APPLICATION OF NEUTROSOPHIC OFFSETS FOR DIGITAL IMAGE PROCESSING

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ABSTRACT
Neutrosophic offsets are neutrosophic sets whose truth-values lie outside the interval [0, 1]. Uninorms are aggregation operators defined in fuzzy logic to generalize t-norms and t-conorms. They satisfy the axioms of symmetry, associativity, monotony and the existence of a neutral element. Fuzzy uninorms have been generalized to intuitionistic fuzzy sets, neutrosophic sets, and neutrosophic offsets, which are called offuninorms in the latter case. This paper aims to demonstrate that offsets and offuninorms can be used in digital image processing, especially for image segmentation and edge detection, moreover algorithms and examples are also provided.

KEYWORDS: neutrosophic set, neutrosophic offset, uninorm, offuninorm, digital image processing.

MSC: 03B60, 03B80, 68U10.

1. INTRODUCTION

According to [19] "Image processing is a science that reveals information about images. Enhancing an image is necessary to perfect the appearance or to highlight some aspect of the information contained in it. Whenever an image is converted from one form to another, for example, acquired, copied, scanned, digitized, transmitted, displayed, printed or compressed, many types of noise or noise-like degradations may occur. For example, when an analog image is digitized, the resulting digital image contains quantization noise; when an image is half-toned for printing, the resulting binary image contains half-tone noise; when an image is transmitted through a communication channel, the received image contains channel noise; when an image is compressed, the decompressed image contains compression errors. Therefore, an important issue is the development of image enhancement algorithms that eliminate (soften) noise artifacts, while retaining the structure of the image."

In this investigation, operators defined in the neutrosophic theory will be applied for digital image processing. Neutrosophy is the branch of philosophy that studies everything related to neutralities, see [10] [15] [16]. In mathematics, it is a generalization of other theories such as fuzzy logic, fuzzy intuitionist logic, among others. For the first time it includes an indeterminacy membership function, in addition to the membership function and the non-membership function, where any of them can be independent of the rest. Indeterminacy models the contradictions, inconsistencies and ignorance of information or knowledge.

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Then the author Florentin Smarandache himself defines for the first time the neutrosophic offsets, which are neutrosophic sets, whose truth values may lie outside the interval [0, 1]. A practical example to explain the usefulness of this theory appears in [17], as set out below.

Assuming we want to study the performance of a group of workers of a certain company, taking into account the number of weekly work hours, then, the set of good workers of the company is defined by the hours worked; then to any worker who fulfills all of his work hours with quality would be assigned a truth-value of 1 of membership to such a group, the worker who did not attend at any time is assigned a truth-value of 0, while the rest is assigned a value in the interval (0, 1), depending on their assistance. However, employees who worked overtime with higher quality than the rest, should have a membership value greater than 1, and those without any assistance and that also caused damage to the company, should have a negative membership value.

This new type of sets has logical operators such as negation, conjunction, disjunction, among others. This leads to the definition of offnegations, offnorms and offconorms.

A very useful aggregation operator is the uninorm, which in fuzzy logic generalizes the idea of t-norm and t-conorm, see [1]. A uninorm is an aggregation operator that satisfies the axioms of symmetry, associativity, monotony and the existence of a neutral element. In the case of t-norms the neutral element is 1, while in t-conorms it is 0.

Uninorms have had great applications in different fields, such as decision-making, as an activation function in artificial neural networks ([18]), among others, including digital image processing, see [4] [7]. They have been generalized to other logics such as Atanassov's fuzzy intuitionist logic ([2]), or the neutrosophic logic ([6]) and the Smarandache's of logic, in the latter case it was called offuninorm when it includes offsets, see [5].

Offuninorms were defined for the first time in [5], with the objective of having an aggregation operator for the neutrosophic offsets. However, the authors of the article described some approaches to the idea of a fuzzy uninorm where intervals outside [0, 1] were admitted, which means that the association between uninorm and offset is very natural, see [1] [18]. These concepts are also linked to the Prospector operator used in the famous MYCIN medical expert system ([13]). Additionally, it is demonstrated that the calculation with offsets is simpler and equal or more interpretable than the use of fuzzy sets.

The aim of this article is to demonstrate that offsets and offnorms can be used as filters in digital image processing. For this purpose, some filters based on these concepts are proposed and the use of this tool for segmenting images ([8]) and for edge detection ([9] [11]) are illustrated with some examples.

This paper is structured as follows: Section 2 Materials and Methods, contains the main definitions from neutrosophic offsets to neutrosophic offuninorms. Section 3 is dedicated to algorithmization and illustration with examples of the use of neutrosophic offsets and offnorms as filters in digital image processing. The article ends with section 4 conclusions.

2. MATERIALS AND METHODS

This section describes the main concepts required to understand this article, which are formally defined below.

**Definition 1.** ([10]) Let X be a universe of discourse. A Neutrosophic Set (NS) is characterized by three membership functions, \( u_A(x), r_A(x), v_A(x) : X \to [0, 1]^* \), which satisfy the condition \( 0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3 \) for all \( x \in X \). \( u_A(x), r_A(x) \) and \( v_A(x) \) denote the functions of truthfulness, indeterminacy and falseness membership of \( x \) in \( A \), respectively, and their images are standard or non-standard subsets of \([0, 1]^*\).

**Single-Value Neutrosophic Sets**, which are defined below, were created in order to apply the NS to nonphilosophical problems.

**Definition 2.** ([10]) Let X be a universe of discourse. A Single-Value-Neutrosophic-Set (SVNS) A over X is an object defined as follows:

\[
A \equiv \{ x, u_A(x), r_A(x), v_A(x) : x \in X \}
\]

(1)

Where \( u_A, r_A, v_A : X \to [0,1] \), satisfy the condition \( 0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3 \) for all \( x \in X \). \( u_A(x), r_A(x) \) and \( v_A(x) \) denote the membership functions of truthfulness, indeterminacy and falseness of \( x \) in \( A \), respectively.

For the sake of convenience, a Single-Value-Neutrosophic Number (SVNN) will be represented as \( A = (a, b, c) \), where a, b, c \( \in [0,1] \) and that satisfies \( 0 \leq a + b + c \leq 3 \).
Definition 3. Let $X$ be a universe of discourse and the neutrosophic set $A\subseteq X$. Let $T(x)$, $I(x)$, $F(x)$ be the membership, indeterminacy and non-membership functions, respectively, of a generic element $x\subseteq X$, with regard to the neutrosophic set $A$:

$$T, I, F: X\sqsubseteq\{0, 1\},$$

where $0 > 1$ is called an *overlimit*, $T(x)$, $I(x)$, $F(x)\subseteq\{0, 1\}$. A Single-Value-Overlimit $A_1$ is defined as $A_1 = \{(x, (T(x), I(x), F(x))), x \in X\}$, such that there is at least one element in $A_1$ that contains at least a neutrosophic component greater than 1, and does not contain any element with components less than 0, see [17].

Definition 4. Let $X$ be a universe of discourse and the neutrosophic set $A\subseteq X$. Let $T(x)$, $I(x)$, $F(x)$ be the membership, indeterminacy and non-membership functions, respectively, of a generic element $x\subseteq X$, with regard to a neutrosophic set $A_2$:

$$T, I, F: X\sqsubseteq\{0, 1\},$$

where $0 < 1$ is called an *underlimit*, $T(x)$, $I(x)$, $F(x)\subseteq\{0, 1\}$. A Single-Value-Underlimit $A_2$ is defined as $A_2 = \{(x, (T(x), I(x), F(x))), x \in X\}$, such that there is at least one element in $A_2$ which contains at least one neutrosophic component that is less than 0, and does not contain any element with components greater than 1, see [17].

Definition 5. Let $X$ be a universe of discourse and the neutrosophic set $A\subseteq X$. Let $T(x)$, $I(x)$, $F(x)$ be the membership, indeterminacy and non-membership functions, respectively, of a generic element $x\subseteq X$, with regard to a neutrosophic set $A_3$:

$$T, I, F: X\sqsubseteq\{0, 1\},$$

where $0 < 1 < 0$ is called an *offset*, $T(x)$, $I(x)$, $F(x)\subseteq\{0, 1\}$. A Single-Value-Offset $A_3$ is defined as $A_3 = \{(x, (T(x), I(x), F(x))), x \in X\}$, such that there is at least one element in $A_3$ which contains a neutrosophic component greater than 1, and contains another neutrosophic component that is less than 0, see [17].

Let $X$ be a universe of discourse, $A = \{(x, (T_A(x), I_A(x), F_A(x))), x \in X\}$ and

$$B = \{(x, (T_B(x), I_B(x), F_B(x))), x \in X\}$$

are two single value oversets /undersets / offsets.

$$T_A, I_A, F_A, T_B, I_B, F_B: X\sqsubseteq\{0, 1\},$$

where $0 < 1 < 0$. $A$ is an underlimit, while $B$ is the overlimit, $T_A(x)$, $I_A(x)$, $F_A(x), T_B(x), I_B(x), F_B(x)\subseteq\{0, 1\}$. Note that in this definition, all three possible cases are taken into account: overset $0 = 0$ and $0 > 1$, underset when $0 < 0$ and $0 = 1$, and offset when $0 < 0$ and $0 > 1$.

Then, the main operations on these sets are defined as follows:

- $A \cup B = \{(x, (\max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x)))), x \in X\}$ is the union.
- $A \cap B = \{(x, (\min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x)))), x \in X\}$ is the intersection.
- $C(A) = \{(x, (F_A(x), \Psi + \Omega - I_A(x), T_A(x))), x \in X\}$ is the neutrosophic complement of the overset / underset / offset.

Definition 6. Let $c$ be a neutrosophic component $(T_0, I_0, F_0)$, $c \subseteq M_0\sqsubseteq\{0, 1\}$, where $0 = 0$ and $0 = 1$. The neutrosophic component $n$-offnorm $N_0^n: [I, I^2] \rightarrow [I, I]$ satisfies the following conditions for any elements $x, y, z\subseteq M_0$:

- i. $N_0^n(c(x), y) = N_0^n(c(x), z) = c(x)$ (Over-boundary conditions),
- ii. $N_0^n(c(x), c(y)) = N_0^n(c(y), c(x))$ (Commutativity),
- iii. $\forall c(x) \leq c(y) \rightarrow N_0^n(c(x), c(z)) \leq N_0^n(c(y), c(z))$ (Monotony),
- iv. $N_0^n(N_0^n(c(x), c(y)), c(z)) = N_0^n(c(x), N_0^n(c(y), c(z)))$ (Associativity).

Definition 7. Let $c$ be a neutrosophic component $(T_0, I_0, F_0)$, $c \subseteq M_0\sqsubseteq\{0, 1\}$, where $0 = 0$ and $0 = 1$. The neutrosophic component $n$-offconorm $N_0^n c: [I, I^2] \rightarrow [I, I]$ satisfies the following conditions for any elements $x, y, z\subseteq M_0$:

- i. $N_0^n(c(x), c(y)) = \Omega, N_0^n(c(x), \Psi) = c(x)$ (Over-boundary conditions),
- ii. $N_0^n(c(x), c(y)) = N_0^n(c(y), c(x))$ (Commutativity),
- iii. $\forall c(x) \leq c(y) \rightarrow N_0^n(c(x), c(z)) \leq N_0^n(c(y), c(z))$ (Monotony), iv.
- $N_0^n(N_0^n(c(x), c(y)), c(z)) = N_0^n(c(x), N_0^n(c(y), c(z)))$ (Associativity).
Example 1. An example of offAND/offOR pair is, 
\( c(x) \wedge_{\mathcal{O}} c(y) = \min(c(x), c(y)) \) and \( c(x) \vee_{\mathcal{O}} c(y) = \max(c(x), c(y)) \), respectively.

Example 2. An offAND/offOR pair is, 
\( c(x) \wedge_{\mathcal{O}} c(y) = \max(\Psi, c(x) + c(y) - \Omega) \) and \( c(x) \vee_{\mathcal{O}} c(y) = \min(\Omega, c(x) + c(y)) \), respectively.

Definition 8. ([5]) Let \( c \) be a neutrosophic component \((T, I, O, F, O, F)\), \( c : M_0 \rightarrow \mathbb{M}_0 \), where \( \mathbb{M}_0 = \{0, 1\} \). The neutrosophic component \( n \)-offuninorm \( N_0^n : \mathbb{M}_0^2 \rightarrow \mathbb{M}_0 \) satisfies the following conditions for any elements \( x, y, z \in \mathbb{M}_0^2 \):

i. There is a \( c(e) \in \mathbb{M}_0 \), such that \( N_0^n(c(x), c(e)) = c(x) \) (Identity),

ii. \( N_0^n(c(x), c(y)) = N_0^n(c(y), c(x)) \) (Commutativity),

iii. If \( c(x) \leq c(y) \), then \( N_0^n(c(x), c(z)) \leq N_0^n(c(y), c(z)) \) (Monotony), iv. 
\( N_0^n(N_0^n(c(x), c(y)), c(z)) = N_0^n(c(x), N_0^n(c(y), c(z))) \) (Associativity).

Example 3. Two examples of \( n \)-offuninorms components are defined as follows:

\[
\begin{align*}
U_{2C}(c(x), c(y)) &= \begin{cases} 
-1 \left( \begin{array}{c} 1 - (c(x)) \wedge_{\mathcal{O}} 1 - (c(y)) \\ \min(c(x), c(y)) \end{array} \right), & \text{if } c(x), c(y) \in [0, c(e)], \\
1 - \left( \begin{array}{c} 1 - (c(x)) \vee_{\mathcal{O}} 1 - (c(y)) \\ \max(c(x), c(y)) \end{array} \right), & \text{otherwise}
\end{cases}
\end{align*}
\]

\[
U_{2D}(c(x), c(y)) = \begin{cases} 
-1 \left( \begin{array}{c} 1 - (c(x)) \wedge_{\mathcal{O}} 1 - (c(y)) \\ \min(c(x), c(y)) \end{array} \right), & \text{if } c(x), c(y) \in [0, c(e)], \\
1 - \left( \begin{array}{c} 1 - (c(x)) \vee_{\mathcal{O}} 1 - (c(y)) \\ \max(c(x), c(y)) \end{array} \right), & \text{otherwise}
\end{cases}
\]

Where \( \mathcal{O} \) and \( \mathcal{Z} \) were defined in Example 1; \( c(e) \in \{0, 1\} \).

For \( \Psi \): \([\Psi, c(e)] \rightarrow [\Psi, \Omega] \), \( \Psi^{-1} : [\Psi, \Omega] \rightarrow [\Psi, c(e)] \), \( \Psi_2 : [c(e), \Omega] \rightarrow [\Psi, \Omega] \) and \( \Psi_2^{-1} : [\Psi, \Omega] \rightarrow [c(e), \Omega] \) defined in Equations 2, 3, 4 and 5, respectively.

\[
1(c(x)) = \left( \frac{\Omega - \Psi}{c(e) - \Psi} \right) (c(x) - \Psi) + \Psi \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (2)
\]

\[
1^{-1}(c(x)) = \left( \frac{c(e) - \Psi}{\Omega - \Psi} \right) (c(x) - \Psi) + \Psi \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (3)
\]

\[
2(c(x)) = \left( \frac{\Omega - \Psi}{\Omega - c(e)} \right) (c(x) - c(e)) + \Psi \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (4)
\]

\[
2^{-1}(c(x)) = \left( \frac{\Omega - \Psi}{\Omega - c(e)} \right) (c(x) - \Psi) + c(e) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (5)
\]

In [5] there are more offuninorms examples and ways to obtain them. Since images are studied in this article, which are basically represented through matrices containing integer values between 0 and 255, then the offuninorm defined from Silvert’s fuzzy uninorm will be used, see [14] and Equation 6:

\[
\begin{align*}
U_\lambda(x, y) &= \frac{\lambda xy}{x + y}, & \text{if } (x, y) \in [0, 1]^2 \setminus \{(0, 1), (1, 0)\} = \left\{ \begin{array}{ll}
\lambda xy + (1 - x)(1 - y), & \text{in another case}
\end{array} \right.
\end{align*}
\]
For $\lambda > 0$ and $e_\lambda = \frac{1}{\lambda + 1}$.

From Equation 6, the offuninorm is defined as shown in Equation 7:
\[
N_0^{[\phi_0(x), \phi_0(y)]} = \varphi_3^{-1}\left(u_3\left(\phi_3(c_0(x)), \phi_3(c_0(y))\right)\right)
\]  

(7)

Where $\varphi_3: [0, 1] \rightarrow [0, 1]$ and its inverse $\varphi_3^{-1}: [0, 1] \rightarrow [\Psi, \Omega]$ are expressed in equations 8 and 9, respectively.
\[
\varphi_3(c(x)) = \frac{c(x) - \Psi}{\Omega - \Psi}
\]  

(8)

\[
\varphi_3^{-1}(c(x)) = c(x)
\]  

(9)

While $c_0(x)$ is a neutrosophic component of n-offuninorm.

In particular, $\square \square = 0$ and $\square = 255$ will be set. The parameter $\square$ will be set taking into account that the neutral element of the offnorm defined in Equation 7, is calculated by $e_\lambda = \varphi_3^{-1}(e_\lambda) = \varphi_3^{-1}\left(\frac{1}{\lambda + 1}\right)$. Let us note that in this case it is approached with oversets instead of offsets, however, they will still be called offsets because it is not discarded that in some cases it might be useful to apply truth values less than 0.

3. RESULTS

In this section, the proposed algorithms will be described and the results of the processing performed on several images will be given. To carry out the experiments, the algorithms were coded in Octave 4.2.1, which is a free software for mathematical calculations, similar to MATLAB, see [3].

We will start by describing the neutrosophic offset segmentation:

**Neutrosophic offset segmentation algorithm**

1. The image is converted to a gray tones image. Therefore, a single matrix of dimension nxm is obtained, whose elements are integer values from 0 to 255. Each pixel will be denoted by $P(i,j)$ to represent its value in row i, column j of the matrix; $1 \leq i \leq n$ and $1 \leq j \leq m$.

2. Each $P(i, j)$ is taken to the neutrosophic domain containing a triplet of elements corresponding to the truthfulness, indeterminacy and falsehood that is a white pixel. This is denoted by $P_{NS}(i,j) = (T(i,j), I(i,j), F(i,j))$. In order to do so, the following formulas are used:

   $T(i,j) = AM\{P(k,l): \max\{i - 1, 0\} \leq k, l \leq \min\{i + 1, 255\}\}$

   AM denotes the arithmetic mean.

   $I(i,j) = |P(i,j) - T(i,j)|$

   (10)

   $F(i,j) = 255 - T(i,j)$

   (11)

3. A threshold value $U$ is set, such that the following formula is defined:

   $\overline{T}(i,j) = \{ T(i,j) \text{ sii } I(i,j) < U \}$

   $\overline{T}(i,j) = \{ T(i,j) \text{ sii } I(i,j) \geq U \}$

   (13)

   $\overline{T}(i,j) = AM\{T(k,l): \max\{i - 1, 0\} \leq k, l \leq \min\{i + 1, 255\}\}$

   Where

   Thus, a new image with values equal to $\overline{T}(i,j)$ is obtained.
4. k-means algorithm is applied to classify each of the values of the new image with k= 3 as its minimum value.

5. Finish.

Next, we apply this algorithm on three original images without noise and on two copies of each one of them, to which two different levels of salt and pepper noise were incorporated, the results can be seen in Figures 1, 2 and 3.

**Figure 1.** Segmentation of a geometric image using the neutrosophic offset segmentation algorithm. From left to right, at the top, it goes from image without noise to image with most noise. In the lower part the result of the segmentation of the upper images.

**Figure 2.** Segmentation of an image of the brain by means of the neutrosophic offset segmentation algorithm. From left to right, at the top, it goes from image without noise to image with most noise. In the lower part the result of the segmentation of the upper images.
Below is the description for the edges detection algorithm based on offuninorms.

**Neutrophic Offset Edge Detection Algorithm**

1. The image is converted to a gray tones image. Therefore, a single matrix of dimension nxm is obtained, whose elements are integer values from 0 to 255. Each pixel is denoted by P(i, j) to represent its value in row i, column j of the matrix; 1 ≤ i ≤ n and 1 ≤ j ≤ m.
2. The image is softened and a method to eliminate noise is applied, see for example [12]. In this article the image is obtained after replacing each P(i, j) with T(i, j) according to formula 10.
3. The values defined by each P(i, j) equal to $V_1 = P(i-1,j-1) - P(i+1,j-1)$, $V_2 = P(i-1,j) - P(i+1,j)$, $V_3 = P(i-1,j+1) - P(i+1,j+1)$, $H_1 = P(i-1,j-1) - P(i-1,j+1)$, $H_2 = P(i,j-1) - P(i,j+1)$ and $H_3 = P(i+1,j-1) - P(i+1,j+1)$ are obtained.
4. A component of the neutrosophic offuninorm is calculated according to Equation 7:
   
   $$(i,j) = \widetilde{N}_{\varnothing}(V_1, V_2, V_3) = \widetilde{H}_{\varnothing}(H_1, H_2, H_3) \quad \text{U}_V \quad \text{and} \quad \text{U}_H.$$ 

   In this investigation we set $\varnothing = 0$, $\varnothing = 255$ and $\varnothing = 25$.

5. Two threshold values $U_1$ and $U_2$ are set, such that $U_1 < U_2$; and the pixels are classified as follows:
   5.1. If $U_1 < U_T(i,j) < U_2$ then P(i, j) is considered as part of a region.
   5.2. If $U_1 < U_T(i,j) > U_2$ then P(i, j) is considered as part of an edge.
   5.3. If $U_1 < U_T(i,j) > U_2$ then the pixel is considered to be indeterminate. In this case it is classified as an edge if it has at least one neighbor classified as an edge, otherwise it is considered a region.

6. The obtained image becomes binary, that is, the pixel classified as border is given a black tone and a white tone for region pixels.
7. Edge lines are thinned with algorithms designed for that purpose.

The previous algorithm was applied to two images, as can be seen in Figure 4.
Figure 4. Edge detection with neutrosophic offset edge detection algorithm. On the left the original images appear, on the right we see images of objects with detected edges.

When applying the algorithm, it was not necessary to apply steps 5, 6 and 7, because the images were obtained in binary form and the lines were sufficiently thin.

4. CONCLUSIONS

In this paper, two algorithms based on offsets were proposed. The first one is based on neutrosophic offsets for image segmentation. The second one uses a neutrosophic offuninorms for edge detection. An advantage of both algorithms is the use of offsets, which simplifies the calculations since normalization is not necessary because the interval of definition may exceed the classic [0, 1]. Examples of segmentation and edge detection of several images were offered and the results were visually acceptable. In the case of edge detection, in the experiments carried out, the algorithm executes the binarization of the image and the thinning of the edges without the need to use additional algorithms. In both algorithms the noise of salt and pepper artificially introduced by the authors to the original images was reduced.

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