Applications of neutrosophic cubic sets in multi-criteria decision making

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Abstract

In this paper, we investigate the concepts of the weighted average operator ($\mathcal{A}_W$) and weighted geometric operator ($\mathcal{G}_W$) on neutrosophic cubic sets (NCSs) to aggregate the neutrosophic cubic information. Moreover, on the basis of $\mathcal{A}_W$ & $\mathcal{G}_W$ and certain functions, including score, certainty and accuracy, we develop our algorithm to multiple-criteria decision making in NCSs, in which the assessment standards of another possibility on the characteristics yield the technique of neutrosophic cubic numbers (NCNs) to choice greatest necessary ones. Finally, we provide a mathematical example of the technique to determine the application and usefulness of the established technique.

Keywords: Neutrosophic cubic set, Average operator, Geometric operator, Accuracy functions, Multi-criteria decision making.

1. Introduction

The whole world is characterized with complicated phenomenon, such as, precariousness is unavoidably implicated in difficulties which begin in multitudinous scopes of our life and all the techniques did not succeed to manipulate the problems of this type. To deal with ambiguous or faulty, data is always the biggest task for a long period. Various models were exhibited with the aim of suitably unite ambiguity into system denomination. In 1965, Zadeh [23] familiarized the concept of a fuzzy set. Zadeh reestablished stipulated characteristic purpose of classical crisp sets whose value always takes place in $\{0, 1\}$ by membership function whose values always belongs to the closed interval [0,1]. Conceptually, the theory of a set called a fuzzy set is a very potent procedure to treat with another way, inspection of faulty information connected to elusiveness and for the multifarious system, it is a modeling tool that can be restraint by the human being but very obstinate to outline precisely. Also, it diminishes the probabilities of modeling failures. Up

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until 1960's precariousness was deliberate solitarily in terms of prospect theory but, Zadeh exposed the relationship between the fuzzy set (FS) theory and probability which has a great approach to treating with different doubts and uncertainties. This theory not only expresses indefinite information into the model but it benefits us in resolving difficult problems and making a decision. In fact, FS-methods are appropriate when it is required to model human being acquaintance or assessment. Furthermore, fuzzy logic is the branch of mathematics with the help of which computers model the actual world in the similar means as that of the public do.

To generalize the basic concepts of algebras, various authors have applied the fuzzy set theory. Mortenson et al. [10] have determined the outstanding investigation of fuzzy semigroups, they discovered the fuzzy semigroups theory alongside with some significant applications of fuzzy semigroups. The fuzzy methodology is functional to the problematic integrated project of the highest speed planar device too. But here the point of our conversation is totally about falsehood or degree of non-membership so this theory doesn't work appropriately and we essential somewhat fresh to contract with it much accurately. Atanassov [2, 3] in 1986 familiarized falsehood (f) and describe the intuitionistic fuzzy sets (IFSs). An IFS is actually a generality of a fuzzy set which can be seen in the perception as an approach to fuzzy set in the situation when we are not delivered with enough information. Usage of IFS is supportive in the outline of extra non-membership into set description and is widely used as an instrument of concentrated exploration by researchers and experts for centuries.

Several concepts, including probability, FS, IFS, rough set theory are regularly being practiced as great beneficial implements to contract with multiform in decisions and fuzziness bounded in multifaceted systems. But these theories do not model undecided information sufficiently. So, because of the actuality of indeterminacy in numerous world glitches, neutrosophy establishes its approach into the current exploration. Neutrosophy is a generality of FS, where every model characterized by three sorts of perceptions that are truthfulness, falsity, and indeterminacy. Neutrosophy is actually Latin word "neuter" - neutral, Greek "sophia" - skill/wisdom). It is an outlet of philosophy, idea familiarized by Smarandache studied the commencement, scope, and nature of neutralities, and their relations with many different ideational ranges. Neutrosophy studies an intention, theory, occasion, perception, or article. Neutrosophy is actually the origin of neutrosophic logic, neutrosophic sets (NSs), neutrosophic statistics and probability etc. For more details, see [4, 6, 7, 8, 9, 12].

Motivating from the realisms of physical life phenomenon, i.e., different sports (win/ tie/ defeat), votes like yes/ NA/ NO and making a decision. In 1999, Smrandache [14, 15] presented a fresh idea of NSs and neutrosophic logic, which is the generality of an FS and IFS, NS is defined by (truth-membership, indeterminacy-membership and falsity-membership degrees). This idea of NS creates the NS theory by providing the illustration to indeterminate. This theory is well thought-out as the whole demonstration of nearly each model of all actual difficulties. Thus, vagueness is complicated in problematical questions we use FS whereas, commerce indeterminacy, we must have a neutrosophic theory. This theory has numerous applications in countless fields such as control theory, records, medicinal judgment difficulties and decision-making questions. Such types of models have been studied by several authors (see [11, 18, 19, 20, 21, 22, 24]).

In this paper, we present the perception of neutrosophic cubic sets (NCSs) which is a generality of the FSs, cubic sets and NSs. Also, we provide different procedures and operators on the NCSs. These procedures and operations actually generalize the procedures and operators of FSs, cubic sets, and NS previously which have been proposed. Thus, in section 2, we suggest a conception of NCS and its different procedures as well as the score, certainty and accuracy functions to associate the NCSs. In the similar sector, also we progressed the weighted average operator on NCSs ($\mathcal{A}_W$) and weighted geometric operator on NCSs ($\mathcal{G}_W$) operator to aggregate the neutrosophic cubic information. In section 3, on the based of ($\mathcal{A}_W$) and ($\mathcal{G}_W$) and the functions like score, certainty and accuracy, we progress an approach to multiple criteria decision-making on NCSs, in which the assessment values of substitutes on the aspects took the form of NCNs to choice the utmost necessary ones and stretch the scientific examples to determine the application and usefulness of the establish technique. We conclude this paper in the last section.

2. Basic concepts

In this section, we give almost different perceptions associated to NSs and cubic sets.

Definition 2.1. [14] *Let Z be a universe of discourse, then*

$$\Psi = \{\langle v, \Lambda_T(v), \Lambda_I(v), \Lambda_F(v)\rangle : v \in Z\}$$

is termed as an NS where $\Lambda_T, \Lambda_I, \Lambda_F : Z \to]0^-, 1^+[$ and the member-ship functions $\Lambda_T, \Lambda_I, \Lambda_F$ are truth, indeterminacy and falsity membership degrees respectively and on the sum of $\Lambda_T(v), \Lambda_I(v), \Lambda_F(v)$ there is no restraint so, $0 \leq \Lambda_T(v) + \Lambda_I(v) + \Lambda_F(v) \leq 3$.

For application in physical technical and different engineering regions, Wang et al. [16] gave the conception of a single valued NS as follows:

Definition 2.2. [16] *A single-valued NS is define as:*

$$\Psi_{NS} = \{\langle v, \Lambda_T(v), \Lambda_I(v), \Lambda_F(v)\rangle : v \in Z\}$$

where, $\Lambda_F(v) : Z \longrightarrow [0,1]$, $\Lambda_I(v) : Z \longrightarrow [0,1]$ and $\Lambda_T(v) : Z \longrightarrow [0,1]$ with $0 \leq \Lambda_F(v) + \Lambda_I(v) + \Lambda_T(v) \leq 3$ for all $v \in Z$. The intervals $\Lambda_F(v), \Lambda_I(v)$ and $\Lambda_T(v)$ denote truth, indeterminacy and falsity membership degrees of Z to A, respectively.

Here, we explain some set-theoretic operations for two single valued NSs [16]. Consider two single valued NSs

$$\Psi_{NS} = \{\langle v, \Lambda_T(v), \Lambda_I(v), \Lambda_F(v)\rangle : v \in Z\}$$

and

$$\Psi'_{NS} = \left\{ \left\langle v, \Lambda'_T(v), \Lambda'_I(v), \Lambda'_F(v) \right\rangle : v \in Z \right\}$$

then set-theoretic operations for these two single valued NSs are given as;

(i) $\Psi_{NS} \subset \Psi'_{NS} \Leftrightarrow \Lambda_T(v) \leq \Lambda'_T(v), \Lambda_I(v) \leq \Lambda'_I(v), \Lambda_F(v) \geq \Lambda'_F(v)$.

(ii) $\Psi_{NS} = \Psi'_{NS} \Leftrightarrow \Lambda_T(v) = \Lambda'_T(v), \Lambda_I(v) = \Lambda'_I(v), \Lambda_F(v) = \Lambda'_F(v)$, for any $v \in Z$.

(iii) The complementation of Ψ_{NS} is represented by Ψ^c_{NS} and is defined as follows

$$\Psi^c_{NS} = \{\langle v, \Lambda_F(v), 1 - \Lambda_I(v), \Lambda_T(v)\rangle / v \in Z\}$$

(iv) The intersection

$$\Psi_{NS} \bigcap \Psi'_{NS} = \left\{ \left\langle v, \min\left\{\Lambda_T(v), \Lambda'_T(v)\right\},\ \max\left\{\Lambda_I(v), \Lambda'_I(v)\right\}, \max\left\{\Lambda_F(v), \Lambda'_F(v)\right\} \right\rangle : v \in Z \right\}$$

(v) The Union

$$\Psi_{NS} \bigcup \Psi'_{NS} = \left\{ \left\langle v, \max\left\{\Lambda_T(v), \Lambda'_T(v)\right\},\ \min\left\{\Lambda_I(v), \Lambda'_I(v)\right\}, \min\left\{\Lambda_F(v), \Lambda'_F(v)\right\} \right\rangle : v \in Z \right\}$$

Note: For convenience, a single-valued NN is denoted by $\Psi = \langle \Lambda_T, \Lambda_I, \Lambda_F \rangle$

Definition 2.3. [11] *Let $\Psi_1 = \langle \Lambda_{T_1}, \Lambda_{I_1}, \Lambda_{F_1} \rangle$ and $\Psi_2 = \langle \Lambda_{T_2}, \Lambda_{I_2}, \Lambda_{F_2} \rangle$ be two single valued NNs. Then, the operations for NNs are defined as below;*

(i) $\lambda\Psi_1 = \left\langle 1 - (1 - \Lambda_{T_1})^\lambda, \Lambda^\lambda_{I_1}, \Lambda^\lambda_{F_1} \right\rangle$

(ii) $\Psi^\lambda_1 = \left\langle \Lambda^\lambda_{T_1}, 1 - (1 - \Lambda_{I_1})^\lambda, 1 - (1 - \Lambda_{F_1})^\lambda \right\rangle$

(iii) $\Psi_1 + \Psi_2 = \langle \Lambda_{T_1} + \Lambda_{T_2} - \Lambda_{T_1}\Lambda_{T_2}, \Lambda_{I_1}\Lambda_{I_2}, \Lambda_{F_1}\Lambda_{F_2} \rangle$

(iv) $\Psi_1\Psi_2 = \langle \Lambda_{T_1}\Lambda_{T_2}, \Lambda_{I_1} + \Lambda_{I_2} - \Lambda_{I_1}\Lambda_{I_2}, \Lambda_{F_1} + \Lambda_{F_2} - \Lambda_{F_1}\Lambda_{F_2} \rangle$, where $\lambda > 0$.

Definition 2.4. [1, 11] *Let $\Psi_1 = \langle \Lambda_{T_1}, \Lambda_{I_1}, \Lambda_{F_1} \rangle$ be a singled-valued NN. Then, the score, accuracy and certainty function of an NNs are define as follows:*

(i) $s(\Psi_1) = \frac{(\Lambda_{T_1} + 1 - \Lambda_{I_1} + 1 - \Lambda_{F_1})}{3}$;

(ii) $a(\Psi_1) = \Lambda_{T_1} - \Lambda_{F_1}$;

(iii) $c(\Psi_1) = \Lambda_{T_1}$.

Theorem 2.5. [12] *Let $\Psi_1 = \langle \Lambda_{T_1}, \Lambda_{I_1}, \Lambda_{F_1} \rangle$ and $\Psi_2 = \langle \Lambda_{T_2}, \Lambda_{I_2}, \Lambda_{F_2} \rangle$ be two single-valued NNs. If $\Psi_1 \subseteq \Psi_2$, then $s(\Psi_1) \leq s(\Psi_2)$.*

Proof. By Definition 2.1, we have that

$$s(\Psi_1) = \frac{(\Lambda_{T_1} + 1 - \Lambda_{I_1} + 1 - \Lambda_{F_1})}{3} \text{ and } s(\Psi_2) = \frac{(\Lambda_{T_2} + 1 - \Lambda_{I_2} + 1 - \Lambda_{F_2})}{3}.$$

Now

$$\begin{aligned} s(\Psi_2) - s(\Psi_1) &= \frac{(\Lambda_{T_2} + 1 - \Lambda_{I_2} + 1 - \Lambda_{F_2})}{3} - \frac{(\Lambda_{T_1} + 1 - \Lambda_{I_1} + 1 - \Lambda_{F_1})}{3} \\ &= \frac{(\Lambda_{T_2} - \Lambda_{T_1}) + (\Lambda_{I_1} - \Lambda_{F_1}) + (\Lambda_{F_1} - \Lambda_{F_2})}{3} \end{aligned}$$

since $\Psi_1 \subseteq \Psi_2$, $\Lambda_{T_1} \leq \Lambda_{T_2}$, $\Lambda_{I_1} \geq \Lambda_{I_2}$, $\Lambda_{F_1} \geq \Lambda_{F_2}$ and hence $(\Lambda_{T_1} - \Lambda_{T_2}) \geq 0, (\Lambda_{I_1} - \Lambda_{I_2}) \geq 0$ and $(\Lambda_{F_1} - \Lambda_{F_2}) \geq 0$. Then it follows that $s(\Psi_2) - s(\Psi_1) \geq 0.\square$

Definition 2.6. [11, 12] *Let* $\Psi_1 = \langle \Lambda_{T_1}, \Lambda_{I_1}, \Lambda_{F_1} \rangle$ *and* $\Psi_2 = \langle \Lambda_{T_2}, \Lambda_{I_2}, \Lambda_{F_2} \rangle$ *be two single valued NNs. Then, the comparison method is defined by:*

(i) if $s(\Psi_1) > s(\Psi_2)$, then $\Psi_1 > \Psi_2$,
(ii) if $s(\Psi_1) = s(\Psi_2)$ and $a(\Psi_1) > a(\Psi_2)$ then Ψ_1 is inferior to ,Ψ_2 denoted by $\Psi_1 < \Psi_2$,
(iii) if $s(\Psi_1) = s(\Psi_2)$, $a(\Psi_1) = a(\Psi_2)$ and $c(\Psi_1) > c(\Psi_2)$ then Ψ_1 is superior to, Ψ_2 denoted by $\Psi_1 > \Psi_2$,
(iv) if $s(\Psi_1) = s(\Psi_2)$, $a(\Psi_1) = a(\Psi_2)$ and $c(\Psi_1) = c(\Psi_2)$ then Ψ_1 is equal to Ψ_2 , that is, Ψ_1 is indifferent to, Ψ_2 denoted by $\Psi_1 = \Psi_2$.

The idea of NCSs was given by Jun et al. in 2015, for details, see [6].

Definition 2.7. [5] *A non-empty set* Z*, then structure of the arrangement*

$$\ddot{\mathtt{C}} = \left\{ v, \check{\maltese}(v), \maltese(v) | v \in Z \right\}$$

is called a cubic set in Z *in which* $\check{\maltese}$ *is an interval-valued fuzzy* (IVF) *set in* Z *and* $\maltese$ *is a fuzzy set* (FS) *in* Z*.*

A cubic set $\ddot{\mathtt{C}} = \left\{ \langle Z, \check{\maltese}(v), \maltese(v) \rangle | v \in Z \right\}$ is simply denoted by $\ddot{\mathtt{C}} = \langle \check{\maltese}, \maltese \rangle$ and the collection of all cubic sets in Z is denoted by $\ddot{\mathtt{C}}^n$ such that $\ddot{\mathtt{C}}^1 = \langle \check{\maltese}^1, \maltese^1 \rangle$ $\check{\maltese}^1(v) = [0,0]$ and $\maltese^1(v) = 1$ $\Big($respectivly $\check{\maltese}^1(v) = [1,1]$ and $\maltese^1(v) = 0\Big)$ $\forall\, v \in Z$ is denoted by $\ddot{0}$ $\Big($respectively $\ddot{1}\Big)$. A cubic set $\ddot{\mathtt{C}}^2 = \langle \check{\maltese}^2, \maltese^2 \rangle$ in which $\check{\maltese}'(v) = [0,0]$ and $\maltese^2(v) = 0$ $\Big($respectively $\check{\maltese}^2(v) = [1,1]$ and $\maltese^2(v) = 1\Big)$ for all $v \in Z$ is denoted by $\widehat{0}$ $\Big($respectively $\widehat{1}\Big)$.

3. Neutrosophic Cubic Sets(NCSs)

In this segment, we outline the NCSs and some operations on it for instance score, certainty and accuracy functions to associate the NCSs. Also, we develop the neutrosophic cubic weight and geometric operators to aggregate the neutrosophic cubic information. Some of it is mentioned from [3, 4, 5, 19, 21].

Definition 3.1. [6] *An NCS in* Z *is a pair* $\mathcal{A} = (\tilde{\Psi}, \Psi)$ *where*

$$\tilde{\Psi} := \left\{ \left\langle v, \tilde{\Phi}_T(v), \tilde{\Phi}_I(v), \tilde{\Phi}_F(v) \right\rangle : v \in Z \right\}$$

is an interval NS in Z *where* $\tilde{\Phi}_T, \tilde{\Phi}_I, \tilde{\Phi}_F : Z \to D[0,1]$ *and*

$$\Psi := \{ \langle v, \Lambda_T(v), \Lambda_I(v), \Lambda_F(v) \rangle : v \in Z \}$$

is an NS in Z*. where* $\Lambda_T, \Lambda_I, \Lambda_F : Z \to [0,1]$.

$\tilde{\Phi}_T(v), \tilde{\Phi}_I(v), \tilde{\Phi}_F(v)$ are interval membership degrees and are described by the respective truth, indeterminate and falsity memberships of an element $v \in Z$ corresponding to an NCS $\mathcal{A}$ and $\Lambda_T(v), \Lambda_I(v), \Lambda_F(v)$ are singled valued memberships described by the respective truth indeterminate and falsity memberships of an element $v \in Z$ to some implicit counter-property equivalent to an NCS $\mathcal{A}$.

Example 3.2. *Let $Z = \{v_1, v_2, v_3\}$. Then*

$$\mathcal{A}_1 = \left\{ \begin{array}{c} (v_1 \langle [0.3, 0.5], [0.2, 0.3], [0.1, 0.5] \rangle, \langle (0.6, 0.4, 0.01) \rangle) \\ (v_2, \langle [0.2, 0.3], [0.1, 0.2], [0.3, 0.7] \rangle, \langle 0.02, 0.003, 0.5 \rangle) \\ (v_3, \langle [0.3, 0.5], [0.2, 0.3], [0.1, 0.4] \rangle, \langle 0.1, 0.5, 0.06 \rangle) \end{array} \right\}$$

is a neutrosophic cubic subset of Z.

The NCS is the generalization of a cubic set which is shown as follows:

Theorem 3.3. [6] *An NCS is the generalization of a cubic set.*

Definition 3.4. [6] *Let*

$$\mathcal{A}_1 = \left(v, \left\langle \tilde{\Phi}_{T_1}(v), \tilde{\Phi}_{I_1}(v), \tilde{\Phi}_{F_1}(v) \right\rangle, \langle \Lambda_{T_1}(v), \Lambda_{I_1}(v), \Lambda_{F_1}(v) \rangle \right)$$

and

$$\mathcal{A}_2 = \left(v, \left\langle \tilde{\Phi}_{T_2}(v), \tilde{\Phi}_{I_2}(v), \tilde{\Phi}_{F_2}(v) \right\rangle, \langle \Lambda_{T_2}(v), \Lambda_{I_2}(v), \Lambda_{F_2}(v) \rangle \right)$$

be two NCSs. Then $\mathcal{A}_1 \subseteq \mathcal{A}_2$ if and only if

$$\tilde{\Phi}_{T_1}(v) \preceq \tilde{\Phi}_{T_2}(v), \tilde{\Phi}_{I_1}(v) \preceq \tilde{\Phi}_{I_2}(v), \tilde{\Phi}_{F_1}(v) \succeq \tilde{\Phi}_{F_2}(v)$$

and

$$\Lambda_{T_1}(v) \geq \Lambda_{T_2}(v), \Lambda_{I_1}(v) \geq \Lambda_{I_2}(v), \Lambda_{F_1}(v) \leq \Lambda_{F_2}(v)$$

for all $v \in Z$.

Example 3.5. *Consider two neutrosophic cubic numbers (NCNs)*

$$\begin{array}{l} \tilde{a}_1 = \{v, \langle [0.3, 0.5], [0.2, 0.3], [0.3, 0.5] \rangle, \langle 0.5, 0.4, 0.6 \rangle\} \text{ and} \\ \tilde{a}_2 = \{v, \langle [0.5, 0.7], [0.4, 0.5], [0.2, 0.4] \rangle, \langle 0.3, 0.2, 0.8 \rangle\} \end{array}$$

such that $\tilde{a}_1 < \tilde{a}_2$,because

$$\begin{array}{l} [0.3, 0.5] \preceq [0.5, 0.7], \;\; [0.2, 0.3] \preceq [0.4, 0.5], \;\; [0.2, 0.4] \succeq [0.3, 0.5] \\ \text{and } 0.5 \geq 0.3, 0.4 \geq 0.2, 0.6 \leq 0.8. \end{array}$$

Definition 3.6. [6] *Let*

$$\mathcal{A}_1 = \left(v, \left\langle \tilde{\Phi}_{T_1}(v), \tilde{\Phi}_{I_1}(v), \tilde{\Phi}_{F_1}(v) \right\rangle, \langle \Lambda_{T_1}(v), \Lambda_{I_1}(v), \Lambda_{F_1}(v) \rangle \right)$$

and

$$\mathcal{A}_2 = \left(v, \left\langle \tilde{\Phi}_{T_2}(v), \tilde{\Phi}_{I_2}(v), \tilde{\Phi}_{F_2}(v) \right\rangle, \langle \Lambda_{T_2}(v), \Lambda_{I_2}(v), \Lambda_{F_2}(v) \rangle \right)$$

be two NCSs. Then $\mathcal{A}_1 = \mathcal{A}_2$ *if and only if*

$$\tilde{\Phi}_{T_1}(v) = \tilde{\Phi}_{T_2}(v), \tilde{\Phi}_{I_1}(v) = \tilde{\Phi}_{I_2}(v), \tilde{\Phi}_{F_1}(v) = \tilde{\Phi}_{F_2}(v)$$

and

$$\Lambda_{T_1}(v) = \Lambda_{T_2}(v), \Lambda_{I_1}(v) = \Lambda_{I_2}(v), \Lambda_{F_1}(v) = \Lambda_{F_2}(v)$$

for all $v \in Z$*.*

Definition 3.7. [6] *Let*

$$\mathcal{A}_1 = \left(v, \left\langle \tilde{\Phi}_{T_1}(v), \tilde{\Phi}_{I_1}(v), \tilde{\Phi}_{F_1}(v) \right\rangle, \langle \Lambda_{T_1}(v), \Lambda_{I_1}(v), \Lambda_{F_1}(v) \rangle \right)$$

and

$$\mathcal{A}_2 = \left(v, \left\langle \Lambda_{T_2}(v), \tilde{\Phi}_{I_2}(v), \tilde{\Phi}_{F_2}(v) \right\rangle, \langle \Lambda_{T_2}(v), \Lambda_{I_2}(v), \Lambda_{F_2}(v) \rangle \right)$$

be two NCSs. Then the union is defined as:

$$(\mathcal{A}_1 \bigcup \mathcal{A}_2)(v) = \left(\begin{array}{c} \max(\tilde{\Phi}_{T_1}(v), \tilde{\Phi}_{T_2}(v)), \frac{\tilde{\Phi}_{I_1}(v)+\tilde{\Phi}_{I_2}(v)}{2}, min(\tilde{\Phi}_{F_1}(v), \tilde{\Phi}_{F_2}(v)), \\ \min(\Lambda_{T_1}(v), \Lambda_{T_2}(v)), \frac{\Lambda_{I_1}(v)+\Lambda_{I_2}(v)}{2}, \max(\Lambda_{F_1}(v), \Lambda_{F_2}(v)) \end{array} \right)$$

for all $v \in Z$.

Example 3.8. *Let* $Z = \{v_1, v_2, v_3\}$*, then*

$$\mathcal{A}_1 = \left\{ \begin{array}{c} (v_1, \langle [0.3, 0.5], [0.1, 0.3], [0.1, 0.6] \rangle, \langle 0.6, 0.4, 0.01 \rangle) \\ (v_2, \langle [0.2, 0.3], [0.2, 0.4], [0.2, 0.7] \rangle, \langle 0.02, 0.003, 0.5 \rangle) \\ (v_3, \langle [0.3, 0.5], [0.3, 0.4], [0.1, 0.2] \rangle, \langle 0.1, 0.5, 0.06 \rangle) \end{array} \right\}$$

and

$$\mathcal{A}_2 = \left\{ \begin{array}{c} (v_1, \langle [0.2, 0.6], [0.2, 0.5], [0.2, 0.4] \rangle, \langle 0.3, 0.5, 0.1 \rangle) \\ (v_2, \langle [0.3, 0.5], [0.1, 0.7], [0.3, 0.6] \rangle, \langle 0.02, 0.03, 0.3 \rangle) \\ (v_3, \langle [0.1, 0.4], [0.4, 0.5], [0.3, 0.7] \rangle, \langle 0.4, 0.6, 0.7 \rangle) \end{array} \right\}$$

are two NCSs in Z*, then their union is given as follows:*

$$(\mathcal{A}_1 \bigcup \mathcal{A}_2)(v) = \left\{ \begin{array}{c} (v_1, \langle [0.3, 0.6], [0.15, 0.4], [0.1, 0.4] \rangle, \langle 0.3, 0.45, 0.1 \rangle) \\ (v_2, \langle [0.3, 0.5], [0.15, 0.55], [0.2, 0.6] \rangle, \langle 0.02, 0.0165, 0.5 \rangle) \\ (v_3, \langle [0.3, 0.5], [0.35, 0.45], [0.1, 0.2] \rangle, \langle 0.1, 0.55, 0.7 \rangle) \end{array} \right\}$$

Definition 3.9. *Let*

$$\mathcal{A}_1 = \left(v, \left\langle \tilde{\Phi}_{T_1}(v), \tilde{\Phi}_{I_1}(v), \tilde{\Phi}_{F_1}(v) \right\rangle, \langle \Lambda_{T_1}(v), \Lambda_{I_1}(v), \Lambda_{F_1}(v) \rangle \right)$$

and

$$\mathcal{A}_2 = \left(v, \left\langle \tilde{\Phi}_{T_2}(v), \tilde{\Phi}_{I_2}(v), \tilde{\Phi}_{F_2}(v) \right\rangle, \langle \Lambda_{T_2}(v), \Lambda_{I_2}(v), \Lambda_{F_2}(v) \rangle \right)$$

be two NCSs. Then the intersection is defined as:

$$(\mathcal{A}_1 \bigcap \mathcal{A}_2)(v) = \left(\begin{array}{c} min(\tilde{\Phi}_{T_1}(v), \tilde{\Phi}_{T_2}(v)), \frac{\tilde{\Phi}_{I_1}(v)+\tilde{\Phi}_{I_2}(v)}{2}, \max(\tilde{\Phi}_{F_1}(v), \tilde{\Phi}_{F_2}(v)), \\ \max(\Lambda_{T_1}(v), \Lambda_{T_2}(v)), \frac{\Lambda_{I_1}(v)+\Lambda_{I_2}(v)}{2}, \min \Lambda_{F_1}(v), \Lambda_{F_2}(v)) \end{array} \right)$$

for all $v \in Z$.

Example 3.10. *Let $Z = \{v_1, v_2, v_3\}$, then*

$$\mathcal{A}_1 = \left\{ \begin{array}{c} (v_1, \langle [0.3, 0.5], [0.1, 0.3], [0.1, 0.6] \rangle, \langle 0.6, 0.4, 0.01 \rangle) \\ (v_2, \langle [0.2, 0.3], [0.2, 0.4], [0.2, 0.7] \rangle, \langle 0.02, 0.003, 0.5 \rangle) \\ (v_3, \langle [0.3, 0.5], [0.3, 0.4], [0.1, 0.2] \rangle, \langle 0.1, 0.5, 0.06 \rangle) \end{array} \right\}$$

and

$$\mathcal{A}_2 = \left\{ \begin{array}{c} (v_1, \langle [0.2, 0.6], [0.2, 0.5], [0.2, 0.4] \rangle, \langle 0.3, 0.5, 0.1 \rangle) \\ (v_2, \langle [0.3, 0.5], [0.1, 0.7], [0.3, 0.6] \rangle, \langle 0.02, 0.03, 0.3 \rangle) \\ (v_3, \langle [0.1, 0.4], [0.4, 0.5], [0.3, 0.7] \rangle, \langle 0.4, 0.6, 0.7 \rangle) \end{array} \right\}$$

are two NCSs in Z, then their intersection is given as follows:

$$(\mathcal{A}_1 \bigcap \mathcal{A}_2)(v) = \left\{ \begin{array}{c} (v_1, \langle [0.2, 0.5], [0.15, 0.4], [0.2, 0.6] \rangle, \langle 0.6, 0.45, 0.01 \rangle) \\ (v_2, \langle [0.2, 0.3], [0.15, 0.55], [0.3, 0.7] \rangle, \langle 0.02, 0.0165, 0.3 \rangle) \\ (v_3, \langle [0.1, 0.4], [0.35, 0.45], [0.3, 0.7] \rangle, \langle 0.4, 0.55, 0.06 \rangle) \end{array} \right\}$$

Definition 3.11. *Let*

$$\mathcal{A} = \left(v, \left\langle \tilde{\Phi}_T(v), \tilde{\Phi}_I(v), \tilde{\Phi}_F(v) \right\rangle, \langle \Lambda_T(v), \Lambda_I(v), \Lambda_F(v) \rangle \right)$$

be an NCS in Z. Then their complement $\mathcal{A}^c$ is defined as

$$\tilde{\Phi}_T^c(v) = [1, 1] - \tilde{\Phi}_T(v), \tilde{\Phi}_I^c(v) = [1, 1] - \tilde{\Phi}_I(v), \tilde{\Phi}_F^c(v) = [1, 1] - \tilde{\Phi}_F(v)$$

and

$$\Lambda_T^c(v) = 1 - \Lambda_T(v), \Lambda_I^c(v) = 1 - \Lambda_I(v), \Lambda_F^c(v) = 1 - \Lambda_F(v)$$

for all $v \in Z$.

Example 3.12. *Consider $Z = \{v_1, v_2, v_3\}$, then*

$$\mathcal{A} = \left\{ \begin{array}{c} (v_1, \langle [0.3, 0.5], [0.1, 0.3], [0.1, 0.6] \rangle, \langle 0.6, 0.4, 0.01 \rangle) \\ (v_2, \langle [0.2, 0.3], [0.2, 0.4], [0.2, 0.7] \rangle, \langle 0.02, 0.003, 0.5 \rangle) \\ (v_3, \langle [0.3, 0.5], [0.3, 0.4], [0.1, 0.2] \rangle, \langle 0.1, 0.5, 0.06 \rangle) \end{array} \right\}$$

is an NCS in Z. Then the complement of $\mathcal{A}$ is given as follows:

$$\mathcal{A}^c = \left\{ \begin{array}{c} (v_1, \langle [0.5, 0.7], [0.7, 0.9], [0.4, 0.9] \rangle, \langle 0.4, 0.6, 0.99 \rangle) \\ (v_2, \langle [0.7, 0.8], [0.6, 0.8], [0.3, 0.8] \rangle, \langle 0.98, 0.997, 0.5 \rangle) \\ (v_3, \langle [0.5, 0.7], [0.6, 0.7], [0.8, 0.9] \rangle, \langle 0.9, 0.5, 0.94 \rangle) \end{array} \right\}$$

Result: For two NCSs, De-Morgan's laws also hold i.e.

$$(i) \quad \left(\mathcal{A}_1 \bigcup \mathcal{A}_2\right)^C = \mathcal{A}_1^C \bigcap \mathcal{A}_2^C;$$

$$(ii) \quad \left(\mathcal{A}_1 \bigcap \mathcal{A}_2\right)^C = \mathcal{A}_1^C \bigcup \mathcal{A}_2^C.$$

NOTE: The set of all NCSs in Z are denoted by Q and NCN by $\tilde{a} = \left(\left\langle \tilde{\Phi}_T, \tilde{\Phi}_I, \tilde{\Phi}_F \right\rangle, \langle \Lambda_T, \Lambda_I, \Lambda_F \rangle \right)$ for suitability.

Definition 3.13. *Let* $\tilde{a}_1 = \left(\left\langle \tilde{\Phi}_{T_1}, \tilde{\Phi}_{I_1}, \tilde{\Phi}_{F_1} \right\rangle, \langle \Lambda_{T_1}, \Lambda_{I_1}, \Lambda_{F_1} \rangle\right)$ *and* $\tilde{a}_2 = \left(\left\langle \tilde{\Phi}_{T_2}, \tilde{\Phi}_{I_2}, \tilde{\Phi}_{F_2} \right\rangle, \langle \Lambda_{T_2}, \Lambda_{I_2}, \Lambda_{F_2} \rangle\right)$ *be two NCNs, then the following operations for NCSs are defined as below:*

(i)

$$\lambda \tilde{a}_1 = \left\{ \begin{array}{c} \left\langle [1,1] - ([1,1] - (\tilde{\Phi}_{T_1}))^{\lambda}, [1,1] - ([1,1] - (\tilde{\Phi}_{I_1}))^{\lambda}, [1,1] - ([1,1] - (\tilde{\Phi}_{F_1}))^{\lambda} \right\rangle, \\ \left\langle (\Lambda_{T_1})^{\lambda}, (\Lambda_{I_1})^{\lambda}, (\Lambda_{F_1})^{\lambda} \right\rangle \end{array} \right\},$$

(ii)

$$\tilde{a}_1^{\lambda} = \left\{ \begin{array}{c} \left\langle (\tilde{\Phi}_{T_1})^{\lambda}, (\tilde{\Phi}_{I_1})^{\lambda}, (\tilde{\Phi}_{F_1})^{\lambda} \right\rangle, \\ \langle 1 - (1 - \Lambda_{T_1})^{\lambda}, 1 - (1 - \Lambda_{I_1})^{\lambda}, 1 - (1 - \Lambda_{F_1})^{\lambda} \rangle \end{array} \right\},$$

(iii)

$$\tilde{a}_1 + \tilde{a}_2 = \left\{ \begin{array}{c} \left\langle \tilde{\Phi}_{T_1} + \tilde{T}_2 - \tilde{\Phi}_{T_1}\tilde{T}_2, \tilde{\Phi}_{I_1} + \tilde{\Phi}_{I_2} - \tilde{\Phi}_{I_1}\tilde{\Phi}_{I_2}), \tilde{\Phi}_{F_1} + \tilde{\Phi}_{F_2} - \tilde{\Phi}_{F_1}\tilde{\Phi}_{F_2} \right\rangle, \\ \langle \Lambda_{T_1}\Lambda_{T_2}, \Lambda_{I_1}\Lambda_{I_2}, \Lambda_{F_1}\Lambda_{F_2} \rangle \end{array} \right\},$$

(iv)

$$\tilde{a}_1 \cdot \tilde{a}_2 = \left\{ \begin{array}{c} \left\langle \tilde{\Phi}_{T_1}\tilde{\Phi}_{T_2}, \tilde{\Phi}_{I_1}\tilde{\Phi}_{I_2}, \tilde{\Phi}_{F_1}\tilde{\Phi}_{F_2}, \right\rangle, \\ \langle \Lambda_{T_1} + \Lambda_{T_2} - \Lambda_{T_1}\Lambda_{T_2}, \Lambda_{I_1} + \Lambda_{I_2} - \Lambda_{I_1}\Lambda_{I_2}, \Lambda_{F_1} + \Lambda_{F_2} - \Lambda_{F_1}\Lambda_{F_2} \rangle \end{array} \right\}$$

where $\lambda > 0$.

Example 3.14. *Let* $Z = \{v_1\}$*, then*

$$\tilde{a}_1 = \{v, \langle [0.3, 0.5], [0.1, 0.3], [0.1, 0.6] \rangle, \langle 0.6, 0.4, 0.1 \rangle\}$$

and

$$\tilde{a}_2 = \{v, \langle [0.2, 0.6], [0.2, 0.5], [0.2, 0.4] \rangle, \langle 0.3, 0.5, 0.1 \rangle\}$$

are two NCNs in Z *and* $\lambda = 2$ *then*

(i) $\lambda \tilde{a}_1$ is given by

$$\lambda \tilde{a}_1 = \left\{ \begin{array}{c} v, \left\langle [1,1] - ([1,1] - (\tilde{\Phi}_{T_1}))^{\lambda}, [1,1] - ([1,1] - (\tilde{\Phi}_{I_1}))^{\lambda}, [1,1] - ([1,1] - (\tilde{\Phi}_{F_1}))^{\lambda} \right\rangle, \\ \left\langle (\Lambda_{T_1})^{\lambda}, (\Lambda_{I_1})^{\lambda}, (\Lambda_{F_1})^{\lambda} \right\rangle \end{array} \right\}$$

$$2\tilde{a}_1 = \left\{ \begin{array}{c} v, \langle [1,1] - ([1,1] - [0.3, 0.5])^2, [1,1] - ([1,1] - [0.1, 0.3])^2, [1,1] - ([1,1] - [0.1, 0.6])^2 \rangle, \\ \langle (0.6)^2, (0.4)^2, (0.1)^2 \rangle \end{array} \right\}$$

$$= \left\{ \begin{array}{c} v, \langle [1,1] - ([0.5, 0.7])^2, [1,1] - ([0.7, 0.9])^2, [1,1] - ([0.4, 0.9])^2 \rangle, \\ \langle 0.36, 0.16, 0.01 \rangle \end{array} \right\}$$

$$= \left\{ \begin{array}{c} v, \langle [1,1] - [0.25, 0.49], [1,1] - [0.49, 0.81], [1,1] - [0.16, 0.81] \rangle, \\ \langle 0.36, 0.16, 0.01 \rangle \end{array} \right\}$$

$$= \left\{ \begin{array}{c} v, \langle [0.51, 0.75], [0.19, 0.51], [0.19, 0.84] \rangle, \\ \langle 0.36, 0.16, 0.01 \rangle \end{array} \right\}.$$

(ii) $\tilde{a}_1^\lambda$ is given by

$$\tilde{a}_1^\lambda = \left\{ \begin{array}{c} \left\langle (\tilde{\Phi}_{T_1})^\lambda, (\tilde{\Phi}_{I_1})^\lambda, (\tilde{\Phi}_{F_1})^\lambda \right\rangle, \\ \left\langle 1-(1-\Lambda_{T_1})^\lambda, 1-(1-(\Lambda_{I_1})^\lambda, 1-(1-\Lambda_{F_1})^\lambda \right\rangle \end{array} \right\}$$

$$\tilde{a}_1^2 = \left\{ \begin{array}{c} \langle [0.3,0.5]^2, [0.1,0.3]^2, [0.1,0.6]^2 \rangle, \\ \langle 1-(1-0.6)^2, 1-(1-0.4)^2, 1-(1-0.1)^2 \rangle \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \langle [0.09,0.25], [0.01,0.09], [0.01,0.36] \rangle, \\ \langle 1-(0.4)^2, 1-(0.6)^2, 1-(0.9)^2 \rangle \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \langle [0.09,0.25], [0.01,0.09], [0.01,0.36] \rangle, \\ \langle 0.84, 0.64, 0.19 \rangle \end{array} \right\}.$$

(iii) $\tilde{a}_1 + \tilde{a}_2$ is given by

$$\tilde{a}_1 + \tilde{a}_2 = \left\{ \begin{array}{c} \left\langle \tilde{\Phi}_{T_1} + \tilde{\Phi}_{T_2} - \tilde{\Phi}_{T_1}\tilde{\Phi}_{T_2}, \tilde{\Phi}_{I_1} + \tilde{\Phi}_{I_2} - \tilde{\Phi}_{I_1}\tilde{\Phi}_{I_2}), \tilde{\Phi}_{F_1} + \tilde{\Phi}_{F_2} - \tilde{\Phi}_{F_1}\tilde{\Phi}_{F_2} \right\rangle, \\ \langle \Lambda_{T_1}\Lambda_{T_2}, \Lambda_{I_1}\Lambda_{I_2}, \Lambda_{F_1}\Lambda_{F_2} \rangle \end{array} \right\}$$

$$= (v_1, \langle [0.44,0.8], [0.28,0.65], [0.28,0.76] \rangle, \langle 0.18, 0.20, 0.001 \rangle).$$

(iv) $\tilde{a}_1 \cdot \tilde{a}_2$ is given by

$$\tilde{a}_1 \cdot \tilde{a}_2 = \left\{ \begin{array}{c} \left\langle \tilde{\Phi}_{T_1}\tilde{\Phi}_{T_2}, \tilde{\Phi}_{I_1}\tilde{I}_2, \tilde{\Phi}_{F_1}\tilde{\Phi}_{F_2}, \right\rangle, \\ \langle \Lambda_{T_1} + \Lambda_{T_2} - \Lambda_{T_1}\Lambda_{T_2}, \Lambda_{I_1} + \Lambda_{I_2} - \Lambda_{I_1}\Lambda_{I_2}, \Lambda_{F_1} + \Lambda_{F_2} - \Lambda_{F_1}\Lambda_{F_2} \rangle \end{array} \right\}$$

$$= (v_1, \langle [0.06,0.30], [0.02,0.15], [0.02,0.24] \rangle, \langle 0.72, 0.7, 0.2 \rangle).$$

Definition 3.15. *Let $\tilde{a}_1 = \left(\left\langle \tilde{\Phi}_{T_1}, \tilde{\Phi}_{I_1}, \tilde{\Phi}_{F_1} \right\rangle, \langle \Lambda_{T_1}, \Lambda_{I_1}, \Lambda_{F_1} \rangle \right)$ be NCNs. Then, the score, accuracy and certainty functions of an NCN are defined as follows:*

(a) $\tilde{s}(\tilde{a}_1) = \left\{ \left\langle \frac{\tilde{\Phi}_{T_1} + [1,1] - \tilde{\Phi}_{I_1} + [1,1] - \tilde{\Phi}_{F_1}}{3} \right\rangle, \left\langle \frac{\Lambda_{T_1} + 1 - \Lambda_{I_1} + 1 - \Lambda_{F_1}}{3} \right\rangle \right\}$,

(b) $\tilde{a}(\tilde{a}_1) = \left\{ \left\langle \tilde{\Phi}_{T_1} - \tilde{\Phi}_{F_1} \right\rangle, \langle \Lambda_{T_1} - \Lambda_{F_1} \rangle \right\}$,

(c) $\tilde{c}(\tilde{a}_1) = \left\{ \left\langle \tilde{\Phi}_{T_1} \right\rangle, \langle \Lambda_{T_1} \rangle \right\}$.

Example 3.16. *Consider $Z = \{v_1\}$, and*

$$\tilde{a}_1 = \{Z, \langle [0.3,0.5], [0.2,0.3], [0.3,0.5] \rangle, \langle 0.5, 0.4, 0.6 \rangle\},$$

then

(a) $s(\tilde{a}_1)$ is given by

$$\tilde{s}(\tilde{a}_1) = \left\{ \left\langle \frac{\tilde{\Phi}_{T_1} + [1,1] - \tilde{\Phi}_{I_1} + [1,1] - \tilde{\Phi}_{F_1}}{3} \right\rangle, \left\langle \frac{\Lambda_{T_1} + 1 - \Lambda_{I_1} + 1 - \Lambda_{F_1}}{3} \right\rangle \right\}$$

$$= \left\{ \left\langle \frac{[0.3,0.5] + [1,1] - [0.2,0.3] + [1,1] - [0.3,0.6]}{3} \right\rangle, \left\langle \frac{0.6 + 1 - 0.4 + 1 - 0.5}{3} \right\rangle \right\}$$

$$= \{\langle [0.46,0.66] \rangle, \langle 0.83 \rangle\}.$$

(b) $\tilde{a}(\tilde{a}_1)$ *is given by*

$$\tilde{a(\tilde{a}_1)} = \left\{\left\langle \tilde{\Phi}_{T_1} - \tilde{\Phi}_{F_1}\right\rangle, \langle \Lambda_{T_1} - \Lambda_{F_1}\rangle\right\}$$
$$= \{\langle [0.3, 0.6] - [0.3, 0.5]\rangle, \langle 0.5 - 0.6\rangle\} = \{[0.0, 0.1], 0.1\}.$$

(c) $\tilde{c}(\tilde{a}_1)$ *is given by*

$$\tilde{c}(\tilde{a}_1) = \left\{\left\langle \tilde{\Phi}_{T_1}\right\rangle, \langle \Lambda_{T_1}\rangle\right\} = \{[0.3, 0.5], 0.5\}.$$

We state the following Theorem without its proof.

Theorem 3.17. *Let* $\mathcal{A}_1 = \left\{\left\langle \tilde{\Phi}_{T_1}, \tilde{\Phi}_{I_1}, \tilde{\Phi}_{F_1}\right\rangle, \langle \Lambda_{T_1}, \Lambda_{I_1}, \Lambda_{F_1}\rangle\right\}$ *and* $\mathcal{A}_2 = \left\{\left\langle \tilde{\Phi}_{T_2}, \tilde{\Phi}_{I_2}, \tilde{\Phi}_{F_2}\right\rangle, \langle \Lambda_{T_2}, \Lambda_{I_2}, \Lambda_{F_2}\rangle\right\}$ *be two single valued NCNs. If* $\mathcal{A}_1 \subseteq \mathcal{A}_2$*, then* $\tilde{s}(\mathcal{A}_1) \leq \tilde{s}(\mathcal{A}_2)$*.*

Definition 3.18. *Let* $\tilde{a}_1 = \left(\left\langle \tilde{\Phi}_{T_1}, \tilde{\Phi}_{I_1}, \tilde{\Phi}_{F_1}\right\rangle, \langle \Lambda_{T_1}, \Lambda_{I_1}, \Lambda_{F_1}\rangle\right)$ *and* $\tilde{a}_2 = \left(\left\langle \tilde{\Phi}_{T_2}, \tilde{\Phi}_{I_2}, \tilde{\Phi}_{F_2}\right\rangle, \langle \Lambda_{T_2}, \Lambda_{I_2}, \Lambda_{F_2}\rangle\right)$ *be two NCNs. Then comparison method for NCNs can be defined as follows:*

(i) if $\tilde{s}(\tilde{a}_1) > \tilde{s}(\tilde{a}_2)$*, then* $\tilde{a}_1$ *is greater than* $\tilde{a}_2$*, and is denoted by* $\tilde{a}_1 > \tilde{a}_2$*,*
(ii) if $\tilde{s}(\tilde{a}_1) = \tilde{s}(\tilde{a}_2)$*, and* $\tilde{a}(\tilde{a}_1) > \tilde{a}(\tilde{a}_2)$ *then* $\tilde{a}_1$ *is greater than* $\tilde{a}_2$*, and is denoted by* $\tilde{a}_1 < \tilde{a}_2$*,*
(iii) if $\tilde{s}(\tilde{a}_1) = \tilde{s}(\tilde{a}_2)$*,* $\tilde{a}(\tilde{a}_1) = \tilde{a}(\tilde{a}_2)$ *and* $\tilde{c}(\tilde{a}_1) > \tilde{c}(\tilde{a}_2)$ *then* $\tilde{a}_1$ *is greater than* $\tilde{a}_2$*,and is denoted by* $\tilde{a}_1 > \tilde{a}_2$*,*
(iv) if $\tilde{s}(\tilde{a}_1) = \tilde{s}(\tilde{a}_2)$*,* $\tilde{a}(\tilde{a}_1) = \tilde{a}(\tilde{a}_2)$ *and* $\tilde{c}(\tilde{a}_1) = \tilde{c}(\tilde{a}_2)$ *then* $\tilde{a}_1$ *is equal to* $\tilde{a}_2$*, that is* $\tilde{a}_1$ *is indifferent to* $\tilde{a}_2$*, and is denoted by* $\tilde{a}_1 = \tilde{a}_2$*.*

Example 3.19. *Consider two NCNs*

$$\tilde{a}_1 = \{v, \langle [0.5, 0.7], [0.4, 0.5], [0.2, 0.4]\rangle, \langle 0.3, 0.2, 0.8\rangle\} \text{ and}$$
$$\tilde{a}_2 = \{v, \langle [0.3, 0.5], [0.2, 0.3], [0.3, 0.5]\rangle, \langle 0.5, 0.4, 0.6\rangle\}$$

such that $\tilde{a}_1 > \tilde{a}_2$*, then,*

(i) We want to show that $\tilde{s}(\tilde{a}_1) > \tilde{s}(\tilde{a}_2)$. Since

$$\tilde{s}(\tilde{a}_1) = \left\langle \frac{(\tilde{\Phi}_{T_1} + [1, 1] - \tilde{\Phi}_{I_1} + [1, 1] - \tilde{\Phi}_{F_1})}{3}\right\rangle, \left\langle \frac{(\Lambda_{T_1} + 1 - \Lambda_{I_1} + 1 - \Lambda_{F_1})}{3}\right\rangle$$
$$= \{v, \langle [0.93, 0.96]\rangle, \langle 0.76\rangle\}$$

and

$$\tilde{s}(\tilde{a}_2) = \left\langle \frac{(\tilde{\Phi}_{T_2} + [1, 1] - \tilde{\Phi}_{I_2} + [1, 1] - \tilde{\Phi}_{F_2})}{3}\right\rangle, \left\langle \frac{(\Lambda_{T_2} + 1 - \Lambda_{I_2} + 1 - \Lambda_{F_2})}{3}\right\rangle$$
$$= \{v, \langle [0.90, 0.93]\rangle, \langle 0.83\rangle\},$$

which clearly shows that $\tilde{s}(\tilde{a}_1) > \tilde{s}(\tilde{a}_2)$. Similarly we can show (ii),(iii) and (iv) hold.

Based on the study given in [17, 22] we define some weighted aggregation operators related to NCSs as follows:

Theorem 3.20. *Let* $\tilde{a}_j = \left(\left\langle \tilde{\Phi}_{T_j}, \tilde{\Phi}_{I_j}, \tilde{\Phi}_{F_j} \right\rangle, \left\langle \Lambda_{T_j}, \Lambda_{I_j}, \Lambda_{F_j} \right\rangle\right)$ $(j = 1, 2, ..., n)$ *be a family of NCNs. A mapping* $\mathcal{A}_w : Q_n \to Q$ *is called neutrosophic cubic weighted average operator if it satisfies*

$$\mathcal{A}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} w_j \tilde{a}_j$$
$$= \left(\left\langle [1,1] - \prod_{j=1}^{n}([1,1] - \tilde{\Phi}_{T_j})^{w_j}, [1,1] - \prod_{j=1}^{n}([1,1] - \tilde{\Phi}_{I_j})^{w_j}, [1,1] - \prod_{j=1}^{n}([1,1] - \tilde{\Phi}_{F_j})^{w_j} \right\rangle, \right.$$
$$\left. \left\langle \prod_{j=1}^{n}(\Lambda_{T_j})^{w_j}, \prod_{j=1}^{n}(\Lambda_{I_j})^{w_j}, \prod_{j=1}^{n}(\Lambda_{F_j})^{w_j}) \right\rangle\right)$$

where w_j *is weight of* $\tilde{a}_j$ $(j = 1, 2, ..., n), w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$.

Proof: Let $\tilde{a}_j = \left(\left\langle \tilde{\Phi}_{T_j}, \tilde{\Phi}_{I_j}, \tilde{\Phi}_{F_j} \right\rangle, \left\langle \Lambda_{T_j}, \Lambda_{I_j}, \Lambda_{F_j} \right\rangle\right)$ $(j = 1, 2, ..., n)$, and let $\mathcal{A}_w : Q_n \to Q$ be a map, $w_j \in [0, 1]$. Let $w_1 = 1$, we can prove this result by induction method. First let for $n = 1$, we have

$$\mathcal{A}_w(\tilde{a}_1) = \sum_{j=1}^{1} w_j \tilde{a}_j = w_1 \tilde{a}_1 = \tilde{a}_1 \sum_{j=1}^{1} w_j = \tilde{a}_1(1) \quad \because \sum_{j=1}^{n} w_j = 1 = \tilde{a}_1$$
$$= \left(\left\langle [1,1] - \prod_{j=1}^{1}([1,1] - \tilde{\Phi}_{T_j})^{w_j}, [1,1] - \prod_{j=1}^{1}([1,1] - \tilde{\Phi}_{I_j})^{w_j}, [1,1] - \prod_{j=1}^{1}([1,1] - \tilde{\Phi}_{F_j})^{w_j} \right\rangle, \right.$$
$$\left. \left\langle \prod_{j=1}^{1}(\Lambda_{T_1})^{w_j}, \prod_{j=1}^{1}(\Lambda_{I_1})^{w_j}, \prod_{j=1}^{1}(\Lambda_{F_1})^{w_j}) \right\rangle\right)$$
$$= \left(\left\langle [1,1] - ([1,1] - \tilde{\Phi}_{T_1})^{w_1}, [1,1] - ([1,1] - \tilde{\Phi}_{I_1})^{w_1}, [1,1] - ([1,1] - \tilde{\Phi}_{F_1})^{w_1} \right\rangle, \right.$$
$$\langle (\Lambda_{T_1})^{w_1}, (\Lambda_{I_1})^{w_1}, (\Lambda_{F_1})^{w_1})\rangle)$$
$$= \left(\left\langle [1,1] - ([1,1] - \tilde{\Phi}_{T_1}), [1,1] - ([1,1] - \tilde{\Phi}_{I_1}), [1,1] - ([1,1] - \tilde{\Phi}_{F_1}) \right\rangle, \langle (\Lambda_{T_1}), (\Lambda_{I_1}), (\Lambda_{F_1}))\rangle\right)$$
$$= \left(\left\langle \tilde{\Phi}_{T_1}, \tilde{\Phi}_{I_1}, \tilde{\Phi}_{F_1} \right\rangle, \langle \Lambda_{T_1}, \Lambda_{I_1}, \Lambda_{F_1} \rangle\right) = \tilde{a}_1.$$

Suppose that this is true for $n = k$, that is

$$\mathcal{A}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_k) = \sum_{j=1}^{k} w_j \tilde{a}_j = w_k \tilde{a}_k = \tilde{a}_k$$
$$= \left(\left\langle [1,1] - \prod_{j=1}^{k}([1,1] - \tilde{\Phi}_{T_j})^{w_j}, [1,1] - \prod_{j=1}^{k}([1,1] - \tilde{\Phi}_{I_j})^{w_j}, [1,1] - \prod_{j=1}^{k}([1,1] - \tilde{\Phi}_{F_j})^{w_j} \right\rangle, \right.$$
$$\left. \left\langle \prod_{j=1}^{k}(\Lambda_{T_j})^{w_j}, \prod_{j=1}^{k}(\Lambda_{I_j})^{w_j}, \prod_{j=1}^{k}(\Lambda_{F_j})^{w_j}) \right\rangle \right).$$

In order to prove for $n = k + 1$, i.e.,

$$\mathcal{A}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_k, \tilde{a}_{k+1}) = \sum_{j=1}^{k+1} w_j \tilde{a}_j = \sum_{j=1}^{k} w_j \tilde{a}_j + w_1 \tilde{a}_1 = \tilde{a}_{k+1}$$
$$= \left(\left\langle [1,1] - \prod_{j=1}^{k+1}([1,1] - \tilde{\Phi}_{T_j})^{w_j}, [1,1] - \prod_{j=1}^{k+1}([1,1] - \tilde{\Phi}_{I_j})^{w_j}, [1,1] - \prod_{j=1}^{k+1}([1,1] - \tilde{\Phi}_{F_j})^{w_j} \right. \right.$$
$$\left. \left\langle \prod_{j=1}^{k+1}(\Lambda_{T_j})^{w_j}, \prod_{j=1}^{k+1}(\Lambda_{I_j})^{w_j}, \prod_{j=1}^{k+1}(\Lambda_{F_j})^{w_j}) \right\rangle \right)$$
$$= \left(\left\langle [1,1] - \prod_{j=1}^{k}([1,1] - \tilde{\Phi}_{T_j})^{w_j}([1,1] - \tilde{\Phi}_{T_1}), [1,1] - \prod_{j=1}^{k}([1,1] - \tilde{\Phi}_{I_j})^{w_j}([1,1] - \tilde{\Phi}_{I_1}) \right. \right.$$
$$\left. [1,1] - \prod_{j=1}^{k}([1,1] - \tilde{\Phi}_{F_j})^{w_j}([1,1] - \tilde{\Phi}_{F_1}) \right\rangle,$$
$$\left. \left\langle \prod_{j=1}^{k}(\Lambda_{T_j})^{w_j}(\Lambda_{T_1}), \prod_{j=1}^{k}(\Lambda_{I_j})^{w_j}(\Lambda_{I_1}), \prod_{j=1}^{k}(\Lambda_{F_j})^{w_j})(\Lambda_{F_1})) \right\rangle \right)$$
$$= \tilde{a}_{k+1}.$$

This completes the proof. □

Theorem 3.21. *Let* $\tilde{a}_j = \left(\left\langle \tilde{\Phi}_{T_j}, \tilde{\Phi}_{I_j}, \tilde{\Phi}_{F_j} \right\rangle, \langle \Lambda_{T_j}, \Lambda_{I_j}, \Lambda_{F_j} \rangle \right)$ $(j = 1, 2, ..., n)$ *be a family of NCNs. Then*

(i) If $\tilde{a}_j = \tilde{a}$ for all $j = 1, 2, ..., n$ then, $\mathcal{A}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}$,

(ii) $\min_{j=1,2,...,n}\{\tilde{a}_j\} \leq \mathcal{A}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq \max_{j=1,2,...,n}\{\tilde{a}_j\}$,

(iii) If $\tilde{a}_j \leq \tilde{a}_j^*$ for all $j = 1, 2, ..., n$ then, $\mathcal{A}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq \mathcal{A}_w(\tilde{a}_1^*, \tilde{a}_2^*, ..., \tilde{a}_n^*)$.

Proof: Let $\tilde{a}_j = \left(\left\langle \tilde{\Phi}_{T_j}, \tilde{\Phi}_{I_j}, \tilde{\Phi}_{F_j} \right\rangle, \langle \Lambda_{T_j}, \Lambda_{I_j}, \Lambda_{F_j} \rangle \right)$ $(j = 1, 2, ..., n)$ be a family of NCNs, and

(i) Let $\tilde{a}_j = \tilde{a}$ for all $j = 1, 2, ..., n$ such that $\tilde{a}_1 = \tilde{a}$, $\tilde{a}_2 = \tilde{a}$, $\tilde{a}_3 = \tilde{a}...\tilde{a}_n = \tilde{a}$. This implies that $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = (\tilde{a}, \tilde{a}, \tilde{a}, ..., \tilde{a})$. Now,

$$\mathcal{A}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} w_j\tilde{a}_j = w_1\tilde{a}_1 + w_2\tilde{a}_2 + w_3\tilde{a}_3 + ... + w_n\tilde{a}_n$$
$$= w_1\tilde{a} + w_2\tilde{a} + w_3\tilde{a} + ... + w_n\tilde{a} \because \tilde{a}_1 = \tilde{a}, \tilde{a}_2 = \tilde{a}, \tilde{a}_3 = \tilde{a}...\tilde{a}_n = \tilde{a}$$
$$\tilde{a}(w_1 + w_2 + w_3 + ... + w_n) = \tilde{a}\sum_{j=1}^{n} w_j$$
$$= \tilde{a}(1) \quad (\sum_{j=1}^{n} w_j = 1 = \tilde{a})$$

Similarly we can prove (ii) and (iii) hold. □

Definition 3.22. *Let* $\tilde{a}_j = \left(\left\langle \tilde{\Phi}_{T_j}, \tilde{\Phi}_{I_j}, \tilde{\Phi}_{F_j} \right\rangle, \left\langle \Lambda_{T_j}, \Lambda_{I_j}, \Lambda_{F_j} \right\rangle\right)$ $(j = 1, 2, ..., n)$ *be a family of NCNs. A mapping* $\mathcal{G}_w : Q_n \to Q$ *is called neutrosophic cubic weighted geometric operator if it satisfies*

$$\mathcal{G}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_j^{w_j} = \left(\left\langle \prod_{j=1}^{n}(\tilde{\Phi}_{T_j})^{w_j}, \prod_{j=1}^{n}(\tilde{\Phi}_{I_j})^{w_j}, \prod_{j=1}^{n}(\tilde{\Phi}_{F_j})^{w_j} \right\rangle,\right.$$
$$\left.\left\langle [1,1] - \prod_{j=1}^{n}([1,1] - \Lambda_{T_j})^{w_j}, [1,1] - \prod_{j=1}^{n}([1,1] - \Lambda_{I_j})^{w_j}, [1,1] - [1,1] - \prod_{j=1}^{n}(\Lambda_{F_j})^{w_j} \right\rangle\right)$$

where w_j *is weight of* $\tilde{a}_j$ $(j = 1, 2, ..., n), w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$.

Example 3.23. *Consider a problem in which we have four alternatives* $\mathcal{A}_i(i = 1, 2, 3, 4)$ *are available and four attributes are* $\mathcal{C}_j(j = 1, 2, 3, 4)$. *Also, the weight vector of the attributes* $\mathcal{C}_j(j = 1, 2, 3, 4)$ *is* $w = (\frac{1}{2}\frac{1}{4}\frac{1}{8}\frac{1}{8})^T$ *then, find the geometric weight by using neutrosophic cubic weighted geometric operator.*

	$\mathcal{C}_1$	$\mathcal{C}_2$
$\mathcal{A}_1$	$(\langle[0.2, 0.5], [0.3.0.7], [0.1, 0.2]\rangle, \langle0.5, 0.7, 0.2\rangle)$	$(\langle[0.2, 0.4], [0.4, 0.5], [0.2, 0.5]\rangle, \langle0.4, 0.4, 0.5\rangle)$
$\mathcal{A}_2$	$(\langle[0.3, 0.9], [0.2, 0.7], [0.3, 0.5]\rangle, \langle0.9, 0.7, 0.5\rangle)$	$(\langle[0.3, 0.7], [0.6, 0.8], [0.2, 0.4]\rangle, \langle0.7, 0.6, 0.8\rangle)$
$\mathcal{A}_3$	$(\langle[0.1, 0.3], [0.4, 0.8], [0.2, 0.6]\rangle, \langle0.3, 0.4, 0.2\rangle)$	$(\langle[0.1, 0.2], [0.2, 0.3], [0.2, 0.5]\rangle, \langle0.2, 0.2, 0.2\rangle)$
$\mathcal{A}_4$	$(\langle[0.5, 0.9], [0.1, 0.8], [0.2, 0.6]\rangle, \langle0.9, 0.7, 0.2\rangle)$	$(\langle[0.3, 0.5], [0.5, 0.7], [0.1, 0.2]\rangle, \langle0.3, 0.5, 0.2\rangle)$

	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{A}_1$	$(\langle[0.2, 0.7], [0.4, 0.9], [0.5, 0.7]\rangle, \langle0.7, 0.7, 0.5\rangle)$	$(\langle[0.1, 0.6], [0.3, 0.4], [0.5, 0.8]\rangle, \langle0.1, 0.5, 0.7\rangle)$
$\mathcal{A}_2$	$(\langle[0.3, 0.9], [0.4, 0.6], [0.6, 0.8]\rangle, \langle0.9, 0.4, 0.6\rangle)$	$(\langle[0.2, 0.5], [0.4, 0.9], [0.5, 0.8]\rangle, \langle0.5, 0.2, 0.7\rangle)$
$\mathcal{A}_3$	$(\langle[0.4, 0.9], [0.1, 0.2], [0.4, 0.5]\rangle, \langle0.9, 0.5, 0.5\rangle)$	$(\langle[0.6, 0.7], [0.3, 0.6], [0.3, 0.7]\rangle, \langle0.7, 0.5, 0.3\rangle)$
$\mathcal{A}_4$	$(\langle[0.5, 0.6], [0.2, 0.4], [0.3, 0.5]\rangle, \langle0.5, 0.4, 0.5\rangle)$	$(\langle[0.3, 0.7], [0.7, 0.8], [0.6, 0.7]\rangle, \langle0.4, 0.2, 0.8\rangle)$

Solution: As

$$\mathcal{G}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_j^{w_j} = \left(\left\langle \prod_{j=1}^{n} (\tilde{\Phi}_{T_j})^{w_j}, \prod_{j=1}^{n} (\tilde{\Phi}_{I_j})^{w_j}, \prod_{j=1}^{n} (\tilde{\Phi}_{F_j})^{w_j} \right\rangle , \right.$$
$$\left. \left\langle [1,1] - \prod_{j=1}^{n} ([1,1] - \Lambda_{T_j})^{w_j}, [1,1] - \prod_{j=1}^{n} ([1,1] - \Lambda_{I_j})^{w_j}, [1,1] - [1,1] - \prod_{j=1}^{n} (\Lambda_{F_j})^{w_j} \right\rangle \right).$$

Now we compute $\mathcal{G}_{w(}\tilde{a}_{i1}, \tilde{a}_{i2}, \tilde{a}_{i3}, \tilde{a}_{i4})$ for each $i = (1, 2, 3, 4)$ as:

$\mathcal{G}(\tilde{a}_1) = \langle ([0.183, 0.50], [0.33, 0.62], [0.177, 0.35]), (0.47, 0.62, 0.41) \rangle$
$\mathcal{G}(\tilde{a}_2) = \langle ([0.28, 0.78], [0.31, 0.73], [0.31, 0.53]), (0.84, 0.60, 0.63) \rangle$
$\mathcal{G}(\tilde{a}_3) = \langle ([0.14, 0.34], [0.27, 0.50], [0.23, 0.57]), (0.49, 0.38, 0.26) \rangle$
$\mathcal{G}(\tilde{a}_4) = \langle ([0.412, 0.71], [0.20, 0.70], [0.20, 0.45]), (0.75, 0.58, 0.36) \rangle$

Theorem 3.24. *Let* $\tilde{a}_j = \left(\left\langle \tilde{\Phi}_{T_j}, \tilde{\Phi}_{I_j}, \tilde{\Phi}_{F_j} \right\rangle, \langle \Lambda_{T_j}, \Lambda_{I_j}, \Lambda_{F_j} \rangle \right)$ $(j = 1, 2, ..., n)$ *be a family of NCNs, then*

(i) If $\tilde{a}_j = \tilde{a}$ for all $j = 1, 2, ..., n$ then, $\mathcal{G}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}$,
(ii) $\min_{j=1,2,...,n} \{\tilde{a}_j\} \leq \mathcal{G}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq \max_{j=1,2,...,n} \{\tilde{a}_j\}$,
(iii) If $\tilde{a}_j \leq \tilde{a}_j^*$ for all $j = 1, 2, ..., n$ then, $\mathcal{G}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq \mathcal{G}_w(\tilde{a}_1^*, \tilde{a}_2^*, ..., \tilde{a}_n^*)$.

Proof: Let $\tilde{a}_j = \left(\left\langle \tilde{\Phi}_{T_j}, \tilde{\Phi}_{I_j}, \tilde{\Phi}_{F_j} \right\rangle, \langle \Lambda_{T_j}, \Lambda_{I_j}, \Lambda_{F_j} \rangle \right)$ $(j = 1, 2, ..., n)$ be a family of NCNs, and

(i) Let If $\tilde{a}_j = \tilde{a}$ for all $j = 1, 2, ..., n$ such that $\tilde{a}_1 = \tilde{a}$, $\tilde{a}_2 = \tilde{a}$, $\tilde{a}_3 = \tilde{a}...\tilde{a}_n = \tilde{a}$, this implies that $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = (\tilde{a}, \tilde{a}, \tilde{a}, ..., \tilde{a})$.

Now,

$$\begin{aligned} \mathcal{G}_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) &= \prod_{j=1}^{n} \tilde{a}_j^{w_j} = \tilde{a}_1^{w_1} \tilde{a}_2^{w_2} \tilde{a}_3^{w_3} ... \tilde{a}_n^{w_n} \\ &= \tilde{a}^{w_1} \tilde{a}^{w_2} \tilde{a}^{w_3} ... \tilde{a}^{w_n} \quad \because \tilde{a}_1 = \tilde{a}, \tilde{a}_2 = \tilde{a}, \tilde{a}_3 = \tilde{a}...\tilde{a}_n = \tilde{a} \\ \tilde{a}^{w_1+w_2+w_3+...+w_n} &= \tilde{a}^{\sum_{j=1}^{n} w_j} = \tilde{a}^1 = \tilde{a} \quad \because \sum_{j=1}^{n} w_j = 1 \end{aligned}$$

Similarly we can prove (ii) and (iii) hold. □
Note: That the aggregation results are still NCNs.

4. NCN-decision making algorithm

In this section, we progress an algorithm by expanding NCNs based on score, accuracy, certainty function of NCNs by the usage of $\mathcal{A}_w$(or $\mathcal{G}$) operator.

If we have some information about NCNs then on the base of this information we can develop some ranking method to deal with multiple decision criteria making problem based on the $A_w($ or $G)$ operator.

Suppose that $A = \{A_1, A_2, ..., A_n\}$ and $C = \{C_1, C_2, ..., C_n\}$ is the set of alternatives and criterions or attributes, respectively. Let the weight vector of attributes be $\omega = (w_1, w_2, ..., w_n)^T$ such that $\sum_{j=1}^{n} w_j = 1, w_j \geq 0$ $(j = 1, 2, ..., n)$and w_j refers to the weight of attribute $C_{j.}$An alternative on criterions is evaluated by the decision maker, and the evaluation values are represented by the form of NCNs. Assume that $(\tilde{a}_{ij})_{m\times n} = \left(\left\langle \tilde{\Phi}_{T_{ij}}, \tilde{\Phi}_{I_{ij}}, \tilde{\Phi}_{F_{ij}} \right\rangle, \left\langle \Lambda_{T_{ij}}, \Lambda_{I_{ij}}, \Lambda_{F_{ij}} \right\rangle\right)_{m\times n}$ is decision matrix provided by the decision maker; $\tilde{a}_{ij}$ is a NCN for alternative A_i associated with the criterions C_j. We have the conditions $\tilde{\Phi}_{T_{ij}}, \tilde{\Phi}_{I_{ij}}, \tilde{\Phi}_{F_{ij}} \in D[0,1]$ and $\Lambda_{T_{ij}}, \Lambda_{I_{ij}}, \Lambda_{F_{ij}} \in [0,1]$.

Now, we develop an algorithm as follows:

Algorithm

Step 1. Construct the decision matrix provided by the decision maker as;$(\tilde{a}_{ij})_{m\times n} = \left(\left\langle \tilde{\Phi}_{T_{ij}}, \tilde{\Phi}_{I_{ij}}, \tilde{\Phi}_{F_{ij}} \right\rangle, \left\langle \Lambda_{T_{ij}}, \Lambda_{I_{ij}}, \Lambda_{F_{ij}} \right\rangle\right)_{m\times n}$

Step 2: Compute $\tilde{a}_i = \mathcal{A}_w(\tilde{a}_{i1}, \tilde{a}_{i2}, ..., \tilde{a}_{in})$(or $G_w(\tilde{a}_{i1}, \tilde{a}_{i2}, ..., \tilde{a}_{in})$ for each $i = 1, ..., m$.

Step 3. Calculate the score values of $\tilde{s}(\tilde{a}_1)(i = 1, ..., m)$ for the collective overall NCNs of $\tilde{a}_i(i = 1, ..., m.)$

Step 4. Rank all the software systems of $\tilde{a}_i(i = 1, ..., m.)$ according to the score values.

Next, we give some numerical examples as follows:

Example 4.1. *Let us study decision-making problem. A passenger wishes to travel to Karachi. Four kinds of vans (alternatives) $\mathcal{A}_i(i = 1, 2, 3, 4)$ are accessible. The customer takes into account four attributes to evaluate the alternatives; $\mathcal{C}_1$=Facility; $\mathcal{C}_2$=Rent Saving; $\mathcal{C}_3$=Comfort; $\mathcal{C}_4$=Safety and use the neutrosophic cubic values to evaluate the four probable alternatives.$\mathcal{A}_i(i = 1, 2, 3, 4)$ under the above four attributes. Also, the weight vector of the attributes $\mathcal{C}_j(j = 1, 2, 3, 4)$ is $w = (\frac{1}{2}\frac{1}{4}\frac{1}{8}\frac{1}{8})^T$ then,*

Step1:. Construct the decision matrix provided by the customer as:

	$\mathcal{C}_1$	$\mathcal{C}_2$
$\mathcal{A}_1$	$(\langle[0.2, 0.5], [0.3.0.7], [0.1, 0.2]\rangle, \langle 0.9, 0.7, 0.2\rangle)$	$(\langle[0.2, 0.4], [0.4, 0.5], [0.2, 0.5]\rangle, \langle 0.7, 0.4, 0.5\rangle)$
$\mathcal{A}_2$	$(\langle[0.3, 0.9], [0.2, 0.7], [0.3, 0.5]\rangle, \langle 0.5, 0.7, 0.5\rangle)$	$(\langle[0.3, 0.7], [0.6, 0.8], [0.2, 0.4]\rangle, \langle 0.7, 0.6, 0.8\rangle)$
$\mathcal{A}_3$	$(\langle[0.3, 0.4], [0.4, 0.8], [0.2, 0.6]\rangle, \langle 0.1, 0.4, 0.2\rangle)$	$(\langle[0.2, 0.4], [0.2, 0.3], [0.2, 0.5]\rangle, \langle 0.2, 0.2, 0.2\rangle)$
$\mathcal{A}_4$	$(\langle[0.5, 0.9], [0.1, 0.8], [0.2, 0.6]\rangle, \langle 0.1, 0.7, 0.2\rangle)$	$(\langle[0.3, 0.5], [0.5, 0.7], [0.1, 0.2]\rangle, \langle 0.3, 0.5, 0.2\rangle)$

	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{A}_1$	$(\langle[0.2, 0.7], [0.4, 0.9], [0.5, 0.7]\rangle, \langle 0.7, 0.7, 0.5\rangle)$	$(\langle[0.1, 0.6], [0.3, 0.4], [0.5, 0.8]\rangle, \langle 0.5, 0.5, 0.7\rangle)$
$\mathcal{A}_2$	$(\langle[0.3, 0.9], [0.4, 0.6], [0.6, 0.8]\rangle, \langle 0.9, 0.4, 0.6\rangle)$	$(\langle[0.2, 0.5], [0.4, 0.9], [0.5, 0.8]\rangle, \langle 0.5, 0.2, 0.7\rangle)$
$\mathcal{A}_3$	$(\langle[0.4, 0.9], [0.1, 0.2], [0.4, 0.5]\rangle, \langle 0.9, 0.5, 0.5\rangle)$	$(\langle[0.6, 0.7], [0.3, 0.6], [0.3, 0.7]\rangle, \langle 0.7, 0.5, 0.3\rangle)$
$\mathcal{A}_4$	$(\langle[0.5, 0.6], [0.2, 0.4], [0.3, 0.5]\rangle, \langle 0.5, 0.4, 0.5\rangle)$	$(\langle[0.3, 0.7], [0.7, 0.8], [0.6, 0.7]\rangle, \langle 0.4, 0.2, 0.8\rangle)$

Step2: Compute $\tilde{a}_i = A_w(\tilde{a}_{i1}\tilde{a}_{i2}, \tilde{a}_{i3}, \tilde{a}_{i4})$ for each $i = (1, 2, 3, 4)$ as:

$\tilde{a}_1 = \langle([0.19, 0.524], [0.341, 0.678], [0.247, 0.473]), (0.716, 0.583, 0.329)\rangle$
$\tilde{a}_2 = \langle([0.29, 0.841], [0.375, 0.757], [0.355, 0.585]), (0.585, 0.536, 0.60)\rangle$
$\tilde{a}_3 = \langle([0.338, 0.56], [0.311, 0.646], [0.24, 0.58]), (0.199, 0.355, 0.235)\rangle$
$\tilde{a}_4 = \langle([0.43, 0.79], [0.335, 0.747], [0.259, 0.53]), (0.19, 0.513, 0.26)\rangle$

Step3: Calculate the score values of $\tilde{s}(\tilde{a}_1)$ $(i = 1, 2, 3, 4)$ for the collective overall NCNs of $\tilde{a}_i$ $(i = 1, 2, ..., m)$ as;

$\tilde{s}(\tilde{a}_1) = \langle([0.346, 0.645]), (0.601)\rangle$
$\tilde{s}(\tilde{a}_2) = \langle([0.316, 0.703]), (0.483)\rangle$
$\tilde{s}(\tilde{a}_3) = \langle([0.37, 0.669]), (0.536)\rangle$
$\tilde{s}(\tilde{a}_4) = \langle([0.38, 0.732]), (0.47)\rangle$

Step4: Rank all the software systems of $\mathcal{A}_i$ $(i = 1, 2, 3, 4.)$ according to the score values as;

$$\mathcal{A}_4 \succ \mathcal{A}_2 \succ \mathcal{A}_3 \succ \mathcal{A}_1$$

and thus $\mathcal{A}_4$ is the utmost desired alternative.

Example 4.2. *A person wishes to purchase finest Samsung Galaxy J Series mobile model and wants to check three models by their specification (alternatives)* $\mathcal{J}_i(i = 1, 2, 3)$ *are offered. The client takes into account three attributes to estimate the alternatives;* $\mathcal{S}_1$ *=Processor;* $\mathcal{S}_2$ *=Camera;* $\mathcal{S}_3$ *=Battery and use the neutrosophic cubic values to evaluate the four possible alternatives*$\mathcal{A}_i(i = 1, 2, 3)$ *under the above four attributes. Also, the weight vector of the attributes* $\mathcal{S}_j(j = 1, 2, 3)$ *is* $w = (\frac{1}{2}\frac{1}{3}\frac{1}{6})^T$ *then,*

Step1:. Construct the decision matrix provided by the customers;

	$\mathcal{S}_1$	$\mathcal{S}_2$
$\mathcal{J}_1$	$(\langle[0.2, 0.7], [0.3.0.7], [0.3, 0.8]\rangle, \langle 0.3, 0.4, 0.1\rangle)$	$(\langle[0.4, 0.7], [0.3.0.7], [0.5, 0.8]\rangle, \langle 0.2, 0.4, 0.5\rangle)$
$\mathcal{J}_2$	$(\langle[0.2, 0.7], [0.3.0.7], [0.4, 0.6]\rangle, \langle 0.9, 0.6, 0.2\rangle)$	$(\langle[0.2, 0.3], [0.3.0.6], [0.1, 0.4]\rangle, \langle 0.6, 0.7, 0.6\rangle)$
$\mathcal{J}_3$	$(\langle[0.2, 0.5], [0.2.0.7], [0.1, 0.2]\rangle, \langle 0.5, 0.7, 0.2\rangle)$	$(\langle[0.1, 0.6], [0.2.0.6], [0.3, 0.4]\rangle, \langle 0.4, 0.5, 0.6\rangle)$

	$\mathcal{S}_3$
$\mathcal{J}_1$	$(\langle[0.2, 0.8], [0.2.0.7], [0.1, 0.6]\rangle, \langle 0.1, 0.3, 0.5\rangle)$
$\mathcal{J}_2$	$(\langle[0.2, 0.7], [0.4.0.7], [0.1, 0.3]\rangle, \langle 0.3, 0.5, 0.7\rangle)$
$\mathcal{J}_3$	$(\langle[0.2, 0.5], [0.3.0.4], [0.1, 0.2]\rangle, \langle 0.2, 0.4, 0.6\rangle)$

Step2: Compute $\tilde{j}_i = A_w(\tilde{j}_{i2}, \tilde{j}_{i2}, \tilde{j}_{i3})$ for each $i = (1, 2, 3)$ as:

$\tilde{j}_1 = \langle([0.27, 0.71], [0.28, 0.70], [0.34, 0.77]), (0.21, 0.38, 0.22)\rangle$
$\tilde{j}_2 = \langle([0.2, 0.60], [0.31, 0.66], [0.26, 0.49]), (0.63, 0.61, 0.35)\rangle$
$\tilde{j}_3 = \langle([0.16, 0.53], [0.21, 0.62], [0.17, 0.27]), (0.40, 0.57, 0.34)\rangle$

Step3: Calculate the score values of $\tilde{s}(\tilde{j}_1)$ $(i = 1, 2, 3)$ for the collective overall NCN of $\tilde{j}_i$ $(i = 1, 2, ..., m)$ as;

$\tilde{s}(\tilde{j}_1) = \langle([0.26, 0.69]), (0.53)\rangle$
$\tilde{s}(\tilde{j}_2) = \langle([0.35, 0.67]), (0.55)\rangle$
$\tilde{s}(\tilde{j}_3) = \langle([0.42, 0.71]), (0.49)\rangle$

Step4: Rank all the software systems of $\mathcal{J}_i$ $(i = 1, 2, 3)$ according to the score values as;

$$\mathcal{J}_3 \succ \mathcal{J}_1 \succ \mathcal{J}_2$$

and thus $\mathcal{J}_3$ is the extreme preferred alternative.

From the above examples it is clear that by using this concept we can solve different problems arise in several areas and can pick finest choice by means of NCSs in various decision making problems. Here we have membership and non-membership so we can elect top choice by means of membership function as well as non-membership function due to which we can select utmost necessary possibility. Membership quota display us that due to these excellence the succeeding alternative is greatest requirement and non-membership displays us that due to following errors the following alternative is fewer desired. Due to this motive our case is more general for the reason that solitary on the origin of positive features of whatever we cannot make exact decision except we know the disadvantage of it.

5. Comparison analysis and discussion

As a variability of FSs, IFSs and NSs have been generated to express to indeterminate, movable, insufficient and contradictory information that occurs in this present reality. Enhanced NSs have been recommended for the principle cause for tending to matters with an arrangement of specific numbers. Be that as it may, there is sure matter with respect to the recent operations of SNSs, and in addition their conglomeration supervisors and the parallel methods.

In this method, Şahin [13] described the novel operations of amended NNs and builds up a parallel technique in light of the correlated investigation of IFNs. On the principle of these operations and the correlation policy, some efficient NNs accretion administrators are proposed. Moreover, a procedure for multi-criteria communal choice making (MCGDM) matters is reconnoitered by applying these accumulation administrators. At last, a case to exemplify the significance of the suggested stratagem is given and a link some different approaches are ready.

In [1], Ali et al. put forth some fresh types of NCSs and they suggested a decision making technique based on resemblance methods of two NCSs by presumptuous that if likeness between the ideal design and sample design is greater than or equivalent to 0.5, then the sample design goes to the family of ideal design in deliberation.

We generalize the idea provided in [1, 13] by familiarizing the perception of the different functions i.e (score, certainty and accuracy) functions to relate the NCSs. Also to aggregate the neutrosophic cubic information we develop the neutrosophic operators i.e. (weight average and weight geometric) operators. On the basis of $(\mathcal{A}_W and \mathcal{G}_w)$ operators and the functions i.e. (score, certainty and accuracy) functions, we progress the multiple criteria decision making methodology in NCSs, in which the estimation standards of substitutions on the features of the form NCNs to select the utmost desired ones and give an arithmetical case to demonstrate the usage and success of this established technique.

6. Conclusion

This paper presents NCSs and different functions i.e., its score, certainty and accuracy functions. Then, the $\mathcal{A}_W$ and $\mathcal{G}_w$ operators were future to aggregate the neutrosophic cubic data. Moreover, on the basis of $\mathcal{A}_W$ and $\mathcal{G}_w$ operators and the functions such as (certainty, score and the accuracy). we have established an approach to the multiple criteria decision making in NCSs, in which the valuation standards of substitutes on the aspects yield the arrangement NCNs.

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