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Complex vague set based concept lattice

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ABSTRACT

Recently, the calculus of concept lattice is extended from unipolar to bipolar fuzzy space for precise measurement of vagueness in the attributes based on their acceptation and rejection part. These extensions still unable to highlight the uncertainty in vague attributes and measurement of fluctuation at given phase of time. To conquer this problem, current paper proposed a method for adequate analysis of vagueness and uncertainty in data with fuzzy attributes using the amplitude and phase term of a defined complex vague set based concept lattice. In addition, the analysis derived from the proposed method is compared with CVSS method through an illustrative example.

1. Introduction

Recent years, much attention has been paid towards precise analysis of uncertainty and vagueness in the given data set with fuzzy attributes. The uncertainty and vagueness in data coexists simultaneously at the given phase of time. One of the suitable example is large number of data generated from medical diagnoses may contain lots of incomplete, uncertain and vague information. Handling these type of complex or dynamic data set is a major issue for the researcher communities. To crush this problem, recently Ye [1] tried to characterize the uncertainty and vagueness in medical diagnoses data set based on its truth, indeterminacy and falsity membership–value. However uncertainty and vagueness in the symptoms of medical diagnoses data set changes at each interval of time [2]. To represent these types of dynamic data set recently properties of complex vague set [3] is introduced using the extensive properties of complex fuzzy logic [4–6]. The current paper put forward effort to discover all the hidden pattern in a given complex data set. To elaborate the proposed method current paper focuses on medical diagnoses data set and its hidden pattern using the properties of Formal Concept Analysis (FCA). The calculus of FCA is already applied in analysis of gene expression data [2], Chinese medicine data [7], Breast cancer data [8], Health care data [9] and TB data [10]. All of these available approaches focused on finding pattern in medical diagnoses data in binary attributes. These methods lacks in handling the data set beyond the binary attributes and their fluctuation at given phase of time. The reason is to measure the vagueness and fluctuation in the uncertainty calculus of complex vague set, complex vague lattice and complex vague graph is required which is at infancy stage. To fill this backdrop, the current paper aimed at depth analysis of complex fuzzy set, its partial ordering visualization in the concept lattice using the extensive properties of FCA.

FCA is one of the well–established mathematical model for data analysis and processing, based on applied abstract algebra [11]. The calculus of FCA provides a an alternative way to discover all the hidden pattern (i.e. formal concept) in a given data set. The generated formal concepts are nothing but a pair of objects (i.e. extent) and their common attributes (i.e. intent) which are closed with Galois connection. It can be considered as a basic unit of thought for knowledge processing tasks [12]. All of the generated formal concepts can be displayed in compact arrangement of their generalization and specialization properties of a given concept lattice. This hierarchical order visualization provides an adequate way to refine the interested pattern in the given data set when compared to its numerical representation. To intensify the knowledge processing tasks, the calculus of FCA expedite with fuzzy [13], interval [14–16], bipolar [17], three–polar [18], possibility [19], rough set [20] and other extensive theory [21–23]. For defining the vagueness in attributes through unipolar [0,1], bipolar$[0, 1]^2$ or three–polar $[0,1]^3$ fuzzy space based on their acceptation and rejection part. However, these available approaches are unable to highlight the fluctuation in uncertainty and vagueness at given phase of time [23]. In general the uncertainty in data occurs at each interval of time whereas vagueness is created due to problem in computational linguistics (like tall, young or bald). The uncertainty may be derived from the factors like inconsistency, incompleteness, or

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Table 1
Inevitable appearance of vagueness in medical data set in form of context.

Conditions	Objects	Attributes	Relation
a	Complete	Complete	Vague (or) Complex
b	Incomplete or Vague	Complete	Vague (or) Complex
c	Complete	Incomplete or Vague	Vague (or) Complex
d	Incomplete or Vague	Incomplete or Vague	Vague (or) Complex

Table 2
Comparison among interval, bipolar, vague and complex vague set.

	Interval	Bipolar	Vague	Complex vague set
Domain	Universe of discourse	Universe of discourse	Universe of discourse	
Co–domain	Unipolar	Bipolar interval- [0,1]	[0,1] [-1,0) × (0, 1]	
Uncertainty	Yes	Yes	Yes	Yes
True	Yes	Yes	Yes	Yes
Falsity	No	No	No	Yes
Positive	[0,1]	[0,1]	(0, 1]	[0,1]
Negative	No	No	[−1, 0)	[0,1]
Sharp boundaries	Yes	Yes	No	No
Unit circle	[0,1]	[0,1]	[0,2]	[0,1]
Amplitude term	Yes	Yes	Yes	Yes
Phase term	No	No	No	Yes

seasonality. It occurs when, the data shows cyclic or periodic pattern in a given phase of time. Temperature is one of the suitable example to define the uncertainty and its fluctuation at given phase of time (i.e. year). The temperature 22 ° is considered as cool in the summer whereas the same temperature 22 ° is considered as warm in the winter season. This type of inconsistency and fluctuation in the given attributes creates uncertainty in the data. The vagueness is somehow related with the fuzziness available in given attributes (i.e. like young, tall or bald). It can be measured based on evidence to support (i.e. true membership –value) or reject (i.e. false membership–value) the attributes for the given context. It means vagueness occurs when a fuzzy attribute cannot be defined via a sharp boundary. This situation generally can be found in medical diagnoses data set, stock market and time series data. As for example, How much hair loss is required to consider a person is bald or not ?, How much loss of vision is required to consider a person is legally blind or not?, What is the point of conception from date of birth to declare a person as human being? These all fuzzy attributes contain vagueness which cannot be defined by a precise boundaries as shown in Table 1. To represent these types of attributes, an expert requires some evidence to support or reject them in a seized scale [0, 1]. For this purpose, recently properties of vague set [24], vague graph [25,26], vague hypergraph [27,28], vague soft set [29], and vague lattice [30–32] is studied to expedite its applications [33]. Subsequently, for measurement of uncertainty and its fluctuation in the attributes [34] properties of complex vague set [3] is introduced using the calculus of complex fuzzy set [4], complex fuzzy logic [5,6] and vague soft set [29]. Motivated from these recent studies current paper focuses on exploring the calculus of complex vague set with concept lattice for handling complex data set. To fulfill this objective interval–valued, bipolar, vague and complex fuzzy set is comparatively studied in Table 2. This table shows that the complex vague set provides more precise representation of vagueness and uncertainty in the fuzzy attributes using the amplitude and phase term.

To achieve the goal, a method is proposed, in this paper to generate all the hidden pattern (i.e. formal concepts) in a given complex vague context using the amplitude and phase of a defined complex vague set [3], vague graph [25,26], and concept lattice [11,12,35–41]. To explore the properties of complex vague relation [41–43] for refining the knowledge processing tasks using technique of concept lattice [23,44,45]. To fulfill this backdrop, the proposed method is applied on a medical diagnoses data set with step by step illustration. The motivation is to improve the medical diagnoses data processing tasks using a mathematical model rather than traditional methods as it affects human life directly. The objective is to provide an accurate result for the adequate analysis of disease and its recovery for a given phase of time. To validate the results, analysis derived from the proposed method is compared with CVSS method [3] with an illustrative example.

Rest of the paper is organized as follows: Section 2 provides a brief background about FCA with the vague setting. Section 3 contains the proposed method for generating the complex vague concepts and its illustration in Section 4. Section 5 provide discussions followed by conclusions, and references.

2. Formal concept analysis with the vague setting

There are many data set like (http://indianalgae.co.in/) which contains vague attributes [34]. Medical data set is one of the suitable example which contains lots of incomplete, inconsistent and vague information. To represent these type of attributes an expert need evidence to accept or reject them in a seized scale [0, 1]. To fill this backdrop, Gau and Buehrer [24] introduced properties of vague set. In this section some basic preliminaries about FCA with vague setting is given for handling the data with vague attributes.

Definition 1. (Formal fuzzy context) [13]: A formal fuzzy context $\mathbf{F} = (X, Y, \tilde{R})$ is a fuzzy matrix having X as set of objects, Y as set of attributes, and L–relation among them i.e. $\tilde{R}$: $X\times Y\rightarrow L$. In general the relation $\tilde{R}$ represents non–zero fuzzy membership value at which the object $x \in X$ has the attribute $y \in Y$ in [0, 1] where L is a support set of some complete residuated lattice $\mathbf{L}$ [13].

Definition 2. (Formal vague context) [24,32]: A formal vague context $\mathbf{F} = (X, Y, \tilde{R})$ represents set of objects (X), set of vague attributes (Y) and a vague relation $\tilde{R}$ between them $\tilde{R} = \{\langle (x, y), t_{\tilde{R}}(x, y), f_{\tilde{R}}(x, y)\rangle | x \in X, y \in Y\}$. As for example, a patient suffer from pneumonia or not can be represented through an evidence for its acceptation (i.e. positive) and rejection (i.e. false membership) value of a defined vague set and vague relation among them.

Definition 3. (Residuated lattice) [35] : It is a basic structure of truth degrees $\mathbf{L} = (L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ in which 1 represents greatest elements and 0 represents least elements respectively. $\mathbf{L}$ is a complete residuated lattice iff :

(1) $(L, \wedge, \vee, 0, 1)$ is a complete lattice.
(2) $(L, \otimes, 1)$ is commutative monoid.
(3) $\otimes$ and $\rightarrow$ are adjoint operators called as multiplication and residuum, respectively i.e. $a\otimes b \leq c$ iff $a \leq b \rightarrow c, \forall a, b, c \in L$.

The operators $\otimes$ and $\rightarrow$ are defined distinctly by Lukasiewicz, G ödel, and Goguen t–norms and their residua as given below [36];

Lukasiewicz:

- $a \otimes b = \max(a+b-1, 0)$,
- $a \rightarrow b = \min(1-a+b, 1)$.

G ödel:

- $a \otimes b = \min(a, b)$,
- $a \rightarrow b = 1$ if $a \leq b$, otherwise b.

Goguen:

- $a \otimes b = a \cdot b$,
- $a \to b = 1$ if $a \leq b$, otherwise b/a.

Recently, it is extensively studied for other L–sets [15,21,30,36].

Definition 4. (Fuzzy concept forming operator) [36]: The operators $(\uparrow, \downarrow)$ for any **L**–set $A \in L^X$ of objects, and **L**–set $B \in L^Y$ of attributes can be defined as follows:

(1) $A^{\uparrow}(y) = \wedge_{x \in X}(A(x) \to \tilde{R}(x,y))$ where $A^{\uparrow} \in L^Y$ of attributes,
(2) $B^{\downarrow}(x) = \wedge_{y \in Y}(B(y) \to \tilde{R}(x,y))$ where $B^{\downarrow} \in L^X$ of objects.

The operators $(\uparrow, \downarrow)$ are known as Galois connection. The set $A^{\uparrow}(y)$ is interpreted as the **L**-set of attributes $y \in Y$ shared by all objects from A. Similarly, $B^{\downarrow}(x)$ is interpreted as the **L**-set of all objects $x \in X$ having the common attributes (from B). The pair $(A, B) \in L^X \times L^Y$ is called as formal fuzzy concept iff: $A^{\uparrow} = B$ and $B^{\downarrow} = A$. In the formal fuzzy concepts fuzzy set of objects A is called as extent and fuzzy set of attributes B is called as intent. Subsequently, these computations are extended with interval–valued fuzzy setting [15,16], bipolar fuzzy setting [17], vague setting [18,30] and possibility theory [19].

Definition 5. (Formal vague concept) [17,30,31] : The set of objects $(A) = (x_i, [t_A(x_i), 1 - f_A(x_i)])$ having the attributes $(B) = (y_j, [t_B(y_j), 1 - f_B(y_j)])$ can be represented as a node in the vague graph. The pair (A, B) is called as a formal vague concept iff: $B^{\downarrow} = (A, [t_A(x), 1 - f_A(x)])$ and $A^{\uparrow} = (B, [t_B(y), 1 - f_B(y)])$. As for example, set of patients having pneumonia shares similar symptoms of acceptation (i.e. positive) and rejection (i.e. false membership) in a defined vague set and vice versa.

Definition 6. (Partial ordering of fuzzy concepts) [13,14]: All the discovered formal fuzzy concepts $FC_{\mathbf{F}}$ are connected via super and sub concept hierarchy principle of partial ordering i.e. $(A_1, B_1) \leq (A_2, B_2) \Longleftrightarrow A_1 \subseteq A_2 (\Longleftrightarrow B_2 \subseteq B_1)$.

Definition 7. (Partial ordering of vague set) [28]. A vague relation $(\leq)$ on a vague set $(\tilde{S})$ is a partial order relation iff, it satisfies reflexive, antisymmetric and transitive conditions. The partial ordering of vague set be defined as follows: Let us suppose two vague sets $I=(t_I(z), f_I(z))$ and $J=(t_J(z), f_J(z))$ then $I \subseteq J$ iff $t_i(z) \leq t_j(z)$ and $1 - f_i(z) \leq 1 - f_j(z)$. This ordering help us to define the super and sub concept hierarchy between the generated formal vague concepts.

Definition 8. Together with partial ordering the formal concepts forms a complete lattice in which there exists an infimum and a supremum among them as follows:

- $\wedge_{j \in J}(A_j, B_j) = (\bigcap_{j \in J} A_j, (\bigcup_{j \in J} B_j)^{\downarrow\uparrow})$,
- $\vee_{j \in J}(A_j, B_j) = ((\bigcup_{j \in J} A_j)^{\uparrow\downarrow}, \bigcap_{j \in J} B_j)$.

Definition 9. (Complete lattice of vague set)[30,31]. A lattice is a partially ordered set $(\tilde{S}, \leq)$ in which for every pair (u, v), there exist a supremum $= u \vee v$ and an infimum $= u \wedge v$. Similarly, let us suppose $I=(t_I, f_I)$ and $J=(t_J, f_J)$ be two vague sets of a complete lattice then its infimum and supremum can be defined as follows:

- $(t_I(z), f_I(z)) \wedge (t_J(z), f_J(z))=(\min(t_I(z), t_J(z)), \min(1\text{-}f_I(z), 1 - f_J(z)))$.
- $(t_I(z), f_I(z)) \vee (t_J(z), f_J(z))=(\max(t_I(z), t_J(z)), \max(1\text{-}f_I(z), 1 - f_J(z)))$.

Definition 10. (Vague graph) [24–26]: A vague graph with an underlying set V is defined to be a pair $G = (I, J)$ where $I = (t_I, f_I)$ is a vague set fuzzy set on V and $J=(t_J, f_J)$ is a vague set on edges $E \subseteq V \times V$ such that:

$t_J(v_1v_2) \leq \min\,(t_A(v_1), t_A(v_2))$, and

Table 3
A vague set of V for Example 1.

	v_1	v_2	v_3
t_I	0.2	0.3	0.4
f_I	0.3	0.4	0.5

Table 4
A vague set of E for Example 1.

	v_1v_2	v_2v_3	v_3v_1
t_I	0.1	0.2	0.1
f_I	0.5	0.7	0.6

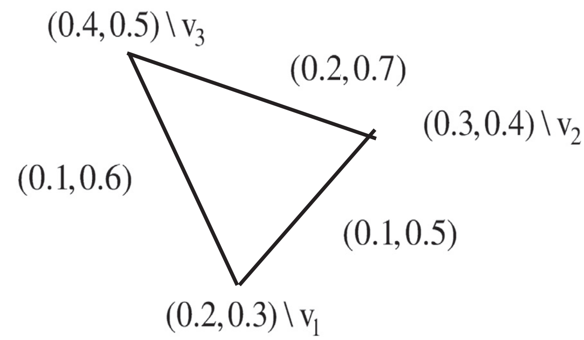

Fig. 1. A vague graph representation based on Tables 3 and 4.

$f_J(v_1v_2) \geq \max\,(t_A(v_1), t_A(v_2))$ for all $v_1, v_2 \in V$ and $(v_1v_2) \in E$.

Example 1. Suppose, a doctor want to write the opinion symptoms of pneumonia available in the patients (v_1, v_2, v_3). For this purpose doctor need an evidence to support (i.e. true membership value) or reject (i.e. false membership–value) his/her opinion. This can be precisely written using the properties of vague set as shown in Table 3. Table 4 represents the corresponding relationship among them. These two sets can be visualized as verices $V=\{v_1, v_2, v_3\}$ and edges set –$E= \{v_1v_2, v_2v_3, v_3v_1\}$ of a defined vague graph G. In which, $I= (t_I, f_I)$ represents the vague set on V as shown in Table 3 and $J=(t_J, f_J)$ represents the vague set on $E \subseteq V \times V$ as shown in Table 4. Fig. 1 shows the vague graph for the vague set of vertices and edges shown in Tables 3 and 4 [27].

A vague graph $G=(I, J)$ is complete iff [27]:
$t_J(v_1v_2)=\min(t_I(v_1), t_I(v_2))$, and
$f_J(v_1v_2)=\max(f_I(v_1), f_I(v_2))$ for all $v_1, v_2 \in V$ and $(v_1v_2) \in E$.

The above given definitions and examples authenticates that properties of vague graph [24–26] provides an effective way to display the vagueness in data with fuzzy attributes. In this case a major problem is addressed while precise measurement of uncertainty and vagueness in the attributes [38–45]. Many times the uncertainty in attribute arises due to inconsistency, incompleteness and its fluctuation in given phase of time. As for example 22 ° temperature is considered as cool in summer whereas warm in winter. To discover such type of pattern in the dynamic or vague context is hard to compute using any of the available approaches [23,45]. To deal with these type of data in the next section a method is proposed based on properties of complex fuzzy logic and its extensive properties.

3. Proposed method

In this section, a method is proposed to generate all the complex vague concepts from a given complex vague context using the properties of Galois connections as given below:

Let a formal vague context $\mathbf{F} = (X, Y, \tilde{R})$ where X is set of objects, Y is set of vague attributes and $\tilde{R} = \{((x, y), r_{\tilde{R}}(x, y)e^{iw_{\tilde{R}}(x,y)}) : \forall x \in X, y \in Y)\}$ represents complex vague relation among them. To represent the uncertainty and its fluctuation in fuzzy attributes using given phase of time whereas the vagueness in fuzzy attributes are represented through amplitude of complex vague set.

Definition 11. (Complex fuzzy set) [4,5]: A complex fuzzy set Z can be defined over a universe of discourse U. The complex–valued grade of membership of an element $z \in U$ can be characterized by $\mu_Z(z)$. The membership–values that $\mu_Z(z)$ may receive all lie within the unit circle in the complex plane in the form $\mu_Z(z) = r_z(x)e^{iw_Z(x)}$, where $i=\sqrt{-1}$, both $r_Z(z)$ and $w_Z(z)$ are real–valued and $r_Z(z) \in [0,1]$. The complex fuzzy set Z may be represented as the set of ordered pairs:

$Z = \{(z, \mu_Z(z)) : z \in U\} = \{(z, r_Z(z)e^{iw_Z(z)}) : zinU\}$

The partial ordering and their subset can be defined using amplitude and phase terms respectively.

Definition 12. (Complex vague set) [3,42,43]: A complex vague set Z can be defined over a universe of discourse U. The complex vague membership of an element $z \in U$ can be characterized by true r_{t_z} and false membership value r_{f_z} for the amplitude term where $0 \le r_{t_z} + r_{f_z} \le 1$. The phase term can be characterized by $w^r_{t_z}$ and $2\pi - w^r_{f_z}$ in real–valued interval $(0, 2\pi]$ and $i = \sqrt{-1}$. It can be represented as follows:

$Z = \{(z, [r_{t_z}, 1 - r_{f_z}] \times e^{w^r_{t_z}, 2\pi - w^r_{f_z}} : z \in U\}$. Similarly, the complex vague relationship among any two elements $z_1, z_2 \in U$ can be represented through true and false membership among their amplitude and phase terms. The partial ordering among two complex vague set can be defined based on properties of vague set for truth and false membership value independently for amplitude and phase term as shown in definitions 7 of this paper. Amplitude represents vagueness in the attributes through a seized scale of define vague set [0, 1]. The phase term is represented by $[0, 2\pi]$ to measure the fluctuation in uncertainty. Now, the concepts from a given complex vague context $\mathbf{F} = (X, Y, \tilde{R})$ can be generated as follows:

Step 1. Let us suppose number of attributes in the given complex vague context is m and number of objects is n which provides 2^m subset of attributes.

Step 2. In general the expert want maximum acceptation of attributes in the given phase of time for refining the knowledge based on his/her requirements. In this case amplitude can be considered as maximum true membership value i.e. 1.0 and minimum false membership value i.e. 0.0 for each of the attributes. This considered membership–value (1.0, 1.0) for amplitude and phase term can be used for each of the subsets $(y_j, [r_{t_{y_j}}, 1 - r_{f_{y_j}}])$ to discover the pattern in complex vague context. Similarly, the phase value can be computed for minimal fluctuation i.e. $(e^{w^r_{t_{y_j}}, 2\pi - w^r_{f_{y_j}}}) = (2\pi, 2\pi)$.

Step 3. Choose any of the attribute set to discover the pattern in given complex vague context using $\downarrow$ of Galois connection. It $\downarrow$ provides an maximal covering objects set while integrating the information from the chosen subset of attributes based on their amplitude and phase term as given below:

$(y_j, [r_{t_{y_j}}, 1 - r_{f_{y_j}}])^{\downarrow} = (x_i, [r_{t_{x_i}}, 1 - r_{f_{x_i}}])$,

$(e^{w^r_{t_{y_j}}, 2\pi - w^r_{f_{y_j}}})^{\downarrow} = (e^{w^r_{t_{x_i}}, 2\pi - w^r_{f_{x_i}}})$ for all $y_j \in Y$ where $j = 1, 2, \ldots, m$ and $i = 1, 2, 3, \ldots, n$.

Step 4. The membership value of the obtained objects set using $\downarrow$ on chosen subset of attributes can be computed for the amplitude and phase term as follows:

Amplitude:
min $(x_i, r_{t_{x_i}})$ for true membership and,
min $(x_i, 1 - r_{f_{x_i}})$ for false membership value.
Phase term:
min $(e^{w^r_{t_{x_i}}})$ for true phase term and,
min $(e^{2\pi - w^r_{f_{x_i}}})$ for false phase term.

Step 5. Now, apply the operator $\uparrow$ on these constituted objects set to find their maximal covering attributes based on amplitude and phase term as follows:

$(x_i, [r_{t_{x_i}}, 1 - r_{f_{x_i}}])^{\uparrow} = (y_j, [r_{t_{y_j}}, 1 - r_{f_{y_j}}])$,

$(e^{w^r_{t_{x_i}}, 2\pi - w^r_{f_{x_i}}})^{\uparrow} = (e^{w^r_{t_{y_j}}, 2\pi - w^r_{f_{y_j}}})$ for all $x_i \in X$ where $i = 1, 2, \ldots, n$ and $j = 1, 2, 3, \ldots, m$.

Step 6. The membership value of the obtained attributes (new attributes) using $\uparrow$ on the constituted objects set can be computed as follows:

Amplitude:
min $(y_j, r_{t_{y_j}})$ for true membership and,
min $(y_j, 1 - r_{f_{y_j}})$ for false membership value.
Phase term:
min $(e^{w^r_{t_{y_j}}})$ for true phase term and,
min $(e^{2\pi - w^r_{f_{y_j}}})$ for false phase term.

Step 7. The finally obtained pair of complex vague set of objects and attributes set (A, B) forms a complex vague concept which is closed with Galois connection.

Step 8. Similarly, other concepts can be generated using remaining subset of attributes.

Step 9. Remove the repeated complex vague concepts. To refine the knowledge adequately using all the distinct concepts.

Step 10. Draw the complex vague concept lattice structure as per their subsets for data analysis and processing tasks.

Table 5 represents the pseudo code of the proposed method to generate all the complex vague concepts using subset of attributes

Table 5
Proposed algorithm for generating the complex vague concepts.

Input: A complex vague context $\mathbf{F} = (X, Y, \tilde{R})$
where number of objects $= n$ and number of attributes $= m$.
Output: Set of complex vague formal concepts:
1. Write the subset of attributes 2^y and represent them as y_j
2. Set the membership value for the attribute set:
   Amplitude = (1.0, 1.0) and Phase = $(2\pi, 2\pi)$
3. Choose any subset of attributes and apply the operator $\downarrow$:
   Amplitude: $(y_j, [r_{t_{y_j}}, 1 - r_{f_{y_j}}])^{\downarrow} = (x_i, [r_{t_{x_i}}, 1 - r_{f_{x_i}}])$,
   Phase: $(e^{w^r_{t_{y_j}}, 2\pi - w^r_{f_{y_j}}})^{\downarrow} = (e^{w^r_{t_{x_i}}, 2\pi - w^r_{f_{x_i}}})$
4. Compute the complex vague membership for the obtained objects:
   Amplitude:
   min $(x_i, r_{t_{x_i}})$ for true membership and,
   min $(x_i, 1 - r_{f_{x_i}})$ for false membership value.
   Phase term:
   min $(e^{w^r_{t_{x_i}}})$ for true phase term and,
   min $(e^{2\pi - w^r_{f_{x_i}}})$ for false phase term.
5. Apply the operator $\uparrow$ on the constituted objects set:
   Amplitude: $(x_i, [r_{t_{x_i}}, 1 - r_{f_{x_i}}])^{\uparrow} = (y_j, [r_{t_{y_j}}, 1 - r_{f_{y_j}}])$,
   Phase: $(e^{w^r_{t_{x_i}}, 2\pi - w^r_{f_{x_i}}})^{\uparrow} = (e^{w^r_{t_{y_j}}, 2\pi - w^r_{f_{y_j}}})$
6. Compute the complex vague membership value for the obtained attributes set:
   Amplitude:
   min $(y_j, r_{t_{y_j}})$ for true membership and,
   min $(y_j, 1 - r_{f_{y_j}})$ for false membership value.
   Phase term:
   min $(e^{w^r_{t_{y_j}}})$ for true phase term and,
   min $(e^{2\pi - w^r_{f_{y_j}}})$ for false phase term.
7. Write the generated complex vague concepts (A, B)
8. Similarly, all complex vague concepts can be generated using other subsets.
9. Remove all the repeated complex vague concepts.
10. Draw the complex vague concept lattice based on their subset.

(2^m) using maximum acceptation for amplitude and phase term as shown in Step 1 and 2. Then find their maximally covering objects set using the operator ↓ on the chosen subset of attributes as shown in Step 3. The membership value for amplitude and phase term of covering objects set can be computed using the properties of complex vague set as shown in Step 4. Now, the proposed method applies the operator ↑ on these constituted objects set to find their maximal covering attributes while integrating the information from them as shown in Step 5. The membership value for the obtained attributes set can be computed using the properties of complex vague set as shown in Step 6. In this way the proposed method gives a pair of objects–attributes set which are closed with Galois connection as shown in Step 7. Similarly, other complex vague concepts can be generated using the remaining subset of attributes as shown in Step 8. Step 9 remove those concepts which are repeated. In last the complex vague concept lattice can be built using their subset hierarchy. It is one of the advantages of proposed method while drawing the hierarchical order among them in concept lattice visualization.

Complexity: Let us suppose m is number of attributes and n is the number of objects in the given complex vague context. Computing the subset of attributes will takes O(2^m) computational time whereas finding its covering objects set will take n time for amplitude and same time for phase term. In this case the proposed method takes total O($2^{m*}n^2$) computation cost to generate the complex vague concepts. The proposed algorithm provides an effective representation of vagueness through amplitude in seized scale [0, 1] and uncertainty through the phase term [0, 2π] of a defined complex vague set. This helps more in refining the knowledge in complex data set with vague attribute at given phase of time. It is one of the favourable output of the proposed method when compared to any other approaches in FCA with vague (or bipolar fuzzy) setting.

4. Illustration

In the last decade, many researchers have tried to analyze the hidden pattern in a given medical data set using the properties of concept lattice [2,7–10]. In general the medical diagnoses data set contains lots of incomplete, vague or uncertain information which values fluctuate at given phase of time. In this case adequate analysis of hidden pattern in medical diagnoses data set and their interpretation is computationally expensive tasks from any of the available approaches. To overcome from this issue, recently Ye [1] tried to characterize the medical diagnoses data set based on truth, false, and indeterminacy membership–values, whereas, Prem Kumar Singh [18] displayed their super–sub concept visualization in the concept lattice. These extensions are also unable to highlight the fluctuation of uncertainty in attributes of medical diagnoses data at given time–interval. To deal with problem precisely, recently, the calculus of complex fuzzy logic [4–6] and its corresponding relationship [42,43] is introduced to represent the medical diagnoses data set using a defined complex vague soft set [3]. The current paper put forward effort to analyze the medical data set using the calculus of complex vague set [3,5,6], concept lattice [11,12] and its extensive properties [13–21]. This research is essential because it affects the human life directly or indirectly. To fulfill this requirement, a method is proposed in Table 5 to generate all the hidden pattern (i.e. formal concepts) in a given medical data set to reveal some interested information as given below:

Example 2. Let us suppose a company manufactures set of medicines–{x_1, x_2, x_3, x_4} to diagnoses the following disease–(y_1=pneumonia, y_2=influenza, y_3=tuberculosis, y_4=asthma). In this case the manufacturing company can take suggestions from different experts on these medicine and their impact on diagnoses of the given disease. The expert provides following analysis to diagnoses the pneumonia based on given medicine:

- The medicine x_1 diagnosis the pneumonia (y_1) 20 to 30% percent in one to four months. The medicine x_2 diagnosis the pneumonia (y_1) 50 to 80% in one to six months.
- The medicine (x_3) diagnosis the pneumonia (y_1)40 to 70 % in four to ten months.
- The medicine (x_4) diagnosis the pneumonia (y_1) 30 to 50% in two to six months.

The obtained evidence (i.e. measuring the vagueness) to diagnose the pneumonia (y_1) and its fluctuation (i.e. uncertainty) based on all the medicine (x_1, x_2, x_3, x_4 can be written via amplitude and phase term of a defined complex vague set. The phase term can be considered as [0, 2π] to represent the diagnoses of disease from the given medicine in a yearly basis. In this case, if the medicine diagnoses the disease in 12 months then, its phase time can be considered as 2π. Similarly the phase value can be computed for other time of interval as shown below:

(a) y_1={$[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/x_1$, $[0.5, 0.8]e^{i[0.2\pi,1.2\pi]}/x_2$, $[0.4, 0.7]e^{i[0.8\pi,1.8\pi]}/x_3$, $[0.3, 0.5]e^{i[0.3\pi,\pi]}/x_4$}.

Subsequently, diagnoses of influenza (y_2), tuberculosis (y_3), and asthma (y_4) based on given medicines (x_1, x_2, x_3, x_4) can be written through a complex vague set as shown below [3]:

(b) y_2 = {$[0.2, 0.4]e^{i[0.4\pi,0.6\pi]}/x_1$, $[0.9, 1]e^{i[0\pi,\pi]}/x_2$, $[0.8, 0.95]e^{i[0.6\pi,0.6\pi]}/x_3$, $[0.5, 0.5]e^{i[0.3\pi,1.5\pi]}/x_4$}.

(c) y_3 = {$[0.6, 0.7]e^{i[0\pi,0.7\pi]}/x_1$, $[0.6, 0.8]e^{i[0.4\pi,1.4\pi]}/x_2$, $[0.4, 0.8]e^{i[0.2\pi,0.6\pi]}/x_3$, $[0.2, 0.5]e^{i[0.1\pi,0.5\pi]}/x_4$}.

(d) y_4={$[0.4, 0.6]e^{i[0.4\pi,0.8\pi]}/x_1$, $[0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/x_2$, $[0.3, 0.9]e^{i[0.2\pi,1.4\pi]}/x_3$, $[0.4, 0.9]e^{i[\pi,2\pi]}/x_4$}.

Now, the problem with doctor is to analyze the most suitable medicine which diagnosis the given disease in given phase of time. The same problem with company to accelerate its manufacturing process. To evaluate this analysis some interested patterns among medicine and its effect on the given disease is required. To achieve this goal shown in Section 3 of this paper can be applied to visualize the above given numerical data in the concept lattice. For this purpose above given data set is represented in form of a complex vague context as shown in Table 6.

Step (1) All the generated complex vague subset of attributes are as follows:

1. {∅},
2. {$[1.0, 1.0]e^{i[2\pi, 2\pi]}/y_1$},
3. {$[1.0, 1.0]e^{i[2\pi, 2\pi]}/y_2$},
4. {$[1.0, 1.0]e^{i[2\pi, 2\pi]}/y_3$},
5. {$[1.0, 1.0]e^{i[2\pi, 2\pi]}/y_4$},
6. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2$},
7. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3$},
8. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4$},
9. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3$},
10. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4$},
11. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4$},
12. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3$},
13. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4$},
14. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4$},
15. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_5$},
16. {$[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4$},

Step (2) Let us consider the first subset of attributes i.e. 1.{∅} to generate the complex vague concepts. To find it apply the operator ↓ to find its maximal covering objects as given below:

Table 6
A complex vague context representation of medicine and its effect on the given disease.

	x_1	x_2	x_3	x_4
y_1	$[0.2, 0.3]e^{i[0.2\pi,\ 0.8\pi]}$	$[0.5, 0.8]e^{i[0.2\pi,\ 1.2\pi]}$	$[0.4, 0.7]e^{i[0.8\pi,\ 1.8\pi]}$	$[0.3, 0.5]e^{i[0.3\pi,\ \pi]}$
y_2	$[0.2, 0.4]e^{i[0.4\pi,\ 0.6\pi]}$	$[0.9, 1]e^{i[0\pi,\ \pi]}$	$[0.8, 0.95]e^{i[0.6\pi,\ 0.6\pi]}$	$[0.5, 0.5]e^{i[0.3\pi,\ 1.5\pi]}$
y_3	$[0.6, 0.7]e^{i[0\pi,\ 0.7\pi]}$	$[0.6, 0.8]e^{i[0.4\pi,\ 1.4\pi]}$	$[0.4, 0.8]e^{i[0.2\pi,\ 0.6\pi]}$	$[0.2, 0.5]e^{i[0.1\pi,\ 0.5\pi]}$
y_4	$[0.4, 0.6]e^{i[0.4\pi,\ 0.8\pi]}$	$[0.1, 0.4]e^{i[0.1\pi,\ 0.8\pi]}$	$[0.3, 0.9]e^{i[0.2\pi,\ 1.4\pi]}$	$[0.4, 0.9]e^{i[\pi,\ 2\pi]}$

$\{\oslash\}^{\downarrow} = \{[1.0, 1.0]e^{i[2\pi,2\pi]}/x_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/x_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/x_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/x_4\}$.

Now apply the operator↑ on these constituted objects set to find their maximal covering attributes as given below:

$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/x_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/x_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/x_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/x_4\}^{\uparrow}=$

$\{[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/y_1 + [0.2, 0.4]e^{i[0\pi,0.6\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/y_4\}$

These provides following complex vague concepts:

1. Extent:
$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/x_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/x_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/x_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/x_4\}$,
Intent:
$\{[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/y_1 + [0.2, 0.4]e^{i[0\pi,0.6\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/y_4\}$.

Step (3) Now consider the second subset of attributes i.e. 2. $\{[1.0,\ 1.0]e^{i[2\pi,\ 2\pi]}/y_1\}$ to generate the complex vague concepts. To find it apply the operator ↓ to find its maximal covering objects as given below:

$\{[1.0,\ 1.0]e^{i[2\pi,\ 2\pi]}/y_1\}^{\downarrow}=$

$\{[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/x_1 + [0.5, 0.8]e^{i[0.2\pi,1.2\pi]}/x_2 + [0.4, 0.7]e^{i[0.8\pi,1.8\pi]}/x_3 + [0.3, 0.5]e^{i[0.3\pi,\pi]}/x_4\}$.

Now apply the operator ↑ on these constituted objects set to find their maximal covering attributes as given below:

$\{[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/x_1 + [0.5, 0.8]e^{i[0.2\pi,1.2\pi]}/x_2 + [0.4, 0.7]e^{i[0.8\pi,1.8\pi]}/x_3 + [0.3, 0.5]e^{i[0.3\pi,\pi]}/x_4\}^{\uparrow}=$

$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[0\pi,0.6\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/y_4\}$.

These provides following complex vague concepts:

2. Extent:
$\{[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/x_1 + [0.5, 0.8]e^{i[0.2\pi,1.2\pi]}/x_2 + [0.4, 0.7]e^{i[0.8\pi,1.8\pi]}/x_3 + [0.3, 0.5]e^{i[0.3\pi,\pi]}/x_4\}$.
Intent:
$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[0\pi,0.6\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/y_4\}$.

Step (4) Similarly following concepts can be generated using other given subset of attribute shown in step 1 :

3. Extent:
$\{[0.2, 0.4]e^{i[0.4\pi,0.6\pi]}/x_1 + [0.9, 1]e^{i[0\pi,\pi]}/x_2 + [0.8, 0.95]e^{i[0.6\pi,0.6\pi]}/x_3 + [0.5, 0.5]e^{i[0.3\pi,1.5\pi]}/x_4\}$.
Intent:
$\{[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/y_4\}$.

4. Extent:
$\{[0.6, 0.7]e^{i[0\pi,0.7\pi]}/x_1 + [0.6, 0.8]e^{i[0.4\pi,1.4\pi]}/x_2 + [0.4, 0.8]e^{i[0.2\pi,0.6\pi]}/x_3, [0.2, 0.5]e^{i[0.1\pi,0.5\pi]}/x_4\}$.
Intent:
$\{[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/y_1 + [0.2, 0.4]e^{i[0\pi,0.6\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/y_4\}$.

5. Extent:
$\{[0.4, 0.6]e^{i[0.4\pi,0.8\pi]}/x_1 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/x_2 + [0.3, 0.9]e^{i[0.2\pi,1.4\pi]}/x_3 + [0.4, 0.9]e^{i[\pi,2\pi]}/x_4\}$.
Intent:
$\{[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/y_1 + [0.2, 0.4]e^{i[0\pi,0.8\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4\}$.

6. Extent:
$\{[0.2, 0.3]e^{i[0.2\pi,0.6\pi]}/x_1 + [0.5, 0.8]e^{i[0\pi,\pi]}/x_2 + [0.4, 0.7]e^{i[0.6\pi,0.6\pi]}/x_3 + [0.3, 0.5]e^{i[0.3\pi,\pi]}/x_4\}$.
Intent:
$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4\}$.

7. Extent:
$\{[0.2, 0.3]e^{i[0\pi,0.7\pi]}/x_1 + [0.5, 0.8]e^{i[0.2\pi,0.4\pi]}/x_2 + [0.4, 0.7]e^{i[0.2\pi,0.6\pi]}/x_3 + [0.2, 0.5]e^{i[0.1\pi,0.5\pi]}/x_4\}$.
Intent:
$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[0\pi,0.6\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/y_4\}$.

8. Extent:
$\{[0.2, 0.3]e^{i[0\pi,0.8\pi]}/x_1 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/x_2 + [0.3, 0.7]e^{i[0.2\pi,1.4\pi]}/x_3 + [0.3, 0.5]e^{i[0.3\pi,\pi]}/x_4\}$.
Intent:
$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4\}$.

9. Extent:
$\{[0.2, 0.4]e^{i[0\pi,0.6\pi]}/x_1 + [0.6, 0.8]e^{i[0\pi,\pi]}/x_2 + [0.4, 0.8]e^{i[0.2\pi,0.6\pi]}/x_3 + [0.2, 0.5]e^{i[0.1\pi,0.5\pi]}/x_4\}$.
Intent:
$\{[0.2, 0.3]e^{i[0.2\pi,0.8\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/y_4\}$.

10. Extent:
$\{[0.2, 0.4]e^{i[0.4\pi,0.6\pi]}/x_1 + [0.1, 0.4]e^{i[0\pi,0.8\pi]}/x_2 + [0.3, 0.9]e^{i[0.2\pi,0.6\pi]}/x_3 + [0.4, 0.5]e^{i[0.3\pi,1.5\pi]}/x_4\}$.
Intent:
$\{[0.2, 0.5]e^{i[0.2\pi,0.8\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4\}$.

11. Extent:
$\{[0.4, 0.6]e^{i[0\pi,0.7\pi]}/x_1 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/x_2 + [0.3, 0.8]e^{i[0.2\pi,0.6\pi]}/x_3 + [0.2, 0.5]e^{i[0.1\pi,0.5\pi]}/x_4\}$.
Intent:
$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [0.2, 0.4]e^{i[0\pi,0.6\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4\}$.

12. Extent:
$\{[0.2, 0.3]e^{i[0\pi,0.6\pi]}/x_1 + [0.5, 0.8]e^{i[0\pi,\pi]}/x_2 + [0.4, 0.7]e^{i[0.2\pi,0.6\pi]}/x_3 + [0.2, 0.5]e^{i[0.1\pi,0.5\pi]}/x_4\}$.
Intent:
$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/y_4\}$.

13. Extent:
$\{[0.2, 0.3]e^{i[0.2\pi,0.6\pi]}/x_1 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/x_2 + [0.3, 0.7]e^{i[0.2\pi,0.6\pi]}/x_3 + [0.3, 0.5]e^{i[0.3\pi,\pi]}/x_4\}$.
Intent:
$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [0.2, 0.5]e^{i[0\pi,0.5\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4\}$.

14. Extent:
$\{[0.2, 0.3]e^{i[0\pi,0.7\pi]}/x_1 + [0.1, 0.4]e^{i[0.1\pi,0.8\pi]}/x_2 + [0.3, 0.7]e^{i[0.2\pi,0.6\pi]}/x_3 + [0.2, 0.5]e^{i[0.1\pi,0.5\pi]}/x_4\}$.
Intent:
$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[0\pi,0.6\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4\}$.

15. Extent:
$\{[0.2, 0.4]e^{i[0\pi,0.6\pi]}/x_1 + [0.1, 0.4]e^{i[0\pi,0.8\pi]}/x_2 + [0.3, 0.8]e^{i[0.2\pi,0.6\pi]}/x_3 + [0.2, 0.5]e^{i[0.1\pi,0.5\pi]}/x_4\}$.
Intent:

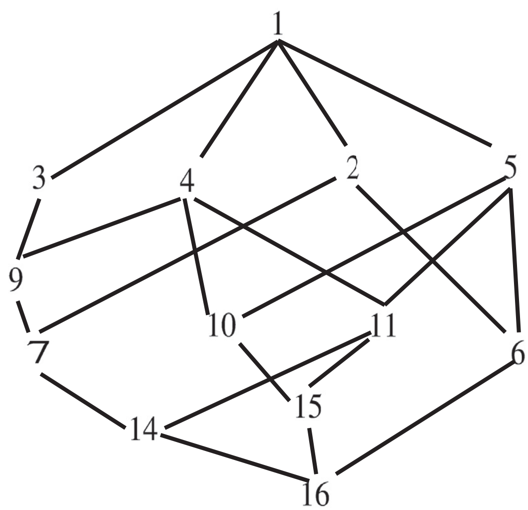

Fig. 2. A complex vague concept lattice generated from context shown in Table 6.

$\{[1.0, 0.3]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4\}$.

16. Extent:

$\{[0.2, 0.3]e^{i[0\pi,0.6\pi]}/x_1 + [0.1, 0.4]e^{i[0\pi,0.8\pi]}/x_2 + [0.3, 0.7]e^{i[0.2\pi,0.6\pi]}/x_3 + [0.2, 0.5]e^{i[0.1\pi,0.3\pi]}/x_4\}$.

Intent:

$\{[1.0, 1.0]e^{i[2\pi,2\pi]}/y_1 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_2 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_3 + [1.0, 1.0]e^{i[2\pi,2\pi]}/y_4\}$.

In the above generated concepts number 6, 8, and 13 are similar. Similarly concept number 12 and 7 are similar. Hence distinct complex vague concepts are 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 14, 15 and 16 which concept lattice is shown in Fig. 2. From that following information can be extracted:

- Concept number 1 represents none of the disease can be diagnoses using all the medicines simultaneously.
- Concept number 2 represents that disease y_1 (pneumonia) and y_2 (influenza) can be recovered by medicine $[0.5, 0.8]e^{i[0.2\pi,\ 1.2\pi]}/x_2$ in five to six month due to its maximal relationship with them.
- Concept number 3 represents that disease y_2 (influenza) has strong relationship with medicine $[0.9, 1]e^{i[0\pi,\ \pi]}/x_2$. It shows that influenza can be recovered using medicine x_2 in five to six months.
- Concept number 4 represents that disease y_3 (tuberculosis) has maximal relationship with medicine $[0.6, 0.8]e^{i[0.4\pi,\ 1.4\pi]}/x_2$. It shows that tuberculosis can be recovered using medicine x_2 in five to six months.
- Concept number 5 represents that disease y_4 (asthma) has maximal relationship with medicine $[0.4, 0.9]e^{i[\pi,\ 2\pi]}/x_4$. It shows that asthma can be recovered using medicine x_4 within six months.
- Concept number 6 represents that disease y_1(pneumonia), y_2 (influenza), y_4 (asthma) have maximal relationship with medicine $[0.5, 0.8]e^{i[0\pi,\ \pi]}/x_2$. It shows that these disease can be recovered using the medicine x_2 within six month.
- Concept number 7 represents that disease y_1(pneumonia), y_2 (influenza), y_3 (tuberculosis) have maximal relationship with $[0.5, 0.8]e^{i[0.2\pi,\ 0.4\pi]}/x_2$. It shows that these disease can be recovered using medicine x_2 within a months.
- Concept number 16 represents that all the disease can be diagnoses 30–70% using medicine $[0.3, 0.7]e^{i[0.2\pi,\ 0.6\pi]}/x_3$ within two months.

Table 7
Comprehension on uses of set theory, lattice, and graph for analyzing pattern in medical data set.

Different medical data	Set theory	Lattice	Graph analytics	Pattern in data
Medical Diagnoses	Neutrosophic Set [1]	*	*	Correlation [1]
Gene Data	Binary Context [2]	Concept Lattice [2]	*	Interval Pattern [2]
Medicine Analysis	Complex Vague set [3]	*	*	CVSS method [3]
China Medicine	Binary Context [7]	Concept Lattice [7]	*	Formal concept [7]
Breast Cancer	Binary Context [8]	Concept Lattice [8]	*	Formal concept [8]
TB data Set	Binary Context [9]	Concept Lattice [9]	*	Formal concept [9]
Health care Data	Binary Context [10]	Concept Lattice [10]	*	Formal concept [10]
Medical Diagnoses	Neutrosophic Set [18]	Concept Lattice [18]	Neutrosophic Graph [18]	Formal Concept [18]

The aforementioned representation and derived analysis from the proposed method resembles with CVSS method [3] with following advantages:

- The proposed method provides many hidden pattern to analyze the best suitable medicine for the diagnoses of particular disease at given phase of time.
- The proposed method provides precise visualization of vagueness and uncertainty in medical attributes via concept lattice rather than its numerical representation.
- The proposed method provides a hierarchical order visualization of generated pattern in a given complex vague context within $O(2^{m*}n^2)$ computational time. This is another advantages of the proposed method while refine the knowledge.

Due to the mentioned advantages of the proposed method it can be useful in various research fields for data analysis and processing tasks [23,42,43]. In future work will be focused on depth analysis of complex vague concept lattice and its several applications.

5. Discussions

Recent years, many researchers have paid attention for discovering some of the interested pattern in medical data set using the properties of concept lattice as shown in Table 7. Among them some notable researches are neutrosophic set based medical diagnoses [1], handling gene expression [2], Chinese medicine discovery [7], Breast cancer [8], TB [9] and Health care [10] data set have received much attention. Each of these available methods unable to highlight the uncertainty and their fluctuation at given phase of time. To deal with this problem recently Prem Kumar Singh [18] characterized it based on acceptation, rejection and uncertain regions of a defined three–way fuzzy concept lattice. Selvachandran et al. [3,43] tried to represent the vagueness and fluctuation in uncertainty of a given medical diagnoses data set based on amplitude and phase term of a defined complex vague soft set, respectively. It can be observe that less attention has been towards handling vagueness and uncertainty in medical data using the properties of graph and concept lattice as marked * in Table 7. However this connection provides adequate and precise analysis of medical diagnoses through graphical visualization rather than its numerical representation. To fulfill this backdrop the current paper put forward effort to analyze the vagueness and uncertainty in medical data set using amplitude and phase term of a defined complex vague concept lattice. To achieve this goal following proposal are made in this paper:

(1) This paper proposed a method to analyze the fluctuation in medical diagnoses using complex vague context as shown in Table 6.
(2) This paper proposed a method to discover all the hidden pattern (i.e. formal concepts) in a given medical data set as shown in Table 5.
(3) To refine the knowledge building the complex vague concept lattice structure is proposed in Section 3 based on subset of attributes.
(4) One application of the proposed method is also discussed in Section 4.
(5) The analysis derived from the proposed method is also compared with CVSS [3] method and shown that obtained results are in agreement with each other. However, the proposed method provides better representation of uncertainty and vagueness in medical data through concept lattice when compare to its numerical representation by CVSS method [3].

Modelling uncertainty and vagueness in non–linear problems is one of the major concern for the expert of any research fields. However the analysis of this paper is focused on medical diagnoses data set. To deal with this notable problem current paper provides an alternative way in Section 3 to represent the vagueness and uncertainty in data using complex vague context. Further to find the hidden pattern in the obtained complex vague context a method is proposed to generate all concepts using their chosen subset of attributes as shown in Table 5. It is shown that Fig. 2 generated by the proposed method provides an effective way to analyze the complex vague context rather than its numerical representation done by CVSS method [3]. However the analysis derived from the proposed method resembles with CVSS method [3] with more depth analysis by their concept lattice visualization. Similarly, it can be observed that the proposed method in this paper is distinct from any of the available approaches shown in Table 7 in following aspects:

(1) The proposed method provides an adequate representation of vagueness in attributes and their fluctuation using amplitude and phase of a defined complex vague set.
(2) The proposed method introduced an alternative way to represent the large number of generated medical diagnoses data set in form of context and their compact display in the concept lattice. To improve the knowledge processing tasks when compare to its numerical representation.
(3) The proposed method established a mathematical model to analyze some hidden pattern in medical diagnoses data set based on extent and intent pair and their hierarchical order visualization rather than traditional way.

Due to above advantages of the proposed methods it can be applied in various research fields for modelling the uncertainty and vagueness in data [23]. In future the research work will be focus on depth analysis of complex vague concept lattice [42,43], reduction [44,45] and its applications in various research fields [23].

6. Conclusions

This paper introduced a method to provide an effective way to analyze the uncertainty and vagueness in complex (or dynamic) data set using complex vague concept lattice. To achieve this goal a method is proposed to discover all the hidden pattern in given complex data set using the properties of complex fuzzy logic and Galois connection within $O(n^2.2^m)$ complexity. The proposed method is demonstrated on a medical diagnoses data set and shown that obtained results are resembled with CVSS method [3]. In addition, the proposed method provides many pattern (i.e. complex vague concept) and their compact display in the concept lattice when compare to its numerical representation.

Disclosure and conflicts of interest

Author discloses that there is no conflict of interest.

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