Comment on "A Novel Method for Solving the Fully Neutrosophic Linear Programming Problems: Suggested Modifications"

Mohamed Abdel-Basset, Mai Mohamed, Florentin Smarandache

Abstract: Some clarifications of a previous paper with the same title are presented here to avoid any reading conflict. Also, corrections of some typo errors are underlined. Each modification is explained with details for making the reader able to understand the main concept of the paper. Also, some suggested modifications advanced by Singh et al. [3] (Journal of Intelligent & Fuzzy Systems, 2019, DOI:10.3233/JIFS-181541) are discussed. It is observed that Singh et al. [3] have constructed their modifications on several mathematically incorrect assumptions. Consequently, the reader must consider only the modifications which are presented in this research.

1. Clarifications and Corrected Errors

In Section 5 and Step 3 of the proposed NLP method [1], the trapezoidal neutrosophic number was presented in the following form:
\[ \tilde{a} = (a^l, a^{m_1}, a^{m_2}, a^u); T_{\tilde{a}}, l_{\tilde{a}}, F_{\tilde{a}} \]
where \(a^l, a^{m_1}, a^{m_2}, a^u\) are the lower bound, the first and second median values and the upper bound for trapezoidal neutrosophic number, respectively. Also, \(T_{\tilde{a}}, l_{\tilde{a}}, F_{\tilde{a}}\) are the truth, indeterminacy and falsity degrees of the trapezoidal neutrosophic number. The ranking function for that trapezoidal neutrosophic number is as follows:
\[ R(\tilde{a}) = \left(\frac{1}{2}(3a^u-a^l) + 2(a^{m_1} - a^{m_2})\right) \times (T_{\tilde{a}} - l_{\tilde{a}} - F_{\tilde{a}}) \]
(8)

The previous ranking function is only for maximization problems.

But, if NLP problem is a minimization problem, then ranking function for that trapezoidal neutrosophic number is as follows:
\[ R(\tilde{a}) = \left( \frac{1}{2}(a^l+a^u) - 3(a^{m_1}+a^{m_2}) \right) \times (T_{\tilde{a}} - l_{\tilde{a}} - F_{\tilde{a}}) \]
(9)

If reader deals with a symmetric trapezoidal neutrosophic number which has the following form:
\[ \tilde{a} = (a^{m_1}, a^{m_2}); \alpha, \beta \]
where \(\alpha = \beta, \alpha, \beta \geq 0\), then the ranking function for that number will be as follows:
\[ R(\tilde{a}) = \left( \frac{1}{2}(a^{m_1}+a^{m_2}) + 2(\alpha+\beta) \right) \times (T_{\tilde{a}} - l_{\tilde{a}} - F_{\tilde{a}}) \]
(10)

We applied Eq. (10) directly in Example 1, but we did not illustrated it in the original work [1], and this caused a reading conflict. After handling typo errors in Example 1, the crisp model of the problem will be as follows:
\[ \text{Maximize } Z = 18x_1 + 19x_2 + 20x_3 \]
Subject to
\[ 12x_1 + 13x_2 + 12x_3 \leq 502, \]
\[ 14x_1 + 13x_3 \leq 486, \]
\[ 12x_1 + 15x_2 \leq 490, \]
\[ x_1, x_2, x_3 \geq 0. \]
The initial simplex form will be as in Table 1.

<table>
<thead>
<tr>
<th>Basic variables</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_4$</th>
<th>$s_5$</th>
<th>$s_6$</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_4$</td>
<td>12</td>
<td>13</td>
<td>12</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>502</td>
</tr>
<tr>
<td>$s_5$</td>
<td>14</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>486</td>
</tr>
<tr>
<td>$s_6$</td>
<td>12</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>490</td>
</tr>
<tr>
<td>$Z$</td>
<td>-18</td>
<td>-19</td>
<td>-20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The optimal simplex form will be as in Table 2.

<table>
<thead>
<tr>
<th>Basic variables</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_4$</th>
<th>$s_5$</th>
<th>$s_6$</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>-12/169</td>
<td>1</td>
<td>0</td>
<td>1/13</td>
<td>-12/169</td>
<td>0</td>
<td>694/169</td>
</tr>
<tr>
<td>$x_3$</td>
<td>14/13</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1/13</td>
<td>0</td>
<td>486/13</td>
</tr>
<tr>
<td>$s_6$</td>
<td>2208/169</td>
<td>0</td>
<td>0</td>
<td>-15/13</td>
<td>180/169</td>
<td>1</td>
<td>72400/169</td>
</tr>
<tr>
<td>$Z$</td>
<td>370/169</td>
<td>0</td>
<td>0</td>
<td>19/13</td>
<td>32/169</td>
<td>0</td>
<td>139546/169</td>
</tr>
</tbody>
</table>

The obtained optimal solution is $x_1 = 0$, $x_2 = 4.11$, $x_3 = 37.38$.

The optimal value of the NLPP is $\tilde{z} \approx (13,15,2,2)x_1 + (12,14,3,3)x_2 + (15,17,2,2)x_3 = (13,15,2,2) \ast 0 + (12,14,3,3) \ast 0.11 + (15,17,2,2) \ast 0.37 = (49.32,57.54,12.33,12.33) + (560.70,635.46,74.76,74.7) = (610.02,693.87,87.09,87.09).

The reader must also note that one can transform the symmetric trapezoidal neutrosophic numbers from Example 1 in [1] to its traditional form, and use Eq. (8) for solving the problem, obtaining the same result. By comparing the result with other existing models mentioned in the original research [1], the proposed model is the best.

By using Eq. (8) and solving Example 2 in [1], the crisp model will be as follows:

Maximize $Z = 25x_1 + 48x_2$

Subject to

$$
13x_1 + 28x_2 \leq 31559,
26x_1 + 9x_2 \leq 16835,
21x_1 + 15x_2 \leq 19624,
x_1, x_2 \geq 0.
$$

The initial simplex form will be as in Table 3.

<table>
<thead>
<tr>
<th>Basic variables</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
<th>$s_5$</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_3$</td>
<td>13</td>
<td>28</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>31559</td>
</tr>
<tr>
<td>$s_4$</td>
<td>26</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>16835</td>
</tr>
<tr>
<td>$s_5$</td>
<td>21</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>19624</td>
</tr>
<tr>
<td>$Z$</td>
<td>-25</td>
<td>-48</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The optimal simplex form will be as in Table 4.
The optimal value of objective function is 54627.

By using Eq. (9) and solving Example 3 in [1], the crisp model will be as follows:

\[
\text{Minimize } Z = 6x_1 + 10x_2 \\
\text{Subject to} \\
2x_1 + 5x_2 \geq 6, \\
3x_1 + 4x_2 \geq 3, \\
x_1, x_2 \geq 0.
\]

The optimal simplex form will be as in Table 5.

<table>
<thead>
<tr>
<th>Basic variables</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_4)</td>
<td>-7/5</td>
</tr>
<tr>
<td>(x_2)</td>
<td>2/5</td>
</tr>
<tr>
<td>(Z)</td>
<td>-2</td>
</tr>
</tbody>
</table>

Hence, the optimal solution has the value of variables:
\(x_1 = 0, x_2 = 1.2, Z = 12\).

The obtained result is better than Saati et al. [2] method.

By correcting typo errors which percolated in the Case study in [1], the problem formulation model will be as follows:

\[
\text{Maximize } \tilde{Z} = 9x_1 + 12x_2 + 13x_3 + 14x_4 \\
\text{Subject to} \\
0.5x_1 + 1.5x_2 + 2.5x_3 + x_4 \leq 1500, \\
3x_1 + x_2 + 2x_3 + 3x_4 \leq 2350, \\
2x_1 + 4x_2 + x_3 + 2x_4 \leq 2600, \\
0.5x_1 + 1x_2 + 0.5x_3 + 0.5x_4 \leq 1200, \\
x_1 \leq 150, \\
x_2 \leq 100, \\
x_3 \leq 300, \\
x_4 \leq 400, \\
x_1, x_2, x_3, x_4 \geq 0.
\]

The values of each trapezoidal neutrosophic number remain the same [1].

By using Eq. (8) and solving the Case study, the crisp model will be as follows:

\[
\text{Maximize } \tilde{Z} = 10x_1 + 10x_2 + 12x_3 + 9x_4 \\
\text{Subject to} \\
0.5x_1 + 1.5x_2 + 1.5x_3 + x_4 \leq 1225, \\
3x_1 + x_2 + 2x_3 + 3x_4 \leq 1680, \\
2x_1 + 4x_2 + x_3 + 2x_4 \leq 2030, \\
0.5x_1 + 1x_2 + 0.5x_3 + 0.5x_4 \leq 945, \\
x_1 \leq 122, \\
x_2 \leq 87, \\
x_3 \leq 227, \\
x_4 \leq 297, \\
x_1, x_2, x_3, x_4 \geq 0.
\]
By solving the previous model using simplex approach, the results are as follows:
\[ x_1 = 122, x_2 = 87, x_3 = 227, x_4 = \frac{773}{3}, Z = 7133. \]

2. A Note on the modifications suggested by Singh et al. [3]


In the second paragraph of the introductory section, Singh et al. [3] assert that “in Abdel-Basset et al.’s method [1], firstly, a neutrosophic linear programming problem (NLPP) is transformed into a crisp linear programming problem (LPP) by replacing each parameter of the NLPP, represented by a trapezoidal neutrosophic number with its equivalent defuzzified crisp value”. However, this is not true, since the neutrosophic linear programming problem (NLPP) is transformed into a crisp linear programming problem (LPP) by replacing each parameter of the NLPP, represented by a trapezoidal neutrosophic number with its equivalent deneutrosophic crisp value. The deneutrosophication process means transforming a neutrosophic value to its equivalent crisp value. In Section 2, Step 1 Singh et al. [3] alleged that Abdel-Basset et al.’s method [1] for comparing two trapezoidal neutrosophic numbers is based on maximization and minimization of problem, which is again not true.

In Section 3 and Definition 4, Abdel-Basset et al. [1] illustrated that the method for comparing two trapezoidal neutrosophic numbers is as follows:

1. If \( R(\tilde{A}) > R(\tilde{B}) \) then \( \tilde{A} > \tilde{B} \).
2. If \( R(\tilde{A}) < R(\tilde{B}) \) then \( \tilde{A} < \tilde{B} \).
3. If \( R(\tilde{A}) = R(\tilde{B}) \) then \( \tilde{A} = \tilde{B} \).

There is well known that if \( a^l = a^{m1} = a^{m2} = a^u \), then the trapezoidal number \( \tilde{a} = (a^l, a^{m1}, a^{m2}, a^u); 1, 0, 0 \) will be transformed into a real number \( a = ((a, a, a), a); 1, 0, 0 \), and hence in this case \( R(a) = a \). We presented this fact to illustrate a great error in the suggested modifications of Singh et al. [3].

In the Suggested modifications section [3], the authors claimed that:

\[
R\left(\sum_{i=1}^{m} (a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i})\right) = \sum_{i=1}^{m} R\left(a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i}\right) - \sum_{i=1}^{m} T_{\tilde{a}_i} + \sum_{i=1}^{m} I_{\tilde{a}_i} + \sum_{i=1}^{m} F_{\tilde{a}_i} \min_{1 \leq j \leq n}\{I_{\tilde{\xi}_j}\} - \max_{1 \leq j \leq n}\{I_{\tilde{\xi}_j}\} - \max_{1 \leq j \leq n}\{F_{\tilde{\xi}_j}\}.
\]

Instead of,

\[
R\left(\sum_{i=1}^{m} (a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i})\right) = \sum_{i=1}^{m} R\left(a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i}\right) .
\]

Let us consider the following example for proving the error in this suggestion [3]

Let \( m = 3 \), which are three trapezoidal neutrosophic numbers \( \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \); since \( \tilde{a}_1 = ((1, 1, 1, 1); 1, 0, 0) \), \( \tilde{a}_2 = ((2, 2, 2, 2); 1, 0, 0) \), \( \tilde{a}_3 = ((3, 3, 3, 3); 1, 0, 0) \), then,

\[
R(\sum_{i=1}^{3} (a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i})) = R(((1, 1, 1, 1); 1, 0, 0) + ((2, 2, 2, 2); 1, 0, 0) + ((3, 3, 3, 3); 1, 0, 0)) \]

\[
= R(((6, 6, 6, 6); 1, 0, 0)),
\]

and according to the previously determined fact “if \( a^l = a^{m1} = a^{m2} = a^u \) then the trapezoidal number \( \tilde{a} = (a^l, a^{m1}, a^{m2}, a^u); 1, 0, 0 \) will be transformed into a real number \( a = ((a, a, a, a); 1, 0, 0) \) and hence in this case \( R(a) = a \) ”, the value of \( R(((6, 6, 6, 6); 1, 0, 0)) = 6 \).

And by calculating the right hand side of Eq. (11), which is \( \sum_{i=1}^{3} R\left(a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i}\right) - \sum_{i=1}^{3} T_{\tilde{a}_i} + \sum_{i=1}^{3} I_{\tilde{a}_i} + \sum_{i=1}^{3} F_{\tilde{a}_i} \min_{1 \leq j \leq n}\{I_{\tilde{\xi}_j}\} - \max_{1 \leq j \leq n}\{I_{\tilde{\xi}_j}\} - \max_{1 \leq j \leq n}\{F_{\tilde{\xi}_j}\} \), we note that,

\[
R((1, 1, 1, 1); 1, 0, 0) + R((2, 2, 2, 2); 1, 0, 0) + R((3, 3, 3, 3); 1, 0, 0) - 3 + 0 + 0 + 1 - 0 - 0 = 1 + 2 + 3 - 3 + 0 + 1 - 0 - 0 = 4.
\]
And then, the left hand side of Eq. (11) does not equal the right hand side, i.e. $6 \neq 4$.

Consequently, the authors [3] built their suggestions on a wrong concept.

Besides Eq. (11), the authors [3] used the expressions $R(a) = 3a + 1$ for maximization problems, and $R(a) = -2a + 1$ for minimization problems, and this shows peremptorily that their assumptions are scientifically incorrect.

There is also a repeated error in all corrected solutions suggested by Singh et al. [3] which contradicts with the basic operations of trapezoidal neutrosophic numbers. This error is iterated in Section 7, as in Example 1, in Step 6. Singh et al. [3] illustrated that the optimal value of the NLPP is calculated using the optimal solution obtained in Step 5 as follows:

$$(11,13,15,17)x_1 + (9,12,14,17)x_2 + (13,15,17,19)x_3 = (11,13,15,17) \times 0 + (9,12,14,17) \times 0 + (13,15,17,19) \times (\frac{245}{18}) = 13 (\frac{245}{18}) + 15 (\frac{245}{18}) + 17 (\frac{245}{18}) + 19 (\frac{245}{18}) = \frac{7840}{9},$$

and because the basic operation of multiplying trapezoidal neutrosophic number by a constant value is as follows:

$$\gamma \mathbb{A} = \mathbb{A} \cdot \gamma = \{(\gamma a_1, \gamma a_2, \gamma a_3, \gamma a_4); T_a, I_a, F_a\} \text{ if } (\gamma \geq 0),$$

then the value of $(11,13,15,17) \times 0 + (9,12,14,17) \times 0 + (13,15,17,19) \times (\frac{245}{18}) = (\frac{3185}{18}, \frac{1225}{18}, \frac{4655}{18}, \frac{4655}{18})$. Then the optimal value of the NLPP is $Z = \frac{3185}{18}, \frac{1225}{18}, \frac{4655}{18}, \frac{4655}{18}$.

The same error appears in Example 4, where the optimal value of the NLPP is calculated by Singh et al. [3] using the optimal solution obtained in Step 5 as follows:

$$(6,8,9,12)x_1 + (9,10,12,14)x_2 + (12,13,15,17)x_3 + (8,9,11,13)x_4 = (6,8,9,12) + (9,10,12,14) + (12,13,15,17) + (8,9,11,13) = 6 (\frac{3700}{21}) + 8 (\frac{3700}{21}) + 9 (\frac{3700}{21}) + 12 (\frac{5200}{7}) + 13 (\frac{6200}{7}) + 15 (\frac{1189700}{21}) = \frac{189700}{21},$$

which is scientifically incorrect and reflects only the weak background of the authors in the neutrosophic field.

Therefore, we concluded that it is scientifically incorrect to use Singh et al.’s modifications [3].

3. Conclusions

Clarifications and corrections of some typo errors are presented here to avoid any reading conflict. Also, the correct results of NLPPs are presented. By using three modified functions for ranking process which were presented by Abdel-Basset et al. [1], the reader will be able to solve all types of linear programming problems with trapezoidal and symmetrical trapezoidal neutrosophic numbers. Also, the mathematically incorrect assumptions used by Singh et al. [3] are discussed and rejected.

Conflict of interest

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

References


Received: November 7, 2019. Accepted: February 3, 2020