

AN INTRODUCTION TO FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce fuzzy neutrosophic topological spaces and its some properties. Also we provide fuzzy continuous and fuzzy compactness of fuzzy neutrosophic topological space and its some properties and examples.

**Keywords:** fuzzy neutrosophic set, fuzzy neutrosophic topology, fuzzy neutrosophic topological spaces, fuzzy continuous and fuzzy compactness.

1. INTRODUCTION

Fuzzy sets were introduced by Zadeh in 1965. The concepts of intuitionistic fuzzy sets by K. Atanassov several researches were conducted on the generalizations of the notion of intuitionistic fuzzy sets. Florentin Smarandache [5, 6] developed Neutrosophic set & logic of A Generalization of the Intuitionistic Fuzzy Logic & set respectively. A.A.Salama & S.A.Alblowi [1] introduced and studied Neutrosophic Topological spaces and its continuous function in [2]. In this paper, we define the notion of fuzzy neutrosophic topological spaces and investigate continuity and compactness by using Coker's intuitionistic topological spaces in [4]. We discuss New examples of FNTS.

2. PRELIMINARIES

Here we shall present the fundamental definitions. The following one is obviously inspired by Haibin Wang and Florentin Smarandache in [7] and A.A.Salama, S.S.Alblow in [1]. Smarandache introduced the neutrosophic set and neutrosophic components. The sets T, I, F are not necessarily intervals but may be any real sub-unitary subsets of  $]^{-}0, 1^{+}[$ . The neutrosophic components T, I, F represents the truth value, indeterminacy value and falsehood value respectively.

**Definition 2.1 [7]:** Let  $X$  be a non-empty fixed set. A fuzzy neutrosophic set (FNS for short)  $A$  is an object having the form  $A = \{(x, \mu_A(x), \sigma_A(x), \nu_A(x)): x \in X\}$  where the functions

$$\mu_A: X \rightarrow ]^{-}0, 1^{+}[ , \quad \sigma_A: X \rightarrow ]^{-}0, 1^{+}[ , \quad \nu_A: X \rightarrow ]^{-}0, 1^{+}[$$

denote the degree of membership function (namely  $\mu_A(x)$ ), the degree of indeterminacy function (namely  $\sigma_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) respectively of each element  $x \in X$  to the set  $A$  and  ${}^{+}0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 1^{+}$ , for each  $x \in X$ .

**Remark 2.2 [7]:** Every fuzzy set  $A$  on a non-empty set  $X$  is obviously a FNS having the form

$$A = \{(x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)): x \in X\}$$

A fuzzy neutrosophic set  $A = \{(x, \mu_A(x), \sigma_A(x), \nu_A(x)): x \in X\}$  can be identified to an ordered triple  $\langle x, \mu_A, \sigma_A, \nu_A \rangle$  in  $]^{-}0, 1^{+}[$  on  $X$ .

**Definition 2.3[1]:** Let  $X$  be a non-empty set and the FNSs  $A$  and  $B$  be in the form  $A = \{(x, \mu_A(x), \sigma_A(x), \nu_A(x)): x \in X$  and  $B = \{(x, \mu_B(x), \sigma_B(x), \nu_B(x)): x \in X\}$  on  $X$  and let  $A_i: i \in J$  be an arbitrary family of FNS's in  $X$ , where  $A_i = \{(x, \mu_{A_i}(x), \sigma_{A_i}(x), \nu_{A_i}(x)): x \in X\}$ .

- a)  $A \subseteq B$  iff  $\mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$ .
- b)  $A = B$  iff  $A \subseteq B$  and  $B \subseteq A$ .
- c)  $\bar{A} = \{(x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x)): x \in X\}$
- d)  $\cup A_i = \{(x, \bigvee_{i \in J} \mu_{A_i}(x), \bigvee_{i \in J} \sigma_{A_i}(x), \bigwedge_{i \in J} \nu_{A_i}(x)): x \in X\}$

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- e)  $\cap A_i = \{ \langle x, \bigwedge_{i \in J} \mu_{A_i}(x), \bigwedge_{i \in J} \sigma_{A_i}(x), \bigvee_{i \in J} \nu_{A_i}(x) \rangle : x \in X \}$
- f)  $A - B = \{ \langle x, \mu_A(x) \wedge \nu_B(x), \sigma_A(x) \wedge (1 - \sigma_B(x)), \nu_A(x) \vee \mu_B(x) \rangle : x \in X \}$
- g)  $\square A = \{ \langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x) \rangle : x \in X \}$
- h)  $\diamond A = \{ \langle x, 1 - \nu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$
- i)  $0_N = \langle x, 0, 0, 1 \rangle$  and  $1_N = \langle x, 1, 1, 0 \rangle$

**Example 2.4:** Let  $X = \{a, b, c\}$  and

$$A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.2}, \frac{c}{0.3}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.2}\right) \rangle,$$

$$B = \langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4}\right) \rangle,$$

$$C = \langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.3}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right) \rangle,$$

$$D = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right) \rangle$$

Here  $\bar{A} = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.8}, \frac{c}{0.7}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}\right) \rangle$  and  $A \subseteq C$  because  $\mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$ . Further,

$$A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x) \rangle$$

$$= \langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.3}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right) \rangle$$

$$A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x) \rangle$$

$$= \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right) \rangle$$

**Corollary 2.5:** Let  $A, B, C$  and  $A_i$  be FNS's in  $X (i \in J)$ . Then

- a)  $A_i \subseteq B$  for each  $i \in J \Rightarrow \cup A_i \subseteq B$
- b)  $B \subseteq A_i$  for each  $i \in J \Rightarrow B \subseteq \cap A_i$
- c)  $\overline{\cup A_i} = \cap \bar{A}_i$  and  $\overline{\cap A_i} = \cup \bar{A}_i$
- d)  $A \subseteq B \Leftrightarrow \bar{B} \subseteq \bar{A}$
- e)  $\overline{(\bar{A})} = A$
- f)  $\overline{0_N} = 1_N$  and  $\overline{1_N} = 0_N$

Now we shall define the image and preimage of FNS's. Let  $X, Y$  be two non-empty sets and  $f: X \rightarrow Y$  a function.

**Definition 2.6[4]:**

- a) If  $B = \{ \langle y, \mu_B(y), \sigma_B(y), \nu_B(y) \rangle : y \in Y \}$  is an FNS in  $Y$  then the preimage of  $B$  under  $f$ , denoted by  $f^{-1}(B)$ , is the FNS in  $X$  defined by

$$f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\nu_B)(x) \rangle : x \in X \}$$

Where  $f^{-1}(\mu_B)(x) = \mu_B f(x), f^{-1}(\sigma_B)(x) = \sigma_B f(x)$  and  $f^{-1}(\nu_B)(x) = \nu_B f(x)$

- b) If  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$  is a FNS in  $X$ , then the image of  $A$  under  $f$ , denoted by  $f(A)$ , is the FNS in  $Y$  defined by

$$f(A) = \{ \langle y, f(\mu_A)(y), f(\sigma_A)(y), (1 - f(1 - \nu_A))(y) \rangle : y \in Y \}$$

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$f(\sigma_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \sigma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$(1 - f(1 - \nu_A))(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

For the sake of simplicity, let us use the symbol  $f_-(\nu_A)$  for  $(1 - f(1 - \nu_A))$ .

**Corollary 2.7[4]:** Let  $A, A_i (i \in J)$  be FNS's in  $X, B, B_j (j \in K)$  be FNS's in  $Y$ , and  $f: X \rightarrow Y$  a function.

- a)  $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2), B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$ ,
- b)  $A \subseteq f^{-1}(f(A))$ , and if  $f$  is injective then  $A = f^{-1}(f(A))$
- c)  $f^{-1}(f(B)) \subseteq B$ , and if  $f$  is surjective, then  $f^{-1}(f(B)) = B$
- d)  $f^{-1}(\cup B_j) = \cup f^{-1}(B_j), f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$ ,
- e)  $f(\cup A_i) = \cup f(A_i), f(\cap A_i) \subseteq \cap f(A_i)$ ; and if  $f$  is injective, then  $f(\cap A_i) = \cap f(A_i)$
- f)  $f^{-1}(1_N) = 1_N, f^{-1}(0_N) = 0_N$
- g)  $f(0_N) = 0_N, f(1_N) = 1_N$ , if  $f$  is surjective

- h) If  $f$  is surjective, then  $\overline{f(A)} \subseteq f(\overline{A})$ . If furthermore,  $f$  is injective, then  $\overline{f(A)} = f(\overline{A})$   
 i)  $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$

### 3. FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES

**Definition 3.1:** A fuzzy neutrosophic topology (FNT for short) a non-empty set  $X$  is a family  $\tau$  of fuzzy neutrosophic subsets in  $X$  satisfying the following axioms.

- (NT1)  $0_N, 1_N \in \tau$   
 (NT2)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$   
 (NT3)  $\cup G_i \in \tau, \forall \{G_i: i \in J\} \subseteq \tau$

In this case the pair  $(X, \tau)$  is called a Fuzzy neutrosophic topological space (FNTP for short) and any fuzzy neutrosophic set in  $\tau$  is known as fuzzy neutrosophic open set (FNOS for short) in  $X$ . The elements of  $\tau$  are called open fuzzy neutrosophic sets.

The complement of FNOS in the FNTP  $(X, \tau)$  is called fuzzy neutrosophic closed set (FNCS for short).

**Example 3.2:** Let  $X = \{a, b, c\}$  and consider the value of  $A, B, C, D$  are defined in Example 2.4, the family  $\tau = \{0_N, 1_N, A, B, C, D\}$  of FNT in  $X$ . Then  $(X, \tau)$  is FNTP on  $X$ .

**Example 3.3:** Let  $(X, \tau)$  be a fuzzy topological space such that  $\tau$  is not indiscrete. Suppose now that  $\tau = \{0, 1\} \cup \{G_i: i \in J\}$ . Then we can construct two FNT's on  $X$  as follows:

- a)  $\tau_1 = \{0_N, 1_N\} \cup \{(x, G_i, \sigma(x), 0): i \in J\}$ , b)  $\tau_2 = \{0_N, 1_N\} \cup \{(x, 0, \sigma(x), G_i): i \in J\}$

**Proposition 3.4:** Let  $(X, \tau)$  be a FNTP on  $X$ . Then we can also construct several FNTP's on  $X$  in the following way  
 $\tau_{0,1} = \{G: G \in \tau\}$ , b)  $\tau_{0,2} = \{G: G \in \tau\}$

**Definition 3.5:** Let  $(X, \tau_1), (X, \tau_2)$  be two FNTP's on  $X$ . Then  $\tau_1$  is said to be contained in  $\tau_2$  (in symbols,  $\tau_1 \subseteq \tau_2$ ), if  $G \in \tau_2$  for each  $G \in \tau_1$ . In this case, we also say that  $\tau_1$  is coarser than  $\tau_2$ .

**Proposition 3.6:** Let  $\{\tau_i: i \in J\}$  be a family of FNT's on  $X$ . Then  $\bigcap_{i \in J} \tau_i$  is FNT on  $X$ . Furthermore,  $\bigcap_{i \in J} \tau_i$  is the coarsest FNT on  $X$  containing all  $\tau_i$ 's.

**Definition 3.7:** Let  $(X, \tau)$  be a FNTP on  $X$ .

- a) A family  $\beta \subseteq \tau$  is called a base for  $(X, \tau)$  iff each member of  $\tau$  can be written as a union of elements of  $\beta$ .  
 b) A family  $\gamma \subseteq \tau$  is called a subbase for  $(X, \tau)$  iff the family of finite intersections of elements in  $\gamma$  forms a base for  $(X, \tau)$ . In this case the FNT  $\tau$  is said to be generated by  $\gamma$ .

**Definition 3.8:** Let  $(X, \tau)$  be a FNTP and  $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$  be a FNS in  $X$ . Then the fuzzy interior and fuzzy closure of  $A$  are defined by

$$cl(A) = \bigcap \{K: K \text{ is a FNCS in } X \text{ and } A \subseteq K\}, int(A) = \bigcup \{G: G \text{ is a FNOS in } X \text{ and } G \subseteq A\}$$

Now that  $cl(A)$  is a FNCS and  $int(A)$  is a FNOS in  $X$ . Further,

- a)  $A$  is a FNCS in  $X$  iff  $cl(A) = A$   
 b)  $A$  is a FNOS in  $X$  iff  $int(A) = A$

**Example 3.9:** Consider the FNTP  $(X, \tau)$  is defined in Example 3.2. If

$$F = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}\right) \rangle$$

Then,  $int(A) = D$  and  $cl(A) = 1_N$ .

**Proposition 3.10:** For any FNS  $A$  in  $(X, \tau)$ , we have  $cl(\overline{A}) = \overline{int(A)}, int(\overline{A}) = \overline{cl(A)}$ .

**Proposition 3.11:** Let  $(X, \tau)$  be a FNTP and  $A, B$  be FNS's in  $X$ . Then the following properties hold:

- a)  $int(A) \subseteq A, A \subseteq cl(A)$   
 b)  $A \subseteq B \Rightarrow int(A) \subseteq int(B), A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$   
 c)  $int(int(A)) = int(A), cl(cl(A)) = cl(A)$   
 d)  $int(A \cap B) = int(A) \cap int(B), cl(A \cup B) = cl(A) \cup cl(B)$   
 e)  $int(1_N) = 1_N, cl(0_N) = 0_N$

**Proposition 3.12:** Let  $(X, \tau)$  be a FNTS. If  $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$  be a FNS in  $X$ . Then we have

- a)  $int(A) \subseteq \langle x, int_{\tau_1}(\mu_A), int_{\tau_1}(\sigma_A), cl_{\tau_2}(\nu_A) \rangle \subseteq A$
  - b)  $A \subseteq \langle x, cl_{\tau_2}(\mu_A), cl_{\tau_2}(\sigma_A), int_{\tau_1}(\nu_A) \rangle \subseteq cl(A)$
- where  $\tau_1$  and  $\tau_2$  are FTS on  $X$  defined by  $\tau_1 = \{\mu_A: G \in \tau\}$ ,  $\tau_2 = \{1 - \nu_A: G \in \tau\}$

**Corollary 3.13:** Let  $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$  be a FNS in  $(X, \tau)$

- a) If  $A$  is a FNCS, then  $\mu_A$  is fuzzy closed in  $(X, \tau_2)$  and  $\nu_A$  is fuzzy open in  $(X, \tau_1)$ .
- b) If  $A$  is a FNOS, then  $\mu_A$  is fuzzy open in  $(X, \tau_1)$  and  $\nu_A$  is fuzzy closed in  $(X, \tau_2)$ .

**Example 3.14:** Consider the FNTS  $(X, \tau)$  is defined in Example 3.2. If

$$F = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}\right) \rangle$$

Then,  $int(A) = D$ . Noting that we have

$$\tau_1 = \left\{0, 1, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\}$$

$$\tau_2 = \left\{0, 1, \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.8}\right), \left(\frac{a}{0.8}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.8}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\}$$

and  $int_{\tau_1}(\mu_A) = \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)$ ,  $cl_{\tau_2}(\nu_A) = \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)$

#### 4. FUZZY NEUTROSOPHIC CONTINUITY

**Definition 4.1:** Let  $(X, \tau)$  and  $(Y, \varphi)$  be two FNTSs and let  $f: X \rightarrow Y$  be a function. Then  $f$  is said to be fuzzy continuous iff the preimage of each FNS in  $\varphi$  is a FNS in  $\tau$ .

**Definition 4.2:** Let  $(X, \tau)$  and  $(Y, \varphi)$  be two FNTSs and let  $f: X \rightarrow Y$  be a function. Then  $f$  is said to be fuzzy open iff the image of each FNS in  $\tau$  is a FNS in  $\varphi$ .

**Example 4.3:** Let  $(X, \tau_0)$  and  $(Y, \varphi_0)$  be two fuzzy topological spaces.

- a) If  $f: X \rightarrow Y$  is fuzzy continuous in the usual sense, then in this case,  $f$  is fuzzy continuous iff the preimage of each FNS in  $\varphi_0$  is a FNS in  $\tau_0$ . Consider the FNTs on  $X$  and  $Y$  respectively, as follows:

$$\tau = \{\langle x, \mu_G, 0, 1 - \mu_G \rangle: \mu_G \in \tau_0\}, \varphi = \{\langle y, \delta_H, 0, 1 - \delta_H \rangle: \delta_H \in \varphi_0\}$$

In this case we have for each  $\langle y, \delta_H, 0, 1 - \delta_H \rangle \in \varphi$ ,  $\delta_H \in \varphi_0$ ,

$$f^{-1}\langle y, \delta_H, 0, 1 - \delta_H \rangle = \langle x, f^{-1}(\delta_H), f^{-1}(0), f^{-1}(1 - \delta_H) \rangle \\ = \langle x, f^{-1}(\delta_H), f^{-1}(0), 1 - f^{-1}(\delta_H) \rangle \in \tau$$

- b) Let  $f: X \rightarrow Y$  be a fuzzy open in the usual sense, then  $f$  is fuzzy open according to Definition 4.2. In this case we have, for each  $\langle x, \mu_G, 0, 1 - \mu_G \rangle \in \tau$ ,  $\mu_G \in \tau_0$ ,

$$f\langle x, \mu_G, 0, 1 - \mu_G \rangle = \langle x, f(\mu_G), 0, f_-(1 - \mu_G) \rangle \\ = \langle x, f(\mu_G), 0, 1 - f_-(\mu_G) \rangle \in \varphi$$

**Proposition 4.4:**  $f: (X, \tau) \rightarrow (Y, \varphi)$  is fuzzy continuous iff the preimage of each FNCS in  $\varphi$  is a FNCS in  $\tau$ .

**Proposition 4.5:** The following are equivalent to each other

- a)  $f: (X, \tau) \rightarrow (Y, \varphi)$  is fuzzy continuous
- b)  $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$  for each FNS  $B$  in  $Y$ .
- c)  $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$  for each FNS  $B$  in  $Y$ .

**Example 4.6:** Let  $(Y, \varphi)$  be a FNTS and  $X$  be a non-empty set and  $f: X \rightarrow Y$  a function. In this case  $\tau = \{f^{-1}(H): H \in \varphi\}$  is a FNT on  $X$ . Indeed,  $t$  is the coarsest FNT on  $X$  which makes the function  $f: X \rightarrow Y$  continuous. One may call the FNT  $t$  on  $X$  the initial FNT with respect to  $f$ .

#### 5. FUZZY NEUTROSOPHIC COMPACTNESS

**Definition 5.1:** Let  $(X, \tau)$  be a FNTS.

- a) If a family  $G_i = \{\langle x, \mu_{G_i}, \sigma_{G_i}, \nu_{G_i} \rangle: i \in J\}$  of FNOS in  $X$  satisfy the condition  $\cup G_i = 1_N$ , then it is called a fuzzy open cover of  $X$ . A finite subfamily of fuzzy open cover  $G_i$  of  $X$ , which is also a fuzzy open cover of  $X$  is called a finite subcover of  $G_i$ .
- b) A family  $K_i = \{\langle x, \mu_{K_i}, \sigma_{K_i}, \nu_{K_i} \rangle: i \in J\}$  of FNCSs in  $X$  satisfies the finite intersection property iff every finite subfamily  $\{\langle x, \mu_{K_i}, \sigma_{K_i}, \nu_{K_i} \rangle: i = 1, 2, \dots, n\}$  of the family satisfies the condition  $\cap_{i=1}^n \{\langle x, \mu_{K_i}, \sigma_{K_i}, \nu_{K_i} \rangle\} \neq 0_N$ .

**Definition 5.2:** A FNTS  $(X, \tau)$  is fuzzy compact iff every fuzzy open cover of  $X$  has a finite subcover.

**Example 5.3:** An FNTS  $(X, \tau)$ , where  $X = \{1, 2, 3\}$ ,

$$G_n = \left\langle x, \left( \frac{1}{n+1}, \frac{2}{n+2}, \frac{3}{n+3} \right), \left( \frac{1}{n+1}, \frac{2}{n+2}, \frac{3}{n+3} \right), \left( \frac{1}{n+2}, \frac{2}{n+3}, \frac{3}{n+4} \right) \right\rangle$$

and  $\tau = \{0_N, 1_N\} \cup \{G_n : n \in \mathbb{N}\}$ . Note that  $\bigcup_{n \in \mathbb{N}} G_n$  is an open cover for  $X$ , but this cover has no finite subcover. Consider

$$G_1 = \left\langle x, \left( \frac{1}{0.5}, \frac{2}{0.3}, \frac{3}{0.25} \right), \left( \frac{1}{0.7}, \frac{2}{0.75}, \frac{3}{0.8} \right), \left( \frac{1}{0.3}, \frac{2}{0.25}, \frac{3}{0.2} \right) \right\rangle$$

$$G_2 = \left\langle x, \left( \frac{1}{0.7}, \frac{2}{0.5}, \frac{3}{0.4} \right), \left( \frac{1}{0.75}, \frac{2}{0.8}, \frac{3}{0.8} \right), \left( \frac{1}{0.25}, \frac{2}{0.2}, \frac{3}{0.2} \right) \right\rangle$$

$$G_3 = \left\langle x, \left( \frac{1}{0.75}, \frac{2}{0.6}, \frac{3}{0.5} \right), \left( \frac{1}{0.8}, \frac{2}{0.8}, \frac{3}{0.9} \right), \left( \frac{1}{0.2}, \frac{2}{0.2}, \frac{3}{0.1} \right) \right\rangle$$

and observe that  $G_1 \cup G_2 \cup G_3 = G_3$ . So, for any finite subcollection  $\{G_{n_i} : i \in I, \text{ where } I \text{ is a finite subset of } \mathbb{N}\}$ ,  $\bigcup_{n_i \in I} G_{n_i} = G_m \neq 1_N$ , where  $m = \max\{n_i : n_i \in I\}$ . therefore the FNTS  $(X, \tau)$  is not compact.

**Proposition 5.4:** Let  $(X, \tau)$  be a FNTS on  $X$ . Then  $(X, \tau)$  is fuzzy compact iff the FNTS  $(X, \tau_{0,1})$  is fuzzy compact.

**Proof:** Let  $(X, \tau)$  be fuzzy compact and consider fuzzy open cover  $\{[ ]G_j : j \in K\}$  of  $X$  in  $(X, \tau_{0,1})$ . Since  $\bigcup ([ ]G_j) = 1_N$ . We obtain  $\bigvee \mu_{G_j} = 1, \bigvee \sigma_{G_j} = 1$  and hence, by  $\nu_{G_j} \leq 1 - \mu_{G_j} \Rightarrow \bigwedge \nu_{G_j} \leq 1 - \bigvee \mu_{G_j} = 0 \Rightarrow \bigwedge \nu_{G_j} = 0$ , we deduce  $\bigcup G_j = 1_N$ . Since  $(X, \tau)$  is fuzzy compact there exist  $G_1, G_2, G_3, \dots, G_n$  such that  $\bigcup_{i=1}^n G_i = 1_N$  from which we obtain  $\bigvee_{i=1}^n \mu_{G_i} = 1, \bigvee_{i=1}^n \sigma_{G_i} = 1$  and  $\bigwedge_{i=1}^n (1 - \mu_{G_i}) = 0$ . That is,  $(X, \tau_{0,1})$  is fuzzy compact.

Suppose that  $(X, \tau_{0,1})$  is fuzzy compact and consider a fuzzy open cover  $\{G_j : j \in K\}$  of  $X$  in  $(X, \tau)$ . Since  $\bigcup G_j = 1_N$ , we obtain  $\bigvee \mu_{G_j} = 1, \bigvee \sigma_{G_j} = 1$  and  $1 - \bigvee \mu_{G_j} = 0$ . Since  $(X, \tau_{0,1})$  is fuzzy compact, there exist  $G_1, G_2, G_3, \dots, G_n$  such that  $\bigcup_{i=1}^n ([ ]G_i) = 1_N$ , that is,  $\bigvee_{i=1}^n \mu_{G_i} = 1, \bigvee_{i=1}^n \sigma_{G_i} = 1$  and  $\bigwedge_{i=1}^n (1 - \mu_{G_i}) = 0$ . Hence  $\mu_{G_i} \leq 1 - \nu_{G_i} \Rightarrow 1 = \bigvee_{i=1}^n \mu_{G_i} \leq 1 - \bigwedge_{i=1}^n \nu_{G_i} = 0 \Rightarrow \bigwedge_{i=1}^n \nu_{G_i} = 0$ . Hence  $\bigcup_{i=1}^n G_i = 1_N$ . Therefore  $(X, \tau)$  is fuzzy compact.

**Corollary 5.5:** Let  $(X, \tau)$  and  $(Y, \varphi)$  be two FNTSs and let  $f : X \rightarrow Y$  be a fuzzy continuous surjection. If  $(X, \tau)$  is fuzzy compact, then so is  $(Y, \varphi)$ .

**Corollary 5.6:** A FNTS  $(X, \tau)$  is fuzzy compact iff every family  $\{\langle x, \mu_{K_i}, \sigma_{K_i}, \nu_{K_i} \rangle : i \in J\}$  of FNCSs in  $X$  having the FNP has a non-empty intersection.

**Definition 5.7:**

- Let  $(X, \tau)$  be a FNTS and  $A$  a FNS in  $X$ . If a family  $G_i = \{\langle x, \mu_{G_i}, \sigma_{G_i}, \nu_{G_i} \rangle : i \in J\}$  of FNOSs in  $X$  satisfies the condition  $A \subseteq \bigcup G_i$ , then it is called a fuzzy open cover of  $A$ . A finite subfamily of the fuzzy open cover  $G_i$  of  $A$ , which is also fuzzy open cover of  $A$ , is called a finite subcover of  $G_i$ .
- A FNS  $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$  in a FNTS  $(X, \tau)$  is called fuzzy compact iff every fuzzy open cover of  $A$  has a finite subcover.

**Corollary 5.8:** A FNS  $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$  in a FNTS  $(X, \tau)$  is fuzzy compact iff for each family  $\mathcal{G} = \{G_i : i \in J\}$  where  $G_i = \{\langle x, \mu_{G_i}, \sigma_{G_i}, \nu_{G_i} \rangle : i \in J\}$ , of FNOS in  $X$  with properties  $\mu_A \leq \bigvee_{i \in J} \mu_{G_i}, \sigma_A \leq \bigvee_{i \in J} \sigma_{G_i}$  and  $1 - \nu_A \leq \bigvee_{i \in J} (1 - \nu_{G_i})$  there exists a finite subfamily  $\{G_i : 1, 2, \dots, n\}$  of  $\mathcal{G}$  such that  $\mu_A \leq \bigvee_{i=1}^n \mu_{G_i}, \sigma_A \leq \bigvee_{i=1}^n \sigma_{G_i}$  and  $1 - \nu_A \leq \bigvee_{i=1}^n (1 - \nu_{G_i})$ .

**Example 5.9:** Let  $X = I$  and consider the FNSs  $(G_n)_{n \in \mathbb{Z}_2}$ , where  $G_n = \langle x, \mu_{G_n}, \sigma_{G_n}, \nu_{G_n} \rangle, n = 2, 3, \dots$  and  $G = \langle x, \mu_G, \sigma_G, \nu_G \rangle$  defined by

$$\mu_{G_n}(x) = \begin{cases} 0.7, & \text{if } x = 0 \\ nx, & \text{if } 0 < x < \frac{1}{n} \\ 1, & \text{if } \frac{1}{n} < x \leq 1 \end{cases}, \sigma_{G_n}(x) = \begin{cases} 0.8, & \text{if } x = 0 \\ (n+1)x, & \text{if } 0 < x < \frac{1}{n} \\ 1, & \text{if } \frac{1}{n} < x \leq 1 \end{cases}, \nu_{G_n} = \begin{cases} 0.1, & \text{if } x = 0 \\ 1 - nx, & \text{if } 0 < x < \frac{1}{n} \\ 0, & \text{if } \frac{1}{n} < x \leq 1 \end{cases}$$

and 
$$\mu_G = \begin{cases} 0.7 & \text{if } x = 0 \\ 1, & \text{otherwise} \end{cases}, \sigma_G = \begin{cases} 0.8 & \text{if } x = 0 \\ 1, & \text{otherwise} \end{cases}$$

$$\nu_G = \begin{cases} 0.1 & \text{if } x = 0 \\ 1, & \text{otherwise} \end{cases}$$

Then  $\tau = \{0_N, 1_N, G\} \cup \{G_n : n \in \mathbb{Z}_2\}$  is a FNT on  $X$ , and consider the FNSs  $C_{\alpha, \beta, \gamma}$  in  $(X, \tau)$  defined by  $C_{\alpha, \beta, \gamma} = \{\langle x, \alpha, \beta, \gamma \rangle : x \in X\}$ , where  $\alpha, \beta, \gamma \in I$  are arbitrary and  $\alpha + \beta + \gamma \leq 2$ . Then the FNSs  $C_{0.75, 0.85, 0.05}, C_{0.65, 0.75, 0.15}, C_{0.85, 0.85, 0.05}$  are all compact.

**Corollary 5.10:** Let  $(X, \tau)$  and  $(Y, \varphi)$  be two FNTSs and let  $f: X \rightarrow Y$  be a fuzzy continuous function. If  $A$  is fuzzy compact in  $(X, \tau)$ , then so is  $f(A)$  in  $(Y, \varphi)$ .

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