BMBJ-neutrosophic ideals in BCK/BCI-algebras

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Abstract: The concepts of a BMBJ-neutrosophic ∘-subalgebra and a (closed) BMBJ-neutrosophic ideal are introduced, and several properties are investigated. Conditions for an MBJ-neutrosophic set to be a BMBJ-neutrosophic ideal in BCK/BCI-algebras are provided. Characterizations of BMBJ-neutrosophic ideal are discussed. Relations between a BMBJ-neutrosophic subalgebra, a BMBJ-neutrosophic ∘-subalgebra and a (closed) BMBJ-neutrosophic ideal are considered.

Keywords: MBJ-neutrosophic set; BMBJ-neutrosophic subalgebra; BMBJ-neutrosophic ideal; BMBJ-neutrosophic ∘-subalgebra.

1 Introduction

Smarandache introduced the notion of neutrosophic set which is a more general platform that extends the notions of classic set, (intuitionistic) fuzzy set and interval valued (intuitionistic) fuzzy set (see [11, 12]). Neutrosophic set theory is applied to various part which is refered to the site

http://fs.gallup.unm.edu/neutrosophy.htm.

Jun and his colleagues applied the notion of neutrosophic set theory to BCK/BCI-algebras (see [4, 5, 6, 7, 10, 13, 14]). Borzooei et al. [2] studied commutative generalized neutrosophic ideals in BCK-algebras. Mohseni et al. [9] introduced the notion of MBJ-neutrosophic sets which is another generalization of neutrosophic set. They introduced the concept of MBJ-neutrosophic subalgebras in BCK/BCI-algebras, and investigated related properties. They gave a characterization of MBJ-neutrosophic subalgebra, and established a new MBJ-neutrosophic subalgebra by using an MBJ-neutrosophic subalgebra of a BCI-algebra. They considered the homomorphic inverse image of MBJ-neutrosophic subalgebra, and discussed translation of MBJ-neutrosophic subalgebra. Bordbar et al. [1] introduced the notion of BMBJ-neutrosophic subalgebras, and investigated related properties.

In this paper, we apply the notion of MBJ-neutrosophic sets to ideals of BCK/BI-algebras. We introduce the concepts of a BMBJ-neutrosophic ∘-subalgebra and a (closed) BMBJ-neutrosophic ideal, and investigate several properties. We provide conditions for an MBJ-neutrosophic set to be a BMBJ-neutrosophic ideal in BCK/BCI-algebras, and discuss characterizations of BMBJ-neutrosophic ideal. We consider relations between a BMBJ-neutrosophic subalgebra, a BMBJ-neutrosophic ∘-subalgebra and a (closed) BMBJ-neutrosophic ideal.

2 Preliminaries

By a BCI-*algebra*, we mean a set X with a binary operation $*$ and a special element 0 that satisfies the following conditions:

(I) $((x*y)*(x*z))*(z*y)=0$,

(II) $(x*(x*y))*y=0$,

(III) $x*x=0$,

(IV) $x*y=0,\ y*x=0\ \Rightarrow\ x=y$

for all $x,y,z\in X$. If a BCI-algebra X satisfies the following identity:

(V) $(\forall x\in X)\ (0*x=0)$,

then X is called a BCK-*algebra*.

By a *weakly* BCK-*algebra* (see [3]), we mean a BCI-algebra X satisfying $0*x\leq x$ for all $x\in X$.

Every BCK/BCI-algebra X satisfies the following conditions:

$$(\forall x\in X)\,(x*0=x)\,, \tag{2.1}$$
$$(\forall x,y,z\in X)\,(x\leq y\ \Rightarrow\ x*z\leq y*z,\ z*y\leq z*x)\,, \tag{2.2}$$
$$(\forall x,y,z\in X)\,((x*y)*z=(x*z)*y)\,, \tag{2.3}$$
$$(\forall x,y,z\in X)\,((x*z)*(y*z)\leq x*y) \tag{2.4}$$

where $x\leq y$ if and only if $x*y=0$. Any BCI-algebra X satisfies the following conditions (see [3]):

$$(\forall x,y\in X)(x*(x*(x*y))=x*y), \tag{2.5}$$
$$(\forall x,y\in X)(0*(x*y)=(0*x)*(0*y)). \tag{2.6}$$

A BCI-algebra X is said to be p-*semisimple* (see [3]) if

$$(\forall x\in X)(0*(0*x)=x). \tag{2.7}$$

In a p-semisimple BCI-algebra X, the following holds:

$$(\forall x,y\in X)(0*(x*y)=y*x,\ x*(x*y)=y). \tag{2.8}$$

A BCI-algebra X is said to be *associative* (see [3]) if

$$(\forall x,y,z\in X)((x*y)*z=x*(y*z)). \tag{2.9}$$

By an (S)-BCK-*algebra*, we mean a BCK-algebra X such that, for any $x,y\in X$, the set

$$\{z\in X\mid z*x\leq y\}$$

has the greatest element, written by $x\circ y$ (see [8]).

A nonempty subset S of a BCK/BCI-algebra X is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$. A subset I of a BCK/BCI-algebra X is called an *ideal* of X if it satisfies:

$$0 \in I, \tag{2.10}$$

$$(\forall x \in X)\,(\forall y \in I)\,(x * y \in I \;\Rightarrow\; x \in I)\,. \tag{2.11}$$

A subset I of a BCI-algebra X is called a *closed ideal* of X (see [3]) if it is an ideal of X which satisfies:

$$(\forall x \in X)(x \in I \;\Rightarrow\; 0 * x \in I). \tag{2.12}$$

By an *interval number* we mean a closed subinterval $\tilde{a} = [a^-, a^+]$ of I, where $0 \leq a^- \leq a^+ \leq 1$. Denote by $[I]$ the set of all interval numbers.

Let X be a nonempty set. A function $A : X \to [I]$ is called an *interval-valued fuzzy set* (briefly, an *IVF set*) in X. Let $[I]^X$ stand for the set of all IVF sets in X. For every $A \in [I]^X$ and $x \in X$, $A(x) = [A^-(x), A^+(x)]$ is called the *degree* of membership of an element x to A, where $A^- : X \to I$ and $A^+ : X \to I$ are fuzzy sets in X which are called a *lower fuzzy set* and an *upper fuzzy set* in X, respectively. For simplicity, we denote $A = [A^-, A^+]$.

Let X be a non-empty set. A *neutrosophic set* (NS) in X (see [11]) is a structure of the form:

$$A := \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\}$$

where $A_T : X \to [0,1]$ is a truth membership function, $A_I : X \to [0,1]$ is an indeterminate membership function, and $A_F : X \to [0,1]$ is a false membership function. For the sake of simplicity, we shall use the symbol $A = (A_T, A_I, A_F)$ for the neutrosophic set

$$A := \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\}.$$

We refer the reader to the books [3, 8] for further information regarding BCK/BCI-algebras, and to the site "http://fs.gallup.unm.edu/neutrosophy.htm" for further information regarding neutrosophic set theory.

Let X be a non-empty set. By an *MBJ-neutrosophic set* in X (see [9]), we mean a structure of the form:

$$\mathcal{A} := \{\langle x; M_A(x), \tilde{B}_A(x), J_A(x)\rangle \mid x \in X\}$$

where M_A and J_A are fuzzy sets in X, which are called a truth membership function and a false membership function, respectively, and $\tilde{B}_A$ is an IVF set in X which is called an indeterminate interval-valued membership function.

For the sake of simplicity, we shall use the symbol $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ for the MBJ-neutrosophic set

$$\mathcal{A} := \{\langle x; M_A(x), \tilde{B}_A(x), J_A(x)\rangle \mid x \in X\}.$$

Let X be a BCK/BCI-algebra. An MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in X is called a ***BMBJ-neutrosophic subalgebra*** of X (see [1]) if it satisfies:

$$(\forall x \in X)(M_A(x) + B_A^-(x) \leq 1,\, B_A^+(x) + J_A(x) \leq 1) \tag{2.13}$$

and

$$(\forall x, y \in X) \begin{pmatrix} M_A(x*y) \geq \min\{M_A(x), M_A(y)\} \\ B_A^-(x*y) \leq \max\{B_A^-(x), B_A^-(y)\} \\ B_A^+(x*y) \geq \min\{B_A^+(x), B_A^+(y)\} \\ J_A(x*y) \leq \max\{J_A(x), J_A(y)\} \end{pmatrix}. \tag{2.14}$$

3 BMBJ-neutrosophic ideals

Definition 3.1. Let X be a BCK/BCI-algebra. An MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in X is called a *BMBJ-neutrosophic ideal* of X if it satisfies (2.13) and

$$(\forall x \in X) \begin{pmatrix} M_A(0) \geq M_A(x) \\ B_A^-(0) \leq B_A^-(x) \\ B_A^+(0) \geq B_A^+(x) \\ J_A(0) \leq J_A(x) \end{pmatrix}, \tag{3.1}$$

$$(\forall x, y \in X) \begin{pmatrix} M_A(x) \geq \min\{M_A(x*y), M_A(y)\} \\ B_A^-(x) \leq \max\{B_A^-(x*y), B_A^-(y)\} \\ B_A^+(x) \geq \min\{B_A^+(x*y), B_A^+(y)\} \\ J_A(x) \leq \max\{J_A(x*y), J_A(y)\} \end{pmatrix}. \tag{3.2}$$

A BMBJ-neutrosophic ideal $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ of a BCI-algebra X is said to be *closed* if

$$(\forall x \in X) \begin{pmatrix} M_A(0*x) \geq M_A(x) \\ B_A^-(0*x) \leq B_A^-(x) \\ B_A^+(0*x) \geq B_A^+(x) \\ J_A(0*x) \leq J_A(x) \end{pmatrix}. \tag{3.3}$$

Example 3.2. Consider a set $X = \{0, 1, 2, a\}$ with the binary operation $*$ which is given in Table 1. Then

Table 1: Cayley table for the binary operation "$*$"

$*$	0	1	2	a
0	0	0	0	a
1	1	0	0	a
2	2	2	0	a
a	a	a	a	0

$(X; *, 0)$ is a BCI-algebra (see [3]). Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X defined by Table 2. It is routine to verify that $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a closed MBJ-neutrosophic ideal of X.

Table 2: MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$

X	$M_A(x)$	$\tilde{B}_A(x)$	$J_A(x)$
0	0.7	$[0.02, 0.08]$	0.2
1	0.5	$[0.02, 0.06]$	0.2
2	0.4	$[0.02, 0.06]$	0.7
a	0.3	$[0.02, 0.06]$	0.7

Proposition 3.3. *Let X be a BCK/BCI-algebra. Then every BMBJ-neutrosophic ideal $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ of X satisfies the following assertion.*

$$x * y \le z \;\Rightarrow \begin{cases} M_A(x) \ge \min\{M_A(y), M_A(z)\}, \\ B_A^-(x) \le \max\{B_A^-(y), B_A^-(z)\}, \\ B_A^+(x) \ge \min\{B_A^+(y), B_A^+(z)\}, \\ J_A(x) \le \max\{J_A(y), J_A(z)\} \end{cases} \tag{3.4}$$

for all $x, y, z \in X$.

Proof. Let $x, y, z \in X$ be such that $x * y \le z$. Then

$$M_A(x * y) \ge \min\{M_A((x * y) * z), M_A(z)\} = \min\{M_A(0), M_A(z)\} = M_A(z),$$

$$B_A^-(x * y) \le \max\{B_A^-((x * y) * z), B_A^-(z)\} = \max\{B_A^-(0), B_A^-(z)\} = B_A^-(z),$$

$$B_A^+(x * y) \ge \min\{B_A^+((x * y) * z), B_A^+(z)\} = \min\{B_A^+(0), B_A^+(z)\} = B_A^+(z),$$

and

$$J_A(x * y) \le \max\{J_A((x * y) * z), J_A(z)\} = \max\{J_A(0), J_A(z)\} = J_A(z).$$

It follows that

$$M_A(x) \ge \min\{M_A(x * y), M_A(y)\} = \min\{M_A(y), M_A(z)\},$$

$$B_A^-(x) \le \max\{B_A^-(x * y), B_A^-(y)\} = \max\{B_A^-(y), B_A^-(z)\},$$

$$B_A^+(x) \ge \min\{B_A^+(x * y), B_A^+(y)\} = \min\{B_A^+(y), B_A^+(z)\},$$

and

$$J_A(x) \le \max\{J_A(x * y), J_A(y)\} = \max\{J_A(y), J_A(z)\}.$$

This completes the proof. □

We provide conditions for an MBJ-neutrosophic set to be a BMBJ-neutrosophic ideal in BCK/BCI-algebras.

Theorem 3.4. *Every MBJ-neutrosophic set in a BCK/BCI-algebra X satisfying* (3.1) *and* (3.4) *is a BMBJ-neutrosophic ideal of X.*

Proof. Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X satisfying (3.1) and (3.4). Note that $x * (x * y) \leq y$ for all $x, y \in X$. It follows from (3.4) that

$$M_A(x) \geq \min\{M_A(x * y), M_A(y)\},$$

$$B_A^-(x) \leq \max\{B_A^-(x * y), B_A^-(y)\},$$

$$B_A^+(x) \geq \min\{B_A^+(x * y), B_A^+(y)\},$$

and

$$J_A(x) \leq \max\{J_A(x * y), J_A(y)\}.$$

Therefore $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X. □

Given an MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in a BCK/BCI-algebra X, we consider the following sets.

$$\begin{aligned} U(M_A; t) &:= \{x \in X \mid M_A(x) \geq t\}, \\ L(B_A^-; \alpha^-) &:= \{x \in X \mid B_A^-(x) \leq \alpha^-\}, \\ U(B_A^+; \alpha^+) &:= \{x \in X \mid B_A^+(x) \geq \alpha^+\}, \\ L(J_A; s) &:= \{x \in X \mid J_A(x) \leq s\} \end{aligned}$$

where $t, s, \alpha^-, \alpha^+ \in [0, 1]$.

Theorem 3.5. *An MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in a BCK/BCI-algebra X is an MBJ-neutrosophic ideal of X if and only if the non-empty sets $U(M_A; t)$, $L(B_A^-; \alpha^-)$, $U(B_A^+; \alpha^+)$ and $L(J_A; s)$ are ideals of X for all $t, s, \alpha^-.\alpha^+ \in [0, 1]$.*

Proof. Suppose that $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic ideal of X. Let $t, s, \alpha^-, \alpha^+ \in [0, 1]$ be such that $U(M_A; t)$, $L(B_A^-; \alpha^-)$, $U(B_A^+; \alpha^+)$ and $L(J_A; s)$ are non-empty. Obviously, $0 \in U(M_A; t) \cap L(B_A^-; \alpha^-) \cap U(B_A^+; \alpha^+) \cap L(J_A; s)$. For any $x, y, a, b, p, q, u, v \in X$, if $x * y \in U(M_A; t)$, $y \in U(M_A; t)$, $a * b \in L(B_A^-; \alpha^-)$, $b \in L(B_A^-; \alpha^-)$, $p * q \in U(B_A^+; \alpha^+)$, $q \in U(B_A^+; \alpha^+)$, $u * v \in L(J_A; s)$ and $v \in L(J_A; s)$, then

$$\begin{aligned} &M_A(x) \geq \min\{M_A(x * y), M_A(y)\} \geq \min\{t, t\} = t, \\ &B_A^-(a) \leq \max\{B_A^-(a * b), B_A^-(b)\} \leq \max\{\alpha^-, \alpha^-\} = \alpha^-, \\ &B_A^+(p) \geq \min\{B_A^+(p * q), B_A^+(q)\} \geq \min\{\alpha^+, \alpha^+\} = \alpha^+, \\ &J_A(u) \leq \max\{J_A(u * v), J_A(v)\} \leq \min\{s, s\} = s, \end{aligned}$$

and so $x \in U(M_A;t)$, $a \in L(B_A^-;\alpha^-)$, $p \in U(B_A^+;\alpha^+)$ and $u \in L(J_A;s)$. Therefore $U(M_A;t)$, $L(B_A^-;\alpha^-)$, $U(B_A^+;\alpha^+)$ and $L(J_A;s)$ are ideals of X.

Conversely, assume that the non-empty sets $U(M_A;t)$, $L(B_A^-;\alpha^-)$, $U(B_A^+;\alpha^+)$ and $L(J_A;s)$ are ideals of X for all $t, s, \alpha^-, \alpha^+ \in [0,1]$. Assume that $M_A(0) < M_A(a)$, $B_A^-(0) > B_A^-(a)$, $B_A^+(0) < B_A^+(a)$ and $J_A(0) > J_A(a)$ for some $a \in X$. Then $0 \notin U(M_A; M_A(a)) \cap L(B_A^-; B_A^-(a)) \cap U(B_A^+; B_A^+(a)) \cap L(J_A; J_A(a)$, which is a contradiction. Hence $M_A(0) \geq M_A(x)$, $B_A^-(0) \leq B_A^-(x)$, $B_A^+(0) \geq B_A^+(x)$ and $J_A(0) \leq J_A(x)$ for all $x \in X$. If $M_A(a_0) < \min\{M_A(a_0 * b_0), M_A(b_0)\}$ for some $a_0, b_0 \in X$, then $a_0 * b_0 \in U(M_A;t_0)$ and $b_0 \in U(M_A;t_0)$ but $a_0 \notin U(M_A;t_0)$ for $t_0 := \min\{M_A(a_0 * b_0), M_A(b_0)\}$. This is a contradiction, and thus $M_A(a) \geq \min\{M_A(a*b), M_A(b)\}$ for all $a, b \in X$. Similarly, we can show that $J_A(a) \leq \max\{J_A(a*b), J_A(b)\}$ for all $a, b \in X$. Suppose that $B_A^-(a_0) > \max\{B_A^-(a_0 * b_0), B_A^-(b_0)\}$ for some $a_0, b_0 \in X$. Taking $\alpha^- = \max\{B_A^-(a_0 * b_0), B_A^-(b_0)\}$ implies that $a_0 * b_0 \in L(B_A^-;\alpha^-)$ and $b_0 \in L(B_A^-;\alpha^-)$ but $a_0 \notin L(B_A^-;\alpha^-)$. This is a contradiction. Thus $B_A^-(x) \leq \max\{B_A^-(x*y), B_A^-(y)\}$ for all $x, y \in X$. Similarly, we obtain $B_A^+(x) \geq \min\{B_A^+(x*y), B_A^+(y)\}$ for all $x, y \in X$. Consequently $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X. □

Theorem 3.6. *An MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in a BCK/BCI-algebra X is a BMBJ-neutrosophic ideal of X if and only if (M_A, B_A^-) and (B_A^+, J_A) are intuitionistic fuzzy ideals of X.*

Proof. Straightforward. □

Theorem 3.7. *Given an ideal I of a BCK/BCI-algebra X, let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X defined by*

$$M_A(x) = \begin{cases} t & \text{if } x \in I, \\ 0 & \text{otherwise,} \end{cases} \quad B_A^-(x) = \begin{cases} \alpha^- & \text{if } x \in I, \\ 1 & \text{otherwise,} \end{cases}$$
$$B_A^+(x) = \begin{cases} \alpha^+ & \text{if } x \in I, \\ 0 & \text{otherwise,} \end{cases} \quad J_A(x) = \begin{cases} s & \text{if } x \in I, \\ 1 & \text{otherwise,} \end{cases}$$

where $t, \alpha^+ \in (0,1]$, $s, \alpha^- \in [0,1)$. Then $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X such that $U(M_A;t) = L(B_A^-;\alpha^-) = U(B_A^+;\alpha^+) = L(J_A;s) = I$.

Proof. It is clear that $U(M_A;t) = L(B_A^-;\alpha^-) = U(B_A^+;\alpha^+) = L(J_A;s) = I$. Let $x, y \in X$. If $x * y \in I$ and $y \in I$, then $x \in I$ and so

$$\begin{aligned} &M_A(x) = t = \min\{M_A(x*y), M_A(y)\} \\ &B_A^-(x) = \alpha^- = \max\{B_A^-(x*y), B_A^-(y)\}, \\ &B_A^+(x) = \alpha^+ = \min\{B_A^+(x*y), B_A^+(y)\}, \\ &J_A(x) = s = \max\{J_A(x*y), J_A(y)\}. \end{aligned}$$

If any one of $x * y$ and y is contained in I, say $x * y \in I$, then $M_A(x*y) = t$, $B_A^-(x*y) = \alpha^-$, $J_A(x*y) = s$, $M_A(y) = 0$, $B_A^-(y) = 1$, $B_A^+(y) = 0$ and $J_A(y) = 1$. Hence

$$\begin{aligned} &M_A(x) \geq 0 = \min\{t, 0\} = \min\{M_A(x*y), M_A(y)\} \\ &B_A^-(x) \leq 1 = \max\{B_A^-(x*y), B_A^-(y)\}, \\ &B_A^+(x) \geq 0 = \min\{B_A^+(x*y), B_A^+(y)\}, \\ &J_A(x) \leq 1 = \max\{s, 1\} = \max\{J_A(x*y), J_A(y)\}. \end{aligned}$$

If $x * y, y \notin I$, then $M_A(x * y) = 0 = M_A(y)$, $B_A^-(x * y) = 1 = B_A^-(y)$, $B_A^+(x * y) = 0 = B_A^+(y)$ and $J_A(x * y) = 1 = J_A(y)$. It follows that

$$\begin{aligned} &M_A(x) \geq 0 = \min\{M_A(x * y), M_A(y)\} \\ &B_A^-(x) \leq 1 = \max\{B_A^-(x * y), B_A^-(y)\}, \\ &B_A^+(x) \geq 0 = \min\{B_A^+(x * y), B_A^+(y)\}, \\ &J_A(x) \leq 1 = \max\{J_A(x * y), J_A(y)\}. \end{aligned}$$

It is obvious that $M_A(0) \geq M_A(x)$, $B_A^-(0) \leq B_A^-(x)$, $B_A^+(0) \geq B_A^+(x)$ and $J_A(0) \leq J_A(x)$ for all $x \in X$. Therefore $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X. □

Theorem 3.8. *For any non-empty subset I of X, let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X which is given in Theorem 3.7. If $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X, then I is an ideal of X.*

Proof. Obviously, $0 \in I$. Let $x, y \in X$ be such that $x * y \in I$ and $y \in I$. Then $M_A(x * y) = t = M_A(y)$, $B_A^-(x * y) = \alpha^- = B_A^-(y)$, $B_A^+(x * y) = \alpha^+ = B_A^+(y)$ and $J_A(x * y) = s = J_A(y)$. Thus

$$\begin{aligned} &M_A(x) \geq \min\{M_A(x * y), M_A(y)\} = t, \\ &B_A^-(x) \leq \max\{B_A^-(x * y), B_A^-(y)\} = \alpha^-, \\ &B_A^+(x) \geq \min\{B_A^+(x * y), B_A^+(y)\} = \alpha^+, \\ &J_A(x) \leq \max\{J_A(x * y), J_A(y)\} = s, \end{aligned}$$

and hence $x \in I$. Therefore I is an ideal of X. □

Theorem 3.9. *In a BCK-algebra, every BMBJ-neutrosophic ideal is a BMBJ-neutrosophic subalgebra.*

Proof. Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be a BMBJ-neutrosophic ideal of a BCK-algebra X. Since $(x * y) * x \leq y$ for all $x, y \in X$, it follows from Proposition 3.3 that

$$\begin{aligned} &M_A(x * y) \geq \min\{M_A(x), M_A(y)\}, \\ &B_A^-(x * y) \leq \max\{B_A^-(x), B_A^-(y)\}, \\ &B_A^+(x * y) \geq \min\{B_A^+(x), B_A^+(y)\}, \\ &J_A(x * y) \leq \max\{J_A(x), J_A(y)\} \end{aligned}$$

for all $x, y \in X$. Hence $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic subalgebra of a BCK-algebra X. □

The converse of Theorem 3.9 may not be true as seen in the following example.

Example 3.10. Consider a BCK-algebra $X = \{0, 1, 2, 3\}$ with the binary operation $*$ which is given in Table 3. Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X defined by Table 4. Then $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic subalgebra of X, but it is not a BMBJ-neutrosophic ideal of X since

$$B_A^+(1) \ngeq \min\{B_A^+(1 * 2), B_A^+(2)\}.$$

We provide a condition for a BMBJ-neutrosophic subalgebra to be a BMBJ-neutrosophic ideal in a BCK-algebra.

Table 3: Cayley table for the binary operation “$*$”

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
3	3	3	3	0

Table 4: MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$

X	$M_A(x)$	$\tilde{B}_A(x)$	$J_A(x)$
0	0.7	$[0.03, 0.08]$	0.2
1	0.4	$[0.02, 0.06]$	0.3
2	0.4	$[0.03, 0.08]$	0.4
3	0.6	$[0.02, 0.06]$	0.5

Theorem 3.11. *Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be a BMBJ-neutrosophic subalgebra of a BCK-algebra X satisfying the condition* (3.4)*. Then $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X.*

Proof. For any $x \in X$, we get

$$M_A(0) = M_A(x * x) \geq \min\{M_A(x), M_A(x)\} = M_A(x),$$

$$B_A^-(0) = B_A^-(x * x) \leq \max\{B_A^-(x), B_A^-(x)\} = B_A^-(x),$$

$$B_A^+(0) = B_A^+(x * x) \geq \min\{B_A^+(x), B_A^+(x)\} = B_A^+(x),$$

and

$$J_A(0) = J_A(x * x) \leq \max\{J_A(x), J_A(x)\} = J_A(x).$$

Since $x * (x * y) \leq y$ for all $x, y \in X$, it follows from (3.4) that

$$\begin{aligned} M_A(x) &\geq \min\{M_A(x * y), M_A(y)\}, \\ B_A^-(x) &\leq \max\{B_A^-(x * y), B_A^-(y)\}, \\ B_A^+(x) &\geq \min\{B_A^+(x * y), B_A^+(y)\}, \\ J_A(x) &\leq \max\{J_A(x * y), J_A(y)\} \end{aligned}$$

for all $x, y \in X$. Therefore $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X. □

Theorem 3.9 is not true in a BCI-algebra as seen in the following example.

Example 3.12. Let $(Y, *, 0)$ be a BCI-algebra and let $(\mathbb{Z}, -, 0)$ be an adjoint BCI-algebra of the additive group $(\mathbb{Z}, +, 0)$ of integers. Then $X = Y \times \mathbb{Z}$ is a BCI-algebra and $I = Y \times \mathbb{N}$ is an ideal of X where $\mathbb{N}$ is the set of all non-negative integers (see [3]). Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X which is given in Theorem 3.7. Then $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X by Theorem 3.7. But it is not a BMBJ-neutrosophic subalgebra of X since

$$M_A((0,0) * (0,1)) = M_A((0,-1)) = 0 < t = \min\{M_A((0,0)), M_A(0,1))\},$$

$$B_A^-((0,0) * (0,2)) = B_A^-((0,-2)) = 1 > \alpha^- = \max\{B_A^-((0,0)), B_A^-(0,2))\},$$

$$B_A^+((0,0) * (0,2)) = B_A^+((0,-2)) = 0 < \alpha^+ = \min\{B_A^+((0,0)), B_A^+(0,2))\},$$

and/or

$$J_A((0,0) * (0,3)) = J_A((0,-3)) = 1 > s = \max\{J_A((0,0)), J_A(0,3))\}.$$

Definition 3.13. A BMBJ-neutrosophic ideal $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ of a BCI-algebra X is said to be *closed* if

$$(\forall x \in X)(M_A(0 * x) \geq M_A(x), B_A^-(0 * x) \leq B_A^-(x), B_A^+(0 * x) \geq B_A^+(x), J_A(0 * x) \leq J_A(x)). \tag{3.5}$$

Theorem 3.14. *In a BCI-algebra, every closed BMBJ-neutrosophic ideal is a BMBJ-neutrosophic subalgebra.*

Proof. Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be a closed BMBJ-neutrosophic ideal of a BCI-algebra X. Using (3.2), (2.3), (III) and (3.3), we have

$$M_A(x * y) \geq \min\{M_A((x * y) * x), M_A(x)\} = \min\{M_A(0 * y), M_A(x)\} \geq \min\{M_A(y), M_A(x)\},$$

$$B_A^-(x * y) \leq \max\{B_A^-((x * y) * x), B_A^-(x)\} = \max\{B_A^-(0 * y), B_A^-(x)\} \leq \max\{B_A^-(y), B_A^-(x)\},$$

$$B_A^+(x * y) \geq \min\{B_A^+((x * y) * x), B_A^+(x)\} = \min\{B_A^+(0 * y), B_A^+(x)\} \geq \min\{B_A^+(y), B_A^+(x)\},$$

and

$$J_A(x * y) \leq \max\{J_A((x * y) * x), J_A(x)\} = \max\{J_A(0 * y), J_A(x)\} \leq \max\{J_A(y), J_A(x)\}$$

for all $x, y \in X$. Hence $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic subalgebra of X. □

Theorem 3.15. *In a weakly BCK-algebra, every BMBJ-neutrosophic ideal is closed.*

Proof. Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be a BMBJ-neutrosophic ideal of a weakly BCK-algebra X. For any $x \in X$, we obtain

$$M_A(0 * x) \geq \min\{M_A((0 * x) * x), M_A(x)\} = \min\{M_A(0), M_A(x)\} = M_A(x),$$

$$B_A^-(0*x) \leq \max\{B_A^-((0*x)*x), B_A^-(x)\} = \max\{B_A^-(0), B_A^-(x)\} = B_A^-(x),$$

$$B_A^+(0*x) \geq \min\{B_A^+((0*x)*x), B_A^+(x)\} = \min\{B_A^+(0), B_A^+(x)\} = B_A^+(x),$$

and

$$J_A(0*x) \leq \max\{J_A((0*x)*x), J_A(x)\} = \max\{J_A(0), J_A(x)\} = J_A(x).$$

Therefore $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a closed BMBJ-neutrosophic ideal of X. □

Corollary 3.16. *In a weakly BCK-algebra, every BMBJ-neutrosophic ideal is a BMBJ-neutrosophic subalgebra.*

The following example shows that any BMBJ-neutrosophic subalgebra is not a BMBJ-neutrosophic ideal in a BCI-algebra.

Example 3.17. Consider a BCI-algebra $X = \{0, a, b, c, d, e\}$ with the $*$-operation in Table 5.

Table 5: Cayley table for the binary operation "$*$"

$*$	0	a	b	c	d	e
0	0	0	c	b	c	c
a	a	0	c	b	c	c
b	b	b	0	c	0	0
c	c	c	b	0	b	b
d	d	b	a	c	0	a
e	e	b	a	c	a	0

Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X defined by Table 6.

Table 6: MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$

X	$M_A(x)$	$\tilde{B}_A(x)$	$J_A(x)$
0	0.7	$[0.14, 0.19]$	0.3
a	0.4	$[0.04, 0.45]$	0.6
b	0.7	$[0.14, 0.19]$	0.3
c	0.7	$[0.14, 0.19]$	0.3
d	0.4	$[0.04, 0.45]$	0.6
e	0.4	$[0.04, 0.45]$	0.6

It is routine to verify that $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic subalgebra of X. But it is not a BMBJ-neutrosophic ideal of X since

$$M_A(d) < \min\{M_A(d * c), M_A(c)\},$$

$$B_A^-(d) > \max\{B_A^-(d * c), B_A^-(c)\},$$

$$B_A^+(d) < \min\{B_A^+(d * c), B_A^+(c)\},$$

and/or

$$J_A(d) > \max\{J_A(d * c), J_A(c)\}.$$

Theorem 3.18. *In a p-semisimple BCI-algebra X, the following are equivalent.*

(1) $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ *is a closed BMBJ-neutrosophic ideal of X.*

(2) $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ *is a BMBJ-neutrosophic subalgebra of X.*

Proof. (1) $\Rightarrow$ (2). See Theorem 3.14.

(2) $\Rightarrow$ (1). Suppose that $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic subalgebra of X. For any $x \in X$, we get

$$M_A(0) = M_A(x * x) \geq \min\{M_A(x), M_A(x)\} = M_A(x),$$

$$B_A^-(0) = B_A^-(x * x) \leq \max\{B_A^-(x), B_A^-(x)\} = B_A^-(x),$$

$$B_A^+(0) = B_A^+(x * x) \geq \min\{B_A^+(x), B_A^+(x)\} = B_A^+(x),$$

and

$$J_A(0) = J_A(x * x) \leq \max\{J_A(x), J_A(x)\} = J_A(x).$$

Hence $M_A(0*x) \geq \min\{M_A(0), M_A(x)\} = M_A(x)$, $B_A^-(0*x) \leq \max\{B_A^-(0), B_A^-(x)\} = B_A^-(x)$ $B_A^+(0*x) \geq \min\{B_A^+(0), B_A^+(x)\} = B_A^+(x)$ and $J_A(0 * x) \leq \max\{J_A(0), J_A(x)\} = J_A(x)$ for all $x \in X$. Let $x, y \in X$. Then

$$\begin{aligned} M_A(x) &= M_A(y * (y * x)) \geq \min\{M_A(y), M_A(y * x)\} \\ &= \min\{M_A(y), M_A(0 * (x * y))\} \\ &\geq \min\{M_A(x * y), M_A(y)\}, \end{aligned}$$

$$\begin{aligned} B_A^-(x) &= B_A^-(y * (y * x)) \leq \max\{B_A^-(y), B_A^-(y * x)\} \\ &= \max\{B_A^-(y), B_A^-(0 * (x * y))\} \\ &\leq \max\{B_A^-(x * y), B_A^-(y)\} \end{aligned}$$

$$\begin{aligned} B_A^+(x) &= B_A^+(y*(y*x)) \geq \min\{B_A^+(y), B_A^+(y*x)\} \\ &= \min\{B_A^+(y), B_A^+(0*(x*y))\} \\ &\geq \min\{B_A^+(x*y), B_A^+(y)\} \end{aligned}$$

and

$$\begin{aligned} J_A(x) &= J_A(y*(y*x)) \leq \max\{J_A(y), J_A(y*x)\} \\ &= \max\{J_A(y), J_A(0*(x*y))\} \\ &\leq \max\{J_A(x*y), J_A(y)\}. \end{aligned}$$

Therefore $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a closed BMBJ-neutrosophic ideal of X. □

Since every associative BCI-algebra is p-semisimple, we have the following corollary.

Corollary 3.19. *In an associative BCI-algebra X, the following are equivalent.*

(1) $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ *is a closed BMBJ-neutrosophic ideal of* X.

(2) $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ *is a BMBJ-neutrosophic subalgebra of* X.

Definition 3.20. Let X be an (S)-BCK-algebra. An MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in X is called a *BMBJ-neutrosophic ∘-subalgebra* of X if the following assertions are valid.

$$\begin{aligned} & M_A(x \circ y) \geq \min\{M_A(x), M_A(y)\}, \\ & B_A^-(x \circ y) \leq \max\{B_A^-(x), B_A^-(y)\}, \\ & B_A^+(x \circ y) \geq \min\{B_A^+(x), B_A^+(y)\}, \\ & J_A(x \circ y) \leq \max\{J_A(x), J_A(y)\} \end{aligned} \tag{3.6}$$

for all $x, y \in X$.

Lemma 3.21. *Every BMBJ-neutrosophic ideal of a BCK/BCI-algebra X satisfies the following assertion.*

$$(\forall x, y \in X)\left(x \leq y \;\Rightarrow\; M_A(x) \geq M_A(y), B_A^-(x) \leq B_A^-(y), B_A^+(x) \geq B_A^+(y), J_A(x) \leq J_A(y)\right). \tag{3.7}$$

Proof. Assume that $x \leq y$ for all $x, y \in X$. Then $x * y = 0$, and so

$$M_A(x) \geq \min\{M_A(x*y), M_A(y)\} = \min\{M_A(0), M_A(y)\} = M_A(y),$$

$$B_A^-(x) \leq \max\{B_A^-(x*y), B_A^-(y)\} = \max\{B_A^-(0), B_A^-(y)\} = B_A^-(y),$$

$$B_A^+(x) \geq \min\{B_A^+(x*y), B_A^+(y)\} = \min\{B_A^+(0), B_A^+(y)\} = B_A^+(y),$$

and

$$J_A(x) \leq \max\{J_A(x*y), J_A(y)\} = \max\{J_A(0), J_A(y)\} = J_A(y).$$

This completes the proof. □

Theorem 3.22. *In an (S)-BCK-algebra, every BMBJ-neutrosophic ideal is a BMBJ-neutrosophic $\circ$-subalgebra.*

Proof. Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be a BMBJ-neutrosophic ideal of an (S)-BCK-algebra X. Note that $(x \circ y) * x \leq y$ for all $x, y \in X$. Using Lemma 3.21 and (3.2) inplies that

$$M_A(x \circ y) \geq \min\{M_A((x \circ y) * x), M_A(x)\} \geq \min\{M_A(y), M_A(x)\},$$

$$B_A^-(x \circ y) \leq \max\{B_A^-((x \circ y) * x), B_A^-(x)\} \leq \max\{B_A^-(y), B_A^-(x)\},$$

$$B_A^+(x \circ y) \geq \min\{B_A^+((x \circ y) * x), B_A^+(x)\} \geq \min\{B_A^+(y), B_A^+(x)\},$$

and

$$J_A(x \circ y) \leq \max\{J_A((x \circ y) * x), J_A(x)\} \leq \max\{J_A(y), J_A(x)\}.$$

Therefore $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic $\circ$-subalgebra of X. □

We provide a characterization of a BMBJ-neutrosophic ideal in an (S)-BCK-algebra.

Theorem 3.23. *Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in an (S)-BCK-algebra X. Then $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X if and only if the following assertions are valid.*

$$\begin{array}{l} M_A(x) \geq \min\{M_A(y), M_A(z)\},\ B_A^-(x) \leq \max\{B_A^-(y), B_A^-(z)\}, \\ B_A^+(x) \geq \min\{B_A^+(y), B_A^+(z)\},\ J_A(x) \leq \max\{J_A(y), J_A(z)\} \end{array} \tag{3.8}$$

for all $x, y, z \in X$ with $x \leq y \circ z$.

Proof. Assume that $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X and let $x, y, z \in X$ be such that $x \leq y \circ z$. Using (3.1), (3.2) and Theorem 3.22, we have

$$\begin{aligned} M_A(x) &\geq \min\{M_A(x * (y \circ z)), M_A(y \circ z)\} \\ &= \min\{M_A(0), M_A(y \circ z)\} \\ &= M_A(y \circ z) \geq \min\{M_A(y), M_A(z)\}, \end{aligned}$$

$$\begin{aligned} B_A^-(x) &\leq \max\{B_A^-(x * (y \circ z)), B_A^-(y \circ z)\} \\ &= \max\{B_A^-(0), B_A^-(y \circ z)\} \\ &= B_A^-(y \circ z) \leq \max\{B_A^-(y), B_A^-(z)\}, \end{aligned}$$

$$\begin{aligned} B_A^+(x) &\geq \min\{B_A^+(x * (y \circ z)), B_A^+(y \circ z)\} \\ &= \min\{B_A^+(0), B_A^+(y \circ z)\} \\ &= B_A^+(y \circ z) \geq \min\{B_A^+(y), B_A^+(z)\}, \end{aligned}$$

and

$$\begin{aligned} J_A(x) &\leq \max\{J_A(x*(y\circ z)), J_A(y\circ z)\} \\ &= \max\{J_A(0), J_A(y\circ z)\} \\ &= J_A(y\circ z) \leq \max\{J_A(y), J_A(z)\}. \end{aligned}$$

Conversely, let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in an (S)-BCK-algebra X satisfying the condition (3.8) for all $x, y, z \in X$ with $x \leq y \circ z$. Sine $0 \leq x \circ x$ for all $x \in X$, it follows from (3.8) that

$$M_A(0) \geq \min\{M_A(x), M_A(x)\} = M_A(x),$$

$$B_A^-(0) \leq \max\{B_A^-(x), B_A^-(x)\} = B_A^-(x),$$

$$B_A^+(0) \geq \min\{B_A^+(x), B_A^+(x)\} = B_A^+(x),$$

and

$$J_A(0) \leq \max\{J_A(x), J_A(x)\} = J_A(x).$$

Note that $x \leq (x*y)\circ y$ for all $x, y \in X$. Hence we have

$$M_A(x) \geq \min\{M_A(x*y), M_A(y)\},\ B_A^-(x) \leq \max\{B_A^-(x*y), B_A^-(y)\},$$
$$B_A^+(x) \geq \min\{B_A^+(x*y), B_A^+(y)\} \text{ and } J_A(x) \leq \max\{J_A(x*y), J_A(y)\}.$$

Therefore $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a BMBJ-neutrosophic ideal of X. □

4 Conclusions

As a generalization of neutrosophic set, Mohseni et al. [9] have introduced the notion of MBJ-neutrosophic sets, and have applied it to BCK/BCI-algebras. BMBJ-neutrosophic set has been introduced in [1] with an application in BCK/BCI-algebras. In this article, we have applied the notion of MBJ-neutrosophic sets to ideals of BCK/BI-algebras. We have introduced the concepts of a BMBJ-neutrosophic ∘-subalgebra and a (closed) BMBJ-neutrosophic ideal, and have investigated several properties. We have provided conditions for an MBJ-neutrosophic set to be a BMBJ-neutrosophic ideal in BCK/BCI-algebras, and have discussed characterizations of BMBJ-neutrosophic ideal. We have considered relations between a BMBJ-neutrosophic subalgebra, a BMBJ-neutrosophic ∘-subalgebra and a (closed) BMBJ-neutrosophic ideal. Using the results and ideas in this paper, our future work will focus on the study of several algebraic structures and substructures.

Acknowledgments

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