Arithmetic Operations of Neutrosophic Sets, Interval Neutrosophic Sets and Rough Neutrosophic Sets

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Abstract In approximation theory, neutrosophic set and logic show an important role. They are generalizations of intuitionistic fuzzy set and logic respectively. Based on neutrosophy, which is a new branch of philosophy, every idea X, has an opposite denoted as anti (X) and their neutral which is denoted as neut (X). These are the main features of neutrosophic set and logic. This chapter is based on the basic concepts of neutrosophic set as well as some of their hybrid structures. In this chapter, we define and study the notion of neutrosophic set and their basic properties. Moreover, interval-valued neutrosophic set are studied with some of their properties. Finally, we define rough neutrosophic sets.

1 Introduction

The data in real life problems like engineering, social, economic, computer, decision making, medical diagnosis etc. are often uncertain and imprecise. This type of data does not have necessarily crisp, precise and deterministic nature because of their fuzziness and vagueness. To handle this kind of data, Zadeh introduced fuzzy sets [1]. Based on fuzzy sets such as interval-valued fuzzy sets [2], intuitionistic fuzzy sets [3], and so on, several types of approaches have been proposed. Fuzzy sets have been successfully applied by researchers in all over the world in several areas such as knowledge representation, artificial intelligence, control, data mining, decision making, stock markets, signal processing, and pattern recognition, etc.

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Atanassov [3] observed that there is some kind of uncertainty in the data which is not handled by fuzzy sets. By inserting the non-membership degree to fuzzy sets, Atanassov [3] introduced intuitionistic fuzzy sets which are the generalization of the ordinary fuzzy sets. An intuitionistic fuzzy set is represented by a degree of membership and a degree of non-membership. Intuitionistic fuzzy sets more appropriately define the fuzzy objects of the real world. A vast amount of research has been shown from different part of intuitionistic fuzzy sets. In several fields intuitionistic fuzzy sets have been successfully applied such as pattern recognition, modeling imprecision, economics, decision making, medical diagnosis, computational intelligence, and so on.

To study the basis, nature, and range of neutralities as well as their contacts with ideational spectra, Smarandache [4] created the theory of neutrosophic set and logic under the neutrosophy, which is a new branch of philosophy. A neutrosophic set can be characterized by a truth membership function 'T', an indeterminacy membership function 'I' and a falsity membership function 'F'. Neutrosophic set is the generalization of fuzzy sets [1], intuitionistic fuzzy sets [3], paraconsistent set [4] etc. Neutrosophic sets can treat uncertain, inconsistent, incomplete, indeterminate and false information. From scientific or engineering point of view, the neutrosophic sets and their associated set theoretic operators need to be identified. In neutrosophic sets indeterminacy are quantified explicitly and *T, I,* and *F* operators are complementally independent which is very important in several applications such as networking, computer, information fusion, information theory, physics, and decision making.

In this chapter, we present the concepts of neutrosophic set and logic. We present a brief introduction in Sect. 1. We study neutrosophic sets with some of their basic properties in Sect. 2. Rough neutrosophic sets and their associated properties and notions have been studied in Sect. 3 and also interval-valued neutrosophic sets have been studied in this section. In Sect. 4, we state the conclusions.

2 Basic Concepts

Definition 2.1 [5] Let $\breve{X}$ be an initial universe of discourse, with a generic element in $\breve{X}$ denoted by $\breve{x}$, the neutrosophic set (NS) is an object having the form

$$\breve{A} = \left\{ \left\langle \breve{x} : \alpha_{\breve{A}}\left(\breve{x}\right), \beta_{\breve{A}}\left(\breve{x}\right), \gamma_{\breve{A}}\left(\breve{x}\right) \right\rangle | \breve{x} \in \breve{X} \right\} \tag{1}$$

where the functions $\alpha, \beta, \gamma : \breve{X} \rightarrow \left]^{-}0, 1^{+}\right[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $\breve{x} \in \breve{X}$ to the set $\breve{A}$ with the condition $0^{-} \leq \alpha_{\breve{A}}\left(\breve{x}\right), \beta_{\breve{A}}\left(\breve{x}\right), \gamma_{\breve{A}}\left(\breve{x}\right) \leq 3^{+}$.

From philosophical point of view, the neutrosophic sets take the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. So instead of $]^{-}0, 1^{+}[$ we need to take the interval [0, 1] for technical applications, because $]^{-}0, 1^{+}[$ will be difficult to apply in the real applications such as in scientific and engineering problems. For two NS,

$$\breve{A} = \left\{\left\langle \breve{x} : \alpha_{\breve{A}}\left(\breve{x}\right), \beta_{\breve{A}}\left(\breve{x}\right), \gamma_{\breve{A}}\left(\breve{x}\right)\right\rangle | \breve{x} \in \breve{X}\right\}$$

and

$$\breve{B} = \left\{\left\langle \breve{x} : \alpha_{\breve{B}}\left(\breve{x}\right), \beta_{\breve{B}}\left(\breve{x}\right), \gamma_{\breve{B}}\left(\breve{x}\right)\right\rangle | \breve{x} \in \breve{X}\right\}$$

The relations are defined as follows:

(a) $\breve{A} \subseteq \breve{B}\ if\ and\ only\ if \alpha_{\breve{A}}\left(\breve{x}\right) \le \alpha_{\breve{B}}\left(\breve{x}\right), \beta_{\breve{A}}\left(\breve{x}\right) \ge \beta_{\breve{B}}\left(\breve{x}\right), \gamma_{\breve{A}}\left(\breve{x}\right) \ge \gamma_{\breve{B}}\left(\breve{x}\right)$.

(b) $\breve{A} = \breve{B}\ if\ and\ only\ if\ \alpha_{\breve{A}}\left(\breve{x}\right) = \alpha_{\breve{B}}\left(\breve{x}\right), \beta_{\breve{A}}\left(\breve{x}\right) = \beta_{\breve{B}}\left(\breve{x}\right), \gamma_{\breve{A}}\left(\breve{x}\right) = \gamma_{\breve{B}}\left(\breve{x}\right)$.

(c) $\breve{A} \cap \breve{B} = \left\{\left\langle \breve{x}, \left(\alpha_{\breve{A}} \wedge \alpha_{\breve{A}}\right)\left(\breve{x}\right), \left(\beta_{\breve{A}} \vee \beta_{\breve{B}}\right)\left(\breve{x}\right), \left(\gamma_{\breve{A}} \vee \gamma_{\breve{B}}\right)\left(\breve{x}\right)\right\rangle | \breve{x} \in \breve{X}\right\}$.

(d) $\breve{A} \cup \breve{B} = \left\{\left\langle \breve{x}, \left(\alpha_{\breve{A}} \vee \alpha_{\breve{A}}\right)\left(\breve{x}\right), \left(\beta_{\breve{A}} \wedge \beta_{\breve{B}}\right)\left(\breve{x}\right), \left(\gamma_{\breve{A}} \wedge \gamma_{\breve{B}}\right)\left(\breve{x}\right)\right\rangle | \breve{x} \in \breve{X}\right\}$.

(e) $\breve{A} = \left\{\left\langle \breve{x}, \gamma_{\breve{A}}\left(\breve{x}\right), 1 - \beta_{\breve{A}}\left(\breve{x}\right), \alpha_{\breve{A}}\left(\breve{x}\right)\right\rangle | \breve{x} \in \breve{X}\right\}$.

(f) $0_{\breve{n}} = (0, 1, 1)\ and\ 1_{\breve{n}} = (1, 0, 0)$.

where $\wedge$ is the t-norm, and $\vee$ is the t-conorm.

We consider the following example for justification.

Example 2.1 [6] Assume that the universe of discourse $\breve{X} = \left\{\breve{x}_1, \breve{x}_2, \breve{x}_3\right\}$, where $\breve{x}_1$ characterizes the capability, $\breve{x}_2$ characterizes the trustworthiness and $\breve{x}_3$ indicates the prices of the objects. It may be further assumed that the values of $\breve{x}_1, \breve{x}_2, \breve{x}_3$ are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose $\breve{A}$ is a neutrosophic set (NS) of $\breve{X}$, such that, $\breve{A} = \left\{\left\langle \breve{x}_1, (0.4, 0.7, 0.8)\right\rangle, \left\langle \breve{x}_2, (0.5, 0.4, 0.3)\right\rangle, \left\langle \breve{x}_3, (0.2, 0.4, 0.6)\right\rangle\right\}$, where the grade of goodness of capability is 0.4, grade of indeterminacy of capability is 0.7 and grade of falsity of capability is 0.8 etc.

Definition 2.2 [5] Let $\breve{X}$ be any non-empty set. Suppose $\breve{R}$ is an equivalence relation over $\breve{X}$. For any non-null subset $\breve{Y}$ of $\breve{X}$, the sets $\breve{A}_1\left(\breve{y}\right) = \left\{\breve{y} : \left[\breve{Y}\right]_{\breve{R}} \subseteq \breve{Y}\right\}$

and $\breve{A}_2\left(\breve{y}\right) = \left\{\breve{y} : \left[\breve{Y}\right]_{\breve{R}} \cap \breve{Y} \neq \emptyset\right\}$ are called the lower approximation and upper approximation, respectively of $\breve{Y}$, where the pair $\breve{T} = \left(\breve{X}, \breve{R}\right)$ is called an approximation space. This equivalent relation $\breve{R}$ is called indiscernibility relation. The pair $\breve{A}\left(\breve{Y}\right) = \left(\breve{A}_1\left(\breve{y}\right), \breve{A}_2\left(\breve{y}\right)\right)$ is called the rough set of $\breve{Y}$ in $\breve{T}$. Here $\left[\breve{Y}\right]_{\breve{R}}$ denotes the equivalence class of $\breve{R}$ containing $\breve{y}$.

Definition 2.3 [5] Let $\breve{A} = \left(\breve{A}_1, \breve{A}_2\right)$ and $\breve{B} = \left(\breve{B}_1, \breve{B}_2\right)$ are two rough sets in the approximation space $\breve{T} = \left(\breve{X}, \breve{R}\right)$. Then,

$$\breve{A} \cup \breve{B} = \left(\breve{A}_1 \cup \breve{A}_2, \breve{B}_1 \cup \breve{B}_2\right),$$

$$\breve{A} \cap \breve{B} = \left(\breve{A}_1 \cap \breve{A}_2, \breve{B}_1 \cap \breve{B}_2\right),$$

$$\breve{A} \subseteq \breve{B}\, if\ and\ only\ if\, \breve{A} \cap \breve{B} = \breve{A},$$

$$\sim \breve{A} = \left\{\breve{X} - \breve{A}_2, \breve{X} - \breve{A}_1\right\}.$$

3 Rough Neutrosophic Sets

In this section, we introduce the notion of rough neutrosophic sets by combining both rough sets and neutrosophic sets and some operations viz. union, intersection, inclusion and equalities over them. Rough neutrosophic set are the generalization of rough fuzzy sets [7] and rough intuitionistic fuzzy sets [8].

Definition 3.1 Let $\breve{X}$ be a non-null set and $\breve{R}$ be an equivalence relation on $\breve{X}$. Let $\breve{K}$ be neutrosophic set in $\breve{X}$ with the membership function $\alpha_{\breve{K}}$, indeterminacy function $\beta_{\breve{K}}$, and non-membership function $\gamma_{\breve{K}}$. The lower and the upper approximations of $\breve{K}$ in the approximation $\left(\breve{X}, \breve{R}\right)$ denoted by $\underline{A}\left(\breve{K}\right)$ and $\overline{A}\left(\breve{K}\right)$ are respectively defined as follows:

$$\underline{A}\left(\breve{K}\right)=\left\{\left\langle \breve{y}:\alpha_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right),\beta_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right),\gamma_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right)\right\rangle|\breve{s}\in\left[\breve{Y}\right]_{\breve{R}},\breve{y}\in\breve{X}\right\},\quad (2)$$

and

$$\overline{A}\left(\breve{K}\right)=\left\{\left\langle \breve{y}:\alpha_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right),\beta_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right),\gamma_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right)\right\rangle|\breve{s}\in\left[\breve{Y}\right]_{\breve{R}},\breve{y}\in\breve{X}\right\},\quad (3)$$

where

$\alpha_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right)=\wedge_{\breve{s}\in\left[\breve{Y}\right]_{\breve{R}}}\alpha_{\breve{K}}\left(\breve{s}\right),\beta_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right)=\vee_{\breve{s}\in\left[\breve{Y}\right]_{\breve{R}}}\beta_{\breve{K}}\left(\breve{s}\right),\gamma_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right)=\vee_{\breve{s}\in\left[\breve{Y}\right]_{\breve{R}}}\gamma_{\breve{K}}\left(\breve{s}\right),$

$\alpha_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right)=\vee_{\breve{s}\in\left[\breve{Y}\right]_{\breve{R}}}\alpha_{\breve{K}}\left(\breve{s}\right),\beta_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right)=\wedge_{\breve{s}\in\left[\breve{Y}\right]_{\breve{R}}}\beta_{\breve{K}}\left(\breve{s}\right),\gamma_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right)=\wedge_{\breve{s}\in\left[\breve{Y}\right]_{\breve{R}}}\gamma_{\breve{K}}\left(\breve{s}\right).$

So $0\le\alpha_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right)+\beta_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right)+\gamma_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right)\le 3$, and $0\le\alpha_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right)+\beta_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right)+\gamma_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right)\le 3$.

where $\wedge$ is the t-norm and $\vee$ is the t-conorm respectively, $\alpha_{\breve{K}}\left(\breve{s}\right)$, $\beta_{\breve{K}}\left(\breve{s}\right)$ and $\gamma_{\breve{K}}\left(\breve{s}\right)$ are the membership, indeterminacy and non-membership of $\breve{s}$ with respect to $\breve{K}$.

It is easy to see that $\overline{A}\left(\breve{K}\right)$ and $\underline{A}\left(\breve{K}\right)$ are two neutrosophic sets in $\breve{X}$, thus $A\breve{T}$ mapping $\overline{A},\underline{A}:\breve{A}\left(\breve{X}\right)\to\breve{T}\left(\breve{X}\right)$, are respectively referred to as the upper and lower rough $A\breve{T}$ approximation operators, and the pair $\underline{A}\left(\breve{K}\right),\overline{A}\left(\breve{K}\right)$ is called the rough neutrosophic set in $\left(\breve{X},\breve{R}\right)$.

From the above definition, we can see that $\underline{A}\left(\breve{K}\right)$ and $\overline{A}\left(\breve{K}\right)$ have constant membership on the equivalence classes of $\breve{X}$. If $\underline{A}\left(\breve{K}\right)=\overline{A}\left(\breve{K}\right)$; i.e., $\alpha_{\underline{A}\left(\breve{K}\right)}=\alpha_{\overline{A}\left(\breve{K}\right)}$, $\beta_{\underline{A}\left(\breve{K}\right)}=\beta_{\overline{A}\left(\breve{K}\right)}$ and $\gamma_{\underline{A}\left(\breve{K}\right)}=\gamma_{\overline{A}\left(\breve{K}\right)}$. For any $\breve{y}\in\breve{X}$, we call $\breve{K}$ a definable neutrosophic set in the approximation $\left(\breve{X},\breve{R}\right)$. It is easily to be proved that zero $0_{\breve{n}}$ neutrosophic set and unite neutrosophic sets $1_{\breve{n}}$ are definable neutrosophic sets.

Example 3.1 Let $\breve{X} = \left\{\breve{h}_1, \breve{h}_2, \breve{h}_3, \breve{h}_4, \breve{h}_5, \breve{h}_6, \breve{h}_7, \breve{h}_8\right\}$ be the universe of discourse. Let $\breve{R}$ be an equivalence relation its partition of $\breve{X}$ is given by $\breve{X}/\breve{R} = \left\{\left\{\breve{h}_1, \breve{h}_2\right\}, \left\{\breve{h}_2, \breve{h}_3, \breve{h}_6\right\}, \left\{\breve{h}_5\right\}, \left\{\breve{h}_7, \breve{h}_8\right\}\right\}$.

Let $\breve{A}\left(\breve{K}\right) = \left\{\left(\breve{h}_1, (0.3, 0.4, 0.5)\right), \left(\breve{h}_4, (0.4, 0.6, 0.5)\right), \left(\breve{h}_5, (0.6, 0.8, 0.4)\right), \left(\breve{h}_7, (0.3, 0.5, 0.7)\right)\right\}$ be a neutrosophic set of $\breve{X}$. By Definition of 3.1, we obtain:

$$\underline{A}\left(\breve{K}\right) = \left\{\left(\breve{h}_1, (0.3, 0.6, 0.5)\right), \left(\breve{h}_4, (0.3, 0.6, 0.5)\right), \left(\breve{h}_5, (0.6, 0.8, 0.4)\right)\right\};$$

$$\overline{A}\left(\breve{K}\right) = \left\{\begin{array}{c}\left(\breve{h}_1, (0.4, 0.4, 0.5)\right), \left(\breve{h}_4, (0.4, 0.4, 0.5)\right), \left(\breve{h}_5, (0.6, 0.8, 0.4)\right), \\ \left(\breve{h}_7, (0.3, 0.5, 0.7)\right), \left(\breve{h}_8, (0.3, 0.5, 0.7)\right)\end{array}\right\}$$

For another neutrosophic set,

$$\breve{A}\left(\breve{L}\right) = \left\{\left(\breve{h}_1, (0.1, 0.2, 0.3)\right), \left(\breve{h}_4, (0.1, 0.2, 0.3)\right), \left(\breve{h}_5, (0.7, 0.6, 0.5)\right)\right\}$$

Then, the lower approximation and upper approximation of $\breve{B}\left(\breve{L}\right)$ is defined as:

$$\underline{A}\left(\breve{L}\right) = \left\{\left(\breve{h}_1, (0.1, 0.2, 0.3)\right), \left(\breve{h}_4, (0.1, 0.2, 0.3)\right), \left(\breve{h}_5, (0.7, 0.6, 0.5)\right)\right\},$$

$$\overline{A}\left(\breve{L}\right) = \left\{\left(\breve{h}_1, (0.1, 0.2, 0.3)\right), \left(\breve{h}_4, (0.1, 0.2, 0.3)\right), \left(\breve{h}_5, (0.7, 0.6, 0.5)\right)\right\}.$$

Obviously, $\underline{A}\left(\breve{L}\right) = \overline{A}\left(\breve{L}\right)$ is a definable neutrosophic set in the approximation space $\left(\breve{X}, \breve{R}\right)$.

Definition 3.2 If $\breve{A}\left(\breve{K}\right) = \left(\underline{A}\left(\breve{L}\right), \overline{A}\left(\breve{L}\right)\right)$ is a rough neutrosophic set in $\left(\hat{X}, \hat{R}\right)$, the rough complement of $\breve{A}\left(\breve{K}\right)$ is the rough neutrosophic set denoted by $\sim \breve{A}\left(\breve{K}\right) = \left(\underline{A}\left(\breve{K}\right)^{C}, \overline{A}\left(\breve{K}\right)^{C}\right)$, where $\underline{A}\left(\breve{K}\right)^{C}, \overline{A}\left(\breve{K}\right)^{C}$ are the complements of neutrosophic sets $\underline{A}\left(\breve{K}\right) and\, \overline{A}\left(\breve{K}\right)$ respectively.

$$\underline{A}\left(\breve{K}\right)^{C} = \left\{\left\langle \breve{y}, \gamma_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right), 1 - \beta_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right), \alpha_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right)\right\rangle | \breve{y} \in \breve{X}\right\}, \tag{4}$$

And

$$\overline{A}\left(\breve{K}\right)^{C} = \left\{\left\langle \breve{y}, \gamma_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right), 1 - \beta_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right), \alpha_{\overline{A}\left(\breve{K}\right)}\left(\breve{y}\right)\right\rangle | \breve{y} \in \breve{X}\right\} \tag{5}$$

Definition 3.3 If $\breve{A}\left(\breve{K}_1\right)$ and $\breve{A}\left(\breve{K}_2\right)$ are two rough neutrosophic set of the neutrosophic sets $\breve{K}_1$ and $\breve{K}_1$ respectively in $\breve{X}$, then we define the following:

(a) $\breve{A}\left(\breve{K}_1\right) = \breve{A}\left(\breve{K}_2\right)$ if and only if $\underline{A}\left(\breve{K}_1\right) = \underline{A}\left(\breve{K}_2\right)$ and $\overline{A}\left(\breve{K}_1\right) = \overline{A}\left(\breve{K}_2\right)$

(b) $\breve{A}\left(\breve{K}_1\right) \subseteq \breve{A}\left(\breve{K}_2\right)$ if and only if $\underline{A}\left(\breve{K}_1\right) \subseteq \underline{A}\left(\breve{K}_2\right)$ and $\overline{A}\left(\breve{K}_1\right) \subseteq \overline{A}\left(\breve{K}_2\right)$

(c) $\breve{A}\left(\breve{K}_1\right) \cup \breve{A}\left(\breve{K}_2\right) = \left\langle \underline{A}\left(\breve{K}_1\right) \cup \underline{A}\left(\breve{K}_2\right), \overline{A}\left(\breve{K}_1\right) \cup \overline{A}\left(\breve{K}_2\right)\right\rangle$

(d) $\breve{A}\left(\breve{K}_1\right) \cap \breve{A}\left(\breve{K}_2\right) = \left\langle \underline{A}\left(\breve{K}_1\right) \cap \underline{A}\left(\breve{K}_2\right), \overline{A}\left(\breve{K}_1\right) \cap \overline{A}\left(\breve{K}_2\right)\right\rangle$

(e) $\breve{A}\left(\breve{K}_1\right) + \breve{A}\left(\breve{K}_2\right) = \left\langle \underline{A}\left(\breve{K}_1\right) + \underline{A}\left(\breve{K}_2\right), \overline{A}\left(\breve{K}_1\right) + \overline{A}\left(\breve{K}_2\right)\right\rangle$

(f) $\breve{A}\left(\breve{K}_1\right) \cdot \breve{A}\left(\breve{K}_2\right) = \left\langle \underline{A}\left(\breve{K}_1\right) \cdot \underline{A}\left(\breve{K}_2\right), \overline{A}\left(\breve{K}_1\right) \cdot \overline{A}\left(\breve{K}_2\right)\right\rangle$

Definition 3.4 Let $\left(\breve{X}, \breve{R}\right)$ be a Pawlak approximation space, for an interval valued neutrosophic set,

$$\breve{A} = \left\{< \breve{y}, \left[\alpha^{L}_{\breve{A}}\left(\breve{y}\right), \alpha^{U}_{\breve{A}}\left(\breve{y}\right)\right], \left[\beta^{L}_{\breve{A}}\left(\breve{y}\right), \beta^{U}_{\breve{A}}\left(\breve{y}\right)\right], \left[\gamma^{L}_{\breve{A}}\left(\breve{y}\right), \gamma^{U}_{\breve{A}}\left(\breve{y}\right)\right] > | \breve{y} \in \breve{X}\right\}$$

be an interval neutrosophic set. The lower approximation $\underline{A}_{\breve{R}}$ and the upper approximation $\overline{A}_{\breve{R}}$ of $\breve{A}$ in the Pawlak approximation space $\left(\breve{X}, \breve{R}\right)$ are defined as:

$$\underline{A}_{\breve{R}} = \Big\{ \Big\langle \breve{y}, \Big[\bigwedge\nolimits_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \alpha_{\breve{A}}^{\breve{L}}(\breve{y}) \big\}, \bigwedge\nolimits_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \alpha_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big],$$
$$\Big[\vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \beta_{\breve{A}}^{\breve{L}}(\breve{y}) \big\} \Big], \Big[\vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \beta_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big],$$
$$\Big[\vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \gamma_{\breve{A}}^{\breve{L}}(\breve{y}) \big\} \}\}, \vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \gamma_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big] \Big] \Big\rangle | \breve{y} \in \breve{X} \Big\} \tag{6}$$

$$\overline{M}_{\hat{R}} = \Big\{ < \breve{y}, \Big[\vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \alpha_{\breve{A}}^{\breve{L}}(\breve{y}) \big\}, \vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \alpha_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big],$$
$$\Big[\bigwedge\nolimits_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \beta_{\breve{A}}^{\breve{L}}(\breve{y}) \big\}, \bigwedge\nolimits_{\breve{t} \in [\breve{X}] \breve{R}} \big\{ \beta_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big],$$
$$\Big[\bigwedge\nolimits_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \gamma_{\breve{A}}^{\breve{L}}(\breve{y}) \big\}, \bigwedge\nolimits_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \gamma_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big] | \breve{y} \in \breve{X} \Big\}. \tag{7}$$

where $\bigwedge$ means t-norm and $\bigvee$ means t-conorm, $\breve{R}$ denote an equivalence relation for interval valued neutrosophic set $\breve{A}$.

Here $\big[\breve{X}\big]_{\breve{R}}$ is the equivalent class of the element $\breve{t}$. It is easy to see that

$$\Big[\bigwedge\nolimits_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \alpha_{\breve{A}}^{\breve{L}}(\breve{y}) \big\}, \bigwedge\nolimits_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \alpha_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big] \subset [0, 1]$$

$$\Big[\vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \beta_{\breve{A}}^{\breve{L}}(\breve{y}) \big\}, \vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \beta_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big] \subset [0, 1]$$

$$\Big[\vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \gamma_{\breve{A}}^{\breve{L}}(\breve{y}) \big\}, \vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \gamma_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big] \subset [0, 1]$$

and

$$0 \le \bigwedge\nolimits_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \alpha_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} + \vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \beta_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} + \vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \gamma_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \le 3.$$

Then, $\underline{A}_{\breve{R}}$ is an interval neutrosophic set.

Similarly, we have

$$\Big[\vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \alpha_{\breve{A}}^{\breve{L}}(\breve{y}) \big\}, \vee_{\breve{t} \in [\breve{X}]_{\breve{R}}} \big\{ \alpha_{\breve{A}}^{\breve{U}}(\breve{y}) \big\} \Big] \subset [0, 1]$$

$$\left[\bigwedge_{\breve{t}\in[\breve{X}]_{\breve{R}}}\left\{\beta^{L}_{\breve{A}}(\breve{y})\right\}, \bigwedge_{\breve{t}\in[\breve{X}]_{\breve{R}}}\left\{\beta^{U}_{\breve{A}}(\breve{y})\right\}\right] \subset [0, 1]$$

$$\left[\bigwedge_{\breve{t}\in[\breve{X}]_{\breve{R}}}\left\{\gamma^{L}_{\breve{A}}(\breve{y})\right\}, \bigwedge_{\breve{t}\in[\breve{X}]_{\breve{R}}}\left\{\gamma^{U}_{\breve{A}}(\breve{y})\right\}\right] \subset [0, 1]$$

and

$$0 \le \vee_{\breve{t}\in[\breve{X}]_{\breve{R}}}\left\{\alpha^{U}_{\breve{A}}(\breve{y})\right\} + \bigwedge_{\breve{t}\in[\breve{X}]_{\breve{R}}}\left\{\beta^{U}_{\breve{A}}(\breve{y})\right\} + \bigwedge_{\breve{t}\in[\breve{X}]\breve{R}}\left\{\gamma^{U}_{\breve{A}}(\breve{y})\right\} \le 3$$

Then, $\overline{A}_{\breve{R}}$ is an interval neutrosophic set.

If $\underline{A}_{\breve{R}} = \overline{A}_{\breve{R}}$, then $\breve{A}$ is a definable set, otherwise $\breve{A}$ is an interval valued neutrosophic rough set. $\underline{A}_{\breve{R}}$ and $\overline{A}_{\breve{R}}$ are called the lower and upper approximations of interval valued neutrosophic set with respect to approximation space $\left(\breve{X}, \breve{R}\right)$, respectively. $\underline{A}_{\breve{R}}$ and $\overline{A}_{\breve{R}}$ are simply denoted by $\underline{A}$ and $\overline{A}$.

Proposition 3.1 *If* $\breve{A}_1, \breve{A}_2$ *and* $\breve{A}_3$ *are rough neutrosophic sets in* $\left(\breve{X}, \breve{R}\right)$,

(a) $\sim\left(\sim \breve{A}_1\right) = \breve{A}_1$,

(b) $\breve{A}_2 \cup \breve{A}_1 = \breve{A}_1 \cup \breve{A}_2$, $\breve{A}_2 \cap \breve{A}_1 = \breve{A}_1 \cap \breve{A}_2$,

(c) $\left(\breve{A}_2 \cup \breve{A}_1\right) \cup \breve{A}_3 = \breve{A}_2 \cup \left(\breve{A}_1 \cup \breve{A}_3\right)$ *and* $\left(\breve{A}_2 \cap \breve{A}_1\right) \cap \breve{A}_3 = \breve{A}_2 \cap \left(\breve{A}_1 \cap \breve{A}_3\right)$,

(d) $\left(\breve{A}_2 \cup \breve{A}_1\right) \cap \breve{A}_3 = \left(\breve{A}_2 \cup \breve{A}_3\right) \cap \left(\breve{A}_1 \cup \breve{A}_3\right)$ *and* $\left(\breve{A}_2 \cap \breve{A}_1\right) \cup \breve{A}_3 = \left(\breve{A}_2 \cap \breve{A}_3\right) \cup \left(\breve{A}_1 \cap \breve{A}_3\right)$.

Proof Straightforward from definition.

For neutrosophic sets De Morgan's law are satisfied:

Proposition 3.2

(a) $\sim\left(\breve{A}\left(\breve{K}_1\right) \cup \breve{A}\left(\breve{K}_2\right)\right) = \left(\sim \breve{A}\left(\breve{K}_1\right)\right) \cap \left(\sim \breve{A}\left(\breve{K}_2\right)\right)$.

(b) $\sim\left(\breve{A}\left(\breve{K}_1\right) \cap \breve{A}\left(\breve{K}_2\right)\right) = \left(\sim \breve{A}\left(\breve{K}_1\right)\right) \cup \left(\sim \breve{A}\left(\breve{K}_2\right)\right)$.

Proof (a)

$$\sim\left(\breve{A}\left(\breve{K}_1\right)\cup\breve{A}\left(\breve{K}_2\right)\right)=\sim\left(\left\{\underline{A}\left(\breve{K}_1\right)\cup\underline{A}\left(\breve{K}_2\right)\right\},\left\{\overline{A}\left(\breve{K}_1\right)\cup\overline{A}\left(\breve{K}_2\right)\right\}\right)$$
$$=\sim\left\{\underline{A}\left(\breve{K}_1\right)\cup\underline{A}\left(\breve{K}_2\right)\right\},\sim\left\{\overline{A}\left(\breve{K}_1\right)\cup\overline{A}\left(\breve{K}_2\right)\right\}$$
$$=\left\{\underline{A}\left(\breve{K}_1\right)\cup\underline{A}\left(\breve{K}_2\right)\right\}^C,\left\{\overline{A}\left(\breve{K}_1\right)\cup\overline{A}\left(\breve{K}_2\right)\right\}^C$$
$$=\left(\sim\left\{\underline{A}\left(\breve{K}_1\right)\cup\underline{A}\left(\breve{K}_2\right)\right\},\sim\left\{\overline{A}\left(\breve{K}_1\right)\cup\overline{A}\left(\breve{K}_2\right)\right\}\right)$$
$$=\sim\breve{A}\left(\breve{K}_1\right)\cap\sim\breve{A}\left(\breve{K}_2\right)$$

(b) Similar to (a).

Proposition 3.3 *If $\breve{K}_1$ and $\breve{K}_2$ are two neutrosophic sets in $\breve{X}$, such that $\breve{K}_1\subseteq\breve{K}_2$, then $\breve{A}\left(\breve{K}_1\right)\subseteq\breve{A}\left(\breve{K}_2\right)$.*

(a) $\breve{A}\left(\breve{K}_1\cup\breve{K}_2\right)\supseteq\breve{A}\left(\breve{K}_1\right)\cup\breve{A}\left(\breve{K}_2\right)$,

(b) $\breve{A}\left(\breve{K}_1\cap\breve{K}_2\right)\supseteq\breve{A}\left(\breve{K}_1\right)\cap\breve{A}\left(\breve{K}_2\right)$.

Proof (a)

$$\alpha_{\underline{A}\left(\breve{K}_1\cup\breve{K}_2\right)}\left(\breve{y}\right)=inf\left\{\alpha_{\underline{A}\left(\breve{K}_1\cup\breve{K}_2\right)}\left(\breve{y}\right)|\breve{y}\in\breve{X}_i\right\}$$
$$=inf\left\{\left(\alpha_{\underline{A}\left(\breve{K}_1\right)}\vee\alpha_{\underline{A}\left(\breve{K}_2\right)}\right)\left(\breve{y}\right)|y\in\breve{X}_i\right\}$$
$$\geq\vee\left\{inf\left\{\alpha_{\underline{A}\left(\breve{K}_1\right)}\left(\breve{y}\right)|\breve{y}\in\breve{X}_i\right\},inf\left\{\alpha_{\underline{A}\left(\breve{K}_2\right)}\left(\breve{y}\right)|\breve{y}\in\breve{X}_i\right\}\right\}$$
$$=\vee\left\{\alpha_{\underline{A}\left(\breve{K}_1\right)}\left(\breve{y}_i\right),\alpha_{\underline{A}\left(\breve{K}_2\right)}\left(\breve{y}_i\right)\right\}$$
$$=\left(\alpha_{\underline{A}\left(\breve{K}_1\right)}\cup\alpha_{\underline{A}\left(\breve{K}_2\right)}\right)\left(\breve{y}_i\right).$$

Similarly,

$$\beta_{\underline{A}\left(\breve{K}_1\cup\breve{K}_2\right)}\left(\breve{y}\right)=\left(\beta_{\underline{A}\left(\breve{K}_1\right)}\cup\beta_{\underline{A}\left(\breve{K}_2\right)}\right)\left(\breve{y}_i\right),\gamma_{\underline{A}\left(\breve{K}_1\cup\breve{K}_2\right)}\left(\breve{y}\right)=\left(\gamma_{\underline{A}\left(\breve{K}_1\right)}\cup\gamma_{\underline{A}\left(\breve{K}_2\right)}\right)\left(\breve{y}_i\right)$$

Thus, $\breve{A}\left(\breve{K}_1 \cup \breve{K}_2\right) \supseteq \breve{A}\left(\breve{K}_1\right) \cup \breve{A}\left(\breve{K}_2\right)$.

We can also see that $\overline{A}\left(\breve{K}_1 \cup \breve{K}_2\right) \supseteq \overline{A}\left(\breve{K}_1\right) \cup \overline{A}\left(\breve{K}_2\right)$. Thus, $\breve{A}\left(\breve{K}_1 \cup \breve{K}_2\right) \supseteq \breve{A}\left(\breve{K}_1\right) \cup \breve{A}\left(\breve{K}_2\right)$. (*b*) Straight forward as (*a*).

Proposition 3.4

(a) $\underline{A}\left(\breve{K}\right) = \sim \overline{A}\left(\sim \breve{K}\right)$
(b) $\overline{A}\left(\breve{K}\right) = \sim \overline{A}\left(\sim \breve{K}\right)$
(c) $\underline{A}\left(\breve{K}\right) \subseteq \overline{A}\left(\breve{K}\right)$.

Proof From definition 3.1, we have that

$$\breve{K} = \left\{\left\langle \breve{y} : \alpha_{\breve{K}}\left(\breve{y}\right), \beta_{\breve{K}}\left(\breve{y}\right), \gamma_{\breve{K}}\left(\breve{y}\right)\right\rangle | \breve{y} \in \breve{X}\right\}$$

$$\sim \breve{K} = \left\{\left\langle \breve{y} : \gamma_{\breve{K}}\left(\breve{y}\right), 1 - \beta_{\breve{K}}\left(\breve{y}\right), \alpha_{\breve{K}}\left(\breve{y}\right)\right\rangle | \breve{y} \in \breve{X}\right\}$$

$$\overline{A}\left(\sim \breve{K}\right) = \left\{\left\langle \breve{y} : \gamma_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right), 1 - \beta_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right), \alpha_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right)\right\rangle | \breve{s} \in \left[\breve{Y}\right]_{\breve{R}}, \breve{y} \in \breve{X}\right\}$$

$$\sim \bar{M}\left(\sim \hat{K}\right) = \left\{\left\langle \breve{y} : \gamma_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right), 1 - \left(1 - \beta_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right)\right), \alpha_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right)\right\rangle | \breve{s} \in \left[\breve{Y}\right]_{\breve{R}}, \breve{y} \in \breve{X}\right\}$$
$$= \left\{\left\langle \breve{y} : \alpha_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right), \beta_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right), \gamma_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right)\right\rangle | \breve{s} \in \left[\breve{Y}\right]_{\breve{R}}, \breve{y} \in \breve{X}\right\}$$

where

$$\alpha_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right) = \wedge_{\breve{s} \in \left[\breve{Y}\right]_{\breve{R}}} \alpha_{\breve{K}}\left(\breve{y}\right), \beta_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right) = \vee_{\breve{s} \in \left[\breve{Y}\right]_{\breve{R}}} \beta_{\breve{K}}\left(\breve{y}\right), \gamma_{\overline{A}\left(\sim \breve{K}\right)}\left(\breve{y}\right) = \vee_{\breve{s} \in \left[\breve{Y}\right]_{\breve{R}}} \gamma_{\breve{K}}\left(\breve{y}\right)$$

Hence $\underline{A}\left(\breve{K}\right) = \sim \overline{A}\left(\sim \breve{K}\right)$.

(*b*) Similar to (*a*).

(c) For any $\breve{t} \in \underline{A}\left(\breve{K}\right)$, we have that $\alpha_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right) = \wedge_{\breve{t}\in\left[\breve{Y}\right]_{\breve{R}}} \alpha_{\breve{K}}\left(\breve{t}\right) \leq \vee_{\breve{t}\in\left[\breve{Y}\right]_{\breve{R}}} \alpha_{\breve{K}}\left(\breve{t}\right)$,

$$\begin{aligned}\beta_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right) &= \vee_{\breve{t}\in\left[\breve{Y}\right]_{\breve{R}}} \beta_{\breve{K}}\left(\breve{t}\right)\\ &\geq \wedge_{\breve{t}\in\left[\breve{Y}\right]_{\breve{R}}} \beta_{\breve{K}}\left(\breve{t}\right), \gamma_{\underline{A}\left(\breve{K}\right)}\left(\breve{y}\right)\\ &= \vee_{\breve{t}\in\left[\breve{Y}\right]_{\breve{R}}} \gamma_{\breve{K}}\left(\breve{t}\right)\\ &\geq \wedge_{\breve{t}\in\left[\breve{Y}\right]_{\breve{R}}} \gamma_{\breve{K}}\left(\breve{t}\right).\end{aligned}$$

Thus $\underline{A}\left(\breve{K}\right) \subseteq \overline{A}\left(\breve{K}\right)$.

Proposition 3.5 *Let $\left(\breve{A}, \breve{B}\right)$ be interval neutrosophic sets and $\underline{A}$ and $\bar{A}$ the lower and upper approximation of interval–valued neutrosophic set $\breve{A}$ with respect to approximation space $\left(\breve{X}, \breve{R}\right)$ respectively.$\underline{B}$ and $\overline{B}$ the lower and upper approximation of interval–valued neutrosophic set $\breve{B}$ with respect to approximation space $\left(\breve{X}, \breve{R}\right)$, respectively. Then, we have*

(a) $\underline{A} \subseteq \breve{A} \subseteq \overline{A}$
(b) $\overline{A \cup B} = \overline{A} \cup \overline{B}$, $\underline{\underline{A \cap B}} = \underline{\underline{A}} \cap \underline{\underline{B}}$
(c) $\underline{\underline{A} \cup \underline{B}} = \underline{A \cup B}$, $\overline{A \cap B} = \overline{\overline{A}} \cap \overline{\overline{B}}$
(d) $\left(\overline{\overline{A}}\right) = \underline{(A)} = \overline{A}$, $\underline{(A)} = \overline{\underline{(A)}} = \underline{A}$
(e) $\underline{\breve{X}} = \breve{X}; \overline{\phi} = \phi$
(f) *If* $\breve{A} \subseteq \breve{B}$, *then* $\underline{A} \subseteq \underline{B}$ *and* $\overline{A} \subseteq \overline{B}$
(g) $\underline{A}^c = \left(\overline{A}\right)^c$, $\overline{A^c} = \left(\underline{A}\right)^c$

Proof We prove parts a, b, c. The others parts are straightforward.

(a) Let $\breve{A} = \left\{\left\langle\breve{y}, \left[\alpha^L_{\breve{A}}\left(\breve{y}\right), \alpha^U_{\breve{A}}\left(\breve{y}\right)\right], \left[\beta^L_{\breve{A}}\left(\breve{y}\right), \beta^U_{\breve{A}}\left(\breve{y}\right)\right], \left[\gamma^L_{\breve{A}}\left(\breve{y}\right)\gamma^U_{\breve{A}}\left(\breve{y}\right)\right]\right\rangle | \breve{y} \in \breve{X}\right\}$ be an interval neutrosophic set.

From definition of $\underline{A}_R$ and $\overline{A}_R$, we have

$$\alpha^L_{\underline{\breve{A}}}\left(\breve{y}\right) \leq \alpha^L_{\breve{A}}\left(\breve{y}\right) \leq \alpha^L_{\overline{\breve{A}}}\left(\breve{y}\right); \alpha^U_{\underline{\breve{A}}}\left(\breve{y}\right) \leq \alpha^U_{\breve{A}}\left(\breve{y}\right) \leq \alpha^U_{\overline{\breve{A}}}\left(\breve{y}\right) \text{ for all } \breve{y} \in \breve{X}$$

$$\beta_{\underline{A}}^{\breve{L}}(\breve{y}) \geq \beta_{\breve{A}}^{\breve{L}}(\breve{y}) \geq \beta_{\bar{A}}^{\breve{L}}(\breve{y}); \beta_{\underline{A}}^{\breve{U}}(\breve{y}) \geq \beta_{\breve{A}}^{\breve{U}}(\breve{y}) \geq \beta_{\bar{A}}^{\breve{U}}(\breve{y}) \text{ for all } \breve{y} \in \breve{X}$$

$$\gamma_{\underline{A}}^{\breve{L}}(\breve{y}) \geq \gamma_{\breve{A}}^{\breve{L}}(\breve{y}) \geq \gamma_{\bar{A}}^{\breve{L}}(\breve{y}); \gamma_{\underline{A}}^{\breve{U}}(\breve{y}) \geq \gamma_{\breve{A}}^{\breve{U}}(\breve{y}) \geq \gamma_{\bar{A}}^{\breve{U}}(\breve{y}) \text{ for all } \breve{y} \in \breve{X}$$

$$\left(\left[\alpha_{\underline{A}}^{\breve{L}}, \alpha_{\underline{A}}^{\breve{U}}\right], \left[\beta_{\underline{A}}^{\breve{L}}, \beta_{\underline{A}}^{\breve{U}}\right], \left[\gamma_{\underline{A}}^{\breve{L}}, \gamma_{\underline{A}}^{\breve{U}}\right]\right) \subseteq \left(\left[\alpha_{\breve{A}}^{\breve{L}}, \alpha_{\breve{A}}^{\breve{U}}\right], \left[\beta_{\breve{A}}^{\breve{L}}, \beta_{\breve{A}}^{\breve{U}}\right], \left[\gamma_{\breve{A}}^{\breve{L}}, \gamma_{\breve{A}}^{\breve{U}}\right]\right) \subseteq \left(\left[\alpha_{\bar{A}}^{\breve{L}}, \alpha_{\bar{A}}^{\breve{U}}\right], \left[\beta_{\bar{A}}^{\breve{L}}, \beta_{\bar{A}}^{\breve{U}}\right], \left[\gamma_{\bar{A}}^{\breve{L}}, \gamma_{\bar{A}}^{\breve{U}}\right]\right).$$

Hence $\underline{A}_{\breve{R}} \subseteq \breve{A} \subseteq \overline{A}_{\breve{R}}$.

(b) Let $\breve{A} = \left\{\left\langle \breve{y}, \left[\alpha_{\breve{A}}^{\breve{L}}(\breve{y}), \alpha_{\breve{A}}^{\breve{U}}(\breve{y})\right], \left[\beta_{\breve{A}}^{\breve{L}}(\breve{y}), \beta_{\breve{A}}^{\breve{U}}(\breve{y})\right], \left[\gamma_{\breve{A}}^{\breve{L}}(\breve{y}), \gamma_{\breve{A}}^{\breve{U}}(\breve{y})\right]\right\rangle | \breve{y} \in \breve{X}\right\}$ and $\breve{B} = \left\{\left\langle \breve{y}, \left[\alpha_{\breve{B}}^{\breve{L}}(\breve{y}), \alpha_{\breve{B}}^{\breve{U}}(\breve{y})\right], \left[\beta_{\breve{B}}^{\breve{L}}(\breve{y}), \beta_{\breve{B}}^{\breve{U}}(\breve{y})\right], \left[\gamma_{\breve{B}}^{\breve{L}}(\breve{y}), \gamma_{\breve{B}}^{\breve{U}}(\breve{y})\right]\right\rangle | \breve{y} \in \breve{X}\right\}$ are two interval valued neutrosophic set and

$$\overline{A \cup B} = \left\{\left\langle \breve{y}, \left[\alpha_{\overline{A \cup B}}^{\breve{L}}(\breve{y}), \alpha_{\overline{M \cup N}}^{\breve{U}}(\breve{y})\right], \left[\beta_{\overline{A \cup B}}^{\breve{L}}(\breve{y}), \beta_{\overline{M \cup N}}^{\breve{U}}(\breve{y})\right], \left[\gamma_{\overline{A \cup B}}^{\breve{L}}(\breve{y}), \gamma_{\overline{M \cup N}}^{\breve{U}}(\breve{y})\right]\right\rangle | \breve{y} \in \breve{X}\right\}$$

$$\overline{A} \cup \overline{B} = \left\{\breve{y}, \left[\left(\alpha_{\bar{A}}^{\breve{L}} \vee \alpha_{\bar{B}}^{\breve{L}}\right)(\breve{y}), \left(\alpha_{\bar{A}}^{\breve{U}} \vee \alpha_{\bar{B}}^{\breve{U}}\right)(\breve{y})\right], \left[\left(\beta_{\bar{A}}^{\breve{L}} \wedge \beta_{\bar{B}}^{\breve{L}}\right)(\breve{y}), \left(\beta_{\bar{A}}^{\breve{U}} \wedge \beta_{\bar{B}}^{\breve{U}}\right)(\breve{y})\right], \left[\left(\gamma_{\bar{A}}^{\breve{L}} \wedge \gamma_{\bar{B}}^{\breve{L}}\right)(\breve{y}), \left(\gamma_{\bar{A}}^{\breve{U}} \wedge \gamma_{\bar{B}}^{\breve{U}}\right)(\breve{y})\right]\right\},$$

for all $\breve{y} \in \breve{X}$ and where $\wedge$ is the t-norm, and $\vee$ is the t-conorm

$$\begin{aligned}
\alpha_{\overline{A \cup B}}^{\breve{L}}(\breve{y}) &= \bigvee\left\{\alpha_{\breve{A} \cup \breve{B}}^{\breve{L}}(\breve{y}) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigvee\left\{\alpha_{\breve{A}}^{\breve{L}}(\breve{y}) \vee \alpha_{\breve{B}}^{\breve{L}}(\breve{y}) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \left(\vee \alpha_{\breve{A}}^{\breve{L}}(\breve{y}) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \bigvee \left(\vee \alpha_{\bar{B}}^{\breve{L}}(\breve{y}) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \\
&= \left(\alpha_{\bar{A}}^{\breve{L}} \bigvee \alpha_{\bar{B}}^{\breve{L}}\right)(\breve{y})
\end{aligned}$$

$$
\begin{aligned}
\alpha^{\breve{U}}_{\overline{A\cup B}}\left(\breve{y}\right) &= \bigvee\left\{\alpha^{\breve{U}}_{\breve{A}\cup\breve{B}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right\}\\
&= \bigvee\left\{\alpha^{\breve{U}}_{\breve{A}}\left(\breve{y}\right)\vee\alpha^{\breve{U}}_{\breve{B}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right\}\\
&= \left(\vee\alpha^{\breve{U}}_{\breve{A}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right)\bigvee\left(\vee\alpha^{\breve{U}}_{B}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right)\\
&= \left(\alpha^{\breve{U}}_{\overline{A}}\bigvee\alpha^{\breve{U}}_{\overline{B}}\right)(\breve{y})
\end{aligned}
$$

$$
\begin{aligned}
\beta^{\breve{L}}_{\overline{A\cup B}}\left(\breve{y}\right) &= \bigwedge\left\{\beta^{\breve{L}}_{\breve{A}\cup\breve{B}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right\}\\
&= \bigwedge\left\{\beta^{\breve{L}}_{\breve{A}}\left(\breve{y}\right)\wedge\beta^{\breve{L}}_{\breve{A}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right\}\\
&= \left(\wedge\beta^{\breve{L}}_{\breve{A}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right)\bigwedge\left(\wedge\beta^{\breve{L}}_{\breve{A}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right)\\
&= \left(\beta^{\breve{L}}_{\breve{A}}\bigwedge\beta^{\breve{L}}_{\breve{A}}\right)\left(\breve{y}\right)
\end{aligned}
$$

$$
\begin{aligned}
\beta^{\breve{U}}_{\overline{A\cup B}}\left(\breve{y}\right) &= \bigwedge\left\{\beta^{\breve{U}}_{\breve{A}\cup\breve{B}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right\}\\
&= \bigwedge\left\{\beta^{\breve{U}}_{\breve{A}}\left(\breve{y}\right)\wedge\beta^{\breve{U}}_{\breve{A}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right\}\\
&= \left(\wedge\beta^{\breve{U}}_{\breve{A}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right)\bigwedge\left(\wedge\beta^{\breve{U}}_{\breve{A}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right)\\
&= \left(\beta^{\breve{U}}_{\breve{A}}\bigwedge\beta^{\breve{U}}_{\breve{A}}\right)\left(\breve{y}\right)
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{\breve{L}}_{\overline{A\cup B}}\left(\breve{y}\right) &= \bigwedge\left\{\gamma^{\breve{L}}_{\breve{A}\cup\breve{B}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right\}\\
&= \bigwedge\left\{\gamma^{\breve{L}}_{\breve{A}}\left(\breve{y}\right)\wedge\gamma^{\breve{L}}_{\breve{B}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right\}\\
&= \left(\wedge\gamma^{\breve{L}}_{\breve{A}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right)\bigwedge\left(\wedge\gamma^{\breve{L}}_{\breve{B}}\left(\breve{y}\right)\mid\breve{y}\in\left[\breve{X}\right]_{\breve{R}}\right)\\
&= \left(\gamma^{\breve{L}}_{\breve{A}}\bigwedge\gamma^{\breve{L}}_{\breve{B}}\right)\left(\breve{y}\right)
\end{aligned}
$$

$$\gamma^{\breve{U}}_{\overline{A\cup B}}(\breve{y}) = \bigwedge\left\{\gamma^{\breve{U}}_{\breve{A}\cup\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\}$$
$$= \bigwedge\left\{\gamma^{\breve{U}}_{\breve{A}}(\breve{y}) \wedge \gamma^{\breve{U}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\}$$
$$= \left(\wedge\gamma^{\breve{U}}_{\breve{A}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \bigwedge \left(\wedge\gamma^{\breve{U}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right)$$
$$= \left(\gamma^{\breve{U}}_{\breve{A}} \bigwedge \gamma^{\breve{U}}_{\breve{B}}\right)(\breve{y}) \text{ Hence, } \overline{A \cup B} = \overline{A} \cup \overline{B}$$

Also for $\underline{A \cap B} = \underline{A} \cap \underline{B}$ for all $\breve{y} \in \breve{X}$

$$\alpha^{\breve{L}}_{\underline{A\cap B}}(\breve{y}) = \bigwedge\left\{\alpha^{\breve{L}}_{\breve{A}\cap\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\}$$
$$= \bigwedge\left\{\alpha^{\breve{L}}_{\breve{A}}(\breve{y}) \wedge \alpha^{\breve{L}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\}$$
$$= \bigwedge\left(\alpha^{\breve{L}}_{\breve{A}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \bigwedge \left(\vee\alpha^{\breve{L}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right)$$
$$= \alpha^{\breve{L}}_{\breve{A}}(\breve{y}) \wedge \alpha^{\breve{L}}_{\breve{B}}(\breve{y})$$
$$= \left(\alpha^{\breve{L}}_{\breve{A}} \wedge \alpha^{\breve{L}}_{\breve{B}}\right)(\breve{y})$$

Also

$$\alpha^{\breve{U}}_{\underline{A\cap B}}(\breve{y}) = \bigwedge\left\{\alpha^{\breve{U}}_{\breve{A}\cap\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\}$$
$$= \bigwedge\left\{\alpha^{\breve{U}}_{\breve{A}}(\breve{y}) \wedge \alpha^{\breve{U}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\}$$
$$= \bigwedge\left(\alpha^{\breve{U}}_{\breve{A}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \bigwedge \left(\vee\alpha^{\breve{U}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right)$$
$$= \alpha^{\breve{U}}_{\breve{A}}(\breve{y}) \wedge \alpha^{\breve{U}}_{\breve{B}}(\breve{y})$$
$$= \left(\alpha^{\breve{U}}_{\breve{A}} \wedge \alpha^{\breve{U}}_{\breve{B}}\right)(\breve{y})$$

$$\beta^{\breve{L}}_{\underline{A\cap B}}(\breve{y}) = \bigvee\left\{\beta^{\breve{L}}_{\breve{A}\cap\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\}$$
$$= \bigvee\left\{\beta^{\breve{L}}_{\breve{A}}(\breve{y}) \vee \beta^{\breve{L}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\}$$

$$
\begin{aligned}
&= \bigvee\left(\beta^{\breve{L}}_{\breve{A}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \bigvee\left(\vee\beta^{\breve{L}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \\
&= \beta^{\breve{L}}_{\breve{A}}(\breve{y}) \vee \beta^{\breve{L}}_{\breve{A}}(\breve{y}) \\
&= \left(\beta^{\breve{L}}_{\breve{A}} \vee \beta^{\breve{L}}_{\breve{A}}\right)(\breve{y})
\end{aligned}
$$

$$
\begin{aligned}
\beta^{\breve{U}}_{\underline{A\cap B}}(\breve{y}) &= \bigvee\left\{\beta^{\breve{U}}_{\breve{A}\cap\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigvee\left\{\beta^{\breve{U}}_{\breve{A}}(\breve{y}) \vee \beta^{\breve{U}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigvee\left(\beta^{\breve{U}}_{\breve{A}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \bigvee\left(\vee\beta^{\breve{U}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \\
&= \beta^{\breve{U}}_{\breve{A}}(\breve{y}) \vee \beta^{\breve{U}}_{\breve{A}}(\breve{y}) \\
&= \left(\beta^{\breve{U}}_{\breve{A}} \vee \beta^{\breve{U}}_{\breve{A}}\right)(\breve{y})
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{\breve{L}}_{\underline{A\cap B}}(\breve{y}) &= \bigvee\left\{\gamma^{\breve{L}}_{\breve{A}\cap\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigvee\left\{\gamma^{\breve{L}}_{\breve{A}}(\breve{y}) \vee \gamma^{\breve{L}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigvee\left(\gamma^{\breve{L}}_{\breve{A}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \bigvee\left(\vee\gamma^{\breve{L}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \\
&= \gamma^{\breve{L}}_{\breve{A}}(\breve{y}) \vee \gamma^{\breve{L}}_{\breve{A}}(\breve{y}) \\
&= \left(\gamma^{\breve{L}}_{\breve{A}} \vee \gamma^{\breve{L}}_{\breve{A}}\right)(\breve{y})
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{\breve{U}}_{\underline{A\cap B}}(\breve{y}) &= \bigvee\left\{\gamma^{\breve{U}}_{\breve{A}\cap\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigvee\left\{\gamma^{\breve{U}}_{\breve{A}}(\breve{y}) \vee \gamma^{\breve{U}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigvee\left(\gamma^{\breve{U}}_{\breve{A}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \bigvee\left(\vee\gamma^{\breve{U}}_{\breve{B}}(\breve{y}) \mid \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \\
&= \gamma^{\breve{U}}_{\breve{A}}(\breve{y}) \vee \gamma^{\breve{U}}_{\breve{A}}(\breve{y}) \\
&= \left(\gamma^{\breve{U}}_{\breve{A}} \vee \gamma^{\breve{U}}_{\breve{A}}\right)(\breve{y})
\end{aligned}
$$

(c)

$$
\begin{aligned}
\alpha^{\breve{U}}_{\overline{\breve{A}\cap\breve{B}}}\left(\breve{y}\right) &= \bigvee\left\{\alpha^{\breve{U}}_{\breve{A}\cap\breve{B}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigvee\left\{\alpha^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) \wedge \alpha^{\breve{U}}_{\breve{B}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigvee\left(\alpha^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \wedge \left(\bigvee\left(\alpha^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right)\right) \\
&= \alpha^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) \vee \alpha^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) \\
&= \left(\alpha^{\breve{U}}_{\breve{A}} \bigvee \alpha^{\breve{U}}_{\breve{A}}\right)\left(\breve{y}\right)
\end{aligned}
$$

$$
\begin{aligned}
\beta^{\breve{U}}_{\overline{\breve{A}\cap\breve{B}}}\left(\breve{y}\right) &= \bigwedge\left\{\beta^{\breve{U}}_{\breve{A}\cap\breve{B}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigwedge\left\{\beta^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) \wedge \beta^{\breve{U}}_{\breve{B}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigwedge\left(\beta^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \vee \left(\bigwedge\left(\beta^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right)\right) \\
&= \beta^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) \vee \beta^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) \\
&= \left(\beta^{\breve{U}}_{\breve{A}} \bigvee \beta^{\breve{U}}_{\breve{A}}\right)\left(\breve{y}\right)
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{\breve{U}}_{\overline{\breve{A}\cap\breve{B}}}\left(\breve{y}\right) &= \bigwedge\left\{\gamma^{\breve{U}}_{\breve{A}\cap\breve{B}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigwedge\left\{\gamma^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) \wedge \gamma^{\breve{U}}_{\breve{B}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right\} \\
&= \bigwedge\left(\gamma^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right) \vee \left(\bigwedge\left(\gamma^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) | \breve{y} \in \left[\breve{X}\right]_{\breve{R}}\right)\right) \\
&= \gamma^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) \vee \gamma^{\breve{U}}_{\breve{A}}\left(\breve{y}\right) \\
&= \left(\gamma^{\breve{U}}_{\breve{A}} \vee \gamma^{\breve{U}}_{\breve{A}}\right)\left(\breve{y}\right).
\end{aligned}
$$

Hence follow that $\overline{A \cap B} = \overline{A} \cap \overline{B}$. We get $\underline{A} \cup \underline{B} = \underline{A \cup B}$ by following the same procedure as above.

4 Conclusion

This chapter is based on the basic concepts of neutrosophic sets as well as some of their hybrid structures. In this chapter, we defined and studied the idea of neutrosophic set and their basic properties. Moreover, interval-valued neutrosophic sets are studied with some of their properties. Finally, we define rough neutrosophic set and studied some of its basic properties.

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