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Decision-Making via Neutrosophic Support Soft Topological Spaces

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Abstract: The concept of interval neutrosophic sets has been studied and the introduction of a new kind of set in topological spaces called the interval valued neutrosophic support soft set has been suggested. We study some of its basic properties. The main purpose of this paper is to give the optimum solution to decision-making in real life problems the using interval valued neutrosophic support soft set.

Keywords: soft sets; support soft sets; interval valued neutrosophic support soft sets

2010 AMS Classification: 06D72; 54A05; 54A40; 54C10

1. Introduction

To deal with uncertainties, many theories have been recently developed, including the theory of probability, the theory of fuzzy sets, the theory of rough sets, and so on. However, difficulties are still arising due to the inadequacy of parameters. The concept of fuzzy sets, which deals with the nonprobabilistic uncertainty, was introduced by Zadeh [1] in 1965. Since then, many researchers have defined the concept of fuzzy topology that has been widely used in the fields of neural networks, artificial intelligence, transportation, etc. The intuitionistic fuzzy set (IFS for short) on a universe X was introduced by K. Atanaasov [2] in 1983 as a generalization of the fuzzy set in addition to the degree of membership and the degree of nonmembership of each element.

In 1999, Molodtsov [3] successfully proposed a completely new theory called soft set theory using classical sets. This theory is a relatively new mathematical model for dealing with uncertainty from a parametrization point of view. After Molodtsov, many researchers have shown interest in soft sets and their applications. Maji [4,5] introduced neutrosophic soft sets with operators, which are free from difficulties since neutrosophic sets [6–9] can handle indeterminate information. However, the neutrosophic sets and operators are hard to apply in real life applications. Therefore, Smarandache [10] proposed the concept of interval valued neutrosophic sets which can represent uncertain, imprecise, incomplete, and inconsistent information.

Nguyen [11] introduced the new concept in a type of soft computing, called the support-neutrosophic set. Deli [12] defined a generalized concept of the interval-valued neutrosophic soft set. In this paper, we combine interval-valued neutrosophic soft sets and support sets to yield the
interval-valued neutrosophic support soft set, and we study some of its basic operations. Our main aim of this paper is to make decisions using interval-valued neutrosophic support soft topological spaces.

## 2. Preliminaries

In this paper, we provide the basic definitions of neutrosophic and soft sets. These are very useful for what follows.

**Definition 1.** ([13]) Let $X$ be a non-empty set. A neutrosophic set, $A$, in $X$ is of the form $A = \{ (x, \mu_A(x), \sigma_A(x), \omega_A(x), \gamma_A(x)) ; x \in X \}$, where $\mu_A : X \rightarrow [0, 1]$, $\sigma_A : X \rightarrow [0, 1]$ and $\gamma_A : X \rightarrow [0, 1]$ represent the degree of membership function, degree of indeterminacy, and degree of non-membership function, respectively and $0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3, \forall x \in X$.

**Definition 2.** ([5]) Let $X$ be a non-empty set, let $P(X)$ be the power set of $X$, and let $E$ be a set of parameters, and $A \subseteq E$. The soft set function, $f_X$, is defined by

$$f_X : A \rightarrow P(X) \text{ such that } f_X(x) = \emptyset \text{ if } x \notin X.$$ 

The function $f_X$ may be arbitrary. Some of them may be empty and may have non-empty intersections. A soft set over $X$ can be represented as the set of order pairs $F_X = \{ (x, f_X(x)) : x \in X, f_X(x) \in P(X) \}$.

**Example 1.** Consider the soft set $\langle F, A \rangle$, where $X$ is a set of six mobile phone models under consideration to be purchased by decision makers, which is denoted by $X = \{ x_1, x_2, x_3, x_4, x_5, x_6 \}$, and $A$ is the parameter set, where $A = \{ y_1, y_2, y_3, y_4, y_5 \} = \{ \text{price, look, camera, efficiency, processor} \}$. A soft set, $F_X$, can be constructed such that $f_X(y_1) = \{ x_1, x_2 \}$, $f_X(y_2) = \{ x_1, x_4, x_5, x_6 \}$, $f_X(y_3) = \emptyset$, $f_X(y_4) = X$, and $f_X(y_5) = \{ x_1, x_2, x_3, x_4, x_5 \}$. Then,

$$F_X = \{ (y_1, x_1, x_2), (y_2, x_1, x_4, x_5, x_6), (y_3, \emptyset), (y_4, X), (y_5, x_1, x_2, x_3, x_4, x_5) \}.$$ 

<table>
<thead>
<tr>
<th>$X$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>$x_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<tr>
<td>$y_2$</td>
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<tr>
<td>$y_3$</td>
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<tr>
<td>$y_4$</td>
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<td>1</td>
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<tr>
<td>$y_5$</td>
<td>1</td>
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<td>0</td>
</tr>
</tbody>
</table>

**Definition 3.** ([14]) Let $X$ be a non-empty set, and $A = \{ y_1, y_2, y_3, \ldots, y_n \}$, the subset of $X$ and $F_X$ is a soft set over $X$. For any $y_j \in A$, $f_X(y_j)$ is a subset of $X$. Then, the choice value of an object, $x_i \in X$, is $C_{V_i} = \sum_j x_{ij}$, where $x_{ij}$ are the entries in the table of $F_X$:

$$x_{ij} = \begin{cases} 1, & \text{if } x_i \in f_X(y_j) \\ 0, & \text{if } x_i \notin f_X(y_j). \end{cases}$$

**Example 2.** Consider Example 2. Clearly, $C_{V_1} = \sum_{j=1}^{5} x_{1j} = 4$, $C_{V_3} = C_{V_5} = \sum_{j=1}^{5} x_{3j} = \sum_{j=1}^{5} x_{5j} = 2$, $C_{V_2} = C_{V_4} = C_{V_6} = \sum_{j=1}^{5} x_{2j} = \sum_{j=1}^{5} x_{4j} = \sum_{j=1}^{5} x_{6j} = 3$.

**Definition 4.** ([13]) Let $F_X$ and $F_Y$ be two soft sets over $X$ and $Y$. Then,

1. The complement of $F_X$ is defined by $F_X^C(x) = X \setminus f_X(x)$ for all $x \in A$;
2. The union of two soft sets is defined by $f_{X \cup Y}(x) = f_X(x) \cup f_Y(x)$ for all $x \in A$;
3. The intersection of two soft sets is defined by $f_{X \cap Y}(x) = f_X(x) \cap f_Y(x)$ for all $x \in A$. 


3. Interval Valued Neutrosophic Support Soft Set

In this paper, we provide the definition of an interval-valued neutrosophic support soft set and perform some operations along with an example.

**Definition 5.** Let $X$ be a non-empty fixed set with a generic element in $X$ denoted by $a$. An interval-valued neutrosophic support set, $A$, in $X$ is of the form

$$A = \{(x, \mu_A(x), \sigma_A(x), \omega_A(x), \gamma_A(x)) / a; a \in X\}.$$  

For each point, $a \in X$, $x, \mu_A(x), \sigma_A(x), \omega_A(x)$, and $\gamma_A(x) \in [0, 1]$.

**Example 3.** Let $X = \{a, b\}$ be a non-empty set, where $a, b \subseteq [0, 1]$. An interval valued neutrosophic support set, $A \subseteq X$, constructed according to the degree of membership function, $\mu_A(x)$, indeterminacy $\sigma_A(x)$, support function $\omega_A(x)$, and non-membership function $\gamma_A(x)$ is as follows:

$$A = \{((0.2, 1.0), (0.2, 0.4), (0.1, 0.7), (0.5, 0.7)) / a, ((0.6, 0.8), (0.8, 1.0), (0.4, 0.6), (0.4, 0.6)) / b\}.$$

**Definition 6.** Let $X$ be a non-empty set; the interval-valued neutrosophic support set $A$ in $X$ is of the form

$$A = \{(x, \mu_A(x), \sigma_A(x), \omega_A(x), \gamma_A(x); x \in X\}.$$

(i) An empty set $A$, denoted by $A = \emptyset$, is defined by

$$\emptyset = \{((0, 0), (1, 1), (0, 0), (1, 1)) / x : x \in X\}.$$

(ii) The universal set is defined by

$$U = \{((1, 1), (0, 0), (1, 1), (0, 0)) / x : x \in X\}.$$

(iii) The complement of $A$ is defined by

$$A^c = \{((\inf \gamma_A(x), \sup \gamma_A(x)), (1 - \sup \sigma_A(x), 1 - \inf \sigma_A(x)), (1 - \sup \omega_A(x), 1 - \inf \omega_A(x)),$$

$$(\inf \mu_A(x), \sup \mu_A(x)) / x : x \in X\}.$$

(iv) $A$ and $B$ are two interval-valued neutrosophic support sets of $X$. $A$ is a subset of $B$ if

$$\mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x), \omega_A(x) \leq \omega_B(x), \gamma_A(x) \geq \gamma_B(x).$$

(v) Two interval-valued neutrosophic support sets $A$ and $B$ in $X$ are said to be equal if $A \subseteq B$ and $B \subseteq A$.

**Definition 7.** Let $A$ and $B$ be two interval-valued neutrosophic support sets. Then, for every $x \in X$

(i) The intersection of $A$ and $B$ is defined by

$$A \cap B = \{\{\min[\inf \mu_A(x), \inf \mu_B(x)], \min[\sup \mu_A(x), \sup \mu_B(x))], (\max[\inf \sigma_A(x), \inf \sigma_B(x)],$$

$$\max[\sup \sigma_A(x), \sup \sigma_B(x)], (\min[\inf \omega_A(x), \inf \omega_B(x)], \min[\sup \omega_A(x), \sup \omega_B(x)],$$

$$(\max[\inf \gamma_A(x), \inf \gamma_B(x)], \max[\sup \gamma_A(x), \sup \gamma_B(x)]) / x : x \in X\}.$$

(ii) The union of $A$ and $B$ is defined by

$$A \cup B = \{\{\max[\inf \mu_A(x), \inf \mu_B(x)], \max[\sup \mu_A(x), \sup \mu_B(x)]], (\min[\inf \sigma_A(x),$$

$$\inf \sigma_B(x)], \min[\sup \sigma_A(x), \sup \sigma_B(x)], (\max[\inf \omega_A(x), \inf \omega_B(x)], \max[\sup \omega_A(x),$$

$$\sup \omega_B(x)], (\min[\inf \gamma_A(x), \inf \gamma_B(x)], \min[\sup \gamma_A(x), \sup \gamma_B(x)]) / x : x \in X\}.$$

(iii) A difference, $B$, is defined by

$$A \setminus B = \{\{\min[\inf \mu_A(x), \inf \mu_B(x)], \min[\sup \mu_A(x), \sup \mu_B(x))], (\max[\inf \sigma_A(x),$$

$$1 - \sup \sigma_B(x)], \max[\sup \sigma_A(x), 1 - \inf \sigma_B(x)], (\min[\inf \omega_A(x), 1 - \sup \omega_B(x)], \min[\sup \omega_A(x), 1 - \inf \omega_B(x)]),$$

$$(\max[\inf \gamma_A(x), \inf \gamma_B(x)], \max[\sup \gamma_A(x), \sup \gamma_B(x)]) / x : x \in X\}.$$

(iv) Scalar multiplication of $A$ is defined by

$$A.a = \{\{\min[\inf \mu_A(x.a, a, 1), \min[\sup \mu_A(x.a, a, 1)], (\min[\inf \sigma_A(x), a, 1], \min[\sup \sigma_A(x), a, 1]),$$

$$(\min[\inf \omega_A(x), a, 1], \min[\sup \omega_A(x), a, 1]), (\min[\inf \gamma_A(x), a, 1], \min[\sup \gamma_A(x), a, 1]) / x : x \in X\}.$$

(v) Scalar division of $A$ is defined by

$$A/a = \{\{\min[\inf \mu_A(x)/a, 1], \min[\sup \mu_A(x)/a, 1]), (\min[\inf \sigma_A(x)/a, 1], \min[\sup \sigma_A(x)/a, 1]),$$

$$(\min[\inf \omega_A(x)/a, 1], \min[\sup \omega_A(x)/a, 1]), (\min[\inf \gamma_A(x)/a, 1], \min[\sup \gamma_A(x)/a, 1]) / x : x \in X\}.$$
Definition 8. Let \( X \) be a non-empty set; \( IVNSS(X) \) denotes the set of all interval-valued neutrosophic support soft sets of \( X \) and a subset, \( A, \) of \( X. \) The soft set function is

\[
g_i : A \to IVNSS(x).
\]

The interval valued neutrosophic support soft set over \( X \) can be represented by

\[
G_i = \{ (y, g_i(y)) : y \in A \}, \text{ such that } g_i(y) = \emptyset \text{ if } x \not\in X.
\]

Example 4. Consider the interval-valued neutrosophic support soft set, \( \langle G_i, A \rangle \), where \( X \) is a set of two brands of mobile phones being considered by a decision maker to purchase, which is denoted by \( X = \{ a, b \} \), and \( A \) is a parameter set, where \( A = \{ y_1 = \text{price}, y_2 = \text{camera specification}, y_3 = \text{Efficiency}, \text{ and } y_4 = \text{size}, \text{ and } y_5 = \text{processor} \}. \) In this case, we define a set \( G_i \) over \( X \) as follows:

\[
G_i = \{ (y, g_i(y)) : y \in A \}, \text{ such that } g_i(y) = \emptyset \text{ if } x \not\in X.
\]

Clearly, we can see that the exact evaluation of each object on each parameter is unknown, while the lower limit and upper limit of such an evaluation are given. For instance, we cannot give the exact membership degree, support, indeterminacy and nonmembership degree of price ‘\( a \)’; however, the price of model ‘\( a \)’ is at least on the membership degree of 0.6 and at most on the membership degree of 0.8.

Definition 9. Let \( G_i \) be a interval valued neutrosophic support soft set of \( X. \) Then, \( G_i \) is known as an empty interval valued neutrosophic support soft set, if \( g_i(y) = \emptyset. \)

Definition 10. Let \( G_i \) be a interval valued neutrosophic support soft set of \( X. \) Then, \( G_i \) is known as the universal interval valued neutrosophic support soft set, if \( g_i(y) = X. \)

Definition 11. Let \( G_i, G_j \) be two interval valued neutrosophic support soft set of \( X. \) Then, \( G_i \) is said to be subset of \( G_j \), if \( g_i(y) \subseteq g_j(y). \)

Example 5. Two interval-valued neutrosophic support soft sets, \( G_i \) and \( G_j \), are constructed as follows:

\[
G_i = \{ (y, g_i(y)) : y \in A \}, \text{ such that } g_i(y) = \emptyset \text{ if } x \not\in X.
\]

Following Definition 11, \( G_i \) is a subset of \( G_j. \)
Definition 12. The two interval valued neutrosophic support soft sets, $G_i$, $G_j$, such that $G_i \subseteq G_j$, is said to be classical subset of X where every element of $G_i$ does not need to be an element of $G_j$.

Proposition 1. Let $G_i$, $G_j$, $G_k$ be an interval valued neutrosophic support soft set of X. Then,

1. Each $G_n$ is a subset of $G_X$, where $n= i,j,k$;
2. Each $G_n$ is a superset of $G_{\emptyset}$, where $n= i,j,k$;
3. If $G_i$ is a subset of $G_j$ and $G_j$ is a subset of $G_k$, then, $G_i$ is a subset of $G_k$.

Proof. The proof of this proposition is obvious.

Definition 13. The two interval valued neutrosophic support soft sets of X are said to be equal, if and only if $g_i = g_j$, for all $i,j \in X$.

Proposition 2. Let $X$ be a non-empty set and $G_i$, $G_j$ be an interval valued neutrosophic support soft set of X. $G_i$ is a subset of $G_j$ and $G_j$ is a subset of $G_i$, if and only if $G_i$ is equal to $G_j$.

Definition 14. The complement of the interval valued neutrosophic support soft set, $G_i$, of X is denoted by $G_i^c$, for all $i \in A$.

(i) The complement of the empty interval valued neutrosophic support soft set of X is the universal interval valued neutrosophic support soft set of X.

(ii) The complement of the universal interval valued neutrosophic support soft set of X is the empty interval valued neutrosophic support soft set of X.

Theorem 1. Let $G_i$, $G_j$ be an interval valued neutrosophic support soft set of X. Then, $G_i$ is a subset of $G_j$ and the complement of $G_j$ is a subset of the complement of $G_i$.

Proof. Let $G_i$ and $G_j$ be an interval valued neutrosophic support soft set of X. By definition, 3.7 $G_i$ is a subset of $G_j$ if $g_i(y) \subseteq g_j(y)$. Then, the complement of $g_i(y) \subseteq g_j(y)$ is $g_i^c(y) \supseteq g_j^c(y)$. Hence, the complement of $G_j$ is a subset of the complement of $G_i$.

Example 6. From Example 4, the complement of $G_i$ is constructed as follows:

$$
\begin{array}{|c|c|c|}
\hline
G_i & a & b \\
\hline
y_1 & [0.1,0.5],[0.1,0.2],[0.4,0.5],[0.6,0.8] & [0.1,0.7],[0.2,0.9],[0.3,0.7],[0.6,0.8] \\
\hline
y_2 & [0.3,0.8],[0.2,0.5],[0.6,0.7],[0.2,0.4] & [0.2,0.3],[0.1,0.4],[0.2,0.5],[0.2,0.8] \\
\hline
y_3 & [0.6,0.8],[0.5,0.8],[0.3,0.5],[0.1,0.9] & [0.5,0.7],[0.4,0.8],[0.4,0.5],[0.4,0.9] \\
\hline
y_4 & [0.8,0.9],[0.1,0.2],[0.3,0.9],[0.6,0.8] & [0.1,0.8],[0.2,0.4],[0.1,0.3],[0.5,0.7] \\
\hline
y_5 & [1.0,1.0],[0.0,0.9],[0.0,0.1],[0.0,0.9] & [0.2,0.5],[0.0,0.2],[0.5,0.7],[0.0,0.8] \\
\hline
\end{array}
$$

Definition 15. The union of the interval valued neutrosophic support soft set of X is denoted by $G_i \cup G_j$ and is defined by $g_i(y) \cup g_j(y) = g_i(y) \cup g_j(y)$ for all $y \in A$.

Proposition 3. Let $G_i,G_j,G_k$ be an interval valued neutrosophic support soft set of X. Then,

(i) $G_i \cup G_{\emptyset} = G_i$.
(ii) $G_i \cup G_X = G_X$.
(iii) $G_i \cup G_j = G_j \cup G_i$.
(iv) $(G_i \cup G_j) \cup G_k = G_i \cup (G_j \cup G_k)$.

Example 7. From Example 4, the union of two sets is represented as follows:
Definition 16. Let $G_i, G_j$ be an interval valued neutrosophic support soft set of $X$. Then, the intersection of two sets denoted by $G_i \cap G_j$ is defined as $g_i(y) \cap g_j(y) = g_j(y) \cap g_i(y)$ for all $y \in A$.

Proposition 4. Let $G_i, G_j, G_k$ be an interval valued neutrosophic support soft set of $X$. Then,

(i) $G_i \cap G_j = G_j \cap G_i$.
(ii) $G_i \cap G_j = G_i \cap G_j$.
(iii) $(G_i \cap G_j) \cap G_k = G_i \cap (G_j \cap G_k)$.

Proof. The proof is obvious. □

Example 8. In accordance with Example 4, the intersection operation is performed as follows:

<table>
<thead>
<tr>
<th>$G_i \cap G_j$</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>$[0.6,0.8]$, $[0.8,0.9]$, $[0.5,0.6]$, $[0.1,0.5]$</td>
<td>$[0.6,0.8]$, $[0.1,0.8]$, $[0.3,0.7]$, $[0.1,0.7]$</td>
</tr>
<tr>
<td>$y_2$</td>
<td>$[0.2,0.4]$, $[0.5,0.8]$, $[0.3,0.4]$, $[0.3,0.8]$</td>
<td>$[0.2,0.8]$, $[0.6,0.9]$, $[0.5,0.8]$, $[0.2,0.3]$</td>
</tr>
<tr>
<td>$y_3$</td>
<td>$[0.1,0.9]$, $[0.2,0.5]$, $[0.5,0.7]$, $[0.6,0.8]$</td>
<td>$[0.4,0.9]$, $[0.2,0.6]$, $[0.5,0.6]$, $[0.5,0.7]$</td>
</tr>
<tr>
<td>$y_4$</td>
<td>$[0.6,0.8]$, $[0.8,0.9]$, $[0.1,0.7]$, $[0.8,0.9]$</td>
<td>$[0.5,0.7]$, $[0.6,0.8]$, $[0.7,0.9]$, $[0.1,0.8]$</td>
</tr>
<tr>
<td>$y_5$</td>
<td>$[0.0,0.9]$, $[0.1,0.9]$, $[0.9,1.0]$, $[1.0,1.0]$</td>
<td>$[0.0,0.8]$, $[0.8,1.0]$, $[0.3,0.5]$, $[0.2,0.5]$</td>
</tr>
</tbody>
</table>

Definition 17. Let $G_i$ be an interval valued neutrosophic support soft set of $X$. Then, the union of interval valued neutrosophic support soft set and its complement is not a universal set and it is not mutually disjoint.

Proposition 5. Let $G_i, G_j$ be an interval valued neutrosophic support soft set of $X$. Then, the D’Margan Laws hold.

(i) $(G_i \cup G_j)^c = G_i^c \cap G_j^c$.
(ii) $(G_i \cap G_j)^c = G_i^c \cup G_j^c$.

Proposition 6. Let $G_i, G_j, G_k$ be an interval valued neutrosophic support soft set of $X$. Then, the following hold.

(i) $G_i \cup (G_j \cap G_k) = (G_i \cup G_j) \cap (G_i \cap G_k)$.
(ii) $G_i \cap (G_j \cup G_k) = (G_i \cap G_j) \cup (G_i \cap G_k)$

Definition 18. Let $G_i, G_j$ be an interval valued neutrosophic support soft set of $X$. Then, the difference between two sets is denoted by $G_i / G_j$ and is defined by

$$g_{i / j}(y) = g_i(y) / g_j(y)$$

for all $y \in A$.

Definition 19. Let $G_i, G_j$ be an interval valued neutrosophic support soft set of $X$. Then the addition of two sets are denoted by $G_i + G_j$ and is defined by

$$g_{i + j}(y) = g_i(y) + g_j(y)$$

for all $y \in A$. 
Definition 20. Let $G_i$ be an interval valued neutrosophic support soft set of $X$. Then, the scalar division of $G_i$ is denoted by $G_i/a$ and is defined by

$$g_{i/a}(y) = g_i(y)/a$$

for all $y \in A$.

4. Decision-Making

In this paper, we provide the definition of relationship between the interval valued neutrosophic support soft set, the average interval valued neutrosophic support soft set and the algorithm to get the optimum decision.

Definition 21. Let $G_i$ be an interval valued neutrosophic support soft set of $X$. Then, the relationship, $R$, for $G_i$ is defined by

$$R_{G_i} = \{r_{G_i}(y,a) : r_{G_i}(y,a) \in \text{interval valued neutrosophic support set}. y \in A, a \in X\}$$

where $r_{G_i} : A \setminus X \Rightarrow \text{interval valued neutrosophic support soft set}(X)$ and $r_{G_i}(y,a) = g_i(y)(a)$ for all $y \in A$ and $a \in X$.

Example 9. From Example 4, the relationship for the interval valued neutrosophic support soft set of $X$ is given below.

$$g_i(y_1)(a) = \{[0.6,0.8],[0.8,0.9],[0.5,0.6],[0.1,0.5]\},$$
$$g_i(y_1)(b) = \{[0.6,0.8],[0.1,0.8],[0.3,0.7],[0.1,0.7]\},$$
$$g_i(y_2)(a) = \{[0.2,0.4],[0.5,0.8],[0.4,0.3],[0.3,0.8]\},$$
$$g_i(y_2)(b) = \{[0.2,0.8],[0.6,0.9],[0.5,0.8],[0.4,0.3]\},$$
$$g_i(y_3)(a) = \{[0.1,0.9],[0.2,0.5],[0.5,0.7],[0.6,0.8]\},$$
$$g_i(y_3)(b) = \{[0.4,0.9],[0.2,0.6],[0.5,0.6],[0.5,0.7]\},$$
$$g_i(y_4)(a) = \{[0.6,0.8],[0.8,0.9],[0.1,0.7],[0.8,0.9]\},$$
$$g_i(y_4)(b) = \{[0.5,0.7],[0.6,0.8],[0.7,0.9],[0.1,0.8]\},$$
$$g_i(y_5)(a) = \{[0.0,0.9],[1.0,0.1],[1.0,0.9],[1.0,1.0]\},$$
$$g_i(y_5)(b) = \{[0.0,0.8],[0.8,1.0],[0.3,0.5],[0.2,0.5]\}.$$

Definition 22. Let $G_i$ be an interval valued neutrosophic support soft set of $X$. For $\mu, \sigma, \omega, \gamma \subseteq [0,1]$, the $(\mu,\sigma,\omega,\gamma)$-level support soft set of $G_i$ defined by $\langle G_i; (\mu,\sigma,\omega,\gamma) \rangle = \{(y_i, \{a_j; a_{ij} \in X, \mu(a_{ij}) = 1\}) : y \in A\}$, where

$$\mu(a_{ij}) = \begin{cases} 1 & \text{if } (\mu,\sigma,\omega,\gamma) \leq g_i(y)(a_j) \\ 0 & \text{if otherwise} \end{cases} \text{ for all } a_j \in X.$$  

Definition 23. Let $G_i$ be an interval valued neutrosophic support soft set of $X$. The average interval valued neutrosophic support soft set is defined by $\langle \mu,\sigma,\omega,\gamma \rangle \text{Avg}_{G_i}(y_i) = \frac{1}{|X|} \sum_{a \in X} g_i(y)(a)$ for all $y \in A$.

Example 10. Considering Example 4, the average interval valued neutrosophic support soft set is calculated as follows:

$$\langle \mu,\sigma,\omega,\gamma \rangle \text{Avg}_{G_i}(y_1) = \frac{2}{2} g_i(y_1)(a) / |X| = \{[0.6,0.8],[0.45,0.85],[0.4,0.65],[0.1,0.6]\}$$

$$\langle \mu,\sigma,\omega,\gamma \rangle \text{Avg}_{G_i}(y_2) = \frac{2}{2} g_i(y_2)(a) / |X| = \{[0.2,0.6],[0.55,0.85],[0.45,0.55],[0.25,0.55]\}$$
Let \( X \) be a non-empty set and \( G_i, G_j \) be an interval valued neutrosophic support soft set of \( X \). \( \{ G_i; (\mu_i, \sigma_i, \omega_i, \gamma_i) \} \) and \( \{ G_j; (\mu_j, \sigma_j, \omega_j, \gamma_j) \} \) are level support soft sets if \( (\mu_1, \sigma_1, \omega_1, \gamma_1) \leq (\mu_2, \sigma_2, \omega_2, \gamma_2) \). Then, \( \{ G_i; (\mu_i, \sigma_i, \omega_i, \gamma_i) \} \leq \{ G_j; (\mu_j, \sigma_j, \omega_j, \gamma_j) \} \).

**Proof.** Let \( G_i \) and \( G_j \) be an interval valued neutrosophic support soft set of \( X \). In accordance with Definition 3.2 (iv), each function is \( \mu_1 \leq \mu_2, \sigma_1 \leq \sigma_2, \omega_1 \leq \omega_2, \gamma_1 \geq \gamma_2 \). Thus, the corresponding interval valued neutrosophic support soft set is \( \{ G_i; (\mu_i, \sigma_i, \omega_i, \gamma_i) \} \leq \{ G_j; (\mu_j, \sigma_j, \omega_j, \gamma_j) \} \). Hence, the proof. □

The following algorithm is used to make decisions in an interval-valued neutrosophic support set.

**Algorithm 1:**

1. Enter the interval valued neutrosophic support soft set, \( G_i \);
2. Enter the average interval valued neutrosophic support soft set, \( \langle \mu, \sigma, \omega, \gamma \rangle \mathcal{A}_{G_i} \), using average-level decision rules to make decisions;
3. Determine the average-level support soft set, \( G_i; \langle \mu, \sigma, \omega, \gamma \rangle \mathcal{A}_{\mathcal{G}_i} \);
4. Present the level support soft set in tabular form;
5. Determine the choice value, \( C_{G_i} \), of \( a \) for any \( a \in X \);
6. Select the optimum value for the optimum decision, \( C_{G_i} = \max_{a \in X} C_{G_i} \).

**Example 11.** People who are affected by cancer, have a combination of treatments, such as surgery with chemotherapy and/or radiation therapy, hormone therapy, and immunotherapy. Our main objective is to find the best treatment from the above mentioned therapies. However, all the treatments can cause side effects. Our goal is to find the best treatment which cause the least side effects, reduce the cost of the treatment, extend the patient’s life, cure the cancer and control its growth using an interval-valued neutrosophic support soft set.

\[
\langle \mu, \sigma, \omega, \gamma \rangle \mathcal{A}_{G_i} (y_3) = \sum_{i=1}^{2} \mathcal{A}_{G_i} (a)/|X| = \langle [0.25, 0.9], [0.2, 0.55], [0.5, 0.65], [0.55, 0.75] \rangle
\]

\[
\langle \mu, \sigma, \omega, \gamma \rangle \mathcal{A}_{G_i} (y_4) = \sum_{i=1}^{2} \mathcal{A}_{G_i} (a)/|X| = \langle [0.55, 0.75], [0.7, 0.85], [0.4, 0.8], [0.45, 0.85] \rangle
\]

\[
\langle \mu, \sigma, \omega, \gamma \rangle \mathcal{A}_{G_i} (y_5) = \sum_{i=1}^{2} \mathcal{A}_{G_i} (a)/|X| = \langle [0.0, 0.85], [0.9, 0.55], [0.65, 0.7], [0.6, 0.75] \rangle
\]

<table>
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<tr>
<th>( a )</th>
<th>( b )</th>
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<table>
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<th>( d )</th>
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</thead>
<tbody>
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<tr>
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<td>([0.1, 0.8][0.8, 0.9][0.2, 0.9][0.3, 0.5])</td>
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<td>([0.1, 0.7][0.4, 0.5][0.2, 0.8][0.8, 0.9])</td>
<td>([0.2, 0.5][0.5, 0.7][0.3, 0.6][0.6, 0.8])</td>
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<tr>
<td>([0.2, 0.4][0.7, 0.9][0.6, 0.8][0.6, 0.9])</td>
<td>([0.3, 0.9][0.6, 0.9][0.2, 0.8][0.3, 0.9])</td>
</tr>
<tr>
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<td>([0.0, 0.1][0.9, 1][0.2, 0.2][0.2, 0.9])</td>
</tr>
</tbody>
</table>
The average interval valued neutrosophic support soft set is determined as follows:

\[
\langle \mu, \sigma, \omega, \gamma \rangle_{\text{Avg}C_0} = \{(0.375, 0.65), (0.55, 0.85), (0.325, 0.775), (0.25, 0.6), (0.125, 0.575), (0.675, 0.825), (0.2, 0.5), (0.3, 0.65)\} / y_1, (0.225, 0.675), (0.35, 0.575), (0.225, 0.675), (0.65, 0.8)) / y_3, (0.4, 0.775), (0.625, 0.875), (0.4, 0.825), (0.4, 0.85)) / y_4, ((0.0, 0.525), (0.825, 1.0), (0.575, 0.625), (0.6, 0.85)) / y_5; \]

2. \( \{G_i; \langle \mu, \sigma, \omega, \gamma \rangle_{\text{Avg}C_0} \} = \{(y_2, b), (y_3, b), (y_4, a), (y_5, b)\} \);

3. The average-level support soft set, \( \{G_i; \langle \mu, \sigma, \omega, \gamma \rangle_{\text{Avg}C_0} \} \) is represented in tabular form.

<table>
<thead>
<tr>
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<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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<td>y_1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y_4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y_5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

4. Compute the choice value, \( C_{y_i} \), of \( a_i \) for all \( a_i \in X \) as

\[
C_{y_3} = C_{y_4} = \sum_{j=1}^{4} a_{3j} = \sum_{j=1}^{4} a_{4j} = 0, \quad C_{y_1} = \sum_{j=1}^{4} a_{1j} = 1, \quad C_{y_2} = \sum_{j=1}^{4} a_{2j} = 3; \]

5. \( C_{y_2} \) gives the maximum value. Therefore \( b \) is the optimum choice.

Now, we conclude that there are a few ways to get rid of cancer, but surgery chemotherapy is preferred by most of the physicians with respect to the cost of treatment and extending the life of the patient with the least side effects. Moreover, side effects will be reduced or vanish completely after finished chemotherapy, and the cancer and its growth will be controlled.

5. Conclusions and Future Work

Fuzzy sets are inadequate for representing some parameters. Therefore, intuitionistic fuzzy sets were introduced to overcome this inadequacy. Further, neutrosophic sets were introduced to represent the indeterminacy. In order to make decisions efficiently, we offer this new research work which does not violate the basic definitions of neutrosophic sets and their properties. In this paper, we add one more function called the support function in interval-valued neutrosophic soft set, and we also provide the basic definition of interval valued neutrosophic support soft set and some of its properties. Further, we framed an algorithm for making decisions in medical science with a real-life problem. Here, we found the best treatment for cancer under some constraints using interval valued neutrosophic support soft set. In the future, motivated by the interval valued neutrosophic support soft set, we aim to develop interval valued neutrosophic support soft set in ideal topological spaces. In addition, weaker forms of open sets, different types of functions and theorems can be developed using interval valued neutrosophic support soft set to allow continuous function. This concept may be applied in operations research, data analytics, medical sciences, etc. Industry may adopt this technique to minimize the cost of investment and maximize the profit.

**Author Contributions:** All authors have contributed equally to this paper. The individual responsibilities and contributions of all authors can be described as follows: the idea of this whole paper was put forward by M.P. and M.K. R.U. and M.K. completed the preparatory work of the paper. F.S. and S.J. analyzed the existing work. The revision and submission of this paper was completed by M.P. and F.S.

**Conflicts of Interest:** The authors declare no conflict of interest.
References


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