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Decision-Making Approach Based on Neutrosophic Rough Information

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Abstract: Rough set theory and neutrosophic set theory are mathematical models to deal with incomplete and vague information. These two theories can be combined into a framework for modeling and processing incomplete information in information systems. Thus, the neutrosophic rough set hybrid model gives more precision, flexibility and compatibility to the system as compared to the classic and fuzzy models. In this research study, we develop neutrosophic rough digraphs based on the neutrosophic rough hybrid model. Moreover, we discuss regular neutrosophic rough digraphs, and we solve decision-making problems by using our proposed hybrid model. Finally, we give a comparison analysis of two hybrid models, namely, neutrosophic rough digraphs and rough neutrosophic digraphs.

Keywords: neutrosophic rough hybrid model; regular neutrosophic rough digraphs; decision-making method

1. Introduction

The concept of a neutrosophic set, which generalizes fuzzy sets [1] and intuitionistic fuzzy sets [2], was proposed by Smarandache [3] in 1998, and it is defined as a set about the degree of truth, indeterminacy, and falsity. A neutrosophic set A in a universal set X is characterized independently by a truth-membership function ($T_A(x)$), an indeterminacy-membership function ($I_A(x)$), and a falsity-membership function ($F_A(x)$). To apply neutrosophic sets in real-life problems more conveniently, Smarandache [3] and Wang et al., [4] defined single-valued neutrosophic sets which take the value from the subset of [0, 1].

Rough set theory was proposed by Pawlak [5] in 1982. Rough set theory is useful to study the intelligence systems containing incomplete, uncertain or inexact information. The lower and upper approximation operators of rough sets are used for managing hidden information in a system. Therefore, many hybrid models have been built such as soft rough sets, rough fuzzy sets, fuzzy rough sets, soft fuzzy rough sets, neutrosophic rough sets and rough neutrosophic sets for handling uncertainty and incomplete information effectively. Dubois and Prade [6] introduced the notions of rough fuzzy sets and fuzzy rough sets. Liu and Chen [7] have studied different decision-making methods. Mordeson and Peng [8] presented operations on fuzzy graphs. Akram et al., [9–12] considered several new concepts of neutrosophic graphs with applications. Rough fuzzy digraphs with applications are presented in [13,14]. To get the extended notion of neutrosophic sets and rough sets, many attempts have been made. Broumi et al., [15] introduced the concept of rough neutrosophic sets. Yang et al., [16] proposed single valued neutrosophic rough sets by combining single valued neutrosophic sets and rough sets, and established an algorithm for the decision-making problem based on single valued neutrosophic rough sets on two universes. Nabeela et al., [17] and Sayed et al., [18] introduced rough neutrosophic digraphs, in which they have approximated neutrosophic set under the influence of a crisp equivalence relation. In this research article, we apply another hybrid set model, neutrosophic rough, to graph theory. We deal with regular neutrosophic rough digraphs and then solve the decision-making problem by using our proposed hybrid model.

Our paper is organized as follows: Firstly, we develop the notion of neutrosophic rough digraphs and present some numerical examples. Secondly, we explore basic properties of neutrosophic rough digraphs. In particular, we investigate the regularity of neutrosophic rough digraphs. We describe novel applications of our proposed hybrid decision-making method. To compare the two notions, rough neutrosophic digraphs and neutrosophic rough digraphs, we give a comparison analysis. Finally, we end the paper by concluding remarks.

2. Neutrosophic Rough Information

Definition 1. *[4] Let Z be a nonempty universe. A neutrosophic set N on Z is defined as follows:*

$$N = \{< x : T_N(x), I_N(x), F_N(x) >: x \in Z\},$$

where the functions T, I, F:$Z \to [0,1]$ represent the degree of membership, the degree of indeterminacy and the degree of falsity.

Definition 2. *[5] Let Z be a nonempty universe and R an equivalence relation on Z. A pair (Z, R) is called an approximation space. Let N^* be a subset of Z and the lower and upper approximations of N^* in the approximation space (Z, R) denoted by $\underline{R}N^*$ and $\overline{R}N^*$ are defined as follows:*

$$\begin{aligned} \underline{R}N^* &= \{x \in Z | [x]_R \subseteq N^*\}, \\ \overline{R}N^* &= \{x \in Z | [x]_R \cap N^* \neq \phi\}, \end{aligned}$$

where $[x]_R$ denotes the equivalence class of R containing x. A pair $(\underline{R}N^*, \overline{R}N^*)$ is called a rough set.

For other notations and applications, readers are referred to [19–32].

Definition 3. *[16] Let X^* be a nonempty universe and R a neutrosophic relation on X^*. Let X be a neutrosophic set on X^*, defined as*

$$X = \{< x, T_X(x), I_X(x), F_X(x) >: x \in X^*\}.$$

Then the lower and upper approximations of X represented by $\underline{R}X$ and $\overline{R}X$, respectively, are characterized as neutrosophic sets in X^* such that, $\forall x \in X^*$

$$\begin{aligned} \underline{R}X &= \{< x, T_{\underline{R}(X)}(x), I_{\underline{R}(X)}(x), F_{\underline{R}(X)}(x) >: y \in X^*\}, \\ \overline{R}X &= \{< x, T_{\overline{R}(X)}(x), I_{\overline{R}(X)}(x), F_{\overline{R}(X)}(x) >: y \in X^*\}, \end{aligned}$$

where

$$\begin{aligned} T_{\underline{R}X}(x) &= \bigwedge_{y \in X} (F_R(x, y) \vee T_X(y)), \\ I_{\underline{R}X}(x) &= \bigvee_{y \in X} (1 - I_R(x, y) \wedge I_X(y)), \\ F_{\underline{R}X}(x) &= \bigvee_{y \in X} (T_R(x, y) \wedge F_X(y)), \end{aligned}$$

and

$$T_{\overline{R}X}(x) = \bigvee_{y \in X} \big(T_R(x,y) \wedge T_X(y)\big),$$
$$I_{\overline{R}X}(x) = \bigwedge_{y \in X} \big(I_R(x,y) \vee I_X(y)\big),$$
$$F_{\overline{R}X}(x) = \bigwedge_{y \in X} \big(F_R(x,y) \vee F_X(y)\big).$$

A pair $(\underline{R}X, \overline{R}X)$ ia called a neutrosophic rough set.

Definition 4. *Let X^* be a nonempty set and R a neutrosophic tolerance relation on X^*. Let X be a neutrosophic set on X^* defined as:*

$$X = \{< x, T_X(x), I_X(x), F_X(x) >: x \in X^*\}.$$

Then the lower and upper approximations of X represented by $\underline{R}X$ and $\overline{R}X$, respectively, are characterized as neutrosophic sets in X^* such that, $\forall\ x \in X^*$

$$\underline{R}X = \{< x, T_{\underline{R}X}(x), I_{\underline{R}X}(x), F_{\underline{R}X}(x) >: y \in X^*\},$$
$$\overline{R}X = \{< x, T_{\overline{R}X}(x), I_{\overline{R}X}(x), F_{\overline{R}X}(x) >: y \in X^*\},$$

where

$$T_{\underline{R}X}(x) = \bigwedge_{y \in X^*} \big(F_R(x,y) \vee T_X(y)\big),$$
$$I_{\underline{R}X}(x) = \bigwedge_{y \in X^*} \big(1 - I_R(x,y) \vee I_X(y)\big),$$
$$F_{\underline{R}X}(x) = \bigvee_{y \in X^*} \big(T_R(x,y) \wedge F_X(y)\big),$$

and

$$T_{\overline{R}X}(x) = \bigvee_{y \in X^*} \big(T_R(x,y) \wedge T_X(y)\big),$$
$$I_{\overline{R}X}(x) = \bigvee_{y \in X^*} \big(I_R(x,y) \wedge I_X(y)\big),$$
$$F_{\overline{R}X}(x) = \bigwedge_{y \in X^*} \big(F_R(x,y) \vee F_X(y)\big).$$

Let $Y^* \subseteq X^* \times X^*$ and S a neutrosophic tolerance relation on Y^* such that

$$T_S((x_1,x_2)(y_1,y_2)) = \min\{T_R(x_1,y_1), T_R(x_2,y_2)\},$$
$$I_S((x_1,x_2)(y_1,y_2) = \min\{I_R(x_1,y_1), I_R(x_2,y_2)\},$$
$$F_S((x_1,x_2)(y_1,y_2)) = \max\{F_R(x_1,y_1), F_R(x_2,y_2)\}.$$

Let Y be a neutrosophic set on Y^* defined as:

$$Y = \{< xy, T_Y(xy), I_Y(xy), F_Y(xy) >: xy \in\ Y^*\},$$

such that

$$T_Y(xy) \leq \min\{T_{\underline{R}X}(x), T_{\underline{R}X}(y)\},$$
$$I_Y(xy) \leq \min\{I_{\underline{R}X}(x), I_{\underline{R}X}(y)\},$$
$$F_Y(xy) \leq \max\{F_{\overline{R}X}(x), F_{\overline{R}X}(y)\} \quad \forall\ x,y \in X^*.$$

Then the lower and the upper approximations of Y represented by $\underline{S}Y$ *and* $\overline{S}Y$*, are defined as follows:*

$$\underline{S}Y = \{< xy, T_{\underline{S}Y}(xy), I_{\underline{S}Y}(xy), F_{\underline{S}Y}(xy) >: xy \in Y^*\},$$
$$\overline{S}Y = \{< xy, T_{\overline{S}Y}(xy), I_{\overline{S}Y}(xy), F_{\overline{S}Y}(xy) >: xy \in Y^*\},$$

where

$$\begin{aligned} T_{\underline{S}Y}(xy) &= \bigwedge_{wz \in Y^*} \big(F_S((xy),(wz)) \vee T_Y(wz)\big), \\ I_{\underline{S}Y}(xy) &= \bigwedge_{wz \in Y^*} \big((1 - I_S((xy),(wz))) \vee I_Y(wz)\big), \\ F_{\underline{S}Y}(xy) &= \bigvee_{wz \in Y^*} \big(T_S((xy),(wz)) \wedge F_Y(wz)\big), \end{aligned}$$

and

$$\begin{aligned} T_{\overline{S}Y}(xy) &= \bigvee_{wz \in Y^*} \big(T_S((xy),(wz)) \wedge T_Y(wz)\big), \\ I_{\overline{S}Y}(xy) &= \bigvee_{wz \in Y^*} \big(I_S((xy),(wz)) \wedge I_Y(wz)\big), \\ F_{\overline{S}Y}(xy) &= \bigwedge_{wz \in Y^*} \big(F_S((xy),(wz)) \vee F_Y(wz)\big). \end{aligned}$$

A pair $SY = (\underline{S}Y, \overline{S}Y)$ *is called neutrosophic rough relation.*

Definition 5. *A neutrosophic rough digraph on a nonempty set* X^* *is a 4-ordered tuple* $G = (R, RX, S, SY)$ *such that*

(a) *R is a neutrosophic tolerance relation on* X^**,*
(b) *S is a neutrosophic tolerance relation on* $Y^* \subseteq X^* \times X^*$*,*
(c) $RX = (\underline{R}X, \overline{R}X)$ *is a neutrosophic rough set on* X^**,*
(d) $SY = (\underline{S}Y, \overline{S}Y)$ *is a neutrosophic rough relation on* X^**,*
(e) (RX, SY) *is a neutrosophic rough digraph where* $\underline{G} = (\underline{R}X, \underline{S}Y)$ *and* $\overline{G} = (\overline{R}X, \overline{S}Y)$ *are lower and upper approximate neutrosophic digraphs of G such that*

$$\begin{aligned} T_{\underline{S}Y}(xy) &\le \min\{T_{\underline{R}X}(x), T_{\underline{R}X}(y)\}, \\ I_{\underline{S}Y}(xy) &\le \min\{I_{\underline{R}X}(x), I_{\underline{R}X}(y)\}, \\ F_{\underline{S}Y}(xy) &\le \max\{F_{\underline{R}X}(x), F_{\underline{R}X}(y)\}, \end{aligned}$$

$$\begin{aligned} T_{\overline{S}Y}(xy) &\le \min\{T_{\overline{R}X}(x), T_{\overline{R}X}(y)\}, \\ I_{\overline{S}Y}(xy) &\le \min\{I_{\overline{R}X}(x), I_{\overline{R}X}(y)\}, \\ F_{\overline{S}Y}(xy) &\le \max\{F_{\overline{R}X}(x), F_{\overline{R}X}(y)\} \;\; \forall\, x, y \in X^*. \end{aligned}$$

Example 1. *Let* $X^* = \{p, q, r, s, t\}$ *be a nonempty set and R a neutrosophic tolerance relation on* X^* *is given as:*

R	p	q	r	s	t
p	(1,1,0)	(0.5,0.2,0.3)	(0.1,0.9,0.4)	(0.6,0.5,0.2)	(0.2,0.1,0.8)
q	(0.5,0.2,0.3)	(1,1,0)	(0.3,0.7,0.5)	(0.1,0.9,0.6)	(0.6,0.5,0.1)
r	(0.1,0.9,0.4)	(0.3,0.7,0.5)	(1,1,0)	(0.2,0.8,0.7)	(0.1,0.9,0.6)
s	(0.6,0.5,0.2)	(0.1,0.9,0.6)	(0.2,0.8,0.7)	(1,1,0)	(0.2,0.3,0.1)
t	(0.2,0.1,0.8)	(0.6,0.5,0.1)	(0.1,0.9,0.6)	(0.2,0.3,0.1)	(1,1,0)

Let $X_1 = \{(p, 0.2, 0.1, 0.7), (q, 0.4, 0.5, 0.6), (r, 0.7, 0.8, 0.9), (s, 0.2, 0.9, 0.1), (t, 0.6, 0.8, 0.4)\}$ *be a neutrosophic set on* X^*. *The lower and upper approximations of* X_1 *are given as:*

$$\underline{R}X_1 = \{(p, 0.2, 0.1, 0.7), (q, 0.3, 0.5, 0.6), (r, 0.4, 0.1, 0.9), (s, 0.2, 0.5, 0.6), (t, 0.2, 0.5, 0.6)\},$$
$$\overline{R}X_1 = \{(p, 0.4, 0.2, 0.8), (q, 0.6, 0.9, 0.4), (r, 0.7, 0.8, 0.6), (s, 0.2, 0.9, 0.1), (t, 0.6, 0.8, 0.1)\}.$$

Let $Y^* = \{pr, qs, rt, sp, tq\} \subseteq X^* \times X^*$ *and S a neutrosophic tolerance relation which is given as:*

S	pr	qs	rt	sp	tq
pr	(1,1,0)	(0.2,0.2,0.7)	(0.1,0.9,0.6)	(0.1,0.5,0.4)	(0.2,0.1,0.8)
qs	(0.2,0.2,0.7)	(1,1,0)	(0.2,0.3,0.5)	(0.1,0.5,0.6)	(0.1,0.5,0.6)
rt	(0.1,0.9,0.6)	(0.2,0.3,0.5)	(1,1,0)	(0.2,0.1,0.8)	(0.1,0.5,0.6)
sp	(0.1,0.5,0.4)	(0.1,0.5,0.6)	(0.2,0.1,0.8)	(1,1,0)	(0.2,0.2,0.3)
tq	(0.2,0.1,0.8)	(0.1,0.5,0.6)	(0.1,0.5,0.6)	(0.2,0.2,0.3)	(1,1,0)

Let $Y_1 = \{(pr, 0.2, 0.1, 0.5), (qs, 0.1, 0.3, 0.3), (rt, 0.2, 0.1, 0.4), (sp, 0.1, 0.1, 0.2), (tq, 0.1, 0.4, 0.3)\}$ *be a neutrosophic set on* Y^*. *The lower and upper approximations of* Y_1 *are given as:*

$$\underline{S}Y_1 = \{(pr, 0.2, 0.1, 0.5), (qs, 0.1, 0.3, 0.3), (rt, 0.2, 0.1, 0.4), (sp, 0.1, 0.1, 0.2), (tq, 0.1, 0.4, 0.3)\},$$
$$\overline{S}Y_1 = \{(pr, 0.2, 0.2, 0.4), (qs, 0.2, 0.4, 0.3), (rt, 0.2, 0.4, 0.4), (sp, 0.2, 0.3, 0.2), (tq, 0.2, 0.4, 0.3)\}.$$

Thus, $\underline{G} = (\underline{R}X_1, \underline{S}Y_1)$ *and* $\overline{G} = (\overline{R}X_1, \overline{S}Y_1)$ *are neutrosophic digraphs as shown in Figure 1.*

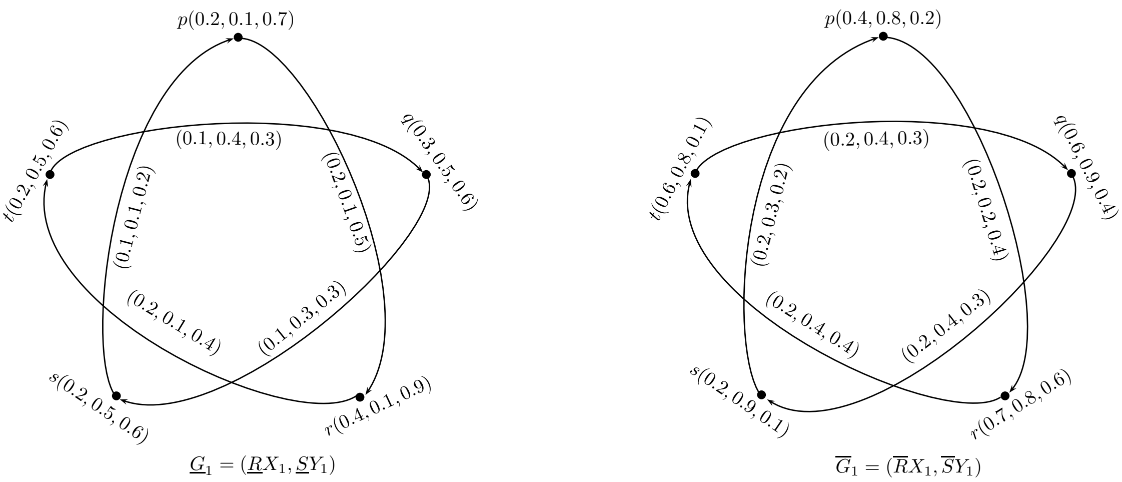


Figure 1. Neutrosophic rough digraph $G_1 = (\underline{G}_1, \overline{G}_1)$.

Example 2. *Let* $X^* = \{u, v, w, x, y, z\}$ *be a crisp set and R a neutrosophic tolerance relation on* X^* *given by*

R	u	v	w	x	y	z
u	(1,1,0)	(0.2,0.3,0.5)	(0.5,0.6,0.9)	(0.3,0.8,0.3)	(0.3,0.2,0.1)	(0.1,0.1,0.5)
v	(0.2,0.3,0.5)	(1,1,0)	(0.9,0.5,0.6)	(0.1,0.5,0.7)	(0.8,0.9,0.1)	(0.8,0.9,0.1)
w	(0.5,0.6,0.9)	(0.9,0.5,0.6)	(1,1,0)	(0.3,0.6,0.8)	(0.2,0.3,0.6)	(0.7,0.6,0.6)
x	(0.3,0.8,0.3)	(0.1,0.5,0.7)	(0.3,0.6,0.8)	(1,1,0)	(0.5,0.1,0.9)	(0.8,0.7,0.2)
y	(0.3,0.2,0.1)	(0.8,0.9,0.1)	(0.2,0.3,0.6)	(0.5,0.1,0.9)	(1,1,0)	(0.6,0.5,0.9)
z	(0.1,0.1,0.5)	(0.8,0.9,0.1)	(0.7,0.6,0.6)	(0.8,0.7,0.2)	(0.6,0.5,0.9)	(1,1,0)

Let $X = \{(u, 0.9, 0.3, 0.1), (v, 0.5, 0.6, 0.2), (w, 0.8, 0.5, 0.3), (x, 0.7, 0.6, 0.9), (y, 0.5, 0.2, 0.1), (z, 0.9, 0.7, 0.3)\}$ *be a neutrosophic set on* X^**. Then the lower and upper approximations of X are given as follows:*

$$\underline{R}X = \{(u, 0.5, 0.3, 0.3), (v, 0.5, 0.2, 0.3), (w, 0.6, 0.4, 0.3), (x, 0.7, 0.3, 0.9), (y, 0.5, 0.2, 0.5), (z, 0.5, 0.5, 0.8)\},$$
$$\overline{R}X = \{(u, 0.9, 0.6, 0.1), (v, 0.8, 0.7, 0.1), (w, 0.8, 0.6, 0.3), (x, 0.8, 0.7, 0.3), (y, 0.6, 0.6, 0.1), (z, 0.9, 0.7, 0.2)\}.$$

Let $Y^* = \{uv, vw, wx, xy, yz, zu, zw, vy\} \subseteq X^* \times X^*$ *and S a neutrosophic tolerance relation on* Y^* *given as*

S	uv	vw	wx	xy	yz	zu	zw	vy
uv	(1,1,0)	(0.2,0.3,0.6)	(0.1,0.5,0.9)	(0.3,0.8,0.3)	(0.3,0.2,0.1)	(0.1,0.1,0.5)	(0.1,0.1,0.6)	(o.2,0.3,0.5)
vw	(0.2,0.3,0.6)	(1,1,0)	(0.3,0.5,0.8)	(0.1,0.3,0.7)	(0.7,0.6,0.6)	(0.5,0.6,0.9)	(0.8,0.9,0.1)	(0.2,0.3,0.6)
wx	(0.1,0.5,0.9)	(0.3,0.5,0.8)	(1,1,0)	(0.3,0.1,0.9)	(0.2,0.3,0.6)	(0.3,0.6,0.6)	(0.3,0.6,0.8)	(0.5,0.1,0.9)
xy	(0.3,0.8,0.3)	(0.1,0.3,0.7)	(0.3,0.1,0.9)	(1,1,0)	(0.5,0.1,0.9)	(0.3,0.2,0.2)	(0.2,0.3,0.6)	(0.1,0.5,0.7)
yz	(0.3,0.2,0.1)	(0.7,0.6,0.6)	(0.2,0.3,0.6)	(0.5,0.1,0.9)	(1,1,0)	(0.1,0.1,0.9)	(0.6,0.5,0.9)	(0.6,0.5,0.9)
zu	(0.1,0.1,0.5)	(0.5,0.6,0.9)	(0.3,0.6,0.6)	(0.3,0.2,0.2)	(0.1,0.1,0.9)	(1,1,0)	(0.5,0.6,0.9)	(0.3,0.3,0.1)
zw	(0.1,0.1,0.6)	(0.8,0.9,0.1)	(0.3,0.6,0.8)	(0.2,0.3,0.6)	(0,6,0.5,0.9)	(0.5,0.6,0.9)	(1,1,0)	(0.2,0.3,0.6)
vy	(0.2,0.3,0.5)	(0.2,0.3,0.6)	(0.5,0.1,0.9)	(0.1,0.5,0.7)	(0.6,0.5,0.9)	(0.3,0.2,0.1)	(0.2,0.3,0.6)	(1,1,0)

Let Y be a neutrosophic set on Y^* *defined as* $Y = \{(uv, 0.5, 0.2, 0.1), (vw, 0.5, 0.2, 0.3), (wx, 0.5, 0.3, 0.3), (xy, 0.5, 0.2, 0.3), (yz, 0.5, 0.2, 0.2), (zu, 0.5, 0.3, 0.2), (zw, 0.5, 0.4, 0.3), (vy, 0.5, 0.2, 0.1)\}$*. Then the lower and upper approximations of Y are given as*

$$\underline{S}Y = \{(uv, 0.5, 0.2, 0.3), (vw, 0.5, 0.2, 0.3), (wx, 0.5, 0.3, 0.3), (xy, 0.5, 0.2, 0.3), (yz, 0.5, 0.2, 0.3), (zu, 0.5, 0.3, 0.3), (zw, 0.5, 0.2, 0.3), (vy, 0.5, 0.2, 0.3)\},$$
$$\overline{S}Y = \{(uv, 0.5, 0.3, 0.1), (vw, 0.5, 0.4, 0.3), (wx, 0.5, 0.4, 0.3), (xy, 0.5, 0.3, 0.3), (yz, 0.5, 0.4, 0.1), (zu, 0.5, 0.4, 0.1), (zw, 0.5, 0.4, 0.3), (vy, 0.5, 0.3, 0.1)\}.$$

Thus, $\underline{G} = (\underline{R}X, \underline{S}Y)$ *and* $\overline{G} = (\overline{R}X, \overline{S}Y)$ *are the neutrosophic digraphs as shown in Figure 2.*

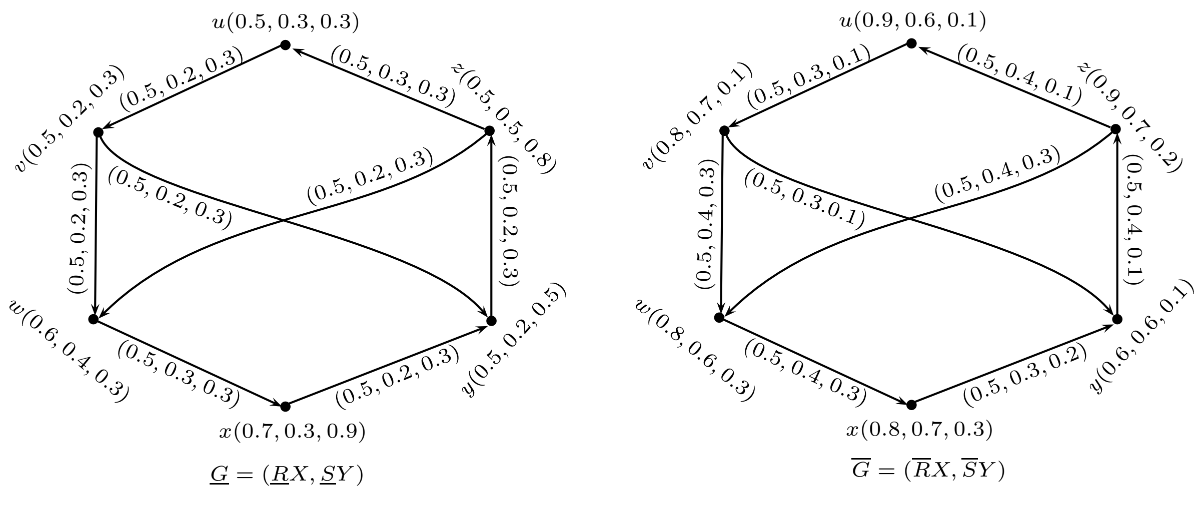


Figure 2. Neutrosophic rough digraph $G = (\underline{G}, \overline{G})$.

Now we discuss regular neutrosophic rough digraphs.

Definition 6. *Let* $G = (\underline{G}, \overline{G})$ *be a neutrosophic rough digraph on a nonempty set* X^**. The indegree of a vertex* $x \in G$ *is the sum of membership degree, indeterminacy and falsity of all edges towards x from other vertices in* $\underline{G}$ *and* $\overline{G}$*, respectively, represented by* $id_G(x)$ *and defined by*

$$id_G(x) = id_{\underline{G}}(x) + id_{\overline{G}}(x),$$

where

$$id_{\underline{G}}(x) = (\sum_{x,y \in \underline{S}Y} T_{\underline{G}}(yx), \sum_{x,y \in \underline{S}Y} I_{\underline{G}}(yx), \sum_{x,y \in \underline{S}Y} F_{\underline{G}}(yx)),$$
$$id_{\overline{G}}(x) = (\sum_{x,y \in \overline{S}Y} T_{\overline{G}}(yx), \sum_{x,y \in \overline{S}Y} I_{\overline{G}}(yx), \sum_{x,y \in \overline{S}Y} F_{\overline{G}}(yx)).$$

The outdegree of a vertex $x \in G$ is the sum of membership degree, indeterminacy and falsity of all edges outward from x to other vertices in $\underline{G}$ and $\overline{G}$, respectively, represented by $od_G(x)$ and defined by

$$od_G(x) = od_{\underline{G}}(x) + od_{\overline{G}}(x),$$

where

$$od_{\underline{G}}(x) = (\sum_{x,y \in \underline{S}Y} T_{\underline{G}}(xy), \sum_{x,y \in \underline{S}Y} I_{\underline{G}}(xy), \sum_{x,y \in \underline{S}Y} F_{\underline{G}}(xy)),$$
$$od_{\overline{G}}(x) = (\sum_{x,y \in \overline{S}Y} T_{\overline{G}}(xy), \sum_{x,y \in \overline{S}Y} I_{\overline{G}}(xy), \sum_{x,y \in \overline{S}Y} F_{\overline{G}}(xy)).$$

$d_G(x) = id_G(x) + od_G(x)$ *is called degree of vertex x.*

Definition 7. *A neutrosophic rough digraph is called a regular neutrosophic rough digraph of degree (m_1, m_2, m_3) if*

$$d_G(x) = (m_1, m_2, m_3)\ , \forall x \in X.$$

Example 3. *Let $X^* = \{p, q, r, s\}$ be a nonempty set and R a neutrosophic tolerance relation on X^* is given as:*

R	p	q	r	s
p	(1,1,0)	(0.1,0.9,0.8)	(0.7,0.5,0.8)	(0.1,0.9,0.8)
q	(0.9,0.8,0.1)	(1,1,0)	(0.1,0.9,0.8)	(0.4,0.3,0.9)
r	(0.7,0.5,0.8)	(0.1,0.9,0.8)	(1,1,0)	(0.1,0.9,0.8)
s	(0.1,0.9,0.8)	(0.4,0.3,0.9)	(0.1,0.9,0.8)	(1,1,0)

Let $X_1 = \{(p, 0.1, 0.4, 0.8), (q, 0.2, 0.3, 0.9), (r, 0.1, 0.6, 0.8), (s, 0.9, 0.6, 0.3)\}$ be a neutrosophic set on X^*. The lower and upper approximations of X_1 are given as:

$$\underline{R}X_1 = \{(p, 0.1, 0.3, 0.8), (q, 0.2, 0.3, 0.9), (r, 0.1, 0.3, 0.8), (s, 0.8, 0.4, 0.4)\},$$
$$\overline{R}X_1 = \{(p, 0.1, 0.6, 0.8), (q, 0.4, 0.6, 0.8), (r, 0.1, 0.6, 0.8), (s, 0.9, 0.6, 0.3)\}.$$

Let $Y^* = \{pq, qr, rs, sp\} \subseteq X^* \times X^*$ and S a neutrosophic tolerance relation on Y^* which is given as:

S	pq	qr	rs	sp
pq	(1,1,0)	(0.1,0.9,0.8)	(0.4,0.3,0.9)	(0.1,0.9,0.8)
qr	(0.1,0.9,0.8)	(1,1,0)	(0.1,0.9,0.8)	(0.4,0.3,0.9)
rs	(0.4,0.3,0.9)	(0.1,0.9,0.8)	(1,1,0)	(0.1,0.9,0.8)
sp	(0.1,0.9,0.8)	(0.4,0.3,0.9)	(0.1,0.9,0.8)	(1,1,0)

Let $Y_1 = \{(pq, 0.1, 0.3, 0.8), (qr, 0.1, 0.3, 0.3), (rs, 0.1, 0.3, 0.8), (sp, 0.1, 0.3, 0.8)\}$ be a neutrosophic set on Y^*. The lower and upper approximations of Y_1 are given as:

$$\underline{S}Y_1 = \{(pq, 0.1, 0.3, 0.8), (qr, 0.1, 0.3, 0.3), (rs, 0.1, 0.3, 0.8), (sp, 0.1, 0.3, 0.8)\},$$
$$\overline{S}Y_1 = \{(pq, 0.1, 0.3, 0.8), (qr, 0.1, 0.3, 0.3), (rs, 0.1, 0.3, 0.8), (sp, 0.1, 0.3, 0.8)\}.$$

Thus, $G_1 = (\underline{G}_1, \overline{G}_1)$ is a regular neutrosophic rough digraph as shown in Figure 3.

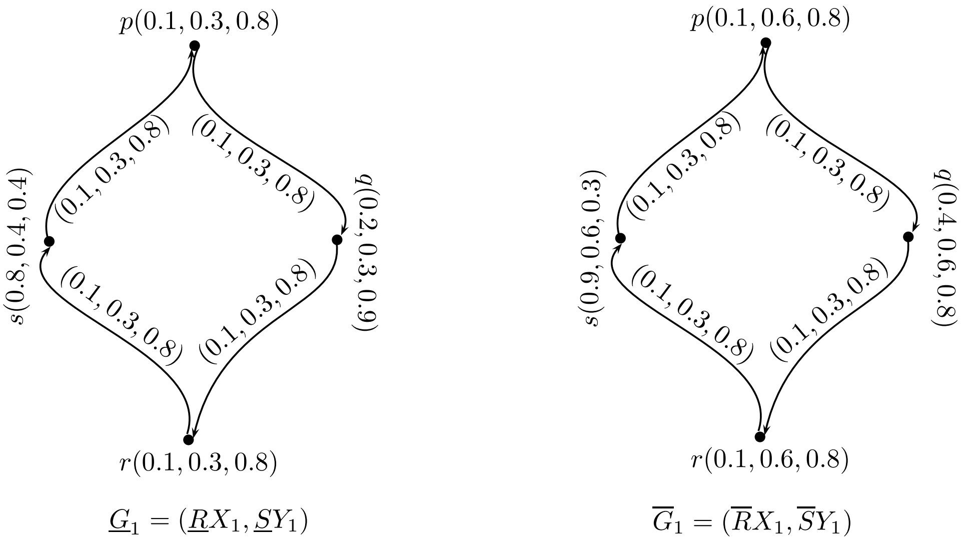


Figure 3. Regular neutrosophic rough digraph $G_1 = (\underline{G}_1, \overline{G}_1)$.

Definition 8. *Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic rough digraphs. Then the direct sum of G_1 and G_2 is a neutrosophic rough digraph $G = G_1 \oplus G_2 = (\underline{G}_1 \oplus \underline{G}_2, \overline{G}_1 \oplus \overline{G}_2)$, where $\underline{G}_1 \oplus \underline{G}_2 = (\underline{R}X_1 \oplus \underline{R}X_2, \underline{S}Y_1 \oplus \underline{S}Y_2)$ and $\overline{G}_1 \oplus \overline{G}_2 = (\overline{R}X_1 \oplus \overline{R}X_2, \overline{S}Y_1 \oplus \overline{S}Y_2)$ are neutrosophic digraphs.*

(1)

$$T_{\underline{R}X_1 \oplus \underline{R}X_2}(x) = \begin{cases} T_{\underline{R}X_1}(x), & \text{if } x \in \underline{R}X_1 - \underline{R}X_2 \\ T_{\underline{R}X_2}(x), & \text{if } x \in \underline{R}X_2 - \underline{R}X_1 \\ \max(T_{\underline{R}X_1}(x), T_{\underline{R}X_2}(x)), & \text{if } x \in \underline{R}X_1 \cap \underline{R}X_2 \end{cases}$$

$$I_{\underline{R}X_1 \oplus \underline{R}X_2}(x) = \begin{cases} I_{\underline{R}X_1}(x), & \text{if } x \in \underline{R}X_1 - \underline{R}X_2 \\ I_{\underline{R}X_2}(x), & \text{if } x \in \underline{R}X_2 - \underline{R}X_1 \\ \max(I_{\underline{R}X_1}(x), I_{\underline{R}X_2}(x)), & \text{if } x \in \underline{R}X_1 \cap \underline{R}X_2 \end{cases}$$

$$F_{\underline{R}X_1 \oplus \underline{R}X_2}(x) = \begin{cases} F_{\underline{R}X_1}(x), & \text{if } x \in \underline{R}X_1 - \underline{R}X_2 \\ F_{\underline{R}X_2}(x), & \text{if } x \in \underline{R}X_2 - \underline{R}X_1 \\ \min(F_{\underline{R}X_1}(x), F_{\underline{R}X_2}(x)), & \text{if } x \in \underline{R}X_1 \cap \underline{R}X_2 \end{cases}$$

$$T_{\underline{S}Y_1 \oplus \underline{S}Y_2}(x, y) = \begin{cases} T_{\underline{S}Y_1}(x, y), & \text{if } (x, y) \in \underline{S}Y_1 \\ T_{\underline{S}Y_2}(x, y), & \text{if } (x, y) \in \underline{S}Y_2 \end{cases}$$

$$I_{\underline{S}Y_1 \oplus \underline{S}Y_2}(x, y) = \begin{cases} I_{\underline{S}Y_1}(x, y), & \text{if } (x, y) \in \underline{S}Y_1 \\ I_{\underline{S}Y_2}(x, y), & \text{if } (x, y) \in \underline{S}Y_2 \end{cases}$$

$$F_{\underline{S}Y_1 \oplus \underline{S}Y_2}(x, y) = \begin{cases} F_{\underline{S}Y_1}(x, y), & \text{if } (x, y) \in \underline{S}Y_1 \\ F_{\underline{S}Y_2}(x, y), & \text{if } (x, y) \in \underline{S}Y_2 \end{cases}$$

(2)

$$T_{\overline{R}X_1 \oplus \overline{R}X_2}(x) = \begin{cases} T_{\overline{R}X_1}(x), & \text{if } x \in \overline{R}X_1 - \overline{R}X_2 \\ T_{\overline{R}X_2}(x), & \text{if } x \in \overline{R}X_2 - \overline{R}X_1 \\ \max(T_{\overline{R}X_1}(x), T_{\overline{R}X_2}(x)), & \text{if } x \in \overline{R}X_1 \cap \overline{R}X_2 \end{cases}$$

$$I_{\overline{R}X_1 \oplus \overline{R}X_2}(x) = \begin{cases} I_{\overline{R}X_1}(x), & \text{if } x \in \overline{R}X_1 - \overline{R}X_2 \\ I_{\overline{R}X_2}(x), & \text{if } x \in \overline{R}X_2 - \overline{R}X_1 \\ \max(I_{\overline{R}X_1}(x), I_{\overline{R}X_2}(x)), & \text{if } x \in \overline{R}X_1 \cap \overline{R}X_2 \end{cases}$$

$$F_{\overline{R}X_1 \oplus \overline{R}X_2}(x) = \begin{cases} F_{\overline{R}X_1}(x), & \text{if } x \in \overline{R}X_1 - \overline{R}X_2 \\ F_{\overline{R}X_2}(x), & \text{if } x \in \overline{R}X_2 - \overline{R}X_1 \\ \min(F_{\overline{R}X_1}(x), F_{\overline{R}X_2}(x)), & \text{if } x \in \overline{R}X_1 \cap \overline{R}X_2 \end{cases}$$

$$T_{\overline{S}Y_1 \oplus \overline{S}Y_2}(x,y) = \begin{cases} T_{\overline{S}Y_1}(x,y), & \text{if } (x,y) \in \overline{S}Y_1 \\ T_{\overline{S}Y_2}(x,y), & \text{if } (x,y) \in \overline{S}Y_2 \end{cases}$$

$$I_{\overline{S}Y_1 \oplus \overline{S}Y_2}(x,y) = \begin{cases} I_{\overline{S}Y_1}(x,y), & \text{if } (x,y) \in \overline{S}Y_1 \\ I_{\overline{S}Y_2}(x,y), & \text{if } (x,y) \in \overline{S}Y_2 \end{cases}$$

$$F_{\overline{S}Y_1 \oplus \overline{S}Y_2}(x,y) = \begin{cases} F_{\overline{S}Y_1}(x,y), & \text{if } (x,y) \in \overline{S}Y_1 \\ F_{\overline{S}Y_2}(x,y), & \text{if } (x,y) \in \overline{S}Y_2 \end{cases}$$

Example 4. *Let $X^* = \{p,q,r,s,t\}$ be a set. Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic rough digraphs on X^* as shown in Figures 1 and 4. The direct sum of G_1 and G_2 is $G = (\underline{G}_1 \oplus \underline{G}_2, \overline{G}_1 \oplus \overline{G}_2)$, where $\underline{G}_1 \oplus \underline{G}_2 = (\underline{R}X_1 \oplus \underline{R}X_2, \underline{S}Y_1 \oplus \underline{S}Y_2)$ and $\overline{G}_1 \oplus \overline{G}_2 = (\overline{R}X_1 \oplus \overline{R}X_2, \overline{S}Y_1 \oplus \overline{S}Y_2)$ are neutrosophic digraphs as shown in Figure 5.*

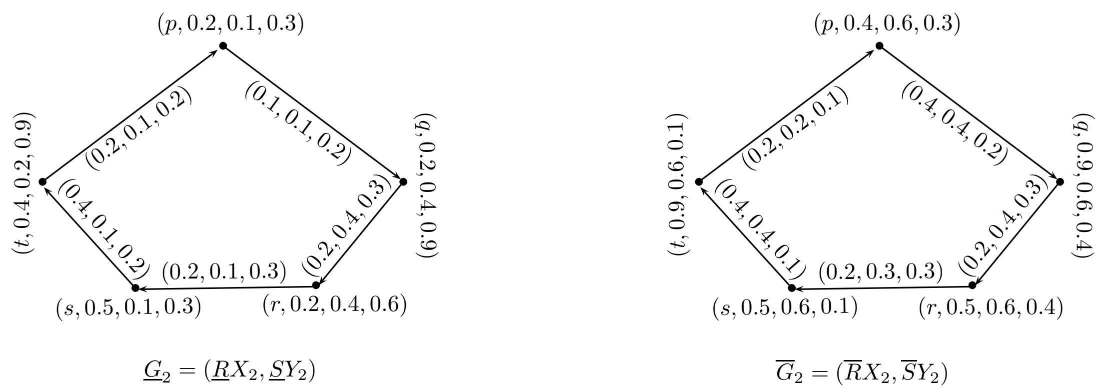
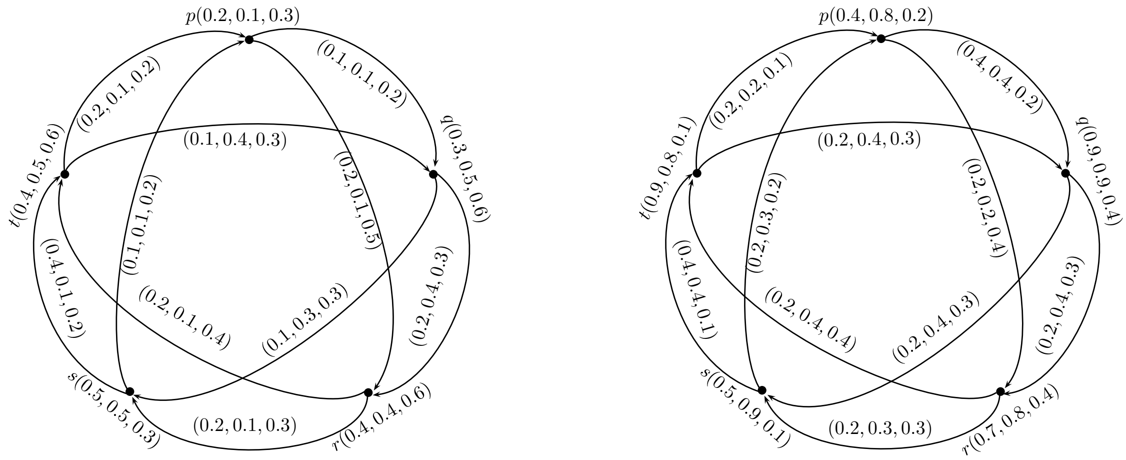


Figure 4. Neutrosophic rough digraph $G = (\underline{G}_2, \overline{G}_2)$.

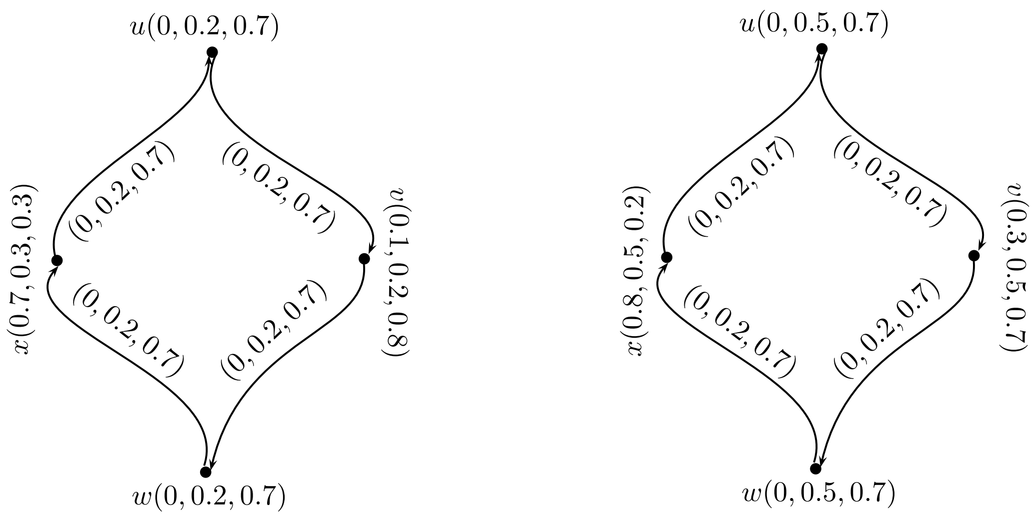


$\underline{G}_1 \oplus \underline{G}_2 = (\underline{R}X_1 \oplus \underline{R}X_2, \underline{S}Y_1 \oplus \underline{S}Y_2)$ $\quad$ $\overline{G}_1 \oplus \overline{G}_2 = (\overline{R}X_1 \oplus \overline{R}X_2, \overline{S}Y_1 \oplus \overline{S}Y_2)$

Figure 5. Neutrosophic rough digraph $G = (\underline{G_1} \oplus \underline{G_2}, \overline{G}_1 \oplus \overline{G}_2)$.

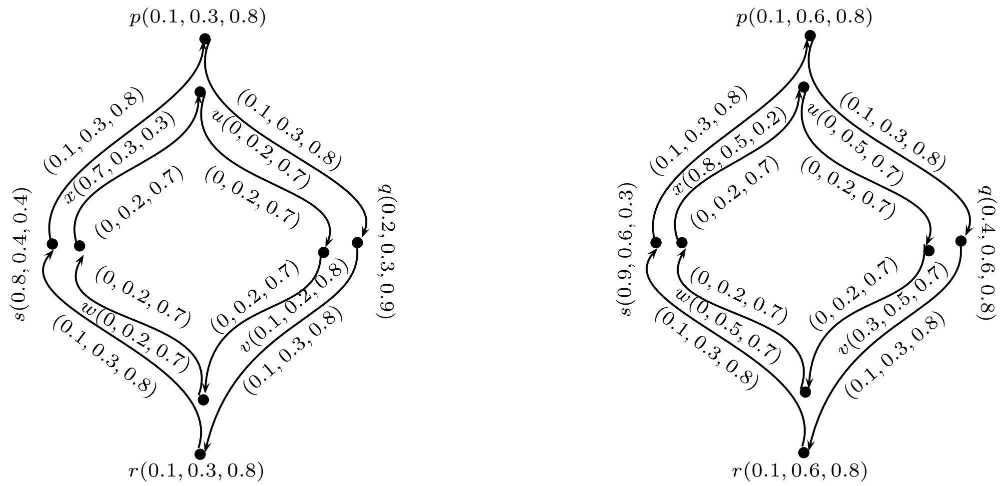
Remark 1. *The direct sum of two regular neutrosophic rough digraphs may not be regular neutrosophic rough digraph, as shown in the following example.*

Example 5. *Consider the two regular neutrosophic rough digraphs* $G_1 = (\underline{G_1}, \overline{G}_1)$ *and* $G_2 = (\underline{G_2}, \overline{G}_2)$ *as shown in Figures 3 and 6, respectively, then the direct sum* $G = (\underline{G_1} \oplus \underline{G_2}, \overline{G}_1 \oplus \overline{G}_2)$ *of* G_1 *and* G_2 *as shown in Figure 7 is not a regular neutrosophic rough digraph.*



$\underline{G}_2 = (\underline{R}X_2, \underline{S}Y_2)$ $\quad$ $\overline{G}_2 = (\overline{R}X_2, \overline{S}Y_2)$

Figure 6. Regular neutrosophic rough digraph $G_2 = (\underline{G_2}, \overline{G}_2)$.



$\underline{G}_1 \oplus \underline{G}_2 = (\underline{R}X_1 \oplus \underline{R}X_2, \underline{S}Y_1 \oplus \underline{S}Y_2)$ $\overline{G}_1 \oplus \overline{G}_2 = (\overline{R}X_1 \oplus \overline{R}X_2, \overline{S}Y_1 \oplus \overline{S}Y_2)$

Figure 7. Neutrosophic rough digraph $G = (\underline{G}_1 \oplus \underline{G}_2, \overline{G}_1 \oplus \overline{G}_2)$.

Remark 2. *If $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ are two regular neutrosophic rough digraphs with degree (m_1, m_2, m_3) and (n_1, n_2, n_3) on X_1^* , X_2^*, respectively, and $X_1^* \cap X_2^* = \phi$, then $G_1 \oplus G_2$ is a regular neutrosophic rough digraph if and only if $(m_1, m_2, m_3) = (n_1, n_2, n_3)$.*

Definition 9. *Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic rough digraphs on crisp sets X_1^* and X_2^* respectively. The residue product of G_1 and G_2 is a neutrosophic rough digraph $G = G_1 * G_2 = (\underline{G}_1 * \underline{G}_2, \overline{G}_1 * \overline{G}_2)$, where $\underline{G}_1 * \underline{G}_2 = (\underline{R}X_1 * \underline{R}X_2, \underline{S}Y_1 * \underline{S}Y_2)$ and $\overline{G}_1 * \overline{G}_2 = (\overline{R}X_1 * \overline{R}X_2, \overline{S}Y_1 * \overline{S}Y_2)$ are neutrosophic digraphs, respectively, such that*

(1)

$$
\begin{aligned}
&T_{\underline{R}X_1 * \underline{R}X_2}(x_1, x_2) = \max\{T_{\underline{R}X_1}(x_1), T_{\underline{R}X_2}(x_2)\},\\
&I_{\underline{R}X_1 * \underline{R}X_2}(x_1, x_2) = \max\{I_{\underline{R}X_1}(x_1), I_{\underline{R}X_2}(x_2)\},\\
&F_{\underline{R}X_1 * \underline{R}X_2}(x_1, x_2) = \min\{F_{\underline{R}X_1}(x_1), F_{\underline{R}X_2}(x_2)\}, \ \forall (x_1, x_2) \in \underline{R}X_1 \times \underline{R}X_2
\end{aligned}
$$

$$
\begin{aligned}
&T_{\underline{S}Y_1 * \underline{S}Y_2}(x_1, x_2)(y_1, y_2) = T_{\underline{S}Y_1}(x_1, y_1),\\
&I_{\underline{S}Y_1 * \underline{S}Y_2}(x_1, x_2)(y_1, y_2) = I_{\underline{S}Y_1}(x_1, y_1),\\
&F_{\underline{S}Y_1 * \underline{S}Y_2}(x_1, x_2)(y_1, y_2) = F_{\underline{S}Y_1}(x_1, y_1), \ \forall (x_1, y_1) \in \underline{S}Y_1, x_1 \neq y_2
\end{aligned}
$$

(2)

$$
\begin{aligned}
&T_{\overline{R}X_1 * \overline{R}X_2}(x_1, x_2) = \max\{T_{\overline{R}X_1}(x_1), T_{\overline{R}X_2}(x_2)\},\\
&I_{\overline{R}X_1 * \overline{R}X_2}(x_1, x_2) = \max\{I_{\overline{R}X_1}(x_1), I_{\overline{R}X_2}(x_2)\},\\
&F_{\overline{R}X_1 * \overline{R}X_2}(x_1, x_2) = \min\{F_{\overline{R}X_1}(x_1), F_{\overline{R}X_2}(x_2)\}, \ \forall (x_1, x_2) \in \overline{R}X_1 \times \overline{R}X_2
\end{aligned}
$$

$$
\begin{aligned}
&T_{\overline{S}Y_1 * \overline{S}Y_2}(x_1, x_2)(y_1, y_2) = T_{\overline{S}Y_1}(x_1, y_1),\\
&I_{\overline{S}Y_1 * \overline{S}Y_2}(x_1, x_2)(y_1, y_2) = I_{\overline{S}Y_1}(x_1, y_1),\\
&F_{\overline{S}Y_1 * \overline{S}Y_2}(x_1, x_2)(y_1, y_2) = F_{\overline{S}Y_1}(x_1, y_1), \ \forall (x_1, y_1) \in \overline{S}Y_1, x_1 \neq y_2
\end{aligned}
$$

Example 6. *Let $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ be two neutrosophic rough digraphs on the two crisp sets $X_1^* = \{p, q\}$ and $X_2^* = \{u, v, w, x\}$ as shown in Figures 8 and 9. Then the residue product of G_1 and G_2 is a neutrosophic rough digraph $G = G_1 * G_2 = (\underline{G}_1 * \underline{G}_2, \overline{G}_1 * \overline{G}_2)$ where $\underline{G}_1 * \underline{G}_2 = (\underline{R}X_1 * \underline{R}X_2, \underline{S}Y_1 * \underline{S}Y_2)$ and $\overline{G}_1 * \overline{G}_2 = (\overline{R}X_1 * \overline{R}X_2, \overline{S}Y_1 * \overline{S}Y_2)$ and the respective figures are shown in Figure 10.*

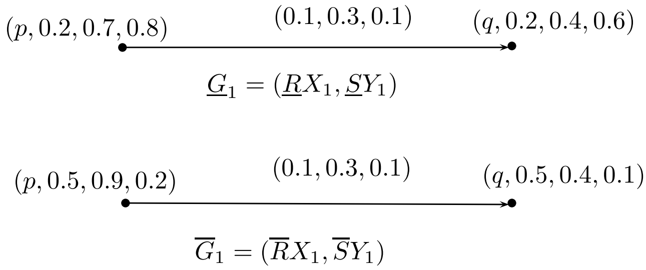


Figure 8. Neutrosophic rough digraph $G_1 = (\underline{G}_1, \overline{G}_1)$.

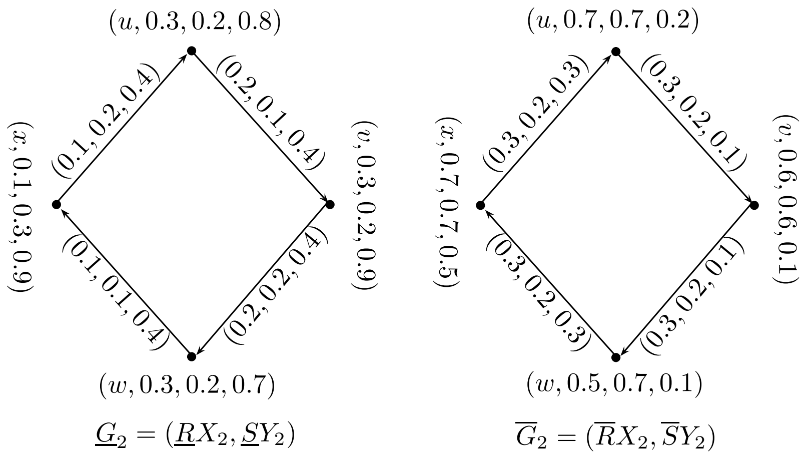


Figure 9. Neutrosophic rough digraph $G_2 = (\underline{G}_2, \overline{G}_2)$.

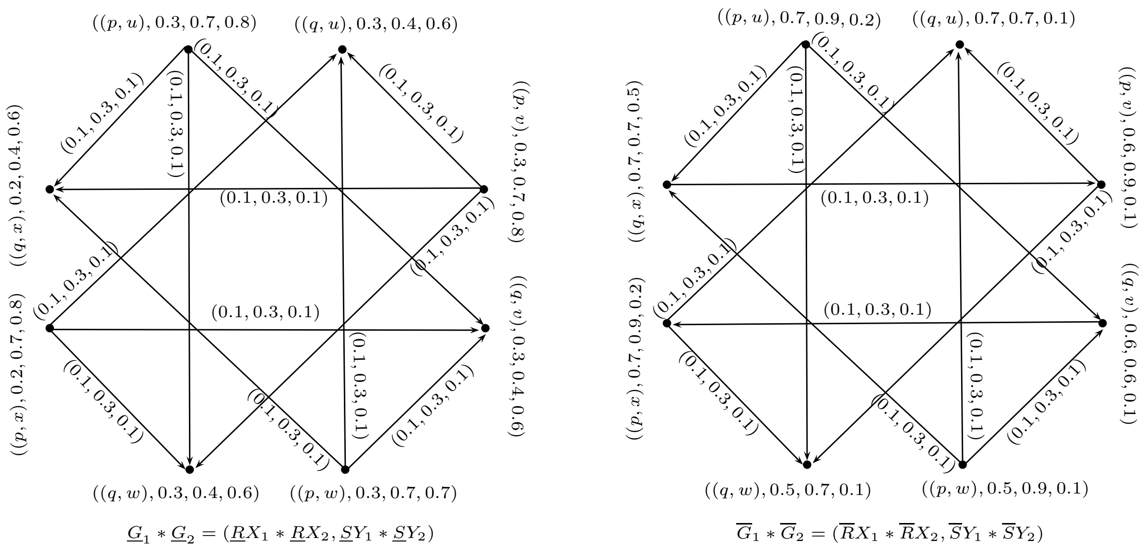


Figure 10. Neutrosophic rough digraph $G = (\underline{G}_1 * \underline{G}_2, \overline{G}_1 * \overline{G}_2)$.

Theorem 1. *If $G_1 = (\underline{G}_1, \overline{G}_1)$ and $G_2 = (\underline{G}_2, \overline{G}_2)$ are two neutrosophic rough digraph such that $|X_2^*| > 1$, then their residue product is regular if and only if G_1 is regular.*

Proof. Let $G_1 * G_2$ be a regular neutrosophic rough digraph.

Then, for any two vertices (x_1, x_2) and (y_1, y_2) in $X_1^* \times X_2^*$,

$$d_{G_1 * G_2}(x_1, x_2) = d_{G_1 * G_2}(y_1, y_2)$$
$$\Rightarrow d_{G_1}(x_1) = d_{G_1}(y_1).$$

This is true for all vertices in X_1^*. Hence G_1 is a regular neutrosophic rough digraph.

Conversely, suppose that $G_1 = (\underline{G}_1, \overline{G}_1)$ is a (m_1, m_2, m_3)-regular neutrosophic rough digraph and $G_2 = (\underline{G}_2, \overline{G}_2)$ is any neutrosophic rough digraph with $|X_2^*| > 1$. If $|X_2^*| > 1$, then $d_{G_1 * G_2}(x_1, x_2) = d_{G_1}(x_1) = (m_1, m_2, m_3)$. This is a constant ordered-triplet for all vertices in $X_1^* \times X_2^*$. Hence $G_1 * G_2$ is a regular neutrosophic rough digraph. □

3. Applications to Decision-Making

In this section, we present some real life applications of neutrosophic rough digraphs in decision making. In decision-making, the selection is facilitated by evaluating each choice on the set of criteria. The criteria must be measurable and their outcomes must be measured for every decision alternative.

3.1. Online Reviews and Ratings

Customer reviews are increasingly available online for a wide range of products and services. As customers search online for product information and to evaluate product alternatives, they often have access to dozens or hundreds of product reviews from other customers. These reviews are very helpful in product selection. However, only considering the good reviews about a product is not very helpful in decision-making. The customer should keep in mind bad and neutral reviews as well. We use percentages of good reviews, bad reviews and neutral reviews of a product as truth membership, false membership and indeterminacy respectively.

Mrs. Sadia wants to purchase a refrigerator. For this purpose she visits web sites of different refrigerator companies. The refrigerator companies and their ratings by other customers are shown in Table 1.

Table 1. Companies and their ratings.

X^*	Good Reviews	Neutral	Bad Reviews
PEL	45%	29%	37%
Dawlance	52%	25%	49%
Haier	51%	43%	45%
Waves	47%	41%	38%
Orient	51%	35%	48%

Here $X^* = \{$Pel(P),Dawlance(D),Haier(H),Waves(W),Orient(O)$\}$ and the neutrosophic set on X^* according to the reviews will be $X = \{(P, 0.45, 0.29, 0.37), (D, 0.52, 0.25, 0.49), (H, 0.51, 0.43, 0.45), (W, 0.47, 0.41, 0.38)(O, 0.51, 0.35, 0.48)\}$. The neutrosophic tolerance relation on X^* is given below

R	P	D	H	W	O
P	(1,1,0)	(0.5,0.6,0.9)	(0.2,0.3,0.6)	(0.1,0.2,0.3)	(0.4,0.6,0.8)
D	(0.5,0.6,0.9)	(1,1,0)	(0.1,0.6,0.9)	(0.4,0.5,0.9)	(0.9,0.8,0.2)
H	(0.2,0.3,0.6)	(0.1,0.6,0.9)	(1,1,0)	(0.2,0.9,0.6)	(0.1,0.9,0.7)
W	(0.1,0.2,0.3)	(0.4,0.5,0.9)	(0.2,0.9,0.6)	(1,1,0)	(0.2,0.5,0.9)
O	(0.4,0.6,0.8)	(0.9,0.8,0.2)	(0.1,0.9,0.7)	(0.2,0.5,0.9)	(1,1,0)

The lower and upper approximations of X are as follows:

$$\underline{R}X = \{(P, 0.45, 0.29, 0.49), (D, 0.51, 0.25, 0.49), (H, 0.51, 0.35, 0.45),$$
$$(W, 0.45, 0.41, 0.40), (O, 0.51, , 0.25, 0.49)\},$$
$$\overline{R}X = \{(P, 0.50, 0.35, 0.37), (D, 0.52, 0.43, 0.48), (H, 0.51, 0.43, 0.45),$$
$$(W, 0.47, 0.43, 0.37), (O, 0.52, 0.43, 0.48)\}.$$

Let $Y^* = \{(P, D), (P, H), (D, H), (D, W), (H, W), (H, O), (W, P), (W, O), (O, P), (O, D)\}$ be the subset of $X^* \times X^*$ and the tolerance relation S on Y^* is given as follows:

S	(P,D)	(P,H)	(D,H)	(D,W)	(H,W)
(P,D)	(1,1,0)	(0.1,0.6,0.9)	(0.1,0.6,0.9)	(0.4,0.5,0.9)	(0.2,0.3,0.9)
(P,H)	(0.1,0.6,0.9)	(1,1,0)	(0.5,0.6,0.9)	(0.2,0.6,0.9)	(0.2,0.3,0.6)
(D,H)	(0.1,0.6,0.9)	(0.5,0.6,0.9)	(1,1,0)	(0.2,0.9,0.6)	(0.1,0.6,0.9)
(D,W)	(0.4,.5,.9)	(0.2,0.6,0.9)	(0.2,0.6,0.9)	(1,1,0)	(0.1,0.6,0.9)
(H,W)	(0.2,0.3,0.9)	(0.2,0.3,0.6)	(0.1,0.6,0.9)	(0.1,0.6,.9)	(1,1,0)
(H,O)	(0.2,0.3,0.6)	(0.1,0.3,0.7)	(0.1,0.6,0.9)	(0.1,0.5,0.9)	(0.2,0.5,0.9)
(W,P)	(0.1,0.2,0.9)	(0.1,0.2,0.6)	(0.2,0.3,0.9)	(0.1,0.2,0.9)	(0.1,0.2,0.6)
(W,O)	(0.1,0.2,0.3)	(0.1,0.2,0.7)	(0.1,0.5,0.9)	(0.2,0.5,0.9)	(0.2,0.5,0.9)
(O,P)	(0.4,0.6,0.9)	(0.2,0.3,0.8)	(0.2,0.3,0.6)	(0.1,0.2,0.3)	(0.1,0.2,0.7)
(O,D)	(0.4,0.6,0.8)	(0.1,0.6,0.9)	(0.1,0.6,0.9)	(0.4,0.5,0.9)	(0.1,0.5,0.9)

S	(H,O)	(W,P)	(W,O)	(O,P)	(O,D)
(P,D)	(0.2,0.3,0.6)	(0.1,0.2,0.9)	(0.1,0.2,0.3)	(0.4,0.6,0.9)	(0.4,0.6,0.8)
(P,H)	(0.1,0.3,0.7)	(0.1,0.2,0.6)	(0.1,0.2,0.7)	(0.2,0.3,0.8)	(0.1,0.6,0.9)
(D,H)	(0.2,0.3,0.9)	(0.1,0.5,0.9)	(0.2,0.3,0.6)	(0.1,0.6,0.9)	(0.1,0.6,0.9)
(D,W)	(0.1,0.2,0.9)	(0.2,0.5,0.9)	(0.1,0.2,0.3)	(0.4,0.5,0.9)	(0.1,0.5,0.9)
(H,W)	(0.1,0.2,0.6)	(0.2,0.5,0.9)	(0.1,0.2,0.7)	(0.1,0.5,0.9)	(0.2,0.5,0.9)
(H,O)	(1,1,0)	(0.2,0.6,0.8)	(0.2,0.9,0.6)	(0.1,0.6,0.8)	(0.1,0.8,0.7)
(W,P)	(0.2,0.6,0.8)	(1,1,0)	(0.4,0.6,0.8)	(0.2,0.5,0.9)	(0.2,0.5,0.9)
(W,O)	(0.2,0.9,0.6)	(0.4,0.6,0.8)	(1,1,0)	(0.2,0.5,0.9)	(0.2,0.5,0.9)
(O,P)	(0.1,0.6,0.8)	(0.2,0.5,0.9)	(0.2,0.5,0.9)	(1,1,0)	(0.5,0.6,0.9)
(O,D)	(0.1,0.8,0.7)	(0.2,0.5,0.9)	(0.2,0.5,0.9)	(0.5,0.6,0.9)	(1,1,0)

Thus, the lower and upper approximations of Y are calculated as follows:

$$\underline{S}Y = \{((P, D), 0.42, 0.23, 0.47), ((P, H), 0.45, 0.28, 0.45), ((D, H), 0.50, 0.21, 0.45),$$
$$((D, W), 0.43, 0.22, 0.45), ((H, W), 0.41, 0.30, 0.44), ((H, O), 0.51, 0.22, 0.46),$$
$$((W, P), 0.42, 0.26, 0.40), ((W, O), 0.42, 0.23, 0.44), ((O, P), 0.43, 0.25, 0.48),$$
$$((O, D), 0.50, 0.22, 0.48)\}$$
$$\overline{S}Y = \{((P, D), 0.42, 0.30, 0.44), ((P, H), 0.50, 0.30, 0.41), ((D, H), 0.50, 0.30, 0.45),$$
$$((D, W), 0.43, 0.30, 0.45), ((H, W), 0.41, 0.30, 0.44), ((H, O), 0.51, 0.30, 0.46),$$
$$((W, P), 0.42, 0.26, 0.37), ((W, O), 0.45, 0.30, 0.44), ((O, P), 0.50, 0.28, 0.45),$$
$$((O, D), 0.50, 0.30, 0.47)\}.$$

Thus, $\underline{G} = (\underline{R}X, \underline{S}Y)$ and $\overline{G} = (\overline{R}X, \overline{SY})$ are the neutrosophic digraphs as shown in Figure 11. To find the best company, we use the following formula:

$$S(v_i) = \sum_{v_i \in X^*} \frac{(T_{\underline{R}X}(v_i) \times T_{\overline{R}X}(v_i)) + (I_{\underline{R}X}(v_i) \times I_{\overline{R}X}(v_i)) - (F_{\underline{R}X}(v_i) \times F_{\overline{R}X}(v_i))}{1 - \{T(v_i v_j) + I(v_i v_j) - F(v_i v_j)\}}$$

where

$$T(v_iv_j) = \max_{v_j \in X^*} T_{\underline{S}Y}(v_iv_j) \times \max_{v_j \in X^*} T_{\overline{S}Y}(v_iv_j),$$
$$I(v_iv_j) = \max_{v_j \in X^*} I_{\underline{S}Y}(v_iv_j) \times \max_{v_j \in X^*} I_{\overline{S}Y}(v_iv_j),$$
$$F(v_iv_j) = \min_{v_j \in X^*} F_{\underline{S}Y}(v_iv_j) \times \min_{v_j \in X^*} F_{\overline{S}Y}(v_iv_j).$$

By direct calculations we have

$$S(P) = 0.167, S(D) = 0.156, S(H) = 0.268, S(W) = 0.272, S(O) = 0.155.$$

From the above calculations it is clear that Waves is the best company for refrigerator.

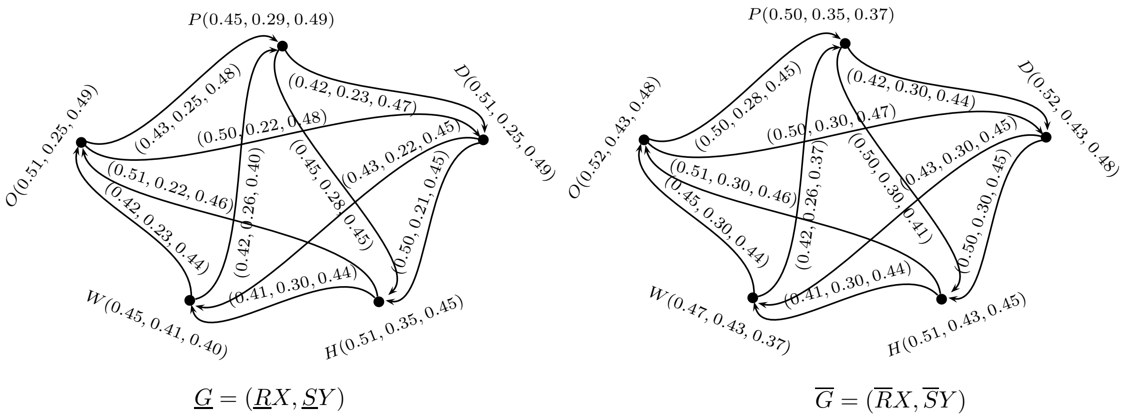


Figure 11. $G = (\underline{G}, \overline{G})$.

3.2. Context of Recruitment

Choosing the right candidate for the position available is not something that should be left to chance or guesswork.

How to choose the right candidate?

In any recruitment process the ability of the candidate is weighed against the suitability of the candidate. Pakistan Telecommunication Company Limited (PTCL) wants to recruit a person for the post of administrator. To keep the procedure simple, the company wants to appoint their employee on the basis of education (Edu) and experience (Exp). Let $X^* = \{(C1, Edu), (C1, Exp), (C2, Edu), (C2, Exp), (C3, Edu), (C3, Exp)\}$ be the set of candidates who applied for the post and their corresponding attributes. Let R be a neutrosophic tolerance on X^* given as follows:

R	(C1,Edu)	(C1,Exp)	(C2,Edu)	(C2,Exp)	(C3,Edu)	(C3,Exp)
(C1,Edu)	(1,1,0)	(0.3,0.6,0.1)	(0.6,0.7,0.2)	(0.6,0.5,0.8)	(0.3,0.2,0.1)	(0.9,0.1,0.1)
(C1,Exp)	(0.3,0.6,0.1)	(1,1,0)	(0.9,0.9,0.3)	(0.8,0.7,0.6)	(0.4,0.5,0.9)	(0.3,0.1,0.1)
(C2,Edu)	(0.6,0.7,0.2)	(0.9,0.9,0.3)	(1,1,0)	(0.6,0.5,0.1)	(0.3,0.2,0.1)	(0.4,0.8,0.7)
(C2,Exp)	(0.6,0.5,0.8)	(0.8,0.7,0.6)	(0.6,0.5,0.1)	(1,1,0)	(0.1,0.1,0.2)	(0.5,0.6,0.7)
(C3,Edu)	(0.3,0.2,0.1)	(0.4,0.5,0.9)	(0.3,0.2,0.1)	(0.1,0.1,0.2)	(1,1,0)	(0.2,0.1,0.2)
(C3,Exp)	(0.9,0.1,0.1)	(0.3,0.1,0.1)	(0.4,0.8,0.7)	(0.5,0.6,0.7)	(0.2,0.1,0.2)	(1,1,0)

Let $X = \{((C1, Edu), 0.9, 0.1, 0.5), ((C1, Exp), 0.2, 0.6, 0.5), ((C2, Edu), 0.7, 0.2, 0.3), ((C2, Exp), 0.1, 0.3, 0.9), ((C3, Edu), 0.4, 0.6, 0.8), ((C3, Exp), 0.8, 0.1, 0.2)\}$ be a neutrosophic set define on X^*. Then the lower and upper approximations of X are given as:

$$\begin{aligned}\underline{R}X = \{&((C1, Edu), 0.2, 0.1, 0.6), ((C1, Exp), 0.2, 0.2, 0.8), ((C2, Edu), 0.1, 0.2, 0.6),\\ &((C2, Exp), 0.1, 0.3, 0.9), ((C3, Edu), 0.2, 0.6, 0.8), ((C3, Exp), 0.2, 0.1, 0.5)\},\\ \overline{R}X = \{&((C1, Edu), 0.9, 0.6, 0.2), ((C1, Exp), 0.7, 0.6, 0.2), ((C2, Edu), 0.7, 0.6, 0.3),\\ &((C2, Exp), 0.6, 0.6, 0.3), ((C3, Edu), 0.4, 0.6, 0.2), ((C3, Exp, 0.9, 0.3, 0.2)\}.\end{aligned}$$

Let $Y^* = \{(C1, Edu)(C1, Exp), (C1, Exp)(C2, Edu), (C1, Edu)(C3, Exp), (C3, Exp)(C1, Exp), (C1, Exp)(C2, Exp), (C2, Exp)(C2, Edu), (C3, Exp)(C3, Edu), (C3, Edu)(C2, Exp), (C3, Exp)(C2, Exp)\} \subseteq X^* \times X^*$ and S be a neutrosophic tolerance relation on Y^* given as follows:

S	(C1,Edu)(C1,Exp)	(C1,Exp)(C2,Edu)	(C1,Edu)(C3,Exp)	(C3,Exp)(C1,Exp)	(C1,Exp)(C2,Exp)
(C1,Edu)(C1,Exp)	(1,1,0)	(0.3,0.6,0.3)	(0.3,0.1,0.1)	(0.9,0.1,0.1)	(0.3,0.6,0.6)
(C1,Exp)(C2,Edu)	(0.3,0.6,0.3)	(1,1,0)	(0.3,0.6,0.7)	(0.3,0.1,0.3)	(0.6,0.5,0.1)
(C1,Edu)(C3,Exp)	(0.3,0.1,0.1)	(0.3,0.6,0.7)	(1,1,0)	(0.3,0.1,0.1)	(0.3,0.6,0.7)
(C3,Exp)(C1,Exp)	(0.9,0.1,0.1)	(0.3,0.1,0.3)	(0.3,0.1,0.1)	(1,1,0)	(0.3,0.1,0.6)
(C1,Exp)(C2,Exp)	(0.3,0.6,0.6)	(0.6,0.5,0.1)	(0.3,0.6,0.7)	(0.3,0.1,0.6)	(1,1,0)
(C2,Exp)(C2,Edu)	(0.6,0.5,0.8)	(0.8,0.7,0.6)	(0.4,0.5,0.8)	(0.5,0.6,0.7)	(0.6,0.5,0.6)
(C3,Exp)(C2,Exp)	(0.8,0.1,0.6)	(0.3,0.1,0.1)	(0.5,0.1,.7)	(0.8,0.7,0.6)	(0.3,0.1,0.1)
(C3,Exp)(C3,Edu)	(0.4,0.1,0.9)	(0.3,0.1,0.1)	(0.2,0.1,.2)	(0.4,0.5,0.9)	(0.1,0.1,0.2)
(C3,Edu)(C2,Exp)	(0.3,0.2,0.6)	(0.4,0.5,0.9)	(0.3,.2,.7)	(0.2,0.1,0.6)	(0.4,0.5,0.9)

S	(C2,Exp)(C2,Edu)	(C3,Exp)(C2,Exp)	(C3,Exp)(C3,Edu)	(C3,Edu)(C2,Exp)
(C1,Edu)(C1,Exp)	(0.6,0.5,0.8)	(0.8,0.1,0.6)	(0.4,0.1,0.9)	(0.3,0.2,0.6)
(C1,Exp)(C2,Edu)	(0.8,0.7,0.6)	(0.3,0.1,0.1)	(0.3,0.1,0.1)	(0.4,0.5,0.9)
(C1,Edu)(C3,Exp)	(0.4,0.5,0.8)	(0.5,0.1,0.7)	(0.2,0.1,0.2)	(0.3,0.2,0.7)
(C3,Exp)(C1,Exp)	(0.5,0.6,0.7)	(0.8,0.7,0.6)	(0.4,0.5,0.9)	(0.2,0.1,0.6)
(C1,Exp)(C2,Exp)	(0.6,0.5,0.6)	(0.3,0.1,0.1)	(0.1,0.1,0.2)	(0.4,0.5,0.9)
(C2,Exp)(C2,Edu)	(1,1,0)	(0.5,0.5,0.7)	(0.3,0.2,0.7)	(0.1,0.1,0.2)
(C3,Exp)(C2,Exp)	(0.5,0.5,0.7)	(1,1,0)	(0.1,0.1,0.2)	(0.2,0.1,0.2)
(C3,Exp)(C3,Edu)	(0.3,0.2,0.7)	(0.1,0.1,0.2)	(1,1,0)	(0.1,0.1,0.2)
(C3,Edu)(C2,Exp)	(0.1,0.1,0.2)	(0.2,0.1,0.2)	(0.1,0.1,0.2)	(1,1,0)

Let $Y = \{((C1, Edu)(C1, Exp), 0.2, 0.1, 0.1), ((C1, Exp)(C2, Edu), 0.1, 0.1, 0.3), ((C1, Edu)(C3, Exp), 0.2, 0.1, 0.2), ((C3, Exp)(C1, Exp), 0.2, 0.1, 0.2), ((C1, Exp)(C2, Exp), 0.1, 0.2, 0.3), ((C2, Exp)(C2, Edu), 0.1, 0.2, 0.3)), ((C3, Exp)(C2, Exp), 0.1, 0.1, 0.3), ((C3, Exp)(C3, Edu), 0.2, 0.1, 0.2), ((C3, Edu)(C2, Exp), 0.1, 0.3, 0.3)\}$ be neutrosophic rough set on Y^*. Then the lower and upper approximations of Y are given as follows:

$$\begin{aligned}\underline{S}Y = \{&((C1, Edu)(C1, Exp), 0.2, 0.1, 0.3), ((C1, Exp)(C2, Edu), 0.1, 0.1, 0.3),\\ &((C1, Edu)(C3, Exp), 0.2, 0.1, 0.3), ((C3, Exp)(C1, Exp), 0.2, 0.1, 0.3),\\ &((C1, Exp)(C2, Exp), 0.1, 0.2, 0.3), ((C2, Exp)(C2, Edu, 0.1, 0.2, 0.3)),\\ &((C3, Exp)(C2, Exp), 0.1, 0.1, 0.3), ((C3, Exp)(C3, Edu), 0.1, 0.1, 0.3),\\ &((C3, Edu)(C2, Exp), 0.1, 0.3, 0.3)\},\\ \overline{S}Y = \{&((C1, Edu)(C1, Exp), 0.2, 0.2, 0.1), ((C1, Exp)(C2, Edu), 0.2, 0.3, 0.2),\\ &((C1, Edu)(C3, Exp), 0.2, 0.2, 0.1), ((C3, Exp)(C1, Exp), 0.2, 0.2, 0.1),\\ &((C1, Exp)(C2, Exp), 0.2, 0.2, 0.1), ((C2, Exp)(C2, Edu, 0.2, 0.2, 0.3)),\\ &((C3, Exp)(C2, Exp), 0.2, 0.2, 0.2), ((C3, Exp)(C3, Edu), 0.2, 0.2, 0.2),\\ &((C3, Edu)(C2, Exp), 0.2, 0.3, 0.2)\}\end{aligned}$$

Thus, $\underline{G} = (\underline{R}X, \underline{S}Y)$ and $\overline{G} = (\overline{R}X, \overline{S}Y)$ are the neutrosophic digraphs as shown in Figures 12 and 13.

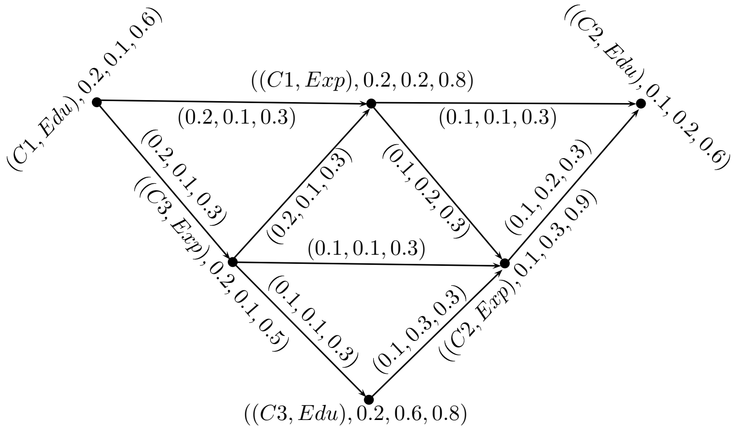
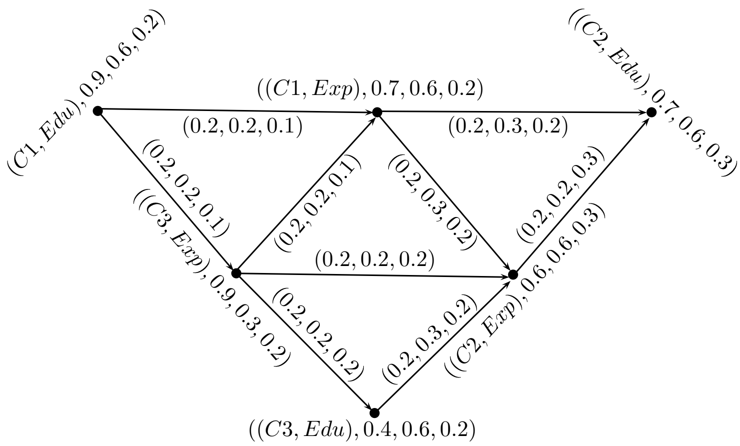


Figure 12. Neutrosophic Digraph $\underline{G} = (\underline{R}X, \underline{S}Y)$



$\overline{G} = (\overline{R}X, \overline{S}Y)$

Figure 13. Neutrosophic Digraph $\overline{G} = (\overline{R}X, \overline{S}Y)$

To find the best employee using the following calculations, we have

$$I_{\overline{R}Y}(C1) = \frac{I_{\overline{R}Y}(C1,Edu)+I_{\overline{R}Y}(C1,Exp)}{2} = \frac{0.9+0.7}{2} = 0.8$$
$$I_{\overline{R}Y}(C2) = \frac{I_{\overline{R}Y}(C2,Edu)+I_{\overline{R}Y}(C2,Exp)}{2} = \frac{0.7+0.6}{2} = 0.65$$
$$I_{\overline{R}Y}(C3) = \frac{I_{\overline{R}Y}(C3,Edu)+I_{\overline{R}Y}(C3,Exp)}{2} = \frac{0.4+0.9}{2} = 0.65$$

$$\max\{I_{\overline{R}Y}(C1), I_{\overline{R}Y}(C2), I_{\overline{R}Y}(C3)\} = \max\{0.8, 0.65, 0.65\} = 0.8$$

Thus, $C1$ is the best employee for the post under consideration. So, PTCL can hire C1 for the post of administrator.

4. Comparative Analysis of Rough Neutrosophic Digraphs and Neutrosophic Rough Digraphs

Rough neutrosophic digraphs and neutrosophic rough digraphs are two different notions on the basis of their construction and approach. In rough neutrosophic digraphs, the relation defined on the universe of discourse is a crisp equivalence relation that only classifies the elements which are related. On the other hand, in neutrosophic rough digraphs the relation defined on the set is neutrosophic tolerance relation. The neutrosophic tolerance relation not only classifies the elements of the set which are related but also expresses their relation in terms of three components, namely truth membership (T), Indeterminacy (I) and falsity (F). This approach leaves room for indeterminacy and incompleteness. Below, we apply the method of rough neutrosophic digraphs to the above presented application (online reviews and ratings).

Here $X^* = \{Pel(P), Dawlance(D), Haier(H), Waves(W), Orient(O)\}$ and the neutrosophic set on X^* according to the reviews will be $X = \{(P, 0.45, 0.29, 0.37), (D, 0.52, 0.25, 0.49), (H, 0.51, 0.43, 0.45), (W, 0.47, 0.41, 0.38), (O, 0.51, 0.35, 0.48)\}$. The equivalence relation on X^* is given below

R	P	D	H	W	O
P	1	0	1	0	1
D	0	1	0	0	0
H	1	0	1	0	1
W	0	0	0	1	0
O	1	0	1	0	1

The lower and upper approximations of X are as follows:

$$\begin{aligned}\underline{R}X = \{&(P, 0.45, 0.29, 0.48), (D, 0.52, 0.25, 0.49), (H, 0.45, 0.29, 0.48),\\ &(W, 0.47, 0.41, 0.38), (O, 0.45, , 0.29, 0.48)\},\\ \overline{R}X = \{&(P, 0.51, 0.43, 0.37), (D, 0.52, 0.25, 0.49), (H, 0.51, 0.43, 0.37),\\ &(W, 0.47, 0.41, 0.38), (O, 0.51, 0.43, 0.37)\}.\end{aligned}$$

Let $Y^* = \{(P, D), (P, H), (D, H), (D, W), (H, W), (H, O), (W, P), (W, O), (O, P), (O, D)\}$ be the subset of $X^* \times X^*$ and the equivalence relation S on Y^* is given as follows:

S	(P,D)	(P,H)	(D,H)	(D,W)	(H,W)	(H,O)	(W,P)	(W,O)	(O,P)	(O,D)
(P,D)	1	0	0	0	0	0	0	0	0	0
(P,H)	0	1	0	0	0	1	0	0	1	1
(D,H)	0	0	1	0	0	0	0	0	0	0
(D,W)	0	0	0	1	0	0	0	0	0	0
(H,W)	0	0	0	0	1	0	0	0	0	0
(H,O)	0	1	0	0	0	1	0	0	1	1
(W,P)	0	0	0	0	0	0	1	1	0	0
(W,O)	0	0	0	0	0	0	1	1	0	0
(O,P)	0	1	0	0	0	1	0	0	1	1
(O,D)	0	1	0	0	0	1	0	0	1	1

Thus, the lower and upper approximations of Y are calculated as follows:

$$\begin{aligned}\underline{S}Y = \{&((P, D), 0.45, 0.25, 0.48), ((P, H), 0.42, 0.24, 0.37), ((D, H), 0.45, 0.25, 0.47),\\ &((D, W), 0.45, 0.24, 0.48), ((H, W), 0.45, 0.29, 0.38), ((H, O), 0.42, 0.24, 0.37),\\ &((W, P), 0.42, 0.22, 0.37), ((W, O), 0.42, 0.22, 0.37), ((O, P), 0.42, 0.24, 0.37),\\ &((O, D), 0.42, 0.24, 0.37)\}\\ \overline{S}Y = \{&((P, D), 0.45, 0.25, 0.48), ((P, H), 0.45, 0.29, 0.37), ((D, H), 0.45, 0.25, 0.47),\\ &((D, W), 0.45, 0.24, 0.48), ((H, W), 0.45, 0.29, 0.38), ((H, O), 0.45, 0.29, 0.37),\\ &((W, P), 0.45, 0.29, 0.35), ((W, O), 0.45, 0.29, 0.35), ((O, P), 0.45, 0.29, 0.37),\\ &((O, D), 0.45, 0.29, 0.37)\}.\end{aligned}$$

To find the best company ratings, we use the following formula:

$$S(v_i) = \sum_{v_i \in X^*} \frac{(T_{\underline{R}X}(v_i) \times T_{\overline{R}X}(v_i)) + (I_{\underline{R}X}(v_i) \times I_{\overline{R}X}(v_i)) - (F_{\underline{R}X}(v_i) \times F_{\overline{R}X}(v_i))}{1 - \{T(v_i v_j) + I(v_i v_j) - F(v_i v_j)\}}$$

where

$$T(v_i v_j) = \max_{v_j \in X^*} T_{\underline{S}Y}(v_i v_j) \times \max_{v_j \in X^*} T_{\overline{S}Y}(v_i v_j),$$
$$I(v_i v_j) = \max_{v_j \in X^*} I_{\underline{S}Y}(v_i v_j) \times \max_{v_j \in X^*} I_{\overline{S}Y}(v_i v_j),$$
$$F(v_i v_j) = \min_{v_j \in X^*} F_{\underline{S}Y}(v_i v_j) \times \min_{v_j \in X^*} F_{\overline{S}Y}(v_i v_j).$$

By direct calculations, we have

$$S(P) = 0.20, S(D) = 0.0971, S(H) = 0.2077, S(W) = 0.2790, S(O) = 0.2011.$$

From the above calculations, we have Waves as the best choice and Dawlance as the least choice for refrigerator. This is because the relation applied in this method is crisp equivalence relation which does not consider the uncertainty between the companies of the same equivalence class. Whereas in our proposed method, least choice for refrigerator is different. So, the results may vary when we apply the method of rough neutrosophic digraphs and neutrosophic rough digraphs on the same application. This means that rough neutrosophic digraphs and neutrosophic rough digraphs have a different approach.

5. Conclusions

Neutrosophic set and rough set are two different theories to deal with uncertainty and imprecise and incomplete information. Due to the limitation of human knowledge to understand the complex problems, it is very difficult to apply only a single type of uncertainty method to deal with such problems. Therefore, it is necessary to develop hybrid models by incorporating the advantages of many other different mathematical models dealing with uncertainty. Thus, by combining these two mathematical tools, we have presented a new hybrid model, namely, neutrosophic rough digraphs. We have escribed regular neutrosophic rough digraphs and we have presented novel applications of our proposed hybrid in decision-making. We have given a comparison of both models, rough neutrosophic digraphs and neutrosophic rough digraphs. We plan to extend our research work to (1) Neutrosophic rough hypergraphs; (2) Bipolar neutrosophic rough hypergraphs; (3) Soft rough neutrosophic graphs; (4) Decision support systems based on neutrosophic rough information.

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References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
3. Smarandache, F. *Neutrosophy: Neutrosophic Probability, Set, and Logic*; American Research Press: Santa Fe, NM, USA, 1998.
4. Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. Single-valued neutrosophic sets. *Multispace Multistructure* **2010**, *4*, 410–413.
5. Pawlak, Z. Rough sets. *Int. J. Comput. Inf. Sci.* **1982**, *11*, 341–356. [CrossRef]
6. Dubois, D.; Prade, H. Rough fuzzy sets and fuzzy rough sets. *Int. J. Gen. Syst.* **1990**, *17*, 191–209. [CrossRef]

7. Liu, P.; Chen, S.M. Group decision-making based on Heronian aggregation operators of intuitionistic fuzzy numbers. *IEEE Trans. Cybern.* **2017**, *47*, 2514–2530. [CrossRef] [PubMed]
8. Mordeson, J.N.; Peng, C.S. Operations on fuzzy graphs. *Inf. Sci.* **1994**, *79*, 159–170. [CrossRef]
9. Akram, M.; Shahzadi, S. Neutrosophic soft graphs with applicatin. *J. Intell. Fuzzy Syst.* **2017**, *32*, 841–858. [CrossRef]
10. Akram, M.; Sarwar, M. Novel multiple criteria decision-making methods based on bipolar neutrosophic sets and bipolar neutrosophic graphs. *Ital. J. Pure Appl. Math.* **2017**, *38*, 368–389.
11. Akram, M.; Siddique, S. Neutrosophic competition graphs with applications. *J. Intell. Fuzzy Syst.* **2017**, *33*, 921–935. [CrossRef]
12. Akram, M.; Sitara, M. Interval-valued neutrosophic graph structures. *Punjab Univ. J. Math.* **2018**, *50*, 113–137.
13. Akram, M.; Zafar, F. Rough fuzzy digraphs with application. *J. Appl. Math. Comput.* **2018**. [CrossRef]
14. Zafer, F.; Akram, M. A novel decision-making method based on rough fuzzy information. *Int. J. Fuzzy Syst.* **2018**, *20*, 1000–1014. [CrossRef]
15. Broumi, S.; Smarandache, F.; Dhar, M. Rough neutrosophic sets. *Neutrosophic Sets Syst.* **2014**, *3*, 62–67.
16. Yang, H.L.; Zhang, C.L.; Guo, Z.L.; Liu, Y.; Liao, X. A hybrid model of single valued neutrosophic sets and rough sets: Single valued neutrosophic rough set model. *Soft Comput.* **2016**., [CrossRef]
17. Nabeela, I.; Sayed, S.; Akram, M.; Smarandache, F. Notions of Rough Neutrosophic Digraphs. *Mathematics* **2018**, *6*, 18.
18. Sidra, S.; Ishfaq, N.; Akram, M.; Smarandache, F. Rough Neutrosophic Digraphs with Application. *Axioms* **2018**, *7*, 5.
19. Akram, M.; Shum, K.P. Bipolar neutrosophic planar graphs. *J. Math. Res. Appl.* **2017**, *36*, 631–648.
20. Peng, H.G.; Zhang, H.Y.; Wang, J.Q. Probability multi-valued neutrosophic sets and its application in multi-criteria group decision-making problems. *Neural Comput. Appl.* **2016**. [CrossRef]
21. Ye, J. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *Int. J. General Syst.* **2013**, *42*, 386–394. [CrossRef]
22. Ye, J. Improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision-making. *J. Intell. Fuzzy Syst.* **2014**, *27*, 2453–2462.
23. Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. In *Neutrosophy*; American Research Press: Santa Fe, NM, USA, 1999.
24. Akram, M.; Malik, H.M.; Shahzadi, S.; Smarandache, F. Neutrosophic Soft Rough Graphs with Application. *Axioms* **2018**, *7*, 14. [CrossRef]
25. Banerjee, M.; Pal, S.K. Roughness of a fuzzy set. *Inf. Sci.* **1996**, *93*, 235–246. [CrossRef]
26. Liu, P.; Chen, S.M.; Junlin, L. Some intuitionistic fuzzy interaction partitioned Bonferroni mean operators and their application to multi-attribute group decision-making. *Inf. Sci.* **2017**, *41*, 98–121. [CrossRef]
27. Liu, P. Multiple attribute group decision-making method based on interval-valued intuitionistic fuzzy power Heronian aggregation operators. *Comput. Ind. Eng.* **2017**, *18*, 199–212. [CrossRef]
28. Liang, R.X.; Wang, J.Q.; Zhang, H.Y. A multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information. *Neural Comput. Appl.* **2017**. [CrossRef]
29. Nie, R.X.; Wang, J.Q.; Zhang, H.Y. Solving solar-wind power station location problem using an extended weighted aggregated sum product assessment (WASPAS) technique with interval neutrosophic sets. *Symmetry* **2017**, *9*, 106. [CrossRef]
30. Luo, S.Z.; Cheng, P.F.; Wang, J.Q.; Huang, Y.J. Selecting project delivery systems based on simplified neutrosophic linguistic preference relations. *Symmetry* **2017**, *9*, 151. [CrossRef]
31. Zhang, X.; Dai, J.; Yu, Y. On the union and intersection operations of rough sets based on various approximation spaces. *Inf. Sci.* **2015**, *292*, 214–229. [CrossRef]
32. Bao, Y.L.; Yang, H.L. On single valued neutrosophic refined rough set model and its application. *J. Intell. Fuzzy Syst.* **2017**, *33*, 1235–1248. [CrossRef]