

Applying Dijkstra Algorithm for Solving Neutrosophic Shortest Path Problem

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Abstract— The selection of shortest path problem is one the classic problems in graph theory. In literature, many algorithms have been developed to provide a solution for shortest path problem in a network. One of common algorithms in solving shortest path problem is Dijkstra's algorithm. In this paper, Dijkstra's algorithm has been redesigned to handle the case in which most of parameters of a network are uncertain and given in terms of neutrosophic numbers. Finally, a numerical example is given to explain the proposed algorithm.

Keywords— *Dijkstra's algorithm; Single valued neutrosophic number; Shortest path problem; Network.*

I. INTRODUCTION

To express indeterminate and inconsistent information which exist in real world, Smarandache [1] originally proposed the concept of a neutrosophic set from a philosophical point of view. The concept of the neutrosophic set (NS for short) is powerful mathematical tool which generalizes the concept of classical sets, fuzzy sets [3], intuitionistic fuzzy sets [4], interval-valued fuzzy sets [5] and interval-valued intuitionistic fuzzy sets [6]. The concept of the neutrosophic has three basic components such that a truth-membership (T), indeterminacy-membership (I) and a falsity membership (F), which are defined independently of one another. But a neutrosophic set So will be more difficult to apply it in real scientific and engineering areas. Thus, Wang et al. [7] proposed the concept of single valued neutrosophic set (for short SVNS), which is an instance of a neutrosophic set, whose functions of truth, indeterminacy and falsity lie in $[0, 1]$ and provided the set theoretic operators and various properties of SVNSs. Some of the recent research works on neutrosophic set theory and its applications in various fields can be found in [8]. In addition, Thamaraiselvi and Santhi [9] introduced a mathematical representation of a transportation problems in neutrosophic

environment based on single valued trapezoidal neutrosophic numbers and also provided the solution method. The operations on neutrosophic sets and the ranking methods are presented in [10]

The shortest path problem (SPP) is one of the most fundamental and well-known combinatorial problems that appear in various fields of science and engineering, e.g. road networks application, transportation and other applications. In a network, the shortest path problem aims at finding the path from one source node to destination node with minimum weight, where some weight is attached to each edge connecting a pair of nodes. The edge length of the network may represent the real life quantities such as, time, cost, etc. In conventional shortest path problem, it is assumed that decision maker is certain about the parameters (distance, time etc) between different nodes. But in real life situations, there always exist uncertainty about the parameters between different nodes. For this purpose, many algorithms have been developed to find the shortest path under different types of input data, including fuzzy set, intuitionistic fuzzy sets, vague sets [11-15]. One of the most used methods to solve the shortest path problem is the Dijkstra's algorithm [16]. Dijkstra's algorithm solves the problem of finding the shortest path from a point in a graph (the source) to a destination.

Recently, numerous papers have been published on neutrosophic graph theory [17-23]. In addition, Broumi et al. [24-26] proposed some algorithms to find the shortest path of a network (graph) where edge weights are characterized by a neutrosophic numbers including single valued neutrosophic numbers, bipolar neutrosophic numbers and interval valued neutrosophic numbers.

The main purpose of this paper is to propose a new version of Dijkstra algorithm for solving shortest path problem on a network where the edge weights are characterized by a single

valued neutrosophic numbers. The proposed method is more efficient due to the fact that the summing operation and the ranking of SVNNs can be done in a easy and straight manner. The rest of the article is organized as follows. Section 2 introduces some basic concepts of neutrosophic sets, single valued neutrosophic sets. In Section 3, a network terminology is presented, In section 4, we propose the new version of Dijkstra'algorithm for solving the shortest path with connected edges in neutrosophic data. Section 5 illustrates a practical example which is solved by the proposed algorithm. Conclusions and further research are given in section 6.

II. PRELIMINARIES

In this section, some basic concepts and definitions on neutrosophic sets and single valued neutrosophic sets are reviewed from the literature.

Definition 2.1 [1]. Let X be a space of points (objects) with generic elements in X denoted by x ; then the neutrosophic set A (NS A) is an object having the form $A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$, where the functions $T, I, F: X \rightarrow]0, 1^+[$ [define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with the condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (1)$$

The functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0, 1^+[$.

Since it is difficult to apply NSs to practical problems, Wang et al. [7] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [7]. Let X be a space of points (objects) with generic elements in X denoted by x . A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$.

For each point x in X $T_A(x), I_A(x), F_A(x) \in [0, 1]$. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\} \quad (2)$$

Definition 2.3 [10]. Let $\tilde{A}_1 = (T_1, I_1, F_1)$ and $\tilde{A}_2 = (T_2, I_2, F_2)$ be two single valued neutrosophic number. Then, the operations for SVNNs are defined as below;

$$i. \quad \tilde{A}_1 \oplus \tilde{A}_2 = < T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2 > \quad (3)$$

$$ii. \quad \tilde{A}_1 \otimes \tilde{A}_2 = < T_1 T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2 > \quad (4)$$

$$iii. \quad \lambda \tilde{A}_1 = < 1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda > \quad (5)$$

$$iv. \quad \tilde{A}_1^\lambda = (T_1^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda) \text{ where } \lambda > 0 \quad (6)$$

Definition 2.4 [10]. 0_n may be defined as follow:

$$0_n = \{< x, (0, 1, 1) >: x \in X\} \quad (7)$$

A convenient method for comparing of single valued neutrosophic number is by use of score function.

Definition 2.5 [11]. Let $\tilde{A}_1 = (T_1, I_1, F_1)$ be a single valued neutrosophic number. Then, the score function $s(\tilde{A}_1)$, accuracy function $a(\tilde{A}_1)$ and certainty function $c(\tilde{A}_1)$ of a SVNN are defined as follows:

$$(i) \quad s(\tilde{A}_1) = \frac{2 + T_1 - I_1 - F_1}{3} \quad (8)$$

$$(ii) \quad a(\tilde{A}_1) = T_1 - F_1 \quad (9)$$

$$(iii) \quad c(\tilde{A}_1) = T_1 \quad (10)$$

Definition 2.6 [11]. Suppose that $\tilde{A}_1 = (T_1, I_1, F_1)$

and $\tilde{A}_2 = (T_2, I_2, F_2)$ are two single valued neutrosophic numbers. Then, we define a ranking method as follows:

- i. If $s(\tilde{A}_1) > s(\tilde{A}_2)$, then \tilde{A}_1 is greater than \tilde{A}_2 , that is, \tilde{A}_1 is superior to \tilde{A}_2 , denoted by $\tilde{A}_1 \succ \tilde{A}_2$
- ii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, and $a(\tilde{A}_1) > a(\tilde{A}_2)$ then \tilde{A}_1 is greater than \tilde{A}_2 , that is, \tilde{A}_1 is superior to \tilde{A}_2 , denoted by $\tilde{A}_1 \succ \tilde{A}_2$
- iii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) > c(\tilde{A}_2)$ then \tilde{A}_1 is greater than \tilde{A}_2 , that is, \tilde{A}_1 is superior to \tilde{A}_2 , denoted by $\tilde{A}_1 \succ \tilde{A}_2$
- iv. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) = c(\tilde{A}_2)$ then \tilde{A}_1 is equal to \tilde{A}_2 , that is, \tilde{A}_1 is indifferent to \tilde{A}_2 , denoted by $\tilde{A}_1 = \tilde{A}_2$

III. NETWORK TERMINOLOGY

Consider a directed network $G = (V, E)$ consisting of a finite set of nodes $V = \{1, 2, \dots, n\}$ and a set of m directed edges $E \subseteq V \times V$. Each edge is denoted by an ordered pair (i, j) where $i, j \in V$ and $i \neq j$. In this network, we specify two nodes, denoted by s and t , which are the source node and the destination node, respectively. We define a path as a sequence $P_{ij} = \{i = i_1, (i_1, i_2), i_2, \dots, i_{l-1}, (i_{l-1}, i_l), i_l = j\}$ of alternating nodes and edges. The existence of at least one path P_{st} in $G(V, E)$ is assumed for every $i \in V - \{s\}$.

d_{ij} Denotes a single valued neutrosophic number associated with the edge (i, j) , corresponding to the length necessary to traverse (i, j) from i to j . In real problems, the lengths correspond to the cost, the time, the distance, etc. Then neutrosophic distance along the path P is denoted as $d(P)$ is defined as

$$d(P) = \sum_{(i,j) \in P} d_{ij} \quad (14)$$

Remark: A node i is said to be predecessor node of node j if

- (i) Node i is directly connected to node j .
- (ii) The direction of path connecting node i and j from i to j .

IV. SINGLE VALUED NEUTROSOPHIC DIJKSTRA ALGORITHM

In this subsection, we slightly modified the fuzzy Dijkstra algorithm adapted from [27] in order to deal on a network with parameters characterized by a single valued neutrosophic numbers.

This algorithm finds the shortest path and the shortest distance between a source node and any other node in the network. The algorithm advances from a node i to an immediately successive node j using a neutrosophic labeling procedure. Let \tilde{u}_i be the shortest distance from node 1 to node i and $s(\tilde{d}_{ij}) \geq 0$ be the length of (i, j) edge. Then, the neutrosophic label for node j is defined as:

$$[\tilde{u}_j, i] = [\tilde{u}_i \oplus \tilde{d}_{ij}, i], \quad S(\tilde{d}_{ij}) \geq 0. \quad (15)$$

Here label $[\tilde{u}_j, i]$ mean we are coming from nodes i after covering a distance \tilde{u}_j from the starting node. Dijkstra's algorithm divides the nodes into two subset groups: **Temporary set (T) and Permanent set (P)**. A temporary neutrosophic label can be replaced with another temporary neutrosophic label, if shortest path to the same neutrosophic node is detected. At the point when no better path can be found, the status of temporary label is changed to permanent. The steps of the algorithm are summarized as follows:

Step 1 Assign to source node (say node 1) the permanent label $[(0,1,1), -]$. Set $i=1$.

Making a node permanent means that it has been included in the short path.

Step 2 Compute the temporary label $[\tilde{u}_i \oplus \tilde{d}_{ij}, i]$ for each node j that can be reached from i , provided j is not permanently labeled. If node j is already labeled as $[\tilde{u}_j, k]$ through another node k , and if $S(\tilde{u}_i \oplus \tilde{d}_{ij}) < S(\tilde{u}_j)$ replace $[\tilde{u}_j, k]$ with $[\tilde{u}_i \oplus \tilde{d}_{ij}, i]$.

Step 3 If all the nodes are permanently labeled, the algorithm terminates. Otherwise, choose the label $[\tilde{u}_r, s]$ with shortest distance (\tilde{u}_r) from the list of temporary labels. Set $i=r$ and repeat step 2.

Step 4 Obtain the shortest path between node 1 and the destination node j by tracing backward through the network using the label's information.

Remark:

At each iteration among all temporary nodes, make those nodes permanent which have smallest distance. Note that at any iteration we can not move to permanent node, however, reverse is possible. After all the nodes have permanent labels and only one temporary node remains, make it permanent.

After describing the proposed algorithm, in next section we solve a numerical example and explain the proposed method completely.

V. ILLUSTRATIVE EXAMPLE

Now we solve an hypothetical example to verify the proposed approach. Consider the network shown in figure1; we want to obtain the shortest path from node 1 to node 6 where edges have a single valued neutrosophic numbers. Let us now apply the extended Dijkstra algorithm to the network given in figure 1.

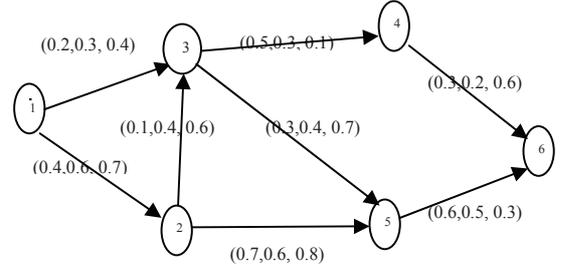


Fig. 1. A network with single valued neutrosophic weights

In this network each edge have been assigned to single valued neutrosophic number as follows:

Edges	Single valued Neutrosophic distance
1-2	(0.4, 0.6, 0.7)
1-3	(0.2, 0.3, 0.4)
2-3	(0.1, 0.4, 0.6)
2-5	(0.7, 0.6, 0.8)
3-4	(0.5, 0.3, 0.1)
3-5	(0.3, 0.4, 0.7)
4-6	(0.3, 0.2, 0.6)
5-6	(0.6, 0.5, 0.3)

Table 1. weights of the graphs

According to Dijkstra's algorithm we start with

Iteration 0: Assign the permanent label $[(0,1,1), -]$ to node 1.

Iteration 1: Node 2 and node 3 can be reached from (the last permanently labeled) node 1. Thus, the list of labeled nodes (temporary and permanent) becomes

Nodes	label	Status
1	$[(0,1,1), -]$	P
2	$[(0.4, 0.6, 0.7), 1]$	T
3	$[(0.2, 0.3, 0.4), 1]$	T

In order to compare the $(0.4, 0.6, 0.7)$ and $(0.2, 0.3, 0.4)$ we use the Eq.8

$$S(0.2, 0.3, 0.4) = \frac{2+T-I-F}{3} = \frac{2+0.2-0.3-0.4}{3} = 0.5$$

$$S(0.4, 0.6, 0.7) = \frac{2+T-I-F}{3} = \frac{2+0.4-0.6-0.7}{3} = 0.36$$

Since the rank of [(0.4, 0.6, 0.7), 1] is less than [(0.2, 0.3, 0.4), 1]. Thus the status of node 2 is changed to permanent.

Iteration 2: Node 3 and 5 can be reached from node 2. Thus, the list of labeled nodes (temporary and permanent) becomes

Nodes	label	Status
1	[(0,1,1), -]	P
2	[(0.4, 0.6, 0.7), 1]	P
3	[(0.2, 0.3, 0.4), 1] or [(0.46, 0.24, 0.42), 2]	T
5	[(0.82, 0.36, 0.56), 2]	T

$$S(0.46, 0.24, 0.42) = \frac{2+0.46-0.24-0.42}{3} = 0.6$$

$$S(0.82, 0.36, 0.56) = \frac{2+0.82-0.36-0.56}{3} = 0.63$$

Among the temporary labels [(0.2, 0.3, 0.4), 1] or [(0.46, 0.24, 0.42), 2], [(0.82, 0.36, 0.56), 2] and since the rank of (0.2, 0.3, 0.4) is less than of (0.46, 0.24, 0.42) and (0.82, 0.36, 0.56), So the status of node 3 is changed to permanent.

Iteration 3: Node 4 and 5 can be reached from node 3. Thus , the list of labeled nodes (temporary and permanent) becomes

Nodes	label	Status
1	[(0,1,1), -]	P
2	[(0.4, 0.6, 0.7), 1]	P
3	[(0.2, 0.3, 0.4), 1]	P
4	[(0.6, 0.09, 0.04), 3]	T
5	[(0.82, 0.36, 0.56), 2] or [(0.44, 0.12, 0.28), 3]	T

$$S(0.6, 0.09, 0.04) = \frac{2+0.6-0.09-0.04}{3} = 0.82$$

$$S(0.44, 0.12, 0.28) = \frac{2+0.44-0.12-0.28}{3} = 0.68$$

Among the temporary labels [(0.6, 0.09, 0.04), 3] or [(0.82, 0.36, 0.56), 2], [(0.44, 0.12, 0.28), 3] and since the rank of (0.82, 0.36, 0.56), is less than of (0.44, 0.12, 0.28) and (0.6, 0.09, 0.04). So the status of node 5 is changed to permanent.

Iteration 4: Node 6 can be reached from node 5. Thus , the list of labeled nodes (temporary and permanent) becomes

Nodes	label	Status
1	[(0,1,1), -]	P
2	[(0.4, 0.6, 0.7), 1]	P
3	[(0.2, 0.3, 0.4), 1]	P
4	[(0.6, 0.09, 0.04), 3]	T

5	[(0.82, 0.36, 0.56), 2]	P
6	[(0.93, 0.18, 0.17), 5]	T

Since, there exit one permanent node from where we can reach at node 6. So, make temporary label [(0.93, 0.18, 0.17), 5] as permanent.

Iteration 5: the only temporary node is 4, this node can be reached from node 3 and 6. Thus , the list of labeled nodes (temporary and permanent) becomes

Nodes	label	Status
1	[(0,1,1), -]	P
2	[(0.4, 0.6, 0.7), 1]	P
3	[(0.2, 0.3, 0.4), 1]	P
4	[(0.6, 0.09, 0.04), 3] or [(0.95, 0.04, 0.10), 6]	T
5	[(0.82, 0.36, 0.56), 2]	P
6	[(0.93, 0.18, 0.17), 5]	P

In order to compare the (0.6, 0.09, 0.04) and (0.95, 0.04, 0.10) we use the Eq.8

$$S(0.6, 0.09, 0.04) = 0.82 \text{ and } S(0.95, 0.04, 0.10) = 0.94$$

Since the rank of [(0.6, 0.09, 0.04), 3] is less than [(0.95, 0.04, 0.10), 6]. And the node 4 is the only one temporary node remains then, the status of node 4 is changed to permanent.

Nodes	label	Status
1	[(0,1,1), -]	P
2	[(0.4, 0.6, 0.7), 1]	P
3	[(0.2, 0.3, 0.4), 1]	P
4	[(0.6, 0.09, 0.04), 3]	P
5	[(0.82, 0.36, 0.56), 2]	P
6	[(0.93, 0.18, 0.17), 5]	P

Based on the step 4, the following sequence determines the shortest path from node 1 to node 6

(6) → [(0.93, 0.18, 0.17), 5] → (5) → [(0.82, 0.36, 0.56), 2] → (2) → [(0.4, 0.6, 0.7), 1] → (1)

Thus, the required shortest path is 1 → 2 → 5 → 6

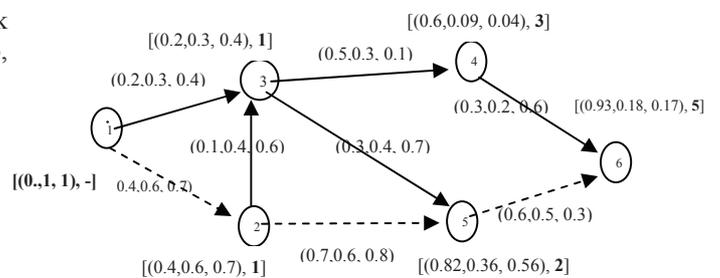


FIG 2. Network with single valued neutrosophic shortest distance of each node from node 1

VI. CONCLUSION

This paper extended the fuzzy Dijkstra's algorithm to find the shortest path of a network with single valued neutrosophic edge weights. The use of neutrosophic numbers as weights in the graph express more uncertainty than fuzzy numbers. The proposed algorithm proposes solution to one issue, this issue is addressed by identification of shortest path in neutrosophic environment. A numerical example was used to illustrate the efficiency of the proposed method. In future, we will research the application of this algorithm.

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