{F+G}(X), \overline{apr}^{O}_{F+G}(X))$ is referred to as an optimistic multigranulation rough set over two universes. Similarly, we call the pair $(\underline{apr}^{P}_{F+G}(X), \overline{apr}^{P}_{F+G}(X))$ a pessimistic multigranulation rough set over two universes.

In the above definition, the word "optimistic" means that in multiple independent granular structures, at least one granular structure must satisfy the inclusion condition between an equivalence class and a target concept. Conversely, the word "pessimistic" means that all granular structures must satisfy the inclusion condition between an equivalence class and a target concept.

3. IVHF multigranulation rough sets over two universes

In this section, based on constructive approach to interval-valued hesitant fuzzy rough sets over two universes, we extend interval-valued hesitant fuzzy relations over two universes into the background of multigranulation rough sets.

3.1. Optimistic IVHF multigranulation rough sets over two universes

Definition 3.1. Let U, V be two universes of discourse and $R_i \in IVHFR(U \times V)(i = 1, 2, \ldots, m)$ be m interval-valued hesitant fuzzy relations over $U \times V$, the pair (U, V, R_i) is called an interval-valued hesitant fuzzy multigranulation approximation space over two universes. For any $A \in IVHF(V)$, the optimistic IVHF multigranulation lower and upper approximations over two universes of A are defined by:

$$\underline{\sum_{i=1}^{m} R_i}^{O}(A) = \left\{\left\langle x, h_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\right\rangle | x \in U\right\};$$
$$\overline{\sum_{i=1}^{m} R_i}^{O}(A) = \left\{\left\langle x, h_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\right\rangle | x \in U\right\},$$

where $h_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \vee_{i=1}^{m} \wedge_{y\in V}\left\{h_{R_i^c}(x, y) \vee h_A(y)\right\}$; $h_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \wedge_{i=1}^{m} \vee_{y\in V}\left\{h_{R_i}(x, y) \wedge h_A(y)\right\}$.

The pair $(\underline{\sum_{i=1}^{m} R_i}^{O}(A), \overline{\sum_{i=1}^{m} R_i}^{O}(A))$ is an optimistic IVHF multigranulation rough set over two universes of A with respect to (U, V, R_i). Additionally, the IVHF multigranulation rough set over two universes will reduce to an IVHF rough set over two universes if $m = 1$.

Theorem 3.1. *Let U, V be two universes of discourse and $R_i \in IVHFR(U \times V)(i = 1, 2, \ldots, m)$ be m interval-valued hesitant fuzzy relations over $U \times V$. For any $A, A' \in IVHF(V)$, the optimistic IVHF multigranulation lower and upper approximations over two universes satisfy the following properties:*

(1) $\underline{\sum_{i=1}^{m} R_i}^{O}(A^c) = (\overline{\sum_{i=1}^{m} R_i}^{O}(A))^c$, $\overline{\sum_{i=1}^{m} R_i}^{O}(A^c) = (\underline{\sum_{i=1}^{m} R_i}^{O}(A))^c$;

(2) $A \subseteq A' \Rightarrow \underline{\sum_{i=1}^{m} R_i}^{O}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A')$, $A \subseteq A' \Rightarrow \overline{\sum_{i=1}^{m} R_i}^{O}(A) \subseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A')$;

(3) $\underline{\sum_{i=1}^{m} R_i}^{O}(A \cap A') = \underline{\sum_{i=1}^{m} R_i}^{O}(A) \cap \underline{\sum_{i=1}^{m} R_i}^{O}(A')$, $\overline{\sum_{i=1}^{m} R_i}^{O}(A \cup A') = \overline{\sum_{i=1}^{m} R_i}^{O}(A) \cup \overline{\sum_{i=1}^{m} R_i}^{O}(A')$;

(4) $\underline{\sum_{i=1}^{m} R_i}^{O}(A \cup A') \supseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A) \cup \underline{\sum_{i=1}^{m} R_i}^{O}(A')$, $\overline{\sum_{i=1}^{m} R_i}^{O}(A \cap A') \subseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A) \cap \overline{\sum_{i=1}^{m} R_i}^{O}(A')$.

Proof.

(1) For all $x \in U$, we have $\overline{\sum_{i=1}^{m} R_i}^{O}(A^c) = \{\langle x, \vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}(x, y) \vee h_{A^c}(y)\}\rangle | x \in U\} = \{\langle x, \vee_{i=1}^{m} \wedge_{y \in V}\{(\sim h_{R_i}(x, y)) \vee (\sim h_A(y))\}\rangle | x \in U\} = \{\langle x, \vee_{i=1}^{m} \wedge_{y \in V}\{\sim (h_{R_i}(x, y) \wedge h_A(y))\}\rangle | x \in U\} = \{\langle x, \sim (\wedge_{i=1}^{m} \vee_{y \in V}\{(h_{R_i}(x, y) \wedge h_A(y))\})\rangle | x \in U\} = (\overline{\sum_{i=1}^{m} R_i}^{O}(A))^c$. $\overline{\sum_{i=1}^{m} R_i}^{O}(A^c) = (\underline{\sum_{i=1}^{m} R_i}^{O}(A))^c$ is similarly obtained.

(2) Since $A \subseteq A'$, by Definition 2.6, we have $h_A(y) \preceq h_{A'}(y) \Leftrightarrow h_A^{\sigma(k)L}(y) \le h_{A'}^{\sigma(k)L}(y)$, $h_A^{\sigma(k)U}(y) \le h_{A'}^{\sigma(k)U}(y)$.
Thus, it follows that $\vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}^{\sigma(k)L}(x, y) \vee h_A^{\sigma(k)L}(y)\} \le \vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}^{\sigma(k)L}(x, y) \vee h_{A'}^{\sigma(k)L}(y)\}$ and $\vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}^{\sigma(k)U}(x, y) \vee h_A^{\sigma(k)U}(y)\} \le \vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}^{\sigma(k)U}(x, y) \vee h_{A'}^{\sigma(k)U}(y)\}$.
Therefore, we have $A \subseteq A' \Rightarrow \underline{\sum_{i=1}^{m} R_i}^{O}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A')$. $A \subseteq A' \Rightarrow \overline{\sum_{i=1}^{m} R_i}^{O}(A) \subseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A')$ is obtained in an identical fashion.

(3) For all $x \in U$, we have $\underline{\sum_{i=1}^{m} R_i}^{O}(A \cap A') = \{\langle x, \vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}(x, y) \vee h_{A \cap A'}(y)\}\rangle | x \in U\} = \{\langle x, \vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}(x, y) \vee (h_A(y) \wedge h_{A'}(y))\}\rangle | x \in U\} = \{\langle x, [\vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}^{\sigma(k)L}(x, y) \vee (h_A^{\sigma(k)L}(y) \wedge h_{A'}^{\sigma(k)L}(y))\}, \vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}^{\sigma(k)U}(x, y) \vee (h_A^{\sigma(k)U}(y) \wedge h_{A'}^{\sigma(k)U}(y))\}]\rangle | x \in U\} = \{\langle x, [\vee_{i=1}^{m} \wedge_{y \in V}\{(h_{R_i^c}^{\sigma(k)L}(x, y) \vee h_A^{\sigma(k)L}(y)) \wedge (h_{R_i^c}^{\sigma(k)L}(x, y) \vee h_{A'}^{\sigma(k)L}(y))\}, \vee_{i=1}^{m} \wedge_{y \in V}\{(h_{R_i^c}^{\sigma(k)U}(x, y) \vee h_A^{\sigma(k)U}(y)) \wedge (h_{R_i^c}^{\sigma(k)U}(x, y) \vee h_{A'}^{\sigma(k)U}(y))\}]\rangle | x \in U\} = \{\langle x, h_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\rangle | x \in U\} \wedge \{\langle x, h_{\underline{\sum_{i=1}^{m} R_i}^{O}(A')}(x)\rangle | x \in U\} = \underline{\sum_{i=1}^{m} R_i}^{O}(A) \cap \underline{\sum_{i=1}^{m} R_i}^{O}(A')$ Similarly, $\overline{\sum_{i=1}^{m} R_i}^{O}(A \cup A') = \overline{\sum_{i=1}^{m} R_i}^{O}(A) \cup \overline{\sum_{i=1}^{m} R_i}^{O}(A')$ is obtained.

(4) Based on above conclusions, it is easy to obtain $\underline{\sum_{i=1}^{m} R_i}^{O}(A \cup A') \supseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A) \cup \underline{\sum_{i=1}^{m} R_i}^{O}(A')$ and $\overline{\sum_{i=1}^{m} R_i}^{O}(A \cap A') \subseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A) \cap \overline{\sum_{i=1}^{m} R_i}^{O}(A')$. □

In this theorem, (1) indicates the complement of optimistic interval-valued hesitant fuzzy multigranulation rough sets over two universes; (2) indicates the monotone of optimistic interval-valued hesitant fuzzy multigranulation rough sets over two universes with respect to a variety of interval-valued hesitant fuzzy targets; (3) and (4) indicate the multiplication and addition of optimistic interval-valued hesitant fuzzy multigranulation rough sets over two universes.

Theorem 3.2. *Let U, V be two universes of discourse and $R_i, R_i' \in IVHFR(U \times V)(i = 1, 2, \ldots, m)$ be two interval-valued hesitant fuzzy relations over $U \times V$. If $R_i \subseteq R_i'$, for any $A \in IVHF(V)$, the following properties are true.*

(1) $\underline{\sum_{i=1}^{m} R_i'}^{O}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A)$, *for all* $A \in IVHF(V)$;

(2) $\overline{\sum_{i=1}^{m} R_i'}^{O}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A)$, *for all* $A \in IVHF(V)$.

Proof. Since $R_i \subseteq R_i'$, according to Definition 2.6, we have $h_{R_i^c}^{\sigma(k)L}(x, y) \ge h_{R_i'^c}^{\sigma(k)L}(x, y)$ and $h_{R_i^c}^{\sigma(k)U}(x, y) \ge h_{R_i'^c}^{\sigma(k)U}(x, y)$ for all $(x, y) \in (U \times V)$. Therefore, it follows that $\underline{\sum_{i=1}^{m} R_i}^{O}(A) = \{\langle x, \vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}(x, y) \vee h_A(y)\}\rangle | x \in U\} = \{\langle x, [\vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}^{\sigma(k)L}(x, y) \vee h_A^{\sigma(k)L}(y)\}, \vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i^c}^{\sigma(k)U}(x, y) \vee h_A^{\sigma(k)U}(y)\}]\rangle | x \in U\} \ge \{\langle x, [\vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i'^c}^{\sigma(k)L}(x, y) \vee h_A^{\sigma(k)L}(y)\}, \vee_{i=1}^{m} \wedge_{y \in V}\{h_{R_i'^c}^{\sigma(k)U}(x, y) \vee h_A^{\sigma(k)U}(y)\}]\rangle | x \in U\} = \underline{\sum_{i=1}^{m} R_i'}^{O}(A)$ Hence, we have $\underline{\sum_{i=1}^{m} R_i'}^{O}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A)$. Similarly, $\overline{\sum_{i=1}^{m} R_i'}^{O}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A)$ is obtained. □

In this theorem, (1) and (2) indicate the lower and upper approximations in optimistic interval-valued hesitant fuzzy multigranulation rough sets over two universes are monotonic with respect to the monotonic forms of multiple interval-valued hesitant fuzzy relations.

3.2. Pessimistic IVHF multigranulation rough sets over two universes

Definition 3.2. Let *U*, *V* be two universes of discourse and $R_i \in IVHFR(U \times V)(i = 1, 2, \ldots, m)$ be *m* interval-valued hesitant fuzzy relations over $U \times V$, the pair (U, V, R_i) is called an interval-valued hesitant fuzzy multigranulation approximation space over two universes. For any $A \in IVHF(V)$, the pessimistic IVHF multigranulation lower and upper approximations over two universes of A are defined by:

$$\underline{\sum_{i=1}^{m} R_i}^{P}(A) = \left\{\left\langle x, h_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\right\rangle \middle| x \in U\right\};$$

$$\overline{\sum_{i=1}^{m} R_i}^{P}(A) = \left\{\left\langle x, h_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\right\rangle \middle| x \in U\right\},$$

where $h_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \wedge_{i=1}^{m} \wedge_{y\in V} \left\{h_{R_i^c}(x,y) \vee h_A(y)\right\}$; $h_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \vee_{i=1}^{m} \vee_{y\in V} \left\{h_{R_i}(x,y) \wedge h_A(y)\right\}$.

The pair $\left(\underline{\sum_{i=1}^{m} R_i}^{P}(A), \overline{\sum_{i=1}^{m} R_i}^{P}(A)\right)$ is a pessimistic IVHF multigranulation rough set over two universes of A with respect to (U, V, R_i).

Theorem 3.3. *Let U, V be two universes of discourse and $R_i \in IVHFR(U \times V)(i = 1, 2, \ldots, m)$ be m interval-valued hesitant fuzzy relations over $U \times V$. For any $A, A' \in IVHF(V)$, the pessimistic IVHF multigranulation lower and upper approximations over two universes satisfy the following properties:*

(1) $\underline{\sum_{i=1}^{m} R_i}^{P}(A^c) = (\overline{\sum_{i=1}^{m} R_i}^{P}(A))^c$, $\overline{\sum_{i=1}^{m} R_i}^{P}(A^c) = (\underline{\sum_{i=1}^{m} R_i}^{P}(A))^c$;
(2) $A \subseteq A' \Rightarrow \underline{\sum_{i=1}^{m} R_i}^{P}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{P}(A')$, $A \subseteq A' \Rightarrow \overline{\sum_{i=1}^{m} R_i}^{P}(A) \subseteq \overline{\sum_{i=1}^{m} R_i}^{P}(A')$;
(3) $\underline{\sum_{i=1}^{m} R_i}^{P}(A \cap A') = \underline{\sum_{i=1}^{m} R_i}^{P}(A) \cap \underline{\sum_{i=1}^{m} R_i}^{P}(A')$, $\overline{\sum_{i=1}^{m} R_i}^{P}(A \cup A') = \overline{\sum_{i=1}^{m} R_i}^{P}(A) \cup \overline{\sum_{i=1}^{m} R_i}^{P}(A')$;
(4) $\underline{\sum_{i=1}^{m} R_i}^{P}(A \cup A') \supseteq \underline{\sum_{i=1}^{m} R_i}^{P}(A) \cup \underline{\sum_{i=1}^{m} R_i}^{P}(A')$, $\overline{\sum_{i=1}^{m} R_i}^{P}(A \cap A') \subseteq \overline{\sum_{i=1}^{m} R_i}^{P}(A) \cap \overline{\sum_{i=1}^{m} R_i}^{P}(A')$.

In this theorem, (1) indicates the complement of pessimistic interval-valued hesitant fuzzy multigranulation rough sets over two universes; (2) indicates the monotone of pessimistic interval-valued hesitant fuzzy multigranulation rough sets over two universes with respect to a variety of interval-valued hesitant fuzzy targets; (3) and (4) indicate the multiplication and addition of pessimistic interval-valued hesitant fuzzy multigranulation rough sets over two universes.

Theorem 3.4. *Let U, V be two universes of discourse and $R_i, R'_i \in IVHFR(U \times V)(i = 1, 2, \ldots, m)$ be two interval-valued hesitant fuzzy relations over $U \times V$. If $R_i \subseteq R'_i$, for any $A \in IVHF(V)$, the following properties are true.*

(1) $\underline{\sum_{i=1}^{m} R'_i}^{P}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{P}(A)$, *for all* $A \in IVHF(V)$;
(2) $\overline{\sum_{i=1}^{m} R'_i}^{P}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{P}(A)$, *for all* $A \in IVHF(V)$.

In this theorem, (1) and (2) indicate the lower and upper approximations in pessimistic interval-valued hesitant fuzzy multigranulation rough sets over two universes are monotonic with respect to the monotonic forms of multiple interval-valued hesitant fuzzy relations.

3.3. Relationships between optimistic and pessimistic IVHF multigranulation rough sets over two universes

Theorem 3.5. *Let U, V be two universes of discourse and $R_i \in IVHFR(U \times V)(i = 1, 2, \ldots, m)$ be m interval-valued hesitant fuzzy relations over $U \times V$. For any $A \in IVHF(V)$, the optimistic and pessimistic IVHF multigranulation lower and upper approximations over two universes satisfy the following properties:*

(1) $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A)$;
(2) $\overline{\sum_{i=1}^{m} R_i}^{P}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A)$.

Proof. For any $x \in U$, $\underline{\sum_{i=1}^{m} R_i}^{O}(A) = \{\langle x, \vee_{i=1}^{m} \wedge_{y\in V}\{h_{R_i^c}(x,y) \vee h_A(y)\}\rangle | x \in U\} \geq \{\langle x, \wedge_{i=1}^{m} \wedge_{y\in V}\{h_{R_i^c}(x,y) \vee h_A(y)\}\rangle | x \in U\} = \underline{\sum_{i=1}^{m} R_i}^{P}(A)$. $\overline{\sum_{i=1}^{m} R_i}^{P}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A)$ is similarly obtained. □

From Theorem 3.5, it is noted that the pessimistic interval-valued hesitant fuzzy multigranulation lower approximation is included in the optimistic interval-valued hesitant fuzzy multigranulation lower approximation, while the optimistic interval-valued hesitant fuzzy multigranulation upper approximation is included in the pessimistic interval-valued hesitant fuzzy multigranulation upper approximation.

4. Steam turbine fault diagnosis approach

In this section, we present a novel method to the decision-making problem of steam turbine fault diagnosis with the aid of IVHF multigranulation rough sets over two universes. Specifically, some key points of the established decision-making approach are summarized in the following subsections.

4.1. Application model

In the domain of electric power production and management, steam turbine generator units are a significant piece of equipment in the power industry. One of the common fault types of a steam turbine generator unit is vibration, which can be triggered by many factors, such as mechanical structure, load, and vacuum degree. To deal with this type of mechanical fault diagnosis, failures of steam turbine generator units have been specified based on the different amplitude ratios of vibration signals in various frequency ranges. Moreover, many researchers have established the relationship between the cause and fault phenomena of steam turbine generator units, and numerous studies have been conducted based on intelligent computational and mathematical methods, such as expert systems, neural networks, and fuzzy approaches. However, few studies have addressed the situation where the vibration signal amplitude ratio value is in the background of hesitant and interval ambiguity. By utilizing the model of IVHF multigranulation rough sets over two universes, we can not only express the vibration signal amplitude ratio value by using interval-valued hesitant fuzzy elements, but can also solve group decision-making problems by providing optimistic and pessimistic strategies.

There are two kinds of objections related to steam turbine fault diagnosis, namely, the fault pattern sets and the frequency range sets of the steam turbine generator unit. Therefore, we let $U = \{x_1, x_2, \dots, x_j\}$ be a set of fault patterns and $V = \{y_1, y_2, \dots, y_k\}$ be a set of frequency ranges. Let $R_i \in IVHFR(U \times V)(i = 1, 2, \dots, m)$ be m interval-valued hesitant fuzzy relations over $U \times V$, which reflects the steam turbine fault diagnosis knowledge base with interval-valued hesitant fuzzy information given by m experts. We also let $A \in IVHF(V)$ be the fault testing sample. Then, we obtain an interval-valued hesitant fuzzy decision information system (U, V, R_i, A) of a typical steam turbine fault diagnosis procedure.

Next, we present a decision-making approach for the above-mentioned problem by using IVHF multigranulation rough sets over two universes. First, according to Definitions 3.1 and 3.2, we separately compute lower and upper approximations of optimistic and pessimistic IVHF multigranulation rough sets over two universes of A. That is, we obtain the sets $\underline{\sum_{i=1}^m R_i}^O(A)$, $\overline{\sum_{i=1}^m R_i}^O(A)$, $\underline{\sum_{i=1}^m R_i}^P(A)$ and $\overline{\sum_{i=1}^m R_i}^P(A)$. Then, according to the operational laws presented in [24]:

$$h_A(x) \oplus h_B(x) = \{[h_A^{\sigma(k)L}(x) + h_B^{\sigma(k)L}(x) - h_A^{\sigma(k)L}(x)h_B^{\sigma(k)L}(x), h_A^{\sigma(k)U}(x) + h_B^{\sigma(k)U}(x) - h_A^{\sigma(k)U}(x)h_B^{\sigma(k)U}(x)]\},$$

we further obtain the sets $\underline{\sum_{i=1}^m R_i}^O(A) \oplus \overline{\sum_{i=1}^m R_i}^O(A)$ and $\underline{\sum_{i=1}^m R_i}^P(A) \oplus \overline{\sum_{i=1}^m R_i}^P(A)$. Based on Definition 2.4, we can calculate the score function values of interval-valued hesitant fuzzy elements belonging to $\underline{\sum_{i=1}^m R_i}^O(A) \oplus \overline{\sum_{i=1}^m R_i}^O(A)$ and $\underline{\sum_{i=1}^m R_i}^P(A) \oplus \overline{\sum_{i=1}^m R_i}^P(A)$. Based on a previously established decision-making strategy [32], we then present the decision rules for steam turbine fault diagnosis using the proposed rough set model. First, to facilitate the decision-making process, we denote:

$$T_1 = \left\{ i \middle| \max_{x_i \in U} \left\{ \underline{\sum_{i=1}^m R_i}^O (A)(x_i) \oplus \overline{\sum_{i=1}^m R_i}^O (A)(x_i) \right\} \right\},$$

$$T_2 = \left\{ j \middle| \max_{x_j \in U} \left\{ \underline{\sum_{i=1}^m R_i}^P (A)\left(x_j\right) \oplus \overline{\sum_{i=1}^m R_i}^P (A)\left(x_j\right) \right\} \right\},$$

$$T_3 = \left\{ k \middle| \max_{x_k \in U} \left\{ \left(\underline{\sum_{i=1}^m R_i}^O (A)(x_k) \oplus \overline{\sum_{i=1}^m R_i}^O (A)(x_k) \right) \oplus \left(\underline{\sum_{i=1}^m R_i}^P (A)(x_k) \oplus \overline{\sum_{i=1}^m R_i}^P (A)(x_k) \right) \right\} \right\}.$$

From the viewpoint of decision-making, optimistic multigranulation rough sets are based on the “seeking common ground while reserving differences” (SCRD) strategy, which implies that one reserves both common decisions and inconsistent decisions at the same time. Thus, this opinion can be viewed as a risk-seeking decision strategy. In contrast, pessimistic multigranulation rough sets are based on the “seeking common ground while eliminating differences” (SCED) strategy, this strategy indicates that one reserves common decisions while deleting inconsistent decisions. Hence, this opinion can be seen as a risk-averse decision strategy. According to the above decision rules, T_1 is the optimistic decision-making criterion, T_2 is the pessimistic decision-making criterion, and T_3 is the weighted decision-making criterion of T_1 and T_2 with the weighted value 0.5. Based on the above definitions, the decision rules can be presented as follows:

(1) If $T_1 \cap T_2 \cap T_3 \neq \emptyset$, then $x_i (i \in T_1 \cap T_2 \cap T_3)$ is the determined fault pattern;
(2) If $T_1 \cap T_2 \cap T_3 = \emptyset$ and $T_1 \cap T_2 \neq \emptyset$, then $x_i (i \in T_1 \cap T_2)$ is the determined fault pattern. Otherwise, if $T_1 \cap T_2 \cap T_3 = \emptyset$ and $T_1 \cap T_2 = \emptyset$, then $x_i (i \in T_3)$ is the determined fault pattern.

4.2. Algorithm for steam turbine fault diagnosis using IVHF multigranulation rough sets over two universes

We present an algorithm for steam turbine fault diagnosis by utilizing IVHF multigranulation rough sets over two universes:

Algorithm 1 Steam turbine fault diagnosis based on IVHF multigranulation rough sets over two universes.

Require: The relation between the universes U and V provided by an expert (U, V, R_i) and a fault testing sample A.
Ensure: The determined fault pattern.

1: Calculate $\underline{\sum_{i=1}^{m} R_i}^O(A)$, $\overline{\sum_{i=1}^{m} R_i}^O(A)$, $\underline{\sum_{i=1}^{m} R_i}^P(A)$ and $\overline{\sum_{i=1}^{m} R_i}^P(A)$, respectively;
2: Calculate $\underline{\sum_{i=1}^{m} R_i}^O(A) \oplus \overline{\sum_{i=1}^{m} R_i}^O(A)$ and $\underline{\sum_{i=1}^{m} R_i}^P(A) \oplus \overline{\sum_{i=1}^{m} R_i}^P(A)$, respectively;
3: Determine the score function values for the sets $\underline{\sum_{i=1}^{m} R_i}^O(A) \oplus \overline{\sum_{i=1}^{m} R_i}^O(A)$ and $\underline{\sum_{i=1}^{m} R_i}^P(A) \oplus \overline{\sum_{i=1}^{m} R_i}^P(A)$, respectively;
4: Compute T_1, T_2, T_3, $T_1 \cap T_2 \cap T_3$ and $T_1 \cap T_2$, and confirm the determined fault pattern.

Table 1
Knowledge of system fault given by expert 1.

R_1	y_1	y_2	y_3	y_4	y_5
x_1	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.85, 0.88], [0.85, 0.93]}
x_2	{[0, 0]}	{[0.27, 0.32], [0.28, 0.33]}	{[0.08, 0.1], [0.09, 0.13]}	{[0.54, 0.6], [0.56, 0.7]}	{[0, 0]}
x_3	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.3, 0.58], [0.4, 0.6]}
x_4	{[0.09, 0.1], [0.1, 0.12]}	{[0.78, 0.8], [0.8, 0.82]}	{[0, 0]}	{[0.08, 0.1], [0.1, 0.11]}	{[0, 0]}
x_5	{[0.09, 0.11], [0.11, 0.12]}	{[0.09, 0.1], [0.1, 0.11]}	{[0.08, 0.1], [0.1, 0.12]}	{[0.09, 0.11], [0.11, 0.12]}	{[0.18, 0.2], [0.19, 0.21]}
x_6	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.18, 0.2], [0.2, 0.22]}
x_7	{[0, 0]}	{[0, 0]}	{[0.08, 0.11], [0.1, 0.12]}	{[0.86, 0.89], [0.88, 0.93]}	{[0, 0]}
x_8	{[0, 0]}	{[0.27, 0.3], [0.29, 0.32]}	{[0.08, 0.1], [0.09, 0.12]}	{[0.54, 0.6], [0.57, 0.62]}	{[0, 0]}
x_9	{[0.85, 0.9], [0.89, 0.93]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
x_{10}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
R_1	y_6	y_7	y_8	y_9	
x_1	{[0.03, 0.05], [0.04, 0.06]}	{[0.04, 0.06], [0.05, 0.07]}	{[0, 0]}	{[0, 0]}	
x_2	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.07, 0.12], [0.08, 0.14]}	
x_3	{[0.4, 0.6], [0.45, 0.62]}	{[0.08, 0.12], [0.1, 0.13]}	{[0, 0]}	{[0, 0]}	
x_4	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	
x_5	{[0.08, 0.12], [0.1, 0.13]}	{[0.08, 0.12], [0.1, 0.13]}	{[0.08, 0.1], [0.1, 0.12]}	{[0.08, 0.1], [0.1, 0.12]}	
x_6	{[0.12, 0.15], [0.15, 0.17]}	{[0.37, 0.4], [0.4, 0.45]}	{[0, 0]}	{[0.22, 0.25], [0.24, 0.28]}	
x_7	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	
x_8	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	
x_9	{[0, 0]}	{[0, 0]}	{[0.08, 0.1], [0.09, 0.12]}	{[0, 0]}	
x_{10}	{[0.77, 0.8], [0.8, 0.83]}	{[0.19, 0.2], [0.2, 0.23]}	{[0, 0]}	{[0, 0]}	

5. Case study

In this section, we illustrate the efficiency of our proposed algorithm by using a steam turbine fault diagnosis problem with IVHFS information. To enhance the accuracy and reliability of the steam turbine fault diagnosis procedure, we aim to solve the problem in the context of group decision-making. In group decision-making, each mechanical engineer might have differing thoughts about the system fault knowledge base, but should also have the common goal to reach a final consensus. Moreover, the knowledge of system faults has been modeled and studied by many scholars utilizing sets such as interval-valued intuitionistic fuzzy sets [3], vague sets [4], and single-valued neutrosophic sets [5]. To express the aforementioned hesitant and interval ambiguity, based on existing knowledge of system faults and following detailed discussions about several previous cases of steam turbine fault diagnosis with mechanical engineers, we obtained the required knowledge of the system fault dataset with respect to this paper from a state-owned scientific and technological enterprise.

Let $U = \{x_1, x_2, \ldots, x_{10}\}$ be a set of fault patterns, where $x_j (j = 1, 2, \ldots, 10)$ represents unbalance, pneumatic force couple, offset center, oil-membrane oscillation, radial impact friction of rotor, symbiosis looseness, damage of antithrust bearing, surge, looseness of bearing block, and nonuniform bearing stiffness. We also let $V = \{y_1, y_2, \ldots, y_k\}$ be a set of frequency ranges, where $y_k (k = 1, 2, \ldots, 9)$ stands for frequency ranges for different spectra: $C_1(0.01 - 0.39f)$, $C_2(0.4 - 0.49f)$, $C_3(0.5f)$, $C_4(0.51 - 0.99f)$, $C_5(f)$, $C_6(2f)$, $C_7(3 - 5f)$, $C_8(\text{odd times of } f)$, and $C_9(\text{high frequency} > 5f)$. The steam turbine fault diagnosis knowledge base with IVHFS information is presented in Tables 1,2 and 3. The fault testing sample is also given as follows. We aim to seek the determined fault pattern by utilizing the proposed model.

In steam turbine fault diagnosis, assume that we take a fault testing sample, which is represented by the following IVHFS information:

$$A = \{\langle y_1, \{[0.39, 0.4], [0.39, 0.41]\}\rangle, \langle y_2, \{[0.07, 0.09], [0.08, 0.1]\}\rangle, \langle y_3, \{[0, 0]\}\rangle, \langle y_4, \{[0.06, 0.07], [0.06, 0.08]\}\rangle,$$
$$\langle y_5, \{[0, 0]\}\rangle, \langle y_6, \{[0.12, 0.14], [0.13, 0.15]\}\rangle, \langle y_7, \{[0, 0]\}\rangle, \langle y_8, \{[0, 0]\}\rangle, \langle y_9, \{[0.33, 0.36], [0.34, 0.37]\}\rangle\}.$$

Next, we calculate lower and upper approximations of optimistic and pessimistic IVHF multigranulation rough sets over two universes of A. $\underline{\sum_{i=1}^{3} R_i}^O(A) = \{\langle x_1, \{[0.07, 0.15], [0.12, 0.15]\}\rangle, \langle x_2, \{[0.3, 0.45], [0.4, 0.46]\}\rangle, \langle x_3, \{[0.38, 0.6], [0.42, 0.6]\}\rangle,. \langle x_4, \{[0.18, 0.22], [0.2, 0.22]\}\rangle, \langle x_5, \{[0.79, 0.82], [0.81, 0.82]\}\rangle, \langle x_6, \{[0.55, 0.63], [0.6,$

Table 2
Knowledge of system fault given by expert 2.

R_2	y_1	y_2	y_3	y_4	y_5
x_1	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.85, 0.93]}
x_2	{[0, 0]}	{[0.28, 0.31]}	{[0.09, 0.11], [0.1, 0.13]}	{[0.55, 0.7]}	{[0, 0]}
x_3	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.3, 0.58]}
x_4	{[0.09, 0.11], [0.1, 0.12]}	{[0.78, 0.82]}	{[0, 0]}	{[0.08, 0.11]}	{[0, 0]}
x_5	{[0.09, 0.12]}	{[0.09, 0.1], [0.1, 0.11]}	{[0.08, 0.12]}	{[0.09, 0.12]}	{[0.18, 0.19], [0.18, 0.21]}
x_6	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.18, 0.22]}
x_7	{[0, 0]}	{[0, 0]}	{[0.08, 0.12]}	{[0.86, 0.88], [0.86, 0.93]}	{[0, 0]}
x_8	{[0, 0]}	{[0.27, 0.32]}	{[0.08, 0.11], [0.09, 0.12]}	{[0.54, 0.62]}	{[0, 0]}
x_9	{[0.85, 0.93]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
x_{10}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}

R_2	y_6	y_7	y_8	y_9
x_1	{[0.03, 0.06], [0.04, 0.07]}	{[0.04, 0.07]}	{[0, 0]}	{[0, 0]}
x_2	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.08, 0.13]}
x_3	{[0.4, 0.58], [0.42, 0.62]}	{[0.08, 0.1], [0.1, 0.13]}	{[0, 0]}	{[0, 0]}
x_4	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
x_5	{[0.08, 0.13]}	{[0.08, 0.12], [0.1, 0.13]}	{[0.08, 0.12]}	{[0.08, 0.09], [0.1, 0.12]}
x_6	{[0.12, 0.16], [0.14, 0.17]}	{[0.37, 0.45]}	{[0, 0]}	{[0.22, 0.26], [0.24, 0.28]}
x_7	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
x_8	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
x_9	{[0, 0]}	{[0, 0]}	{[0.08, 0.12]}	{[0, 0]}
x_{10}	{[0.77, 0.79], [0.79, 0.83]}	{[0.19, 0.23]}	{[0, 0]}	{[0, 0]}

Table 3
Knowledge of system fault given by expert 3.

R_3	y_1	y_2	y_3	y_4	y_5
x_1	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.85, 0.9], [0.88, 0.93]}
x_2	{[0, 0]}	{[0.28, 0.3], [0.29, 0.32]}	{[0.09, 0.12]}	{[0.55, 0.65], [0.6, 0.7]}	{[0, 0]}
x_3	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.3, 0.55], [0.31, 0.58]}
x_4	{[0.09, 0.11]}	{[0.78, 0.81], [0.8, 0.82]}	{[0, 0]}	{[0.08, 0.1], [0.8, 0.11]}	{[0, 0]}
x_5	{[0.09, 0.1], [0.1, 0.12]}	{[0.09, 0.11]}	{[0.08, 0.09], [0.08, 0.12]}	{[0.09, 0.1], [0.1, 0.12]}	{[0.18, 0.21]}
x_6	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.18, 0.2], [0.18, 0.22]}
x_7	{[0, 0]}	{[0, 0]}	{[0.08, 0.1], [0.1, 0.12]}	{[0.86, 0.93]}	{[0, 0]}
x_8	{[0, 0]}	{[0.27, 0.31], [0.29, 0.32]}	{[0.08, 0.12]}	{[0.54, 0.58], [0.55, 0.62]}	{[0, 0]}
x_9	{[0.85, 0.88], [0.88, 0.93]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
x_{10}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}

R_3	y_6	y_7	y_8	y_9
x_1	{[0.03, 0.06]}	{[0.04, 0.05], [0.05, 0.07]}	{[0, 0]}	{[0, 0]}
x_2	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0.08, 0.1], [0.1, 0.13]}
x_3	{[0.4, 0.62]}	{[0.08, 0.13]}	{[0, 0]}	{[0, 0]}
x_4	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
x_5	{[0.08, 0.1], [0.09, 0.13]}	{[0.08, 0.13]}	{[0.08, 0.1], [0.1, 0.12]}	{[0.08, 0.12]}
x_6	{[0.12, 0.17]}	{[0.36, 0.44], [0.37, 0.45]}	{[0, 0]}	{[0.22, 0.28]}
x_7	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
x_8	{[0, 0]}	{[0, 0]}	{[0, 0]}	{[0, 0]}
x_9	{[0, 0]}	{[0, 0]}	{[0.08, 0.1], [0.09, 0.12]}	{[0, 0]}
x_{10}	{[0.77, 0.83]}	{[0.19, 0.21], [0.2, 0.23]}	{[0, 0]}	{[0, 0]}

0.64]]}⟩, $\langle x_7$, {[0.07, 0.14], [0.12, 0.14]}⟩, . .$\langle x_8$, {[0.38, 0.46], [0.42, 0.46]}⟩, $\langle x_9$, {[0.39, 0.4], [0.39, 0.41]}⟩, $\langle x_{10}$, {[0.17, 0.23], [0.21, 0.23]}⟩}; $\overline{\sum_{i=1}^{3} R_i}^{O}(A) = \{\langle x_1, \{[0.03, 0.05], [0.03, 0.06]\}\rangle, \langle x_2, \{[0.07, 0.12], [0.08, 0.14]\}\rangle, \langle x_3, \{[0.12, 0.14], [0.13, 0.15]\}\rangle, .\langle x_4$, {[0.09, 0.1], [0.09, 0.11]}⟩, $\langle x_5$, {[0.09, 0.12], [0.1, 0.13]}⟩, $\langle x_6$, {[0.22, 0.25], [0.22, 0.28]}⟩, $\langle x_7$, {[0.06, 0.07], [0.06, 0.08]}⟩, . .$\langle x_8$, {[0.07, 0.09], [0.08, 0.1]}⟩, $\langle x_9$, {[0.39, 0.4], [0.39, 0.41]}⟩, $\langle x_{10}$, {[0.12, 0.14], [0.13, 0.15]}⟩}; $\underline{\sum_{i=1}^{3} R_i}^{P}(A) = \{\langle x_1, \{[0.07, 0.12], [0.07, 0.15]\}\rangle, \langle x_2, \{[0.3, 0.4], [0.3, 0.45]\}\rangle, \langle x_3, \{[0.38, 0.55], [0.38, 0.6]\}\rangle, .\langle x_4$, {[0.18, 0.2], [0.18, 0.22]}⟩, $\langle x_5$, {[0.79, 0.81], [0.79, 0.82]}⟩, $\langle x_6$, {[0.55, 0.6], [0.55, 0.63]}⟩, . .$\langle x_7$, {[0.07, 0.12], [0.07, 0.14]}⟩, $\langle x_8$, {[0.38, 0.43], [0.38, 0.46]}⟩, $\langle x_9$, {[0.39, 0.4], [0.39, 0.41]}⟩, $\langle x_{10}$, {[0.17, 0.2], [0.17, 0.23]}⟩}; $\overline{\sum_{i=1}^{3} R_i}^{P}(A) = \{\langle x_1, \{[0.03, 0.06], [0.04, 0.07]\}\rangle, \langle x_2, \{[0.08, 0.13], [0.1, 0.14]\}\rangle, \langle x_3, \{[0.12, 0.14], [0.13, 0.15]\}\rangle, .\langle x_4$, {[0.09, 0.11], [0.1, 0.12]}⟩, $\langle x_5$, {[0.09, 0.13], [0.11, 0.13]}⟩, $\langle x_6$, {[0.22, 0.28], [0.24, 0.28]}⟩, . .$\langle x_7$, {[0.06, 0.07], [0.06, 0.08]}⟩, $\langle x_8$, {[0.07, 0.09], [0.08, 0.1]}⟩, $\langle x_9$, {[0.39, 0.4], [0.39, 0.41]}⟩, $\langle x_{10}$, {[0.12, 0.14], [0.13, 0.15]}⟩}.

Then, we further obtain $\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)$ and $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)$ as follows: $\underline{\sum_{i=1}^{3} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{O}(A) = \{\langle x_1, \{[0.0979, 0.1925], [0.1464, 0.201]\}\rangle, \langle x_2, \{[0.349, 0.516], [0.448, 0.5356]\}\rangle,.\langle x_3, \{[0.4544, 0.656], [0.4954, 0.66]\}\rangle, \langle x_4, \{[0.2538, 0.298], [0.272, 0.3058]\}\rangle, \langle x_5, \{[0.8089, 0.8416], [0.829, 0.8434]\}\rangle, \langle x_6, \{[0.649, 0.7225], [0.688, 0.7408]\}\rangle, \langle x_7, \{[0.1258, 0.2002], [0.1728, 0.2088]\}\rangle, \langle x_8, \{[0.4234, 0.5086], [0.4664, 0.514]\}\rangle., \langle x_9, \{[0.6279, 0.64], [0.6279, 0.6519]\}\rangle, \langle x_{10}, \{[0.2696, 0.3378], [0.3127, 0.3455]\}\rangle\}$; $\underline{\sum_{i=1}^{3} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{P}(A) = \{\langle x_1, \{[0.0979, 0.1728], [0.1072, 0.2095]\}\rangle, \langle x_2, \{[0.356, 0.478], [0.37, 0.527]\}\rangle, \langle x_3, \{[0.4544, 0.613], [0.4606, 0.66]\}\rangle, \langle x_4, \{[0.2538, 0.288], [0.262, 0.3136]\}\rangle, \langle x_5, \{[0.8089, 0.8347], [0.8131, 0.8434]\}\rangle, \langle x_6, \{[0.649, 0.712], [0.658, 0.7336]\}\rangle, \langle x_7, \{[0.1258, 0.1816], [0.1258, 0.2088]\}\rangle, \langle x_8, \{[0.4234, 0.4813], [0.4296, 0.514]\}\rangle., \langle x_9, \{[0.6279, 0.64], [0.6279, 0.6519]\}\rangle, \langle x_{10}, \{[0.2696, 0.312], [0.2779, 0.3455]\}\rangle\}$.

Finally, according to Definition 2.4, we calculate the score function values of interval-valued hesitant fuzzy elements belonging to $\underline{\sum_{i=1}^{3} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{O}(A)$ and $\underline{\sum_{i=1}^{3} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{P}(A)$, respectively. The ranking results for optimistic and pessimistic IVHF multigranulation rough sets on two universes of A are identical. That is, the ranking order of all faults is: $x_5 > x_6 > x_9 > x_3 > x_8 > x_2 > x_{10} > x_4 > x_7 > x_1$. According to the diagnostic order of the above results, it is not difficult to obtain $T_1 \cap T_2 \cap T_3 = \{5\}$. Thus, from the arguments of the above results, it is easy to find that the turbine vibration fault is initiated by rotor radial impact friction.

To validate the effectiveness of the proposed model based on IVHF multigranulation rough sets over two universes, a comparison analysis is conducted by utilizing the most commonly used aggregation operators for interval-valued hesitant fuzzy information. As established in previous work [14], we let $h_j (j = 1, 2, \ldots, n)$ be a collection of IVHFEs and $w = (1/n, 1/n, \ldots, 1/n)^T$ be the weight vector of of h_j with equal weight. Then, we have the following special aggregation operators.

(1) The interval-valued hesitant fuzzy averaging (IVHFA) operator $IVHFA(h_1, h_2, \ldots, h_n) = \oplus_{j=1}^{n}(\frac{1}{n}h_j) = \{[1 - \prod_{j=1}^{n}(1-\gamma_j^L)^{1/n}, 1 - \prod_{j=1}^{n}(1-\gamma_j^U)^{1/n}] | \gamma_1 \in h_1, \gamma_2 \in h_2, \ldots, \gamma_n \in h_n\}$;

(2) The interval-valued hesitant fuzzy geometric (IVHFG) operator $IVHFG(h_1, h_2, \ldots, h_n) = \otimes_{j=1}^{n} {h_j}^{1/n} = \{[\prod_{j=1}^{n}(\gamma_j^L)^{1/n}, \prod_{j=1}^{n}(\gamma_j^U)^{1/n}] | \gamma_1 \in h_1, \gamma_2 \in h_2, \ldots, \gamma_n \in h_n\}$.

Through utilizing the above two aggregation operators, we can aggregate the IVHF relations R_1, R_2 and R_3 in Tables 1, 2 and 3 to a single IVHF relation R. Then, we calculate the score function values of interval-valued hesitant fuzzy elements belonging to $\underline{R}(A) \oplus \overline{R}(A)$ within the background of IVHF rough sets over two universes. The ranking order of all faults for the IVHFA and IVHFG operators is $x_5 > x_6 > x_9 > x_3 > x_8 > x_2 > x_{10} > x_4 > x_7 > x_1$, the ranking order is consistent with that obtained by Algorithm 1.

By comparing the aggregation methods of the IVHFA and IVHFG operators and of IVHF multigranulation rough sets on two universes model utilized in steam turbine fault diagnosis, the main fault of the testing sample is rotor radial impact friction. Compared to the other aggregation operators, the IVHF multigranulation rough set over two universes model takes full advantages of interval-valued hesitant fuzzy sets and multigranulation rough sets in steam turbine fault diagnosis procedures. On one hand, the hesitancy membership function in interval-valued hesitant fuzzy sets provides mechanical engineers with more flexible vehicles by which to convey understanding about the system fault knowledge base. On the other hand, the multigranulation rough set method can be viewed as a superior information fusion strategy that allows synthesis of each engineer's view to form a final conclusion by providing optimistic and pessimistic decision-making strategies. If the ranking order of all faults for the optimistic and pessimistic versions of IVHF multigranulation rough sets over two universes are different from each other, the mechanical engineer could refer to results according to risk preference. Thus, the ranking result of optimistic IVHF multigranulation rough sets over two universes is often adopted when the decision-maker is more prone to pursue risk. Conversely, the pessimistic counterpart is adopted when the decision-maker intends to select a conservative fault diagnosis strategy. In light of the above, the superiorities of IVHF multigranulation rough sets over two universes could greatly decrease uncertainties and increase diagnostic accuracies of steam turbine fault diagnosis.

6. Conclusions

The focal point of interest in the paper is to propose a novel rough set approach, called an interval-valued hesitant fuzzy multigranulation rough set over two universes, for solving steam turbine fault diagnosis problems. In the framework of this study, basic definitions and some useful theorems of optimistic and pessimistic interval-valued hesitant fuzzy multigranulation rough sets over two universes were explored. Then, we proposed a general approach to decision-making related to steam turbine fault diagnosis. Lastly, case study outcomes show that the proposed fault diagnosis method could effectively handle steam turbine fault diagnosis problems. There still exist some challenging issues in the theoretical and practical researches of the proposed rough set model. First, it is meaningful to further study attribute reduction approaches, uncertainty measures, and topological structures for interval-valued hesitant fuzzy multigranulation rough sets over two universes. Second, how to apply the proposed approach to other complex and uncertain decision-making situations will also be taken into account in the future.

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