Certain Notions of Single-Valued Neutrosophic K-Algebras

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Abstract

We apply the notion of single-valued neutrosophic sets to K-algebras. We develop the concept of single-valued neutrosophic K-subalgebras, and present some of their properties. Moreover, we study the behavior of single-valued neutrosophic K-subalgebras under homomorphism. Finally, we discuss $(\in, \in \vee q)$-single-valued neutrosophic K-algebras.

Keywords: Single-valued neutrosophic sets, K-algebras, homomorphism, $(\in, \in \vee q)$-single-valued neutrosophic K-algebras.

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1 Introduction

A new kind of logical algebra, known as K-algebra, was introduced by Dar and Akram [9]. A K-algebra was built on a group G by adjoining the induced binary operation on G. The group G is particularly of the type in which each non-identity element is not of order 2. This algebraic structure is, in general, non-commutative and non-associative with right identity element [5,10,11]. Akram et.al [2–4] introduced fuzzy K-algebras. They then developed fuzzy K-algebras with other researchers worldwide. The concepts and results of K-algebras have been broadened to the fuzzy setting frames by applying Zadeh's fuzzy set theory and its generalizations, namely, interval- valued fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, bipolar fuzzy sets and vague sets.

In handling information regarding various aspects of uncertainty, non-classical logic (a great extension and development of classical logic) is considered to be a more powerful technique than the classical logic. The non-classical logic has nowadays become a useful tool in computer science. Moreover, non-classical logic deals with fuzzy information and uncertainty. In 1998, Smarandache [15] introduced neutrosophic sets as a generalization of fuzzy sets [19] and intuitionistic fuzzy sets [6]. A neutrosophic set is identified by three functions called truth-membership (T), indeterminacy-membership (I) and falsity-membership (F) whose values are real standard or non-standard subset of unit interval $]^{-}0, 1^{+}[$, where $^{-}0 = 0 - \epsilon$, $1^{+} = 1 + \epsilon$, ϵ is an infinitesimal number. To apply neutrosophic set in real-life problems more conveniently, Smarandache [15] and Wang et al. [16] defined single-valued neutrosophic sets which takes the value from the subset of [0, 1]. Thus, a single-valued neutrosophic set is an instance of neutrosophic set, and can be used expediently to deal with real-world problems, especially in decision support. Algebraic structures have a vital place with vast applications in various disciplines. Neutrosophic set theory has been applied to algebraic structures [1,8,13]. In this research article, we introduce the notion of single-valued neutrosophic K-subalgebra and investigate some of their properties. We discuss K-subalgebra in terms of level sets using neutrosophic environment. We study the homomorphisms between the single-valued neutrosophic K-subalgebras. We discuss characteristic K-subalgebras and fully invariant K-subalgebras. Finally, we discuss $(\in, \in \vee q)$-single-valued neutrosophic K-algebras.

2 Single-Valued Neutrosophic K-algebras

The concept of K-algebra was developed by Dar and Akram in [14].

Definition 2.1. Let $(G, \cdot, e)$ be a group in which each non-identity element is not of order 2. Then a K- *algebra* is a structure $\mathcal{K} = (G, \cdot, \odot, e)$ on a group G in which induced binary operation $\odot : G \times G \to G$ is defined by $\odot(x, y) = x \odot y = x.y^{-1}$ and satisfies the following axioms:

(i) $(x \odot y) \odot (x \odot z) = (x \odot ((e \odot z) \odot (e \odot y))) \odot x$,

(ii) $x \odot (x \odot y) = (x \odot (e \odot y)) \odot x$,

(iii) $(x \odot x) = e$,

(iv) $(x \odot e) = x$,

(v) $(e \odot x) = x^{-1}$,

for all x, y, $z \in G$.

Definition 2.2. [16] Let Z be a space of objects with a general element $z \in Z$. A single-valued neutrosophic set $\mathcal{A}$ in Z is characterized by three membership functions, $\mathcal{T}_{\mathcal{A}}$-truth membership function, $\mathcal{I}_{\mathcal{A}}$-indeterminacy membership function and $\mathcal{F}_{\mathcal{A}}$-falsity membership function, where $\mathcal{T}_{\mathcal{A}}(z), \mathcal{I}_{\mathcal{A}}(z), \mathcal{F}_{\mathcal{A}}(z) \in [0, 1]$, for all $z \in Z$. That is $\mathcal{T}_{\mathcal{A}} : Z \to [0, 1], \mathcal{I}_{\mathcal{A}} : Z \to [0, 1], \mathcal{F}_{\mathcal{A}} : Z \to [0, 1]$ with no restriction on the sum of these three components. $\mathcal{A}$ can also be written as $\mathcal{A} = \{< z, \mathcal{T}_{\mathcal{A}}(z), \mathcal{I}_{\mathcal{A}}(z), \mathcal{F}_{\mathcal{A}}(z) > \ | \ z \in Z\}$.

Definition 2.3. A single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in a K-algebra $\mathcal{K}$ is called a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if it satisfy the following conditions:

$$
\begin{aligned}
&(a)\ \mathcal{T}_{\mathcal{A}}(s \odot t) \ \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},\\
&(b)\ \mathcal{I}_{\mathcal{A}}(s \odot t) \ \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},\\
&(c)\ \mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}, \text{ for all } s, t \in G.
\end{aligned}
$$

Note that $\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s)$, $\mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G$.

Example 2.1. Consider $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given as:

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8
e	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^8	x^7	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^8	x^7	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^8	x^7	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^8	x^7	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e	x^8	x^7
x^7	x^7	x^6	x^5	x^4	x^3	x^2	x	e	x^8
x^8	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x	e

We define a single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in K-algebra as follows:
$\mathcal{T}_{\mathcal{A}}(e) = 0.8, \mathcal{I}_{\mathcal{A}}(e) = 0.7, \mathcal{F}_{\mathcal{A}}(e) = 0.4$,
$\mathcal{T}_{\mathcal{A}}(s) = 0.2, \mathcal{I}_{\mathcal{A}}(s) = 0.3, \mathcal{F}_{\mathcal{A}}(s) = 0.6$, for all $s \neq e \in G$.
Clearly, $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Example 2.2. Consider $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra on dihedral group $D4$ given as $G = \{e, a, b, c, x, y, u, v\}$, where $c = ab, x = a^2, y = a^3$, $u = a^2b, v = a^3b$ and Caley's table for $\odot$ is given as:

$\odot$	e	a	b	c	x	y	u	v
e	e	y	b	c	x	a	u	v
a	a	e	c	u	y	x	v	b
b	b	c	e	y	u	v	x	a
c	c	u	a	e	v	b	y	x
x	x	a	u	v	e	y	b	c
y	y	x	v	b	a	e	c	u
u	u	v	x	a	b	c	e	y
v	v	b	y	x	c	u	a	e

We define a single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in K-algebra as follows:
$\mathcal{T}_{\mathcal{A}}(e) = 0.9, \mathcal{I}_{\mathcal{A}}(e) = 0.3, \mathcal{F}_{\mathcal{A}}(e) = 0.3$,
$\mathcal{T}_{\mathcal{A}}(s) = 0.6, \mathcal{I}_{\mathcal{A}}(s) = 0.2, \mathcal{F}_{\mathcal{A}}(s) = 0.4$, for all $s \neq e \in G$.
By routine calculations, it can be verified that $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra ok $\mathcal{K}$.

Proposition 2.1. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then

1. $(\forall s, t \in G), (\mathcal{T}_{\mathcal{A}}(s \odot t) = \mathcal{T}_{\mathcal{A}}(t) \Rightarrow \mathcal{T}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(e))$.
   $(\forall s, t \in G)(\mathcal{T}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(e) \Rightarrow \mathcal{T}_{\mathcal{A}}(s \odot t) \geq \mathcal{T}_{\mathcal{A}}(t))$.
2. $(\forall s, t \in G), (\mathcal{I}_{\mathcal{A}}(s \odot t) = \mathcal{I}_{\mathcal{A}}(t) \Rightarrow \mathcal{I}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(e))$.
   $(\forall s, t \in G)(\mathcal{I}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(e) \Rightarrow \mathcal{I}_{\mathcal{A}}(s \odot t) \geq \mathcal{I}_{\mathcal{A}}(t))$.
3. $(\forall s, t \in G), (\mathcal{F}_{\mathcal{A}}(s \odot t) = \mathcal{F}_{\mathcal{A}}(t) \Rightarrow \mathcal{F}_{\mathcal{A}}(s) = \mathcal{F}_{\mathcal{A}}(e))$.
   $(\forall s, t \in G)(\mathcal{F}_{\mathcal{A}}(s) = \mathcal{F}_{\mathcal{A}}(e) \Rightarrow \mathcal{F}_{\mathcal{A}}(s \odot t) \leq \mathcal{F}_{\mathcal{A}}(t))$.

Proof.

1. Assume that $\mathcal{T}_{\mathcal{A}}(s \odot t) = \mathcal{T}_{\mathcal{A}}(t)$, for all $s, t \in G$. Taking $t = e$ and using (iii) of Definition 2.1, we have $\mathcal{T}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(s \odot e) = \mathcal{T}_{\mathcal{A}}(e)$. Let for $s, t \in G$ be such that $\mathcal{T}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(e)$.
   Then $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\} = \min\{\mathcal{T}_{\mathcal{A}}(e), \mathcal{T}_{\mathcal{A}}(t)\} = \mathcal{T}_{\mathcal{A}}(t)$.
2. Again assume that $\mathcal{I}_{\mathcal{A}}(s \odot t) = \mathcal{I}_{\mathcal{A}}(t)$, for all $s, t \in G$. Taking $t = e$ and by (iii) of Definition 2.1, we have $\mathcal{I}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(s \odot e) = \mathcal{I}_{\mathcal{A}}(e)$. Also let $s, t \in G$ be such that $\mathcal{I}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(e)$.
   Then $\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), I_(t)\} = \min\{\mathcal{I}_{\mathcal{A}}(e), \mathcal{I}_{\mathcal{A}}(t)\} = \mathcal{I}_{\mathcal{A}}(t)$.
3. Consider that $\mathcal{F}_{\mathcal{A}}(s \odot t) = \mathcal{F}_{\mathcal{A}}(t)$, for all $s, t \in G$. Taking $t = e$ and again by (iii) of Definition 2.1, we have $\mathcal{F}_{\mathcal{A}}(s) = \mathcal{F}_{\mathcal{A}}(s \odot e) = \mathcal{F}_{\mathcal{A}}(e)$. Let $s, t \in G$ be such that $\mathcal{F}_{\mathcal{A}}(s) = F_(e)$.
   Then $\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\} = \max\{\mathcal{F}_{\mathcal{A}}(e), \mathcal{F}_{\mathcal{A}}(t)\} = \mathcal{F}_{\mathcal{A}}(t)$.
   This completes the proof.

□

Definition 2.4. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in a K-algebra $\mathcal{K}$ and let $(\alpha, \beta, \gamma) \in [0,1] \times [0,1] \times [0,1]$ with $\alpha + \beta + \gamma \leq 3$. Then level subsets of $\mathcal{A}$ are defined as:

$$\mathcal{A}_{(\alpha,\beta,\gamma)} = \{s \in G \mid \mathcal{T}_{\mathcal{A}}(s) \geq \alpha, \mathcal{I}_{\mathcal{A}}(s) \geq \beta, \mathcal{F}_{\mathcal{A}}(s) \leq \gamma\}$$
$$\mathcal{A}_{(\alpha,\beta,\gamma)} = \{s \in G \mid \mathcal{T}_{\mathcal{A}}(s) \geq \alpha\} \cap \{s \in G \mid \mathcal{I}_{\mathcal{A}}(s) \geq \beta\} \cap \{s \in G \mid \mathcal{F}_{\mathcal{A}}(s) \leq \gamma\}$$
$$\mathcal{A}_{(\alpha,\beta,\gamma)} = \cup(\mathcal{T}_{\mathcal{A}}, \alpha) \cap \cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta) \cap L(\mathcal{F}_{\mathcal{A}}, \gamma).$$

are called (α, β, γ) -level subsets of single-valued neutrosophic set $\mathcal{A}$.
The set of all $(\alpha, \beta, \gamma) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}})$ is known as image of $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$.
The set $\mathcal{A}_{(\alpha,\beta,\gamma)} = \{s \in G \mid \mathcal{T}_{\mathcal{A}}(s) > \alpha, \mathcal{I}_{\mathcal{A}}(s) > \beta, \mathcal{F}_{\mathcal{A}}(s) < \gamma\}$ is known as strong (α, β, γ)- level subset of $\mathcal{A}$.

Proposition 2.2. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then the level subsets $\cup(\mathcal{T}_{\mathcal{A}}, \alpha) = \{s \in G \mid \mathcal{T}_{\mathcal{A}}(s) \geq \alpha\}$, $\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta) = \{s \in G \mid \mathcal{I}_{\mathcal{A}}(s) \geq \beta\}$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma) = \{s \in G \mid \mathcal{F}_{\mathcal{A}}(s) \leq \gamma\}$ are k-subalgebras of $\mathcal{K}$, for every $(\alpha, \beta, \gamma) \in \text{Im}(\mathcal{T}_{\mathcal{A}}) \times \text{Im}(\mathcal{I}_{\mathcal{A}}) \times \text{Im}(\mathcal{F}_{\mathcal{A}}) \subseteq [0, 1]$, where $\text{Im}(\mathcal{T}_{\mathcal{A}})$, $\text{Im}(\mathcal{I}_{\mathcal{A}})$ and $\text{Im}(\mathcal{F}_{\mathcal{A}})$ are sets of values of $T_(\mathcal{A})$, $\mathcal{I}_(\mathcal{A})$ and $F_(\mathcal{A})$, respectively.

Proof. Assume that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and let $(\alpha, \beta, \gamma) \in \text{Im}(\mathcal{T}_{\mathcal{A}}) \times \text{Im}(\mathcal{I}_{\mathcal{A}}) \times \text{Im}(\mathcal{F}_{\mathcal{A}})$ be such that $\cup(\mathcal{T}_{\mathcal{A}}, \alpha) \neq \emptyset, \cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta) \neq \emptyset$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma) \neq \emptyset$. Now to prove that $\cup, \cup^{'}$ and L are level K-subalgebras. Let for $s, t \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha)$, $\mathcal{T}_{\mathcal{A}}(s) \geq \alpha$ and $\mathcal{T}_{\mathcal{A}}(t) \geq \alpha$. It follows from Definition 3.1 that $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\} \geq \alpha$. It implies that $s \odot t \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha)$. Hence $\cup(\mathcal{T}_{\mathcal{A}}, \alpha)$ is a level K-subalgebra of $\mathcal{K}$. Similar result can be proved for $\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta)$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma)$. $\square$

Theorem 2.1. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in K-algebra $\mathcal{K}$. Then $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if and only if $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-sublagebra of $\mathcal{K}$, for every $(\alpha, \beta, \gamma) \in \text{Im}(\mathcal{T}_{\mathcal{A}}) \times \text{Im}(\mathcal{I}_{\mathcal{A}}) \times \text{Im}(\mathcal{F}_{\mathcal{A}})$ with $\alpha + \beta + \gamma \leq 3$.

Proof. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in a K-algebra $\mathcal{K}$.Assume that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}$, i.e., the following three conditions of Definition 3.1 hold.

- $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\}$,
- $\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\}$,
- $\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}$, for all $s, t \in G$.
  $\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s)$, $\mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G$.

Let for $(\alpha, \beta, \gamma) \in \text{Im}(\mathcal{T}_{\mathcal{A}}) \times \text{Im}(\mathcal{I}_{\mathcal{A}}) \times \text{Im}(\mathcal{F}_{\mathcal{A}})$ with $\alpha + \beta + \gamma \leq 3$ be such that $\mathcal{A}_{(\alpha,\beta,\gamma)} \neq \emptyset$. Let $s, t \in \mathcal{A}_{(\alpha,\beta,\gamma)}$ be such that

$$\begin{aligned} \mathcal{T}_{\mathcal{A}}(s) \geq \alpha, \mathcal{T}_{\mathcal{A}}(t) \geq \alpha^{'}, \\ \mathcal{I}_{\mathcal{A}}(s) \geq \beta, \mathcal{I}_{\mathcal{A}}(t) \geq \beta^{'}, \\ \mathcal{F}_{\mathcal{A}}(s) \leq \gamma, \mathcal{F}_{\mathcal{A}}(t) \leq \gamma^{'}. \end{aligned}$$

Without loss of generality we can assume that $\alpha \leq \alpha^{'}$, $\beta \leq \beta^{'}$ and $\gamma \geq \gamma^{'}$. It follows from Definition 3.1 that

$$\begin{aligned} \mathcal{T}_{\mathcal{A}}(s \odot t) \geq \alpha = \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\}, \\ \mathcal{I}_{\mathcal{A}}(s \odot t) \geq \beta = \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\}, \\ \mathcal{F}_{\mathcal{A}}(s \odot t) \leq \gamma = \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}. \end{aligned}$$

It implies that $s \odot t \in \mathcal{A}_{(\alpha,\beta,\gamma)}$. So, $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$.
Conversely, we suppose that $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$. If the condition of the Definition 3.1 is not true, then there exist $u, v \in G$ such that

$$\begin{aligned} \mathcal{T}_{\mathcal{A}}(u \odot v) < \min\ \{\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)\}, \\ \mathcal{I}_{\mathcal{A}}(u \odot v) < \min\ \{\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)\}, \\ \mathcal{F}_{\mathcal{A}}(u \odot v) > \max\{\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)\}. \end{aligned}$$

Taking
$\alpha_1 = \frac{1}{2}(\mathcal{T}_{\mathcal{A}}(u \odot v) + \min\{\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)\})$,
$\beta_1 = \frac{1}{2}(\mathcal{I}_{\mathcal{A}}(u \odot v) + \min\{\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)\})$,
$\gamma_1 = \frac{1}{2}(\mathcal{F}_{\mathcal{A}}(u \odot v) + \min\{\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)\})$.
We have $\mathcal{T}_{\mathcal{A}}(u \odot v) < \alpha_1 < \min\{\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)\}$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < \beta_1 < \min\{\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)\}$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) > \gamma_1 > \max\{\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)\}$. It implies that $u, v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$ and $u \odot v \notin \mathcal{A}_{(\alpha,\beta,\gamma)}$, a contradiction. Therefore, the condition of Definition 3.1 is true. Hence $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic k-subalgebra of $\mathcal{K}$. □

Theorem 2.2. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic k-subalgebra and $(\alpha_1, \beta_1, \gamma_1), (\alpha_2, \beta_2, \gamma_2) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ with $\alpha_j + \beta_j + \gamma_j \leq 3$ for $j = 1, 2$. Then $\mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$ if $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$.

Proof. If $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$, then clearly $\mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$.
Assume that $\mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$. Since $(\alpha_1, \beta_1, \gamma_1) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$, there exist $s \in G$ such that $\mathcal{T}_{\mathcal{A}}(s) = \alpha_1, \mathcal{I}_{\mathcal{A}}(s) = \beta_1$ and $\mathcal{F}_{\mathcal{A}}(s) = \gamma_1$. It follows that $s \in \mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$. So that
$\alpha_1 = \mathcal{T}_{\mathcal{A}}(s) \geq \alpha_2, \beta_1 = \mathcal{I}_{\mathcal{A}}(s) \geq \beta_2$ and $\gamma_1 = \mathcal{F}_{\mathcal{A}}(s) \leq \gamma_2$.
Also $(\alpha_2, \beta_2, \gamma_2) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$, there exist $t \in G$ such that $\mathcal{T}_{\mathcal{A}}(t) = \alpha_2, \mathcal{I}_{\mathcal{A}}(t) = \beta_2$ and $\mathcal{F}_{\mathcal{A}}(t) = \gamma_2$. It follows that $t \in \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)} = \mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)}$. So that $\alpha_2 = \mathcal{T}_{\mathcal{A}}(t) \geq \alpha_1$, $\beta_2 = \mathcal{I}_{\mathcal{A}}(t) \geq \beta_1$ and $\gamma_2 = \mathcal{F}_{\mathcal{A}}(t) \leq \gamma_1$.
Hence $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$. □

Theorem 2.3. Let H be a K-subalgebra of K-algebra $\mathcal{K}$. Then there esixt a single-valued neutrosophic K-subalgebra $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ of K-algebra $\mathcal{K}$ such that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}) = H$, for some $\alpha, \beta \in (0, 1], \gamma \in [0, 1)$.

Proof. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in K-algebra $\mathcal{K}$ given by

$$\mathcal{T}_{\mathcal{A}}(s) = \begin{cases} \alpha \in (0,1] & if s \in H, \\ 0 & otherwise. \end{cases}$$
$$\mathcal{I}_{\mathcal{A}}(s) = \begin{cases} \beta \in (0,1] & if s \in H, \\ 0 & otherwise. \end{cases}$$
$$\mathcal{F}_{\mathcal{A}}(s) = \begin{cases} \gamma \in [0,1) & if s \in H, \\ 0 & otherwise. \end{cases}$$

Let $s, t \in G$. If $s, t \in H$, then $s \odot t \in H$ and so
$\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\}$,
$\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\}$,
$\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}$.
But if $s \notin H$ or $t \notin H$, then $\mathcal{T}_{\mathcal{A}}(s) = 0$ or $\mathcal{T}_{\mathcal{A}}(t), \mathcal{I}_{\mathcal{A}}(s) = 0$ or $\mathcal{I}_{\mathcal{A}}(t)$ and $\mathcal{F}_{\mathcal{A}}(s) = 0$ or $\mathcal{F}_{\mathcal{A}}(t)$. It follows that $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\}$, $\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\}$, $\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}$.
Hence $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a SVN K-subalgebra of $\mathcal{K}$. Consequently $\mathcal{A}_{(\alpha,\beta,\gamma)} = H$.
The above Theorem shows that any K-subalgebra of $\mathcal{K}$ can be perceived as a level K-subalgebra of some single-valued neutrosophic K-subalgebras of $\mathcal{K}$. □

Theorem 2.4. Let $\mathcal{K}$ be a K-algebra. Given a chain of K-subalgebras: $\mathcal{A}_0 \subset \mathcal{A}_1 \subset \mathcal{A}_2 \subset ... \subset A_n = G$. Then there exist a single-valued neutrosophic K-subalgebra whose level K-subalgebras are exactly the K-subalgebras in this chain.

Proof. Let $\{\alpha_k \mid k = 0, 1, ..., n\}, \{\beta_k \mid k = 0, 1, ..., n\}$ be finite decreasing sequences and $\{\gamma_k \mid k = 0, 1, ..., n\}$ be finite increasing sequence in $[0, 1]$ such that $\alpha_i + \beta_i + \gamma_i \leq 3$, for $i = 0, 1, 2, ..., n$. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in $\mathcal{K}$ defined by $\mathcal{T}_{\mathcal{A}}(\mathcal{A}_0) = \alpha_0, \mathcal{I}_{\mathcal{A}}(\mathcal{A}_0) = \beta_0, \mathcal{F}_{\mathcal{A}}(\mathcal{A}_0) = \gamma_0$, $\mathcal{T}_{\mathcal{A}}(\mathcal{A}_k \setminus \mathcal{A}_{k-1}) = \alpha_k, \mathcal{I}_{\mathcal{A}}(\mathcal{A}_k \setminus \mathcal{A}_{k-1}) = \beta_k$ and $\mathcal{F}_{\mathcal{A}}(\mathcal{A}_k \setminus \mathcal{A}_{k-1}) = \gamma_k$, for $0 < k \leq n$. We claim that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Let $s, t \in G$. If $s, t \in \mathcal{A}_k \setminus \mathcal{A}_{k-1}$, then it implies that $\mathcal{T}_{\mathcal{A}}(s) = \alpha_k = \mathcal{T}_{\mathcal{A}}(t), \mathcal{I}_{\mathcal{A}}(s) = \beta_k = \mathcal{I}_{\mathcal{A}}(t)$ and $\mathcal{F}_{\mathcal{A}}(s) = \gamma_k = \mathcal{F}_{\mathcal{A}}(t)$. Since each $\mathcal{A}_k$ is a K-subalgebra, it follows that $s \odot t \in \mathcal{A}_k$. So that either $s \odot t \in \mathcal{A}_k \setminus \mathcal{A}_{k-1}$ or $s \odot t \in \mathcal{A}_{k-1}$. In any case, we conclude that

$$\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \alpha_k = \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},$$
$$\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \beta_k = \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},$$
$$\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \gamma_k = \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.$$

For $i > j$, if $s \in \mathcal{A}_i \setminus \mathcal{A}_{i-1}$ and $t \in \mathcal{A}_j \setminus \mathcal{A}_{j-1}$, then $\mathcal{T}_{\mathcal{A}}(s) = \alpha_i$, $\mathcal{T}_{\mathcal{A}}(t) = \alpha_j, \mathcal{I}_{\mathcal{A}}(s) = \beta_i, \mathcal{I}_{\mathcal{A}}(t) = \beta_j$ and $\mathcal{F}_{\mathcal{A}}(s) = \gamma_i, \mathcal{F}_{\mathcal{A}}(t) = \gamma_j$ and $s \odot t \in \mathcal{A}_i$ because $\mathcal{A}_i$ is a K-subalgebra and $\mathcal{A}_j \subset \mathcal{A}_i$. It follows that

$$\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \alpha_i = \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},$$
$$\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \beta_i = \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},$$
$$\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \gamma_i = \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.$$

Thus, $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and all its non empty level subsets are level K-subalgebras of $\mathcal{K}$.
Since $\mathrm{Im}(\mathcal{T}_{\mathcal{A}}) = \{\alpha_0, \alpha_1, ..., \alpha_{\mathrm{n}}\}, \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) = \{\beta_0, \beta_1, ..., \beta_{\mathrm{n}}\}$, $\mathrm{Im}(\mathcal{F}_{\mathcal{A}}) = \{\gamma_0, \gamma_1, ..., \gamma_{\mathrm{n}}\}$. Therefore, the level K-subalgebras of $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ are given by the chain of K-subalgebras:

$$\cup(\mathcal{T}_{\mathcal{A}}, \alpha_0) \subset \cup(\mathcal{T}_{\mathcal{A}}, \alpha_1) \subset ... \subset \cup(\mathcal{T}_{\mathcal{A}}, \alpha_n) = G,$$
$$\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta_0) \subset \cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta_1) \subset ... \subset \cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta_n) = G,$$
$$\mathrm{L}(\mathcal{F}_{\mathcal{A}}, \gamma_0) \subset \mathrm{L}(\mathcal{F}_{\mathcal{A}}, \gamma_1) \subset ... \subset \mathrm{L}(\mathcal{F}_{\mathcal{A}}, \gamma_n) = G,$$

respectively. Indeed,

$$\cup(\mathcal{T}_{\mathcal{A}}, \alpha_0) = \{s \in G \mid \mathcal{T}_{\mathcal{A}}(s) \geq \alpha_0\} = \mathcal{A}_0,$$
$$\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta_0) = \{s \in G \mid \mathcal{I}_{\mathcal{A}}(s) \geq \beta_0\} = \mathcal{A}_0,$$
$$\mathrm{L}(\mathcal{F}_{\mathcal{A}}, \gamma_0) = \{s \in G \mid \mathcal{F}_{\mathcal{A}}(s) \leq \gamma_0\} = \mathcal{A}_0.$$

Now we prove that $\cup(\mathcal{T}_{\mathcal{A}}, \alpha_k) = \mathcal{A}_k, \cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta_k) = \mathcal{A}_k$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma_k) = \mathcal{A}_k$, for $0 < k \leq n$. Clearly,
$\mathcal{A}_k \subseteq \cup(\mathcal{T}_{\mathcal{A}}, \alpha_k), \mathcal{A}_k \subseteq \cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta_k)$ and $\mathcal{A}_k \subseteq L(\mathcal{F}_{\mathcal{A}}, \gamma_k)$. If $s \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha_k)$, then $\mathcal{T}_{\mathcal{A}}(s) \geq \alpha_k$ and so $s \notin \mathcal{A}_i$, for $i > k$.
Hence $\mathcal{T}_{\mathcal{A}}(s) \in \{\alpha_0, \alpha_1, ..., \alpha_k\}$ which implies that $s \in \mathcal{A}_i$, for some $i \leq k$ since $\mathcal{A}_i \subseteq \mathcal{A}_k$. It follows that $s \in \mathcal{A}_k$. Consequently, $\cup(\mathcal{T}_{\mathcal{A}}, \alpha_k) = \mathcal{A}_k$ for some $0 < k \leq n$. Similar case can be proved for $\cup^{'}(\mathcal{I}_{\mathcal{A}}, \beta_k) = \mathcal{A}_k$. Now if $t \in L(\mathcal{F}_{\mathcal{A}}, \gamma_k)$, then $\mathcal{F}_{\mathcal{A}}(s) \leq \gamma_k$ and so $t \notin \mathcal{A}_i$, for some $j \leq k$.Thus, $\mathcal{F}_{\mathcal{A}}(s) \in \{\gamma_0, \gamma_1, ..., \gamma_k\}$ which implies that $s \in \mathcal{A}_j$, for some $j \leq k$. Since $\mathcal{A}_j \subseteq \mathcal{A}_k$. It follows that $t \in \mathcal{A}_k$.
Consequently, $L(\mathcal{F}_{\mathcal{A}}, \gamma_k) = \mathcal{A}_k$, for some $0 < k \leq n$. Hence the proof. □

2.1 Homomorphism of single-valued neutrosophic K-algebras

Definition 2.5. Let $\mathcal{K}_1 = (G_1, \cdot, \odot, e_1)$ and $\mathcal{K}_2 = (G_2, \cdot, \odot, e_2)$ be two K-algebras and let ϕ be a function from $\mathcal{K}_1$ into $\mathcal{K}_2$. If $\mathcal{B} = (\mathcal{T}_{\mathcal{B}}, \mathcal{I}_{\mathcal{B}}, \mathcal{F}_{\mathcal{B}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$, then the *preimage* of $\mathcal{B} = (\mathcal{T}_{\mathcal{B}}, \mathcal{I}_{\mathcal{B}}, \mathcal{F}_{\mathcal{B}})$ under ϕ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$ defined by $\phi^{-1}(\mathcal{T}_{\mathcal{B}})(s) = \mathcal{T}_{\mathcal{B}}(\phi(s))$, $\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s) = \mathcal{I}_{\mathcal{B}}(\phi(s))$ and $\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s) = \mathcal{F}_{\mathcal{B}}(\phi(s))$, for all $s \in G_1$.

Theorem 2.5. Let $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. If $\mathcal{B} = (\mathcal{T}_{\mathcal{B}}, \mathcal{I}_{\mathcal{B}}, \mathcal{F}_{\mathcal{B}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$, then $\phi^{-1}(\mathcal{B})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

Proof. It is easy to see that $\phi^{-1}(\mathcal{T}_{\mathcal{B}})(e) \geq \phi^{-1}(\mathcal{T}_{\mathcal{B}})(s)$, $\phi^{-1}(\mathcal{I}_{\mathcal{B}})(e) \geq \phi^{-1}(\mathcal{I}_{\mathcal{B}})(s)$ and $\phi^{-1}(\mathcal{F}_{\mathcal{B}})(e) \leq \phi^{-1}(\mathcal{F}_{\mathcal{B}})(s)$ for all $s \in G_1$. Let $s, t \in G_1$, then

$$\begin{aligned}
\phi^{-1}(\mathcal{T}_{\mathcal{B}})(s \odot t) &= \mathcal{T}_{\mathcal{B}}(\phi(s \odot t)) \\
\phi^{-1}(\mathcal{T}_{\mathcal{B}})(s \odot t) &= \mathcal{T}_{\mathcal{B}}(\phi(s) \odot \phi(t)) \\
\phi^{-1}(\mathcal{T}_{\mathcal{B}})(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{B}}(\phi(s)), \mathcal{T}_{\mathcal{B}}(\phi(t))\} \\
\phi^{-1}(\mathcal{T}_{\mathcal{B}})(s \odot t) &\geq \min\{\phi^{-1}(\mathcal{T}_{\mathcal{B}})(s), \phi^{-1}(\mathcal{T}_{\mathcal{B}})(t)\},
\end{aligned}$$

$$\begin{aligned}
\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s \odot t) &= \mathcal{I}_{\mathcal{B}}(\phi(s \odot t)) \\
\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s \odot t) &= \mathcal{I}_{\mathcal{B}}(\phi(s) \odot \phi(t)) \\
\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{B}}(\phi(s)), \mathcal{I}_{\mathcal{B}}(\phi(t))\} \\
\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s \odot t) &\geq \min\{\phi^{-1}(\mathcal{I}_{\mathcal{B}})(s), \phi^{-1}(\mathcal{I}_{\mathcal{B}})(t)\},
\end{aligned}$$

$$\begin{aligned}
\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s \odot t) &= \mathcal{F}_{\mathcal{B}}(\phi(s \odot t)) \\
\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s \odot t) &= \mathcal{F}_{\mathcal{B}}(\phi(s) \odot \phi(t)) \\
\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{B}}(\phi(s)), \mathcal{F}_{\mathcal{B}}(\phi(t))\} \\
\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s \odot t) &\leq \max\{\phi^{-1}(\mathcal{F}_{\mathcal{B}})(s), \phi^{-1}(\mathcal{F}_{\mathcal{B}})(t)\}.
\end{aligned}$$

Hence $\phi^{-1}(\mathcal{B})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

□

Theorem 2.6. $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. If $\mathcal{B} = (\mathcal{T}_{\mathcal{B}}, \mathcal{I}_{\mathcal{B}}, \mathcal{F}_{\mathcal{B}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$ and $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is the *preimage* of $\mathcal{B}$ under ϕ. Then $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

Proof. It is easy to see that $\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s)$ and $\mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G_1$. Now for any $s, t \in G_1$,

$$\begin{aligned}
\mathcal{T}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{B}}(\phi(s \odot t)) \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{B}}(\phi(s) \odot \phi(t)) \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{B}}(\phi(s)), \mathcal{T}_{\mathcal{B}}(\phi(t))\} \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{B}}(\phi(s \odot t)) \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{B}}(\phi(s) \odot \phi(t)) \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{B}}(\phi(s)), \mathcal{I}_{\mathcal{B}}(\phi(t))\} \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{B}}(\phi(s \odot t)) \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{B}}(\phi(s) \odot \phi(t)) \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{B}}(\phi(s)), \mathcal{F}_{\mathcal{B}}(\phi(t))\} \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.
\end{aligned}$$

Hence $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

□

Definition 2.6. Let a mapping $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ from $\mathcal{K}_1$ into $\mathcal{K}_2$ of K-algebras and let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set of $\mathcal{K}_2$. The map $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is called the *preimage* of $\mathcal{A}$ under ϕ, if $\mathcal{T}^{\phi}_{\mathcal{A}}(s) = \mathcal{T}_{\mathcal{A}}(\phi(s))$, $\mathcal{I}^{\phi}_{\mathcal{A}}(s) = \mathcal{I}_{\mathcal{A}}(\phi(s))$ and $\mathcal{F}^{\phi}_{\mathcal{A}}(s) = \mathcal{F}_{\mathcal{A}}(\phi(s))$ for all $s \in G_1$.

Proposition 2.3. Let $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$, then $\mathcal{A}^{\phi} = (\mathcal{T}^{\phi}_{\mathcal{A}}, \mathcal{I}^{\phi}_{\mathcal{A}}, \mathcal{F}^{\phi}_{\mathcal{A}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

Proof. For any $s \in G_1$, we have

$$\begin{aligned}
\mathcal{T}^{\phi}_{\mathcal{A}}(e_1) &= \mathcal{T}_{\mathcal{A}}(\phi(e_1)) = \mathcal{T}_{\mathcal{A}}(e_2) \geq \mathcal{T}_{\mathcal{A}}(\phi(s)) = \mathcal{T}^{\phi}_{\mathcal{A}}(s),\\
\mathcal{I}^{\phi}_{\mathcal{A}}(e_1) &= \mathcal{I}_{\mathcal{A}}(\phi(e_1)) = \mathcal{I}_{\mathcal{A}}(e_2) \geq \mathcal{I}_{\mathcal{A}}(\phi(s)) = \mathcal{I}^{\phi}_{\mathcal{A}}(s),\\
\mathcal{F}^{\phi}_{\mathcal{A}}(e_1) &= \mathcal{F}_{\mathcal{A}}(\phi(e_1)) = \mathcal{F}_{\mathcal{A}}(e_2) \leq \mathcal{F}_{\mathcal{A}}(\phi(s)) = \mathcal{F}^{\phi}_{\mathcal{A}}(s).
\end{aligned}$$

For any $s, t \in G_1$, since $\mathcal{A}$ ia a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$

$$\begin{aligned}
\mathcal{T}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{A}}(\phi(s \odot t))\\
\mathcal{T}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{A}}(\phi(s) \odot \phi(t))\\
\mathcal{T}^{\phi}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{A}}(\phi(s)), \mathcal{T}_{\mathcal{A}}(\phi(t))\}\\
\mathcal{T}^{\phi}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}^{\phi}_{\mathcal{A}}(s), \mathcal{T}^{\phi}_{\mathcal{A}}(s)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{A}}(\phi(s \odot t))\\
\mathcal{I}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{A}}(\phi(s) \odot \phi(t))\\
\mathcal{I}^{\phi}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{A}}(\phi(s)), \mathcal{I}_{\mathcal{A}}(\phi(t))\}\\
\mathcal{I}^{\phi}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}^{\phi}_{\mathcal{A}}(s), \mathcal{I}^{\phi}_{\mathcal{A}}(s)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{A}}(\phi(s \odot t))\\
\mathcal{F}^{\phi}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{A}}(\phi(s) \odot \phi(t))\\
\mathcal{F}^{\phi}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{A}}(\phi(s)), \mathcal{F}_{\mathcal{A}}(\phi(t))\}\\
\mathcal{F}^{\phi}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}^{\phi}_{\mathcal{A}}(s), \mathcal{F}^{\phi}_{\mathcal{A}}(s)\}.
\end{aligned}$$

Hence $\mathcal{A}^{\phi} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$. □

Proposition 2.4. Let $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. If $\mathcal{A}^{\phi} = (\mathcal{T}^{\phi}_{\mathcal{A}}, \mathcal{I}^{\phi}_{\mathcal{A}}, \mathcal{F}^{\phi}_{\mathcal{A}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$, then $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

Proof. Since there exist $s \in G_1$ such that $t = \phi(s)$, for any $t \in G_2$

$$\begin{aligned}
\mathcal{T}_{\mathcal{A}}(t) &= \mathcal{T}_{\mathcal{A}}(\phi(s)) = \mathcal{T}^{\phi(s)}_{\mathcal{A}} \leq \mathcal{T}^{\phi(e_1)}_{\mathcal{A}} = \mathcal{T}_{\mathcal{A}}(\phi(e_1)) = \mathcal{T}_{\mathcal{A}}(e_2),\\
\mathcal{I}_{\mathcal{A}}(t) &= \mathcal{I}_{\mathcal{A}}(\phi(s)) = \mathcal{I}^{\phi(s)}_{\mathcal{A}} \leq \mathcal{I}^{\phi(e_1)}_{\mathcal{A}} = \mathcal{I}_{\mathcal{A}}(\phi(e_1)) = \mathcal{I}_{\mathcal{A}}(e_2),\\
\mathcal{F}_{\mathcal{A}}(t) &= \mathcal{F}_{\mathcal{A}}(\phi(s)) = \mathcal{F}^{\phi(s)}_{\mathcal{A}} \geq \mathcal{F}^{\phi(e_1)}_{\mathcal{A}} = \mathcal{F}_{\mathcal{A}}(\phi(e_1)) = \mathcal{F}_{\mathcal{A}}(e_2).
\end{aligned}$$

for any $s, t \in G_2$, $u, v \in G_1$ such that $s = \phi(u)$ and $t = \phi(v)$. It follows that

$$\begin{aligned}
\mathcal{T}_{\mathcal{A}}(s \odot t) &= \mathcal{T}_{\mathcal{A}}(\phi(u \odot v)) \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &= \mathcal{T}^{\phi}_{\mathcal{A}}(u \odot v) \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}^{\phi}_{\mathcal{A}}(u), \mathcal{T}^{\phi}_{\mathcal{A}}(v)\} \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{A}}(\phi(u)), \mathcal{T}_{\mathcal{A}}(\phi(v))\} \\
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_{\mathcal{A}}(s \odot t) &= \mathcal{I}_{\mathcal{A}}(\phi(u \odot v)) \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &= \mathcal{I}^{\phi}_{\mathcal{A}}(u \odot v) \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}^{\phi}_{\mathcal{A}}(u), \mathcal{I}^{\phi}_{\mathcal{A}}(v)\} \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{A}}(\phi(u)), \mathcal{I}_{\mathcal{A}}(\phi(v))\} \\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{\mathcal{A}}(s \odot t) &= \mathcal{F}_{\mathcal{A}}(\phi(u \odot v)) \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &= \mathcal{F}^{\phi}_{\mathcal{A}}(u \odot v) \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}^{\phi}_{\mathcal{A}}(u), \mathcal{F}^{\phi}_{\mathcal{A}}(v)\} \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{A}}(\phi(u)), \mathcal{F}_{\mathcal{A}}(\phi(v))\} \\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.
\end{aligned}$$

Hence $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$. □

From above two propositions we obtain the following theorem.

Theorem 2.7. Let $\phi : \mathcal{K}_1 \to \mathcal{K}_2$ be an epimorphism of K-algebras. Then $\mathcal{A}^{\phi} = (\mathcal{T}^{\phi}_{\mathcal{A}}, \mathcal{I}^{\phi}_{\mathcal{A}}, \mathcal{F}^{\phi}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$ if and only if $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$.

Definition 2.7. A single-valued neutrosophic K-subalgebra $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ of a K-algebra $\mathcal{K}$ is called *characteristic* if $\mathcal{T}_{\mathcal{A}}(\phi(s)) = \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(\phi(s)) = \mathcal{I}_{\mathcal{A}}(s)$ and $\mathcal{F}_{\mathcal{A}}(\phi(s)) = \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G$ and $\phi \in Aut(\mathcal{K})$.

Definition 2.8. A K-subalgebra S of a K-algebra $\mathcal{K}$ is said to be *fully invariant* if $\phi(S) \subseteq S$, for all $\phi \in End(\mathcal{K})$, where $End(\mathcal{K})$ is the set of all endomorphisms of a K-algebra $\mathcal{K}$. A single-valued neutrosophic K-subalgebra $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ of a K-algebra $\mathcal{K}$ is called *fully invariant* if $\mathcal{T}_{\mathcal{A}}(\phi(s)) \leq \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(\phi(s)) \leq \mathcal{I}_{\mathcal{A}}(s)$ and $\mathcal{F}_{\mathcal{A}}(\phi(s)) \leq \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G$ and $\phi \in End(\mathcal{K})$.

Definition 2.9. Let $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ and $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$ be single-valued neutrosophic K-subalgebras of $\mathcal{K}$. Then $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, _{\mathcal{A}_1})$ is said to be the same type of $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$ if there exist $\phi \in Aut(\mathcal{K})$ such that $\mathcal{A}_1 = \mathcal{A}_2 \circ \phi$, i.e., $\mathcal{T}_{\mathcal{A}_1}(s) = \mathcal{T}_{\mathcal{A}_2}(\phi(s))$, $\mathcal{I}_{\mathcal{A}_1}(s) = \mathcal{I}_{\mathcal{A}_2}(\phi(s))$ and $\mathcal{F}_{\mathcal{A}_1}(s) = \mathcal{F}_{\mathcal{A}_2}(\phi(s))$, for all $s \in G$.

Theorem 2.8. Let $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ and $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$ be single-valued neutrosophic K-subalgebras of $\mathcal{K}$. Then $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ is a single-valued neutrosophic K-subalgebra having the same type of $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$ if and only if $\mathcal{A}_1$ is isomorphic to $\mathcal{A}_2$.

Proof. Sufficient condition holds trivially so we only need to prove the necessary condition. Let $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ be a single-valued neutrosophic K-subalgebra having same type of $\mathcal{A}_2 = (\mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$. Then there exist $\phi \in Aut(\mathcal{K})$ such that $\mathcal{T}_{\mathcal{A}_1}(s) = \mathcal{T}_{\mathcal{A}_2}(\phi(s))$, $\mathcal{I}_{\mathcal{A}_1}(s) = \mathcal{I}_{\mathcal{A}_2}(\phi(s))$ and $\mathcal{F}_{\mathcal{A}_1} = \mathcal{F}_{\mathcal{A}_2}(\phi(s))$, for all $s \in G$.
Let $f : \mathcal{A}_1(K) \to \mathcal{A}_2(K)$ be a mapping defined by $f(\mathcal{A}_1(s)) = \mathcal{A}_2(\phi(s))$, for all $s \in G$, that is,

$f(\mathcal{T}_{\mathcal{A}_1}(s)) = \mathcal{T}_{\mathcal{A}_2}(\phi(s))$, $f(\mathcal{I}_{\mathcal{A}_1}(s)) = \mathcal{I}_{\mathcal{A}_2}(\phi(s))$ and $f(\mathcal{F}_{\mathcal{A}_1}(s)) = \mathcal{F}_{\mathcal{A}_2}(\phi(s))$, for all $s \in G$.
Clearly, f is surjective. Also, f is injective because if $f(\mathcal{T}_{\mathcal{A}_1}(s)) = f(\mathcal{T}_{\mathcal{A}_1}(t))$, for all $s, t \in G$, then $\mathcal{T}_{\mathcal{A}_2}(\phi(s)) = \mathcal{T}_{\mathcal{A}_2}(\phi(t))$ and we have $\mathcal{T}_{\mathcal{A}_1}(s) = \mathcal{T}_{\mathcal{A}_1}(t)$. Similarly, $\mathcal{I}_{\mathcal{A}_1}(s) = \mathcal{I}_{\mathcal{A}_1}(t)$, $\mathcal{F}_{\mathcal{A}_1}(s) = \mathcal{F}_{\mathcal{A}_1}(t)$.
Therefore, f is a homomorphism, for $s, t \in G$

$$f(\mathcal{T}_{\mathcal{A}_1}(s \odot t)) = \mathcal{T}_{\mathcal{A}_2}(\phi(s \odot t)) = \mathcal{T}_{\mathcal{A}_2}(\phi(s) \odot \phi(t)),$$
$$f(\mathcal{I}_{\mathcal{A}_1}(s \odot t)) = \mathcal{I}_{\mathcal{A}_2}(\phi(s \odot t)) = \mathcal{I}_{\mathcal{A}_2}(\phi(s) \odot \phi(t)),$$
$$f(\mathcal{F}_{\mathcal{A}_1}(s \odot t)) = \mathcal{F}_{\mathcal{A}_2}(\phi(s \odot t)) = \mathcal{F}_{\mathcal{A}_2}(\phi(s) \odot \phi(t)).$$

Hence $\mathcal{A}_1 = (\mathcal{T}_{\mathcal{A}_1}, \mathcal{I}_{\mathcal{A}_1}, \mathcal{F}_{\mathcal{A}_1})$ is isomorphic to $\mathcal{A}_2 = \mathcal{T}_{\mathcal{A}_2}, \mathcal{I}_{\mathcal{A}_2}, \mathcal{F}_{\mathcal{A}_2})$. Hence the proof. □

3 $(\tilde{a}, \tilde{b})$-Single-Valued Neutrosophic K-Algebras

Definition 3.1. A single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in a set G is called an $(\tilde{a}, \tilde{b})-$single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if it satisfy the following conditions:

- $u_{(\alpha_1,\beta_1,\gamma_1)}\ \tilde{a}\ \mathcal{A},\ v_{(\alpha_2,\beta_2,\gamma_2)}\ \tilde{a}\ \mathcal{A} \Rightarrow (u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))}\ \tilde{b}\ \mathcal{A}$,

  for all $u, v \in G, \alpha_1, \alpha_2 \in (0, 1], \beta_1, \beta_2 \in (0, 1], \gamma_1, \gamma_2 \in [0, 1)$.

Twelve different types of single-valued neutrosophic K-subalgebras can be obtained by replacing the values of $\tilde{a}(\neq \in \wedge q)$ and $\tilde{b}$ by any two values in the set $\{\in, q, \in \vee q, \in \wedge q\}$ in Definition 1.1.

Remark 3.1. Every $(\in, \in)$-single-valued neutrosophic K-subalgebra is in fact, a single-valued neutrosophic K-subalgebra.

Proposition 3.1. Every $(\in, \in)$-single-valued neutrosophic K-subalgebra is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra.

Proof. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Let $u, v \in G$ and $\alpha_1, \alpha_2 \in (0, 1], \beta_1, \beta_2 \in (0, 1], \gamma_1, \gamma_2 \in [0, 1)$ be such that $u_{(\alpha_1,\beta_1,\gamma_1)} \in \mathcal{A}$, $v_{(\alpha_2,\beta_2,\gamma_2)} \in \mathcal{A}$. Then $u_{(\alpha_1,\beta_1,\gamma_1)} \in \mathcal{A}$, $v_{(\alpha_2,\beta_2,\gamma_2)} \in \mathcal{A} \Rightarrow (u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2))} \in \vee q\ \mathcal{A}$. Hence $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$. □

Proposition 3.2. Every $(\in \vee q, \in \vee q)$-single-valued neutrosophic K-subalgebra is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Definition 3.2. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic set in G. The set $\underline{\mathcal{A}}=\{u \in G \mid \mathcal{T}_{\mathcal{A}}(u) \neq 0,\ \mathcal{I}_{\mathcal{A}}(u) \neq 0,\ \mathcal{F}_{\mathcal{A}}(u) \neq 0\}$ is called the *support* of $\mathcal{A}$.

Lemma 3.1. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero $(\in, \in)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.

Proof. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero $(\in, \in)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and let $u, v \in \underline{\mathcal{A}}$. Then $\mathcal{T}_{\mathcal{A}}(u) \neq 0$ and $\mathcal{T}_{\mathcal{A}}(v) \neq 0$, $\mathcal{I}_{\mathcal{A}}(u) \neq 0$ and $\mathcal{I}_{\mathcal{A}}(v) \neq 0$ and $\mathcal{F}_{\mathcal{A}}(u) \neq 0$, $\mathcal{F}_{\mathcal{A}}(v) \neq 0$. If $\mathcal{T}_{\mathcal{A}}(u \odot v) = 0, \mathcal{I}_{\mathcal{A}}(u \odot v) = 0$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) = 0$. Since $u_{\mathcal{T}_{\mathcal{A}}}(u) \in \mathcal{A}$ and $v_{\mathcal{T}_{\mathcal{A}}}(v) \in \mathcal{A}$, $u_{\mathcal{I}_{\mathcal{A}}}(u) \in \mathcal{A}$ and $v_{\mathcal{I}_{\mathcal{A}}}(v) \in \mathcal{A}$, $u_{\mathcal{F}_{\mathcal{A}}}(u) \in \mathcal{A}$ and $v_{\mathcal{F}_{\mathcal{A}}}(v) \in \mathcal{A}$ but

$(u \odot v)_{(\min(\mathcal{T}_{\mathcal{A}}(u),\mathcal{T}_{\mathcal{A}}(v)),\min(\mathcal{I}_{\mathcal{A}}(u),\mathcal{I}_{\mathcal{A}}(v)),\max(\mathcal{F}_{\mathcal{A}}(u),\mathcal{F}_{\mathcal{A}}(v)))} \notin \mathcal{A}$.

Since $\mathcal{T}_{\mathcal{A}}(u\odot v) = 0, \mathcal{I}_{\mathcal{A}}(u\odot v) = 0$ and $\mathcal{F}_{\mathcal{A}}(u\odot v) = 0$. A contradiction. Hence $\mathcal{T}_{\mathcal{A}}(u\odot v) \neq 0, \mathcal{I}_{\mathcal{A}}(u\odot v) \neq 0$ and $\mathcal{F}_{\mathcal{A}}(u\odot v) \neq 0$ which shows that $(u \odot v) \in \underline{\mathcal{A}}$, consequently $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{A}$. □

Lemma 3.2. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero $(\in, q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.

Lemma 3.3. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero $(q, \in)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.

Lemma 3.4. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero (q, q)-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.
The proof of above three lemmas is followed by Definitions.

Theorem 3.1. If $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a non-zero $(\tilde{a}, \tilde{b})$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then $\underline{\mathcal{A}}$ is a K-subalgebra of $\mathcal{K}$.

Definition 3.3. A neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in a K-algebra $\mathcal{K}$ is called an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if it satisfy the following conditions:

(a) $e_{(\alpha,\beta,\gamma)} \in \mathcal{A} \Rightarrow (u)_{(\alpha,\beta,\gamma)} \in \vee q\ \mathcal{A}$,
(b) $u_{(\alpha_1,\beta_1,\gamma_1)} \in \mathcal{A}, v_{(\alpha_2,\beta_2,\gamma_2)} \in \mathcal{A} \Rightarrow (u \odot v)_{(\min(\alpha_1,\alpha_2),\min(\beta_1,\beta_2),\max(\gamma_1,\gamma_2)} \in \vee q\ \mathcal{A}$,

For all $u, v \in G, \alpha, \alpha_1, \alpha_2 \in (0, 1], \beta, \beta_1, \beta_2 \in (0, 1], \gamma, \gamma_1, \gamma_2 \in [0, 1)$.

Example 3.1. Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where
$G = \{e, x, x^2, x^3, x^4, x^5, x^6\}$ is the cyclic group of order 7 and Caley's table for $\odot$ is given as:

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6
e	e	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e

We define a single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in $\mathcal{K}$ as follows:

$$\mathcal{T}_{\mathcal{A}}(u) = \begin{cases} 1 & \text{if } u = e, \\ 0.7 & \text{otherwise} \end{cases}$$

$$\mathcal{I}_{\mathcal{A}}(u) = \begin{cases} 1 & \text{if } u = e, \\ 0.6 & \text{otherwise} \end{cases}$$

$$\mathcal{F}_{\mathcal{A}}(u) = \begin{cases} 0 & \text{if } u = e, \\ 0.5 & \text{otherwise} \end{cases}$$

Now take
$\alpha = 0.4, \alpha_1 = 0.5, \alpha_2 = 0.3$,
$\beta = 0.5, \beta_1 = 0.6, \beta_2 = 0.3$,
$\gamma = 0.6, \gamma_1 = 0.6, \gamma_2 = 0.5$, where
$\alpha, \alpha_1, \alpha_2 \in (0, 1], \beta, \beta_1, \beta_2 \in (0, 1], \gamma, \gamma_1, \gamma_2 \in [0, 1)$.
By direct calculations, it is easy to see that $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Theorem 3.2. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in $\mathcal{K}$. Then $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if and only if

$$\text{(i) } \mathcal{T}_{\mathcal{A}}(u) \geq \min(\mathcal{T}_{\mathcal{A}}(e), 0.5),\quad \mathcal{I}_{\mathcal{A}}(u) \geq \min(\mathcal{I}_{\mathcal{A}}(e), 0.5),\quad \mathcal{F}_{\mathcal{A}}(u) \leq \max(\mathcal{F}_{\mathcal{A}}(e), 0.5).$$

$$\text{(ii) } \mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5),\quad \mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), 0.5),\quad \mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5), \text{ for all } u, v \in G.$$

Proof. Let us assume that $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra.
(ii)$\Rightarrow$ (i): Let for $u, v \in G$. Assume that $\mathcal{T}_{\mathcal{A}}(u \odot v) < \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5)$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), 0.5)$, $\mathcal{F}_{\mathcal{A}}(u \odot v) > \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5)$. Then $\mathcal{T}_{\mathcal{A}}(u \odot v) < \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v))$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v))$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) > \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v))$. Take α, β, γ such that
$\mathcal{T}_{\mathcal{A}}(u \odot v) < \alpha < \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < \beta < \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)$, $\mathcal{F}_{\mathcal{A}}(u \odot v) > \gamma > \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)$.
Then $u_\alpha, v_\alpha \in \mathcal{T}_{\mathcal{A}}$, $u_\beta, v_\beta \in \mathcal{I}_{\mathcal{A}}$ and $u_\gamma, v_\gamma \in \mathcal{F}_{\mathcal{A}}$ but $(u \odot v)_{(\min(\alpha_1,\alpha_2), \min(\beta_1,\beta_2), \max(\gamma_1,\gamma_2))} \overline{\in \vee q}\ \mathcal{A}$,
a contradiction.
Assume that $\mathcal{T}_{\mathcal{A}}(u \odot v) < 0.5$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < 0.5$, $\mathcal{F}_{\mathcal{A}}(u \odot v) > 0.5$. Then $u_{(0.5,0.5,0.5)}, v_{(0.5,0.5,0.5)} \in \mathcal{A}$, but $(u \odot v)_{(0.5,0.5,0.5)} \overline{\in \vee q} \mathcal{A}$ which is also a contradiction. Hence (i) holds.
Let $u_{(\alpha_1,\beta_1,\gamma_1)}, v_{(\alpha_2,\beta_2,\gamma_2)} \in \mathcal{A}$ which means that $\mathcal{T}_{\mathcal{A}}(u) \geq \alpha_1, \mathcal{T}_{\mathcal{A}}(v) \geq \alpha_2, \mathcal{I}_{\mathcal{A}}(u) \geq \beta_1, \mathcal{I}_{\mathcal{A}}(v) \geq \beta_2$,
$\mathcal{F}_{\mathcal{A}}(u) \leq \gamma_1, \mathcal{F}_{\mathcal{A}}(v) \leq \gamma_2$. We have $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5) \geq \min(\alpha_1, \alpha_2, 0.5)$,
$\mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v), 0.5) \geq \min(\beta_1, \beta_2, 0.5)$, $\mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5) \leq \max(\gamma_1, \gamma_2, 0.5)$.
If $\min(\alpha_1, \alpha_2) > 0.5, \min(\beta_1, \beta_2) > 0.5, \max(\gamma_1, \gamma_2) < 0.5$, then $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq 0.5 \Rightarrow \mathcal{T}_{\mathcal{A}}(u \odot v) + \min(\alpha_1, \alpha_2) > 1$,
$\mathcal{I}_{\mathcal{A}}(u \odot v) \geq 0.5 \Rightarrow \mathcal{I}_{\mathcal{A}}(u \odot v) + \min(\beta_1, \beta_2) > 1$, $\mathcal{F}_{\mathcal{A}}(u \odot v) \leq 0.5 \Rightarrow \mathcal{F}_{\mathcal{A}}(u \odot v) + \max(\gamma_1, \gamma_2) < 1$.
But if $\min(\alpha_1, \alpha_2) \leq 0.5, \min(\beta_1, \beta_2) \leq 0.5, \max(\gamma_1, \gamma_2) \geq 0.5$, then $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\alpha_1, \alpha_2)$,
$\mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\beta_1, \beta_2)$, $\mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\gamma_1, \gamma_2)$. Hence $(u \odot v)_{(\min(\alpha_1,\alpha_2), \min(\beta_1,\beta_2), \max(\gamma_1,\gamma_2))} \in \vee q \mathcal{A}$. Which completes the proof. □

Theorem 3.3. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in $\mathcal{K}$. Then $\mathcal{A}$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if and only if each non-empty $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$. For $\alpha, \beta \in (0.5, 1], \gamma \in [0.5, 1)$.

Proof. Assume that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is an $(\in, \in \vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and let $\alpha, \beta \in (0.5, 1], \gamma \in [0.5, 1)$. To prove that $\mathcal{A}_{(\alpha,\beta,\gamma)} = \{u \in G \mid \mathcal{T}_{\mathcal{A}}(u) \geq \alpha, \mathcal{I}_{\mathcal{A}}(u) \geq \beta, \mathcal{F}_{\mathcal{A}}(u) \leq \gamma\}$ is a K-subalgebra of $\mathcal{K}$. If $u, v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$, then $\mathcal{T}_{\mathcal{A}}(u) \geq \alpha, \mathcal{T}_{\mathcal{A}}(v) \geq \alpha, \mathcal{I}_{\mathcal{A}}(u) \geq \beta, \mathcal{I}_{\mathcal{A}}(v) \geq \beta, \mathcal{F}_{\mathcal{A}}(u) \leq \gamma, \mathcal{F}_{\mathcal{A}}(v) \leq \gamma$.
Thus, $\mathcal{T}_{\mathcal{A}}(e) \geq \min(\mathcal{T}_{\mathcal{A}}(u), 0.5) \geq \min(\alpha, 0.5) = \alpha, \mathcal{I}_{\mathcal{A}}(e) \geq \min(\mathcal{I}_{\mathcal{A}}(u), 0.5) \geq \min(\beta, 0.5) = \beta$,
$\mathcal{F}_{\mathcal{A}}(e) \leq \max(\mathcal{F}_{\mathcal{A}}(u), 0.5) \geq \min(\gamma, 0.5) = \gamma$ and $\mathcal{T}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5) \geq \min(\alpha, 0.5) = \alpha$,
$\mathcal{I}_{\mathcal{A}}(u \odot v) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v), 0.5) \geq \min(\beta, 0.5) = \beta$, $\mathcal{F}_{\mathcal{A}}(u \odot v) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v), 0.5) \leq \max(\gamma, 0.5) = \gamma$.
Thus, $u \odot v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$. Hence $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$. Converse part is obvious. □

Theorem 3.4. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in $\mathcal{K}$. Then $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$ if and only if

$$\text{(a) } \max(\mathcal{T}_{\mathcal{A}}(u \odot v), 0.5) \geq \min(\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)),\quad \max(\mathcal{I}_{\mathcal{A}}(u \odot v), 0.5) \geq \min(\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)),\quad \min(\mathcal{F}_{\mathcal{A}}(u \odot v), 0.5) \leq \max(\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)),$$

$$\text{(b) } \max(\mathcal{T}_{\mathcal{A}}(e), 0.5) \geq (\mathcal{T}_{\mathcal{A}}(u),\quad \max(\mathcal{I}_{\mathcal{A}}(e), 0.5) \geq (\mathcal{I}_{\mathcal{A}}(u),\quad \min(\mathcal{F}_{\mathcal{A}}(e), 0.5) \leq (\mathcal{F}_{\mathcal{A}}(u), \text{ for all } u, v \in G.$$

Proof. Suppose that $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$ and let
$\max(\mathcal{T}_\mathcal{A}(u\odot v),0.5) < \min(\mathcal{T}_\mathcal{A}(u),\mathcal{T}_\mathcal{A}(v)) = \alpha$, $\max(\mathcal{I}_\mathcal{A}(u\odot v),0.5) < \min(\mathcal{I}_\mathcal{A}(u),\mathcal{I}_\mathcal{A}(v)) = \beta$,
$\min(\mathcal{F}_\mathcal{A}(u\odot v),0.5) > \max(\mathcal{F}_\mathcal{A}(u),\mathcal{F}_\mathcal{A}(v)) = \gamma$. Then for $\alpha,\beta\in(0.5,1]$ and $\gamma\in[0.5,1)$ and $u,v\in\mathcal{A}_{(\alpha,\beta,\gamma)}$,
$\mathcal{T}_\mathcal{A}(u\odot v) < \alpha, \mathcal{I}_\mathcal{A}(u\odot v) < \beta, \mathcal{F}_\mathcal{A}(u\odot v) > \gamma$. Since $u,v\in\mathcal{A}_{(\alpha,\beta,\gamma)}$ and $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$, so $u,v\in\mathcal{A}_{(\alpha,\beta,\gamma)}$ or $\mathcal{T}_\mathcal{A}(u\odot v)\geq\alpha, \mathcal{I}_\mathcal{A}(u\odot v)\geq\beta, \mathcal{F}_\mathcal{A}(u\odot v)\leq\gamma$. Which is a contradiction.
Conversely, suppose that conditions (a) and (b) holds. Assume that $\alpha,\beta\in(0.5,1], \gamma\in[0.5,1)$, for $u,v\in\mathcal{A}_{(\alpha,\beta,\gamma)}$. Then we have $0.5<\alpha\leq\min(\mathcal{T}_\mathcal{A}(u),\mathcal{T}_\mathcal{A}(v))\leq\max(\mathcal{T}_\mathcal{A}(u\odot v),0.5)\Rightarrow\mathcal{T}_\mathcal{A}(u\odot v)\geq\alpha$,
$0.5<\beta\leq\min(\mathcal{I}_\mathcal{A}(u),\mathcal{I}_\mathcal{A}(v))\leq\max(\mathcal{I}_\mathcal{A}(u\odot v),0.5)\Rightarrow\mathcal{I}_\mathcal{A}(u\odot v)\geq\beta$,
$0.5>\gamma\geq\max(\mathcal{F}_\mathcal{A}(u),\mathcal{F}_\mathcal{A}(v))\geq\min(\mathcal{F}_\mathcal{A}(u\odot v),0.5)\Rightarrow\mathcal{F}_\mathcal{A}(u\odot v)\leq\gamma$.
$0.5<\alpha\leq\mathcal{T}_\mathcal{A}(u)\leq\max(\mathcal{T}_\mathcal{A}(e),0.5)\Rightarrow\mathcal{T}_\mathcal{A}(mu)\geq\alpha$, $0.5<\beta\leq\mathcal{I}_\mathcal{A}(u)\leq\max(\mathcal{I}_\mathcal{A}(e),0.5)\Rightarrow\mathcal{I}_\mathcal{A}(mu)\geq\beta$,
$0.5>\gamma\geq\mathcal{F}_\mathcal{A}(u)\geq\min(\mathcal{F}_\mathcal{A}(e),0.5)\Rightarrow\mathcal{F}_\mathcal{A}(mu)\leq\gamma$, for some $m\in G$ $u\odot v\in\mathcal{A}_{(\alpha,\beta,\gamma)}$. Hence $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$. □

Theorem 3.5. The intersection of any family of $(\in,\in\vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$ is an $(\in,\in\vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Proof. Let $\{\mathcal{A}_j : j\in I\}$ be a family of $(\in,\in\vee q)$-single-valued neutrosophic K-subalgebras of $\mathcal{K}$.
Let $\mathcal{A} = \bigcap\limits_{j\in I}\mathcal{A}_j = (\sup\limits_{j\in I}\mathcal{T}_{\mathcal{A}_i}, \sup\limits_{j\in I}\mathcal{I}_{\mathcal{A}_i}, \inf\limits_{j\in I}\mathcal{F}_{\mathcal{A}_i})$, for $u,v\in G$ we have

$$\mathcal{T}_\mathcal{A}(u\odot v)\geq\min(\mathcal{T}_\mathcal{A}(u),\mathcal{T}_\mathcal{A}(v),0.5),\ \mathcal{I}_\mathcal{A}(u\odot v)\geq\min(\mathcal{I}_\mathcal{A}(u),\mathcal{I}_\mathcal{A}(v),0.5),\ \mathcal{F}_\mathcal{A}(u\odot v)\leq\max(\mathcal{F}_\mathcal{A}(u),\mathcal{F}_\mathcal{A}(v),0.5).$$

$$\begin{gathered}
\mathcal{T}_\mathcal{A}(u\odot v) = \sup_{j\in I}\mathcal{T}_{\mathcal{A}_i}(u\odot v)\\
\mathcal{T}_\mathcal{A}(u\odot v) \geq \sup_{j\in I}\min(\mathcal{T}_{\mathcal{A}_i}(u),\mathcal{T}_{\mathcal{A}_i}(v),0.5)\\
\mathcal{T}_\mathcal{A}(u\odot v) = \min(\sup_{j\in I}\mathcal{T}_{\mathcal{A}_i}(u),\sup_{j\in I}\mathcal{T}_{\mathcal{A}_i}(v),0.5)\\
\mathcal{T}_\mathcal{A}(u\odot v) = \min(\bigcap_{j\in I}\mathcal{T}_{\mathcal{A}_i}(u),\bigcap_{j\in I}\mathcal{T}_{\mathcal{A}_i}(v),0.5)\\
\mathcal{T}_\mathcal{A}(u\odot v) = \min(\mathcal{T}_\mathcal{A}(u),\mathcal{T}_\mathcal{A}(v),0.5),\\
\mathcal{I}_\mathcal{A}(u\odot v) = \sup_{j\in I}\mathcal{I}_{\mathcal{A}_i}(u\odot v)\\
\mathcal{I}_\mathcal{A}(u\odot v) \geq \sup_{j\in I}\min(\mathcal{I}_{\mathcal{A}_i}(u),\mathcal{I}_{\mathcal{A}_i}(v),0.5)\\
\mathcal{I}_\mathcal{A}(u\odot v) = \min(\sup_{j\in I}\mathcal{I}_{\mathcal{A}_i}(u),\sup_{j\in I}\mathcal{I}_{\mathcal{A}_i}(v),0.5)\\
\mathcal{I}_\mathcal{A}(u\odot v) = \min(\bigcap_{j\in I}\mathcal{I}_{\mathcal{A}_i}(u),\bigcap_{j\in I}\mathcal{I}_{\mathcal{A}_i}(v),0.5)\\
\mathcal{I}_\mathcal{A}(u\odot v) = \min(\mathcal{I}_\mathcal{A}(u),\mathcal{I}_\mathcal{A}(v),0.5),\\
\mathcal{F}_\mathcal{A}(u\odot v) = \inf_{j\in I}\mathcal{F}_{\mathcal{A}_i}(u\odot v)\\
\mathcal{F}_\mathcal{A}(u\odot v) \leq \inf_{j\in I}\max(\mathcal{F}_{\mathcal{A}_i}(u),\mathcal{F}_{\mathcal{A}_i}(v),0.5)\\
\mathcal{F}_\mathcal{A}(u\odot v) = \max(\inf_{j\in I}\mathcal{F}_{\mathcal{A}_i}(u),\inf_{j\in I}\mathcal{F}_{\mathcal{A}_i}(v),0.5)\\
\mathcal{F}_\mathcal{A}(u\odot v) = \max(\bigcap_{j\in I}\mathcal{F}_{\mathcal{A}_i}(u),\bigcap_{j\in I}\mathcal{F}_{\mathcal{A}_i}(v),0.5)\\
\mathcal{F}_\mathcal{A}(u\odot v) = \max(\mathcal{F}_\mathcal{A}(u),\mathcal{F}_\mathcal{A}(v),0.5).
\end{gathered}$$

It follows that $\mathcal{A}$ is an $(\in,\in\vee q)$-single-valued neutrosophic K-subalgebra of $\mathcal{K}$. □

Definition 3.4. Let $\epsilon_1,\epsilon_2\in[0,1]$ and $\epsilon_1<\epsilon_2$. Let $\mathcal{A}=(\mathcal{T}_\mathcal{A},\mathcal{I}_\mathcal{A},\mathcal{F}_\mathcal{A})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Then $\mathcal{A}$ is called a single-valued neutrosophic K-subalgebra with thresholds (ϵ_1,ϵ_2) of $\mathcal{K}$ if

$$\begin{aligned}
&\max(\mathcal{T}_\mathcal{A}(u\odot v),\epsilon_1)\geq\min(\mathcal{T}_\mathcal{A}(u),\mathcal{T}_\mathcal{A}(v),\epsilon_2),\\
&\max(\mathcal{I}_\mathcal{A}(u\odot v),\epsilon_1)\geq\min(\mathcal{I}_\mathcal{A}(u),\mathcal{I}_\mathcal{A}(v),\epsilon_2),\\
&\min(\mathcal{F}_\mathcal{A}(u\odot v),\epsilon_1)\leq\max(\mathcal{F}_\mathcal{A}(u),\mathcal{F}_\mathcal{A}(v),\epsilon_2),\ \text{ for all }\ u,v\in G.
\end{aligned}$$

Example 3.2. Using example 2.1, it is easy to see that $\mathcal{A} = (\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A})$ is a single-valued neutrosophic K-subalgebra with thresholds $(\epsilon_1 = 0.3, \epsilon_2 = 0.56)$ and for $(\epsilon_1 = 0.55, \epsilon_2 = 0.78)$.

Remark 3.2. Let for $\epsilon_1, \epsilon_2 \in [0,1]$ and $\epsilon_1 < \epsilon_2$ unless otherwise specified.
(i) When $\epsilon_1 = 0$ and $\epsilon_2 = 1$ in single-valued neutrosophic K-subalgebra with thresholds (ϵ_1, ϵ_2), $\mathcal{A}$ is an ordinary single-valued neutrosophic K-subalgebra.
(2) When $\epsilon_1 = 0$ and $\epsilon_2 = 0.5$ in single-valued neutrosophic K-subalgebra with thresholds (ϵ_1, ϵ_2), $\mathcal{A}$ is an $(\in, \in \vee q)$single-valued neutrosophic K-subalgebra.

Theorem 3.6. A single-valued neutrosophic set $\mathcal{A}$ in $\mathcal{K}$ is a single-valued neutrosophic K-subalgebra with thresholds (ϵ_1, ϵ_2) if and only if
$\cup(\mathcal{T}_\mathcal{A}, \alpha)$, $\cup^{'}(\mathcal{I}_\mathcal{A}, \beta)$, $L(\mathcal{F}_\mathcal{A}, \gamma)(\neq \phi), \alpha, \beta, \gamma \in (\epsilon_1, \epsilon_2]$ is a K-subalgebra of $\mathcal{K}$.

Proof. Assume that $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra with thresholds (ϵ_1, ϵ_2). First to prove that $\cup(\mathcal{T}_\mathcal{A}, \alpha)$ is a K-subalgebra of $\mathcal{K}$, let $u, v \in \cup(\mathcal{T}_\mathcal{A}, \alpha)$. Then $\mathcal{T}_\mathcal{A}(u) \geq \alpha$ and $\mathcal{T}_\mathcal{A}(v) \geq \alpha$, $\alpha \in (\epsilon_1, \epsilon_2]$. Since $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra. It follows that

$$\max(\mathcal{T}_\mathcal{A}(u \odot v), \epsilon_1) \geq \min(\mathcal{T}_\mathcal{A}(u), \mathcal{T}_\mathcal{A}(v), \epsilon_2) = \alpha,$$

so that $u \odot v \in \cup(\mathcal{T}_\mathcal{A}, \alpha)$. Hence $\cup(\mathcal{T}_\mathcal{A}, \alpha)$ is a k-subalgebra of $\mathcal{K}$. Similarly, we can proof for $\cup^{'}(\mathcal{I}_\mathcal{A}, \beta)$ and $L(\mathcal{F}_\mathcal{A}, \gamma)$. Hence $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$.
Conversely, consider that a single-valued neutrosophic set $\mathcal{A}$ be such that $\mathcal{A}_{(\alpha,\beta,\gamma)} \neq \phi$ is a K-subalgebra of $\mathcal{K}$ for $(\epsilon_1,\ epsilon_2) \in [0,1]$ and $(\epsilon_1 < \epsilon_2)$. Suppose that $\max(\mathcal{T}_\mathcal{A}(u \odot v), \epsilon_1) < \min(\mathcal{T}_\mathcal{A}(u), \mathcal{T}_\mathcal{A}(v), \epsilon_2) = \alpha$, then $\mathcal{T}_\mathcal{A}(u \odot v) < \alpha$, $u \in \cup(\mathcal{T}_\mathcal{A}, \alpha), v \in \cup(\mathcal{T}_\mathcal{A}, \alpha), \alpha \in (\epsilon_1, \epsilon_2]$. Since $u, v \in \cup(\mathcal{T}_\mathcal{A}, \alpha)$ and $\cup(\mathcal{T}_\mathcal{A}, \alpha)$ is a K-subalgebra, $u \odot v \in \cup(\mathcal{T}_\mathcal{A}, \alpha)$, i.e., $\mathcal{T}_\mathcal{A}(u \odot v) \geq \alpha$, a contradiction. Similar results can be obtained for $\cup^{'}(\mathcal{I}_\mathcal{A}, \beta)$ and $L(\mathcal{F}_\mathcal{A}, \gamma)$. □

References

[1] A. A. A. Agboola and B. Davvaz, *Introduction to neutrosophic BCI/BCK-algebras*, International Journal of Mathematics and Mathematical Sciences, Article ID 370267, (2015) 1 − 6.

[2] M. Akram, K. H. Dar and P. K. Shum, *Interval-valued (α, β)-fuzzy K-algebras*, Applied Soft Computing, **11** (1) (2011) 1213 − 1222.

[3] M. Akram, B. Davvaz and F. Feng, *Intutionistic fuzzy soft K-algebras*, Mathematics in Computer Science, **7** (3) (2013) 353 − 365.

[4] M. Akram, K. H. Dar, Y. B. Jun and E. H. Roh, *Fuzzy structures of $K(G)$-algebra*, Southeast Asian Bulletin of Mathematics, **31** (4) (2007) 625 − 637.

[5] M. Akram and K. H. Dar, *Generalized fuzzy K-algebras*, VDM Verlag Dr. Miller, 2010, ISBN-13: 978-3639270952.

[6] Atanassov, K., *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1) (1986) 87 − 96.

[7] Bakhat, S. K. and P. Das, $(\in, \in \vee q)$*-fuzzy subgroup*, Fuzzy Sets and Systems **80** (3) (1996) 359 − 368.

[8] R.A. Borzooei, H. Farahani and M. Moniri, *Neutrosophic deductive filters on BL-algebras*, Journal of Intelligent & Fuzzy Systems,**26**(6)(2014), 2993 − 3004.

[9] K.H. Dar and M. Akram, *On a K-algebra built on a group*, Southeast Asian Bulletin of Mathematics, **29** (1) (2005) 41 − 49.

[10] K.H. Dar and M. Akram, *Characterization of a $K(G)$-algebra by self maps*, Southeast Asian Bulletin of Mathematics, **28** (4) (2004) 601 − 610.

[11] K.H. Dar and M. Akram, *On K-homomorphisms of K-algebras*, International Mathematical Forum, **46** (2007) 2283 – 2293.

[12] D. Coker and M. Demirci, *On intuitionistic fuzzy points*, Notes on intuitionistic fuzzy sets, **1**(2) (1995) 79 – 84.

[13] Y. B. Jun, S.-Z. Song, F. Smarandache and H. Bordbar, *Neutrosophic quadruple BCK/BCI-algebras*, Axioms, **7** (2) (2018) 41.

[14] P. M. Pu and Y. M. Liu, *Fuzzy topology, I. Neighbourhood structure of a fuzzy point and Moore-Smith convergence*, Journal of Mathematical Analysis and Applications, **76** (2) (1980) 571 – 599.

[15] F. Smarandache, *Neutrosophy neutrosophic probability, set, and logic*, American Research Press, Rehoboth, USA, (1998).

[16] H. Wang, F. Smarandache, Y.Q. Zhang and R. Sunderraman, *Single valued neutrosophic sets*, Multispace and Multistruct, **4** (2010) 410 – 413.

[17] C. K. Wong, *Fuzzy points and local properties of fuzzy topology*, Jounal of Mathematical Analysis and Applications, **46** (1974), 316 – 328.

[18] X. Yuan, C. Zhang and Y. Ren, *Generalized fuzzy groups and many-valued implications*, Fuzzy sets and Systems, **138** (1) (2003) 205 – 211.

[19] L.A. Zadeh, *Fuzzy sets*, Information and Control, **8**(3) (1965), 338 – 353.