

Correlation and ranking analysis using Neutrosophic sets in a Multi-Criteria single-valued decision-making problems

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Abstract: In this Paper, we investigate the group decision making problems in which all the information provided by the decision-makers is presented as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by interval-valued intuitionistic fuzzy number (IVIFN), and the information about attribute weights is partially known. We use the interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator and interval-valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator to aggregate all individual interval-valued intuitionistic fuzzy decision matrices provided by the decision-makers into the collective interval-valued intuitionistic fuzzy decision matrix, and then we use the score function to calculate the score of each attribute value and construct the score matrix of the collective interval-valued intuitionistic fuzzy decision matrix. From the score matrix and the given attribute weight information, we establish an optimization model.

Keywords: Multi-criteria decision-making, a single-valued Neutrosophic sets fuzzy correlation coefficient. Ranking method

I. INTRODUCTION

In the real world, the decision-making problems with incomplete or inaccurate information are difficult to be precisely expressed by decision-makers. Under these circumstances, Zadeh [1] firstly proposed the theory of fuzzy sets (FSs), where the membership degree is presented using a crisp value between zero and one, have been applied successfully in many different fields. However, FSs only have a membership and lack non-membership degree. In order to solve the problem, Atanassov [1] proposed the intuitionistic fuzzy sets (IFSs), which is an extension of Zadeh's FSs. IFSs have been widely extended and got more attention in solving MCDM problems [3]. Although the theories of FSs and IFSs have been generalized, it can not handle all kinds of uncertainties in many cases. The indeterminate information and inconsistent information existing commonly in the real world can not be deal with by FSs and IFSs. For example, during a voting process, forty percent vote "yes", thirty percent vote "no", twenty percent are not sure, and ten percent give up. This issue is beyond the scope of IFSs, which cannot distinguish the information between unsure and giving up. Therefore, on the basis of IFSs, Smarandache introduced neutrosophic logic and neutrosophic sets (NSs) by adding an independent indeterminacy-membership. Then, the aforementioned example can be expressed as $x(0.4,0.2,0.3)$ with respect to NSs. Moreover, true-membership, indeterminacy-membership and false-membership in NSs are completely independent, whereas the uncertainty is dependent on the true-membership and false-membership in IFSs. So the notion of NSs is more general and overcomes the aforementioned issues. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators will be difficult to apply in the real application without specific description. Therefore, a single-valued Neutrosophic set (SVNS) is proposed [8], which is an extension of NSs, and some properties of SVNS are also provided. Ye [9] proposed the correlation coefficient and weighted correlation coefficient of SVNSs, and proved that the cosine similarity degree is a special case of the correlation coefficient in SVNS. Majumdar [11] defined similarity measures between two SVNSs and introduced a measure of entropy of SVNSs. Ye [12] proposed the cross-entropy of SVNSs. Furthermore, Ye [13] introduced the concept of simplified neutrosophic sets (SNSs), and proposed a MCDM method using a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator. Liu [14] proposed a multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Wang [15] proposed the concept of interval neutrosophic set (INS) and gave the set-theoretic operators of INS. Zhang [16] defined the operations for INSs, and developed two interval neutrosophic number aggregation operators. Ye [17] defined the Hamming and Euclidean distances between INSs, and proposed the similarity measures between INSs based on the relationship between similarity measures and distances. Liu [18] proposed some Hamacher aggregation operators for the interval-valued intuitionistic fuzzy numbers. Peng [19] defined multi-valued NSs, and discussed operations based on Einstein. Liu [20] proposed the concept of the interval neutrosophic hesitant fuzzy set, presented the operations and developed generalized hybrid weighted aggregation

F. Definition 5:

$x = (T, I_1, F_1)$ and $y = (T_2, I_2, F_2)$ any two SVNNS, then the hamming distance between x and y can be defined as follows:

$$d(x,y) = |T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2| \text{ ----- (1)}$$

G. DEFINITION 6

Let $x = (T, I, F)$ be a SVNNS, and the cosine similarity measure $S(x)$ between SVNNS, x and the ideal alternative $(1,0,0)$ can be defined as follows:

$$S(x) = \frac{T}{\sqrt{T^2 + I^2 + F^2}} \text{ ----- (2)}$$

III. RANK TECHNIQUES IN MULTI-CRITERIA DECISION-MAKING METHOD

A. Definition 3.1

Let $\tilde{a}_j = (t_j, 1 - f_j), j = 1, \dots, n$ be a collection of vague values, and let the vague fuzzy weighted averaging operator VWA is defined as VWA: $Q_n \rightarrow Q$ if $VMA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \sum_{j=1}^n w_j \tilde{a}_j = (1 - \prod_{j=1}^n (1 - t_j)^{w_j}, \prod_{j=1}^n (1 - f_j)^{w_j})$ where the weight vector $w = (w_1, w_2, \dots, w_m)^T$ of the attributes can be determined in advance. Note that $w_i > 0$ for each $i = 1$ to n , and $\sum_{j=1}^n w_j = 1$.

B. Definition 3.2

Let $\tilde{a}_j = (t_j, f_j), j = 1, 2, \dots, n$ be a collection of vague values, and let the vague fuzzy hybrid weighting average operator VHA is defined as: VHA: $Q_n \rightarrow Q$ if $VHA_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = (1 - \prod_{j=1}^n (1 - t_j)^{w_j}, 1 - \prod_{j=1}^n (1 - f_j)^{w_j})$ where the weight vector $w = (w_1, w_2, \dots, w_m)^T$ of the attributes can be determined in advance. Note that $w_i > 0$ for each $i = 1$ to n , and $\sum_{j=1}^n w_j = 1$.

C. Model Assumptions And Procedures 3.3

Let $A = \{A_1, A_2, \dots, A_n\}$ be a set of alternatives, $G = \{G_1, G_2, \dots, G_n\}$ be the set of alternatives, $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ is the weighting vector of the attribute $G_j, j=1,2,\dots,n$, where $\omega_j \in [0,1], \sum_{j=1}^n \omega_j = 1$. Let $D = \{D_1, D_2, \dots, D_t\}$ be the set of decision

makers, $V = (V_1, V_2, \dots, V_n)$ be the weighting vector of the decision makers, with $V_k \in [0,1], \sum_{k=1}^t V_k = 1$. Let

$\tilde{R}_k = \left(\tilde{r}_{ij}^{(k)} \right)_{m \times n} = \left(t_{ij}^{(k)}, f_{ij}^{(k)} \right)_{m \times n}$ be the vague decision matrix, where $t_{ij}^{(k)}$ is the degree of the truth membership value that the alternative A_i satisfies the attribute G_j given by the decision maker D_k and $f_{ij}^{(k)}$ is the degree of false membership value that the alternative for the alternative A_i , where $t_{ij}^{(k)}, f_{ij}^{(k)} \in [0,1]$ and $t_{ij}^{(k)} + f_{ij}^{(k)} \leq 1, i = 1, 2, \dots, m, j = 1, 2, \dots, n$, and $k = 1, 2, \dots, t$.

D. An algorithm for a Developed Model of Magdm

Here the steps mentioned below are studied for a model of MAGDM.

E. Algorithm: the Following Steps are Now Given

1) Step 1: Utilize the vague decision matrix $R^k = (\tilde{r}_{ij}^{(k)})_{m \times n} = ((t_{ij}^{(k)}, 1 - (f_{ij}^{(k)}))_{m \times n}$, and the FWA operator which has the associated weighting vector $w = (w_1, w_2, \dots, w_m)^T$ generated from the definition (3.3). Let $(\tilde{r}_{ij}^{(k)})^k = (t_{ij}^{(k)}, 1 - f_{ij}^{(k)}), i = 1, \dots, m; j = 1, 2, \dots, n$ be a matrix of vague values for each $k = 1$ to t . Let $R^k = ((\tilde{r}_{ij}^{(k)})^k)$ be the collection of t number of $m \times n$ matrices of each the form $R^k = ((\tilde{r}_{ij}^{(k)})^k)$ where $k = 1, 2, \dots, t$. Then the operator FWA: $[(M_{m \times n})^k \rightarrow M_{m \times n}] R^1, R^2, \dots, R^k \rightarrow R(r_{ij})$ is defined by VWA $((\tilde{r}_{ij}^{(1)})^k, (\tilde{r}_{ij}^{(2)})^k, \dots, (\tilde{r}_{ij}^{(k)})^k)$ which is found due to the definition (3.1). Here $V = (V_1, V_2, \dots, V_t)$ be the weighting vector of the decision maker or generated from the definition (3.3).

- 2) *STEP 2:* Utilizing the information from the collective decision matrix $R = (C_{ij}, D_{ij})_{m \times n}$ found in the step 1. Then NFHWA operator $R = \tilde{r}_i = (t_i, 1 - f_i)$ is defined by $(1 - (\prod_{j=1}^n (1 - c_{ij})^{w_j}), 1 - (\prod_{j=1}^n (1 - c_{ij})^{w_j}), i = 1, 2, \dots, m$ derive the collective overall preference values of the alternative A_i , which have weight w_i in such a way that the weighting vector as $w = (w_1, w_2, \dots, w_m)^T$ generated from the definition (3.3).
- 3) *STEP 3:* Calculate the distance between the collective overall preference values and the positive ideal vague value \tilde{r}^+ , or the negative ideal vague value \tilde{r}^- , where $\tilde{r}^+ = (1,0)$ and $\tilde{r}^- = (0,1)$. Using the Euclidean distance function we can find the distances between the collective overall preference values \tilde{r}_i and the positive ideal vague value \tilde{r}^+ as follows:

$$d(\tilde{r}_i, \tilde{r}^+) = \sqrt{\frac{1}{2} \sum_{i=1}^n \left[(t_{r_i}(x_i) - t_{r^+}(x_i))^2 + ((1 - f_{r_i}(x_i)) - (1 - f_{r^+}(x_i)))^2 \right]}$$

- 4) *STEP 4:* Rank all the alternatives A_i , where $i = 1, 2, \dots, m$ and select the best one in accordance with the distance obtained in step 3.

F. Numerical Illustration

Suppose an investment company, wanting to invest a sum of money in the best option, and there is a panel with five possible alternatives to invest the money; A_1 is an IT company; A_2 is a multinational company; A_3 is a tools company, A_4 is an airlines company and A_5 is an automobile company. The investment company must take a decision according to the four following attributes; G_1 is the risk analysis, G_2 is the growth analysis, G_3 is the socio-political impact analysis and G_4 is the environmental impact analysis. The five possible alternatives A_i , where $i = 1, 2, \dots, m$, are to be evaluated by three decision makers whose weighting vector is $V = (0.12, 0.16, 0.20, 0.24, 0.28)^T$ under the method in definition (3.3) with $r = 1$, $\alpha = 0.4$, & $n = 5$, and above said four attributes whose weighting vector is $w = (0.16, 0.22, 0.28, 0.34)^T$, which is generated from the method in (3.3) with $r = 1$, $\alpha = 0.4$, & $n = 4$:

$$R_1 = \begin{bmatrix} (0.4873, 0.7256) & (0.5221, 0.7222) & (0.6286, 0.8312) & (0.4427, 0.9986) \\ (0.3271, 0.9001) & (0.6676, 0.5413) & (0.4261, 0.8126) & (0.7710, 0.9442) \\ (0.5238, 0.8011) & (0.4278, 0.5261) & (0.5527, 0.6216) & (0.5687, 0.7981) \\ (0.7218, 0.6283) & (0.7213, 0.8912) & (0.8311, 0.9219) & (0.6626, 0.8215) \\ (0.6257, 0.7983) & (0.8321, 0.9426) & (0.6256, 0.7119) & (0.4136, 0.6295) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} (0.4351, 0.7846) & (0.5121, 0.7221) & (0.1009, 0.6221) & (0.2217, 0.7184) \\ (0.6321, 0.8221) & (0.6226, 0.8108) & (0.3009, 0.5129) & (0.6225, 0.9105) \\ (0.5387, 0.9105) & (0.4124, 0.7216) & (0.5010, 0.7101) & (0.4491, 0.5426) \\ (0.7317, 0.8119) & (0.5221, 0.8001) & (0.2091, 0.4104) & (0.2101, 0.4110) \\ (0.5273, 0.6217) & (0.3125, 0.7278) & (0.4728, 0.7182) & (0.6210, 0.8109) \end{bmatrix}$$

$$R_3 = \begin{bmatrix} (0.3198, 0.8279) & (0.4419, 0.9816) & (0.2211, 0.5221) & (0.6661, 0.7027) \\ (0.7726, 0.8901) & (0.6245, 0.7815) & (0.6216, 0.8225) & (0.7101, 0.9005) \\ (0.5201, 0.7287) & (0.5821, 0.6286) & (0.7117, 0.9211) & (0.6105, 0.9117) \\ (0.3247, 0.4821) & (0.7139, 0.8148) & (0.4212, 0.5334) & (0.5529, 0.7217) \\ (0.7351, 0.9113) & (0.8001, 0.9112) & (0.2221, 0.6121) & (0.4214, 0.5005) \end{bmatrix}$$

$$R_4 = \begin{bmatrix} (0.3269,0.9111) & (0.4575,0.8222) & (0.5527,0.8686) & (0.8421,0.9526) \\ (0.6321,0.8108) & (0.5010,0.8126) & (0.7317,0.9191) & (0.7351,0.9005) \\ (0.3247,0.4821) & (0.7227,0.9444) & (0.5529,0.8287) & (0.1553,0.9891) \\ (0.8112,0.9238) & (0.2091,0.9545) & (0.4494,0.8898) & (0.6522,0.7377) \\ (0.1983,0.9916) & (0.3125,0.8278) & (0.7428,0.8482) & (0.6217,0.9197) \end{bmatrix}$$

$$R_5 = \begin{bmatrix} (0.2686,0.8812) & (0.4427,0.8986) & (0.6676,0.8126) & (0.7717,0.9552) \\ (0.5218,0.7918) & (0.4278,0.7176) & (0.8311,0.9519) & (0.4163,0.7295) \\ (0.5122,0.9100) & (0.2091,0.5515) & (0.2101,0.8126) & (0.3125,0.8287) \\ (0.3198,0.9728) & (0.4491,0.9861) & (0.6261,0.8522) & (0.7101,0.9552) \\ (0.5210,0.6268) & (0.7711,0.9211) & (0.6195,0.7119) & (0.7513,0.9311) \end{bmatrix}$$

G. Explanation: the steps For the Given Algorithm Are as Follows

Utilizing the decision information given in the matrix $\bar{R}_k = \left(\tilde{r}_{ij}^{(k)} \right)_{5 \times 4}$, $k = 1, 2, 3, 4, 5$ and the VWA operator which has the associated weighting vector $w = (0.28, 0.24, 0.2, 0.16, 0.12)^T$

a collective decision matrix $\bar{R} = \left(\tilde{r}_{ij}^{(k)} \right)_{5 \times 4}$ is obtained as follows:

$$R = \begin{bmatrix} (0.3948,0.8059) & (0.4850,0.8048) & (0.4587,0.7095) & (0.5999,0.8490) \\ (0.5917,0.8510) & (0.6001,0.7085) & (0.5777,0.7582) & (0.6901,0.8921) \\ (0.4980,0.7588) & (0.4994,0.6887) & (0.5497,0.7507) & (0.4723,0.7767) \\ (0.6554,0.7103) & (0.5392,0.8729) & (0.5841,0.6702) & (0.5680,0.6785) \\ (0.5702,0.7763) & (0.6829,0.8594) & (0.5562,0.7118) & (0.5570,0.7115) \end{bmatrix}$$

Utilizing the VFHWA operator, the collective overall preference values of the alternatives A_j , $j = 1, 2, \dots, 5$ are found mentioned below:

Using the weighting vector $w = (0.34, 0.28, 0.22, 0.16)$

$$\tilde{r}_1 = (0.4718, 0.7960);$$

$$\tilde{r}_2 = (0.6087, 0.8101);$$

$$\tilde{r}_3 = (0.5064, 0.7423);$$

$$\tilde{r}_4 = (0.5961, 0.7594);$$

$$\tilde{r}_5 = (0.006, 0.7837);$$

Calculating the distances between the collective overall preference values \tilde{r}_i and the positive ideal vague value $\tilde{r} = (1, 0, 0)$. The distances calculated from the following distance function:

$$d(\tilde{r}_i, \tilde{r}) = \sqrt{\frac{1}{2} \sum_{i=1}^n [(t_{\tilde{r}_i} - t_{\tilde{r}})^2 + ((1 - f_{\tilde{r}_i}) - (1 - f_{\tilde{r}}))^2]}$$

$$\text{Thus } d(\tilde{r}_1, \tilde{r}) = 0.6755;$$

$$d(\tilde{r}_2, \tilde{r}) = 0.6361;$$

$$d(\tilde{r}_3, \tilde{r}) = 0.3937 ;$$

$$d(\tilde{r}_4, \tilde{r}) = 0.3324;$$

$$d(\tilde{r}_5, \tilde{r}) = 0.6219;$$

is obtained as follows: Thus $A_1 > A_2 \succcurlyeq A_5 > A_3 > A_4$ Thus A_1 is the best alternative.

From the comparison, it can be observed that there is a change in the ranking of the best alternatives. In the proposed method with a distance function, A_1 is the best alternative, and with the replacement of step-3 in the algorithm with methods as proposed by Robinson & Amirtharaj [2011b], it can be seen that A_1 is the best alternative.

IV. CONCLUSION

As a generalization of intuitionistic fuzzy sets, neutrosophic sets (NSs) can be better handle the incomplete, indeterminate and inconsistent information, which have attracted the widespread concerns for researchers. In this paper, some new correlation coefficient and ranking method are introduced using neutrosophic fuzzy operators and fuzzy average operator in a single-valued neutrosophic environment. Firstly, the definition and operational laws of single-valued neutrosophic numbers (SVNNs) are introduced. Then, the single-valued neutrosophic average operator and the single-valued ranking techniques in neutrosophic are developed, and few properties of on these operators are also analyzed. Furthermore, a method for solving multi-criteria decision-making (MCDM) problems is explored based on the correlation coefficient and raking technique.. Finally, an illustrative example is shown to verify the effectiveness and practicality of the proposed method.

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