Bipolar Neutrosophic Sets and Their Application Based on Multi-Criteria Decision Making Problems

Irfan Deli, Mumtaz Ali and Florentin Smarandache

Abstract—In this paper, we introduce concept of bipolar neutrosophic set and its some operations. Also, we propose score, certainty and accuracy functions to compare the bipolar neutrosophic sets. Then, we develop the bipolar neutrosophic weighted average operator (\(A_w\)) and bipolar neutrosophic weighted geometric operator (\(G_w\)) to aggregate the bipolar neutrosophic information. Furthermore, based on the (\(A_w\)) and (\(G_w\)) operators and the score, certainty and accuracy functions, we develop a bipolar neutrosophic multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of bipolar neutrosophic numbers to select the most desirable one(s). Finally, a numerical example of the method was given to demonstrate the application and effectiveness of the developed method.

Index Terms—Neutrosophic set, bipolar neutrosophic set, average operator, geometric operator, score, certainty and accuracy functions, multi-criteria decision making.

I. INTRODUCTION

To handle with imprecision and uncertainty, concept of fuzzy sets and intuitionistic fuzzy sets originally introduced by Zadeh [26] and Atanassov [1], respectively. Then, Smarandache [17] proposed concept of neutrosophic set which is generalization of fuzzy set theory and intuitionistic fuzzy sets. These sets models have been studied by many authors; on application [4-6,10-12,15,16], theory [18-20,21-25,27,28], and so on.

Bosc and Pivert [2] said that “Bipolarity refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. Positive information states what is possible, satisfactory, permitted, desired, or considered as being acceptable. On the other hand, negative statements express what is impossible, rejected, or forbidden.

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Negative preferences correspond to constraints, since they specify which values or objects have to be rejected (i.e., those that do not satisfy the constraints), while positive preferences correspond to wishes, as they specify which objects are more desirable than others (i.e., satisfy user wishes) without rejecting those that do not meet the wishes.” Therefore, Lee [8,9] introduced the concept of bipolar fuzzy sets which is an generalization of the fuzzy sets. Recently, bipolar fuzzy models have been studied by many authors on algebraic structures such as; Chen et. al. [3] studied of \(m\)-polar fuzzy set and illustrates how many concepts have been defined based on bipolar fuzzy sets. Then, they examined many results which are related to these concepts can be generalized to the case of \(m\)-polar fuzzy sets. They also proposed numerical examples to show how to apply \(m\)-polar fuzzy sets in real world problems. Bosc and Pivert [2] introduced a study is called bipolar fuzzy relations where each tuple is associated with a pair of satisfaction degrees. Manemaran and Chellappa [14] gave some applications of bipolar fuzzy sets in groups are called the bipolar fuzzy groups, fuzzy d-ideals of groups under (T-S) norm. They investigate some related properties of the groups and introduced relations between a bipolar fuzzy group and bipolar fuzzy d-ideals. Majumder [13] proposed bipolar valued fuzzy subsemigroup, bipolar valued fuzzy bi-ideal, bipolar valued fuzzy (1,2)- ideal and bipolar valued fuzzy ideal. Zhou and Li [29] introduced a new framework of bipolar fuzzy subsemirings and bipolar fuzzy ideals which is a generalization of fuzzy subsemirings and bipolar fuzzy ideals in semirings and bipolar fuzzy ideals, respectively, and related properties are examined by the authors.

In this paper, we introduced the concept of bipolar neutrosophic sets which is an extension of the fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets. Also, we give some operations and operators on the bipolar neutrosophic sets. The operations and operators generalize the operations and operators of fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets which have been previously proposed. Therefore, in section 2, we introduce concept of bipolar neutrosophic set and its some operations including the score, certainty and accuracy functions to compare the bipolar neutrosophic sets. In the same section, we also develop the bipolar neutrosophic weighted average operator (\(A_w\)) and bipolar neutrosophic weighted geometric operator (\(G_w\))
operator to aggregate the bipolar neutrosophic information. In section 3, based on the \((A_w)\) and \((G_w)\) operators and the score, certainty and accuracy functions, we develop a bipolar neutrosophic multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of bipolar neutrosophic numbers to select the most desirable one(s) and give a numerical example of the to demonstrate the application and effectiveness of the developed method. In last section, we conclude the paper.

II. BASIC AND FUNDAMENTAL CONCEPTS

In this section, we give some concepts related to neutrosophic sets and bipolar sets.

**Definition 2.1.** [17] Let \(X\) be a universe of discourse. Then a neutrosophic set is defined as:
\[
A = \{(x, F_A(x), T_A(x), I_A(x)) : x \in X\},
\]
which is characterized by a truth-membership function \(T_A : X \to \{0^+, 1^+\}\), an indeterminacy-membership function \(I_A : X \to \{0^-, 1^-\}\) and a falsity-membership function \(F_A : X \to \{0^+, 1^+\}\). There is no restriction on the sum of \(T_A(x), I_A(x)\) and \(F_A(x)\), so \(0^+ \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+\). For application in real scientific and engineering areas, Wang et al. [18] proposed the concept of an single valued neutrosophic set as follows:

**Definition 2.1.** [18] Let \(X\) be a universe of discourse. Then a single valued neutrosophic set is defined as:
\[
A_{NS} = \{(x, F_A(x), T_A(x), I_A(x)) : x \in X\},
\]
which is characterized by a truth-membership function \(T_A : X \to [0, 1]\), an indeterminacy-membership function \(I_A : X \to [0, 1]\) and a falsity-membership function \(F_A : X \to [0, 1]\). There is not restriction on the sum of \(T_A(x), I_A(x)\) and \(F_A(x)\), so \(0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3\).

Set-theoretic operations, for two single valued neutrosophic set:
\[
\begin{align*}
A_{NS} \cap B_{NS} &= \{x, F_A(x), T_A(x), I_A(x), F_B(x) : x \in X\} \\
A_{NS} \cup B_{NS} &= \{x, \min\{T_A(x), T_B(x)\}, \max\{I_A(x), I_B(x)\}, \min\{F_A(x), F_B(x)\} : x \in X\}
\end{align*}
\]

A single valued neutrosophic number is denoted by \(A = \{T, I, F\}\) for convenience.

**Definition 2.2.** [15] Let \(A_1 = (T_1, I_1, F_1)\) and \(A_2 = (T_2, I_2, F_2)\) be two single valued neutrosophic number. Then, the operations for NNs are defined as below:
\[
\begin{align*}
&i. \quad A_1 \land A_2 = (T_1 \land F_1, I_1 \land I_2, T_2 \land F_2), \\
&ii. \quad A_1 \lor A_2 = (T_1 \lor F_1, I_1 \lor I_2, T_2 \lor F_2), \\
&iii. \quad A_1 = A_2 = (T_1 = F_1, I_1 = I_2, T_2 = F_2), \\
&iv. \quad 0 < \lambda < 1 \\
\end{align*}
\]

**Definition 2.3.** [15] Let \(A_1 = (T_1, I_1, F_1)\) be a single valued neutrosophic number. Then, the score function \(s(A_1)\), accuracy function \(a(A_1)\) and certainty function \(c(A_1)\) of an SNN are defined as follows:
\[
\begin{align*}
i. & \quad s(A_1) = (T_1) + 1 - (1 - I_1)^2, \\
&ii. \quad a(A_1) = T_1 - F_1, \\
&iii. \quad c(A_1) = T_1
\end{align*}
\]

**Definition 2.4.** [15] Let \(A_1 = (T_1, I_1, F_1)\) and \(A_2 = (T_2, I_2, F_2)\) be two single valued neutrosophic number. The comparison method can be defined as follows:
\[
\begin{align*}
i. & \quad s(A_1) > s(A_2), \text{then } A_1 \text{ is greater than } A_2, \\
&ii. \quad a(A_1) > a(A_2), \text{then } A_1 \text{ is greater than } A_2, \\
&iii. & \quad c(A_1) > c(A_2), \text{then } A_1 \text{ is greater than } A_2
\end{align*}
\]

**III. BIPOLAR NEUTROSOPHIC SET**

In this section, we introduce concept of bipolar neutrosophic set and its some operations including the score, certainty and accuracy functions to compare the bipolar neutrosophic sets. We also develop the bipolar neutrosophic weighted average operator \((A_w)\) and bipolar neutrosophic weighted geometric operator \((G_w)\) operator to aggregate the bipolar neutrosophic information. Some of it is quoted from [2,6,8,9,14,17,18,20,24,26].
**Definition 3.1.** A bipolar neutrosophic set \( A \) in \( X \) is defined as an object of the form
\[
A = \{(x, T^+(x), I^-(x), F^-(x), T^-(x), I^+(x), F^+(x)) : x \in X\},
\]
where \( T^+, I^-, F^- : X \rightarrow [0,1] \) and \( T^-, I^+, F^+ : X \rightarrow [-1,0] \).

The positive membership degree \( T^+(x), I^-(x), F^-(x) \) denotes the truth membership, indeterminate membership and false membership of an element \( x \in X \) corresponding to a bipolar neutrosophic set \( A \) and the negative membership degree \( T^-(x), I^+(x), F^+(x) \) denotes the truth membership, indeterminate membership and false membership of an element \( x \in X \) to some implicit counter-property corresponding to a bipolar neutrosophic set \( A \).

**Example 3.2.** Let \( X = \{x_1, x_2, x_3\} \). Then
\[
A = \begin{cases}
{x_1, 0.5, 0.3, 0.1, -0.6, -0.4, -0.01}, \\
{x_2, 0.3, 0.2, 0.7, -0.02, -0.003, -0.5}, \\
{x_3, 0.8, 0.05, 0.4, -0.1, -0.5, -0.06}
\end{cases}
\]
is a bipolar neutrosophic subset of \( X \).

**Theorem 3.4.** A bipolar neutrosophic set is the generalization of a bipolar fuzzy set.

**Proof:** Suppose that \( X \) is a bipolar neutrosophic set. Then by setting the positive components \( T^+, I^-, F^- \) equal to zero as well as the negative components \( T^-, I^+, F^+ \) equals to zero reduces the bipolar neutrosophic set to bipolar fuzzy set.

**Definition 3.5.** Let \( A_1 = (x, T^+_1(x), I^-_1(x), F^-_1(x), T^-_1(x), I^+_1(x), F^+_1(x)) \) and \( A_2 = (x, T^+_2(x), I^-_2(x), F^-_2(x), T^-_2(x), I^+_2(x), F^+_2(x)) \) be two bipolar neutrosophic sets. Then \( A_1 \subseteq A_2 \) if and only if
\[
T^+_1(x) \leq T^+_2(x), \quad I^-_1(x) \leq I^-_2(x), \quad F^-_1(x) \leq F^-_2(x),
\]
and
\[
T^-_1(x) \geq T^-_2(x), \quad I^+_1(x) \geq I^+_2(x), \quad F^+_1(x) \leq F^+_2(x)
\]
for all \( x \in X \).

**Definition 3.6.** Let \( A_1 = (x, T^+_1(x), I^-_1(x), F^-_1(x), T^-_1(x), I^+_1(x), F^+_1(x)) \) and \( A_2 = (x, T^+_2(x), I^-_2(x), F^-_2(x), T^-_2(x), I^+_2(x), F^+_2(x)) \) be two bipolar neutrosophic set. Then \( A_1 = A_2 \) if and only if
\[
T^+_1(x) = T^+_2(x), \quad I^-_1(x) = I^-_2(x), \quad F^-_1(x) = F^-_2(x),
\]
and
\[
T^-_1(x) = T^-_2(x), \quad I^+_1(x) = I^+_2(x), \quad F^+_1(x) = F^+_2(x)
\]
for all \( x \in X \).

**Definition 3.7.** Let \( A_1 = (x, T^+_1(x), I^-_1(x), F^-_1(x), T^-_1(x), I^+_1(x), F^+_1(x)) \) and \( A_2 = (x, T^+_2(x), I^-_2(x), F^-_2(x), T^-_2(x), I^+_2(x), F^+_2(x)) \) be two bipolar neutrosophic set. Then their union is defined as:
\[
(A_1 \cup A_2)(x) = \begin{cases}
\max(T^+_1(x), T^+_2(x), I^-_1(x), I^-_2(x), F^-_1(x), F^-_2(x)), \\
\min(T^-_1(x), T^-_2(x), I^+_1(x), I^+_2(x), F^+_1(x), F^+_2(x))
\end{cases}
\]
for all \( x \in X \).

**Example 3.8.** Let \( X = \{x_1, x_2, x_3\} \). Then
\[
A_1 = \begin{cases}
{x_1, 0.5, 0.3, 0.1, -0.6, -0.4, -0.01}, \\
{x_2, 0.3, 0.2, 0.7, -0.02, -0.003, -0.5}, \\
{x_3, 0.8, 0.05, 0.4, -0.1, -0.5, -0.06}
\end{cases}
\]
and
\[
A_2 = \begin{cases}
{x_1, 0.4, 0.6, 0.3, -0.3, -0.5, -0.1}, \\
{x_2, 0.5, 0.1, 0.4, -0.2, -0.3, -0.3}, \\
{x_3, 0.2, 0.5, 0.6, -0.4, -0.6, -0.7}
\end{cases}
\]
are two bipolar neutrosophic sets in \( X \).

Then their union is given as follows:
\[
A_1 \cup A_2 = \begin{cases}
{x_1, 0.5, 0.45, 0.1, -0.6, -0.5, -0.1}, \\
{x_2, 0.5, 0.15, 0.7, -0.2, -0.15, 0.5}, \\
{x_3, 0.8, 0.47, 0.6, -0.4, -0.55, 0.7}
\end{cases}
\]

**Definition 3.9.** Let \( A_1 = (x, T^+_1(x), I^-_1(x), F^-_1(x), T^-_1(x), I^+_1(x), F^+_1(x)) \) and \( A_2 = (x, T^+_2(x), I^-_2(x), F^-_2(x), T^-_2(x), I^+_2(x), F^+_2(x)) \) be two bipolar neutrosophic set. Then their intersection is defined as:
\[
(A_1 \cap A_2)(x) = \begin{cases}
\min(T^+_1(x), T^+_2(x), I^-_1(x), I^-_2(x), F^-_1(x), F^-_2(x)), \\
\max(T^-_1(x), T^-_2(x), I^+_1(x), I^+_2(x), F^+_1(x), F^+_2(x))
\end{cases}
\]
for all \( x \in X \).

**Definition 3.10.** Let \( A = \{(x, T^+(x), I^-(x), F^-(x), T^-(x), I^+(x), F^+(x)) : x \in X\} \) be a bipolar neutrosophic set in \( X \). Then the complement of \( A \) is denoted by \( A' \) and is defined by
\[
T^+_A(x) = \{1\} - T^+_A(x), \quad I^-_A(x) = \{1\} - I^-_A(x), \quad F^-_A(x) = \{1\} - F^-_A(x)
\]
and
\[
T^-_A(x) = \{1\} - T^-_A(x), \quad I^+_A(x) = \{1\} - I^+_A(x), \quad F^+_A(x) = \{1\} - F^+_A(x)
\]
for all \( x \in X \).

**Example 3.11.** Let \( X = \{x_1, x_2, x_3\} \). Then
\[
A = \begin{cases}
{x_1, 0.5, 0.3, 0.1, -0.6, -0.4, -0.01}, \\
{x_2, 0.3, 0.2, 0.7, -0.02, -0.003, -0.5}, \\
{x_3, 0.8, 0.05, 0.4, -0.1, -0.5, -0.06}
\end{cases}
\]
is a bipolar neutrosophic number in $X$. Then the complement of $A$ is given as follows:

$$A^c = \left\{ \left( x_1, 0.5, 0.7, 0.9, -0.4, -0.6, -0.99 \right), \left( x_2, 0.7, 0.8, 0.3, -0.08, -0.997, -0.5 \right), \left( x_3, 0.2, 0.95, 0.6, -0.9, -0.5, -0.94 \right) \right\}.$$

We will denote the set of all the bipolar neutrosophic sets (BNSs) in $X$ by $Q$. A bipolar neutrosophic number (BN) is denoted by $\tilde{a} = (T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^-)$ for convenience.

**Definition 3.12.** Let $\tilde{a}_1 = (T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^-)$ and $\tilde{a}_2 = (T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^-)$ be two bipolar neutrosophic numbers. Then the operations for NNSs are defined as below:

i. $\lambda \tilde{a}_1 = \left( 1 - (1 - T_1^+) \lambda, (T_1^+) \lambda, (F_1^+) \lambda \right), (-1 - (1 - F_1^+) \lambda), (1 - (1 - F_1^+) \lambda))$

ii. $\tilde{a}_1 \pm \tilde{a}_2 = (T_1^+ T_2^-, T_1^+ T_2^+, T_1^+ I_2^+, T_1^+ F_2^+, T_1^- T_2^-, T_1^- I_2^-, T_1^- F_2^-)$

iii. $\tilde{a}_1 \cdot \tilde{a}_2 = (T_1^+ T_2^+, I_1^+ I_2^-, F_1^+ F_2^-, T_1^- T_2^-, I_1^- I_2^-, F_1^- F_2^-)$

where $\lambda > 0$.

**Definition 3.14.** Let $\tilde{a}_1 = (T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^-)$ be a bipolar neutrosophic number. Then, the score function $s(\tilde{a}_1)$, accuracy function $\alpha(\tilde{a}_1)$, and certainty function $\xi(\tilde{a}_1)$ of a NBN are defined as follows:

i. $s(\tilde{a}_1) = (T_1^+ + 1 - I_1^+ + 1 - F_1^+ + 1 + T_1^- - Y_1^- - F_1^-)$

ii. $\alpha(\tilde{a}_1) = T_1^+ + F_1^+ + T_1^- - F_1^-$

iii. $\xi(\tilde{a}_1) = T_1^+ - F_1^-$

**Definition 3.15.** Let $\tilde{a}_1 = (T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^-)$ and $\tilde{a}_2 = (T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^-)$ be two bipolar neutrosophic numbers. The comparison method can be defined as follows:

i. if $s(\tilde{a}_1) > s(\tilde{a}_2)$, then $\tilde{a}_1$ is greater than $\tilde{a}_2$, that is, $\tilde{a}_1$ is superior to $\tilde{a}_2$, denoted by $\tilde{a}_1 \succ \tilde{a}_2$.

ii. if $s(\tilde{a}_1) = s(\tilde{a}_2)$ and $\alpha(\tilde{a}_1) > \alpha(\tilde{a}_2)$, then $\tilde{a}_1$ is greater than $\tilde{a}_2$, that is, $\tilde{a}_1$ is superior to $\tilde{a}_2$, denoted by $\tilde{a}_1 \succ \tilde{a}_2$.

iii. if $s(\tilde{a}_1) = s(\tilde{a}_2)$, $\alpha(\tilde{a}_1) = \alpha(\tilde{a}_2)$ and $\xi(\tilde{a}_1) > \xi(\tilde{a}_2)$, then $\tilde{a}_1$ is greater than $\tilde{a}_2$, that is, $\tilde{a}_1$ is superior to $\tilde{a}_2$, denoted by $\tilde{a}_1 \succ \tilde{a}_2$.

iv. if $s(\tilde{a}_1) = s(\tilde{a}_2)$, $\alpha(\tilde{a}_1) = \alpha(\tilde{a}_2)$ and $\xi(\tilde{a}_1) = \xi(\tilde{a}_2)$, then $\tilde{a}_1$ is equal to $\tilde{a}_2$, that is, $\tilde{a}_1$ is indifferent to $\tilde{a}_2$, denoted by $\tilde{a}_1 \equiv \tilde{a}_2$.

Based on the study given in [15,20] we define some weighted aggregation operators related to bipolar neutrosophic sets as follows:

**Definition 3.16.** Let $\tilde{a}_j = (T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^-)$ ($j = 1, 2, ..., n$) be a family of bipolar neutrosophic numbers. A mapping $A_{\omega}: Q_n \rightarrow Q$ is called bipolar neutrosophic weighted average operator if it satisfies

$$A_{\omega}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} \omega_j \tilde{a}_j = \left( 1 - \prod_{j=1}^{n} (1 - T_j^+)^{\omega_j}, \prod_{j=1}^{n} T_j^{+\omega_j}, \prod_{j=1}^{n} F_j^{+\omega_j}, \prod_{j=1}^{n} (-T_j^-)^{\omega_j}, -1 \right)$$

where $\omega_j$ is the weight of $\tilde{a}_j$ ($j = 1,2, ..., n$), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Based on the study given in [15,20] we give the theorem related to bipolar neutrosophic sets as follows:

**Theorem 3.17.** Let $\tilde{a}_j = (T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^-)$ ($j = 1, 2, ..., n$) be a family of bipolar neutrosophic numbers. A mapping $G_{\omega}: Q_n \rightarrow Q$ is called bipolar neutrosophic weighted geometric operator if it satisfies

$$G_{\omega}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_j = \left( (1 - T_j^+)^{\omega_j}, \prod_{j=1}^{n} T_j^{+\omega_j}, \prod_{j=1}^{n} F_j^{+\omega_j}, \prod_{j=1}^{n} (-T_j^-)^{\omega_j}, -1 \right)$$

where $\omega_j$ is the weight of $\tilde{a}_j$ ($j = 1,2, ..., n$), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Based on the study given in [15,20] we define some weighted aggregation operators related to bipolar neutrosophic sets as follows:

**Definition 3.18.** Let $\tilde{a}_j = (T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^-)$ ($j = 1, 2, ..., n$) be a family of bipolar neutrosophic numbers. A mapping $G_{\omega}: Q_n \rightarrow Q$ is called bipolar neutrosophic weighted geometric operator if it satisfies

$$G_{\omega}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_j = \left( (1 - T_j^+)^{\omega_j}, \prod_{j=1}^{n} T_j^{+\omega_j}, \prod_{j=1}^{n} F_j^{+\omega_j}, \prod_{j=1}^{n} (-T_j^-)^{\omega_j}, -1 \right)$$

where $\omega_j$ is the weight of $\tilde{a}_j$ ($j = 1,2, ..., n$), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Based on the study given in [15,20] we give the theorem related to bipolar neutrosophic sets as follows:

**Theorem 3.19.** Let $\tilde{a}_j = (T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^-)$ ($j = 1, 2, ..., n$) be a family of bipolar neutrosophic numbers. Then,
Step 1. Construct the decision matrix provided by the decision maker as:

\[
\begin{array}{cccc}
& C_1 & C_2 & C_3 & C_4 \\
A_1 & (0.5,0.7,0.2,-0.7,-0.3,-0.6) & (0.4,0.4,0.5,-0.7,-0.8,-0.4) & (0.7,0.7,0.5,-0.8,-0.7,-0.6) & (0.1,0.5,0.7,-0.5,-0.2,-0.8) \\
A_2 & (0.9,0.7,0.5,-0.7,-0.7,-0.1) & (0.7,0.6,0.8,-0.7,-0.5,-0.1) & (0.9,0.4,0.6,-0.1,-0.7,-0.5) & (0.5,0.2,0.7,-0.5,-0.1,-0.9) \\
A_3 & (0.3,0.4,0.2,-0.6,-0.3,-0.7) & (0.2,0.2,0.2,-0.4,-0.7,-0.4) & (0.9,0.5,0.5,-0.6,-0.5,-0.2) & (0.7,0.5,0.3,-0.4,-0.2,-0.2) \\
A_4 & (0.9,0.7,0.2,-0.8,-0.6,-0.1) & (0.3,0.5,0.2,-0.5,-0.5,-0.2) & (0.5,0.4,0.5,-0.1,-0.7,-0.2) & (0.4,0.2,0.8,-0.5,-0.5,-0.6) \\
\end{array}
\]

Step 2. Compute \( \tilde{a}_i = A_w(\tilde{a}_{i1}, \tilde{a}_{i2}, \ldots, \tilde{a}_{im}) \) for each \( i = 1,2,3,4 \) as:

\[
\tilde{a}_1 = (0.471,0.583,0.329,-0.682,-0.531,-0.594) \\
\tilde{a}_2 = (0.839,0.536,0.600,-0.526,-0.608,-0.364) \\
\tilde{a}_3 = (0.489,0.355,0.235,-0.515,-0.447,-0.544) \\
\tilde{a}_4 = (0.751,0.513,0.266,-0.517,-0.580,-0.221).
\]

Step 3. Calculate the score values of \( s(\tilde{a}_i) \) \( (i = 1,2,3,4) \) for the collective overall bipolar neutrosophic number of \( \tilde{a}_i \) \( (i = 1,2,3,4, \ldots, m) \) as:

\[
s(\tilde{a}_1) = 0.50 \\
s(\tilde{a}_2) = 0.52 \\
s(\tilde{a}_3) = 0.56 \\
s(\tilde{a}_4) = 0.54.
\]

Step 4. Rank all the software systems of \( \tilde{a}_i \) \( (i = 1,2,3,4) \) according to the score values as:

\( A_2 > A_4 > A_2 > A_1 \)

and thus \( A_3 \) is the most desirable alternative.

V. CONCLUSION

This paper presented a bipolar neutrosophic set and its score, certainty and accuracy functions. Then, the \( A_w \) and \( G_w \) operators were proposed to aggregate the bipolar neutrosophic information. Furthermore, based on the \( A_w \) and \( G_w \) operators and the score, certainty and accuracy functions, we have developed a bipolar neutrosophic multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of bipolar neutrosophic numbers. The \( A_w \) and \( G_w \) operators are utilized available. The customer takes into account four attributes to evaluate the alternatives: \( C_1 = \text{Fuel economy}; C_2 = \text{Aerod}; C_3 = \text{Comfort}; C_4 = \text{Safety} \) and use the bipolar neutrosophic values to evaluate the four possible alternatives \( A_1(1 = 1,2,3,4) \) under the above four attributes. Also, the weight vector of the attributes \( C_j(j = 1,2,3,4) \) is \( \omega = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})^T \).

Algorithm

Step 1. Construct the decision matrix provided by the customer as:

\[
\begin{array}{cccc}
& C_1 & C_2 & C_3 & C_4 \\
A_1 & (0.5,0.7,0.2,-0.7,-0.3,-0.6) & (0.4,0.4,0.5,-0.7,-0.8,-0.4) & (0.7,0.7,0.5,-0.8,-0.7,-0.6) & (0.1,0.5,0.7,-0.5,-0.2,-0.8) \\
A_2 & (0.9,0.7,0.5,-0.7,-0.7,-0.1) & (0.7,0.6,0.8,-0.7,-0.5,-0.1) & (0.9,0.4,0.6,-0.1,-0.7,-0.5) & (0.5,0.2,0.7,-0.5,-0.1,-0.9) \\
A_3 & (0.3,0.4,0.2,-0.6,-0.3,-0.7) & (0.2,0.2,0.2,-0.4,-0.7,-0.4) & (0.9,0.5,0.5,-0.6,-0.5,-0.2) & (0.7,0.5,0.3,-0.4,-0.2,-0.2) \\
A_4 & (0.9,0.7,0.2,-0.8,-0.6,-0.1) & (0.3,0.5,0.2,-0.5,-0.5,-0.2) & (0.5,0.4,0.5,-0.1,-0.7,-0.2) & (0.4,0.2,0.8,-0.5,-0.5,-0.6) \\
\end{array}
\]
to aggregate the bipolar neutrosophic information corresponding to each alternative to obtain the collective overall values of the alternatives, and then the alternatives are ranked according to the values of the score, certainty and accuracy functions to select the most desirable one(s). Finally, a numerical example of the method was given to demonstrate the application and effectiveness of the developed method.

REFERENCES