symmetry

MDPI

Article

Design of Variable Sampling Plan for Pareto Distribution Using Neutrosophic Statistical Interval Method

Muhammad Aslam [1,*], Nasrullah Khan [2] and Ali Hussein AL-Marshadi [1]

[1] Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia; aalmarshadi@kau.edu.sa

[2] Department of Statistics, University of Veterinary and Animal Sciences, Lahore 54000, Jhang campus, Lahore, Pakistan; nas_shan1@hotmail.com

* Correspondence: aslam_ravian@hotmail.com or magmuhammad@kau.edu.sa; Tel.: +966-59-3329841

Received: 18 December 2018; Accepted: 5 January 2019; Published: 11 January 2019

check for updates

Abstract: The sampling plans have been widely used for the inspection of a lot of the product. In practice, the measurement data may be imprecise, uncertain, unclear or fuzzy. When there is uncertainty in the observations, the sampling plans designed using classical statistics cannot be applied for the inspection of a lot of the product. The neutrosophic statistic, which is the generalization of the classical statistics, can be used when data is not precise, uncertain, unclear or fuzzy. In this paper, we will design the variable sampling plan under the Pareto distribution using the neutrosophic statistics. We used the symmetry property of the normal distribution. We assume uncertainty in measurement data and sample size required for the inspection of a lot of the product. We will determine the neutrosophic plan parameters using the neutrosophic optimization problem. Some tables are given for practical use and are discussed with the help of an example.

Keywords: uncertainty; Pareto distribution; classical statistics; optimization problem; inspection

1. Introduction

To maintain the high-quality level of the product, industries are very careful in their acceptance of a lot of raw material to a finished a lot of the product. Therefore, a strict inspection of the material is done at each stage of the manufacturing process. At each stage, it may not be possible to inspect each item from a lot of raw material to a lot of final product. Therefore, a well-designed acceptance sampling plan is used for the inspection of a lot of the product. At each stage, a random sample of some specific size is selected from a lot of the product for making a decision about the submitted lot. A lot of the product is accepted if the number of non-conforming items are smaller than the allowed number of non-conforming items, otherwise a lot of the product is rejected. Therefore, a well- designed sampling plan is helpful in minimizing the producer's risk (the chance of rejecting a good lot) and consumer's risk (the chance of accepting a bad lot) [1].

Usually, the industrial data can be classified as attribute data and variable data. The attribute data is obtained from the go-no-go or counting process while variable data is obtained when the quality of interest is measurable. For the first case, the attribute sampling plans are applied for the inspection of the submitted lot of the product. In the latter case, the variable sampling plans are applied for the inspection of the submitted lot of the product [2]. Both sampling plans are very popular in the industry to maintain the high level of the quality product [3]. Collani [4] pointed out that variable sampling plan can be used in the industry when the purpose is to see the non-conforming items. Seidel [5] claimed that the variable sampling plan is more optimal than the attribute sampling plan. However, the attribute sampling plan is easy to apply when containing less information than the variable sampling plans. Several authors presented the work on variable sampling plans, including for example References [6–14].

The traditional acceptance sampling plans are applied under the assumption that the percent defective is crisp value. However, in practice, it is not always true that the proportion of defective is certain before inspection. The fuzzy-based sampling plans have been widely used for the inspection or lot sentencing of a lot of product when percent defective is fuzzy or unclear. Several authors contributed in this area and designed sampling plans based on the fuzzy approach. Cheng et al. [15] proposed fuzzy testing for an efficient process. Jamkhaneh and Gildeh [16] worked on a double sampling plan using the fuzzy approach. The sequential fuzzy plan can be seen in Reference [17]. References [18,19] designed fuzzy multiple dependent state sampling (MDS) plans. Afshari et al. [20] worked fuzzy attribute MDS plan. Elango et al. [21] presented some mathematics for fuzzy sampling plan. Some more details on such types of sampling plans can be seen in [22–25].

The existing variable sampling plans are designed using the classical statistics and can only be applied for the lot sentencing under the assumption that the measurement data is clear, precise and certain. Viertl [26] pointed out that "all observations and measurements of continuous variables are not precise numbers but more or less non-precise. This imprecision is different from variability and errors. Therefore, also lifetime data are not precise numbers but are more or less fuzzy. The best up-to-date mathematical model for this imprecision is so-called "non-precise numbers". Smarandache [27] mentioned that "neutrosophic logic as the generalization of fuzzy logic". A generalization of classical statistics is called the "neutrosophic statistics"; it was introduced by Reference [28] and can be applied when measurement data is not precise, uncertain, unclear or fuzzy. References [29,30] applied the neutrosophic statistics in the rock joint roughness. Recently, Aslam [31] introduced the neutrosophic statistics. More details can be seen in References [31–36].

Sathya Narayanan and Rajarathinam [37] designed a variable sampling plan based on the Pareto distribution. By exploring the literature and to the best of our knowledge, there is no work on the variable sampling plan based on the Pareto distribution using the neutrosophic statistics. In this paper, we will design the variable sampling plan under the Pareto distribution using the neutrosophic statistics. We assume uncertainty in measurement data and sample size required for the inspection of a lot of the product. We will determine the neutrosophic plan parameters using the neutrosophic optimization problem. Some tables are given for the practical use and are discussed with the help of an example.

2. Design of Variable Plan under Neutrosophic Statistics

It is assumed that the neutrosophic random variable $X_{Ni} \in \{X_L, X_U\} = i = 1, 2, 3, \ldots, n$ follows the neutrosophic Pareto distribution $F_N(X_N)$. Suppose $\mu_N \in \{\mu_L, \mu_U\}$ is the population neutrosophic average and $\sigma_N \in \{\sigma_L, \sigma_U\}$ is the population neutrosophic standard deviation. Let $\overline{X}_N \in \{\overline{X}_L, \overline{X}_U\}$ be the neutrosophic sample average which is the best linear unbiased estimator (BLUE) of $\mu_N \in \{\mu_L, \mu_U\}$ and $s_N = \{s_L, s_U\}$ is the BLUE neutrosophic standard deviation of $\sigma_N \in \{\sigma_L, \sigma_U\}$. L and U are the lower specification limit (LSL) and upper specification limit (USL), respectively, and an item is labeled as non-conforming if $X_N > U$ or $X_N < L$. Based on these assumptions, the variable plan under the neutrosophic statistics is stated as follows

Step-1: Select a neutrosophic random sample of size $n_N \in \{n_L, n_U\}$ from the lot of the product and record the observations for $X_{Ni} \in \{X_L, X_U\} = i = 1, 2, 3, \ldots, n$.

Step-2: Accept a lot of the product if $Y_N = \overline{X}_N + k_N s_N \leq U$ or $Y_N = \overline{X}_N + k_N s_N \geq L$, where $k_N \in \{k_{aL}, k_{aU}\}$ is the neutrosophic acceptance number, where $\overline{X}_L = \sum_{i=1}^{n} x_i^L / n_L$.

$$\overline{X}_U = \sum_{i=1}^{n} x_i^U / n_U, s_L = \sqrt{\sum_{i=1}^{n} \left(x_i^L - \overline{X}_L\right)^2 / n_L} \text{ and } s_U = \sqrt{\sum_{i=1}^{n} \left(x_i^U - \overline{X}_U\right)^2 / n_U}$$

The neutrosophic statistic Y_N has the approximate normal distribution with mean $\mu_{Y_N} = \mu_N + k_N\sigma_N; \mu_{Y_N} \in \{\mu_{Y_L}, \mu_{Y_U}\}$ and $\sigma_{Y_N} = \sqrt{\frac{\sigma^2}{n}\left[1 + \frac{k^2}{4}(\alpha_4 - 1) + k\alpha_3\right]}; \sigma_{Y_N} \in \{\sigma_{Y_L}, \sigma_{Y_U}\}$.

The proposed variable sampling plan based on neutrosophic statistics has two neutrosophic plan parameters, namely $n_N \in \{n_L, n_U\}$ and $k_N \in \{k_{aL}, k_{aU}\}$. These neutrosophic plan parameters will be determined through a neutrosophic non-linear optimization solution. The proposed plan reduces to Reference [37] plan based on the classical statistics when no uncertainty or fuzziness is found in observations that $n_L = n_U$ and $k_{aL} = k_{aU}$.

Assume that $X_{Ni} \in \{X_L, X_U\} = i = 1, 2, 3, \ldots, n$ is a neutrosophic random variable which follows the neutrosophic Pareto distribution (NPD) with a neutrosophic shape parameter $a_N \in \{a_L, a_U\}$ and neutrosophic scale parameter $b_N \in \{b_L, b_U\}$. The neutrosophic cumulative distribution function (NCDF) of the Pareto distribution is defined by

$$F_N(X_N; a_N, b_N) = \frac{a_N b_N^{a_N}}{X_N^{a_N+1}};\ X_N \in \{X_L, X_U\}, a_N \in \{a_L, a_U\}, b_N \in \{b_L, b_U\} \tag{1}$$

The NPD reduces to the classical Pareto distribution when $a_L = a_U$ and $b_L = b_U$. The neutrosophic mean and variance of NPD are $\mu_N = a_N b_N/(a_N - 1)$ for $a_N \in \{a_L, a_U\} > 1$ and $\sigma_N^2 = a_N b_N^2/(a_N - 1)^2(a_N - 2)$ for $a_N \in \{a_L, a_U\} > 2$, respectively. The neutrosophic skeweness and neutrosophic kurtosis of NPD are $\alpha_{N3} = (2(1 + a_N)/(a_N - 3))\sqrt{(a_N - 2)/a_N}$ for $a_N \in \{a_L, a_U\} > 3$ and $\alpha_{N4} = 3(a_N - 2)\left(3a_N^3 + a_N + 2\right)/a_N(a_N - 3)(a_N - 4)$, respectively. As mentioned above, an item is called non-conforming if $X_N > U$ or $X_N < L$. The probability of the non-conforming item is defined by $p_U = P\{X_N > U\}$; $X_N \in \{X_L, X_U\}$. The neutrosophic operating characteristic (NOC) is derived as follows

According to the proposed plan, a lot of accept will be accepted if $\overline{X}_N + k_N s_N \le U$, so the lot acceptance probability is given by;

$$L_{Na}(p) = P_r\left[\overline{X}_N + k_N s_N \le U \middle| p\right] = P\left(\boldsymbol{U} - (\mu_N + \boldsymbol{k_N}\sigma_{Y_N})/\left(\frac{\sigma_{Y_N}}{\sqrt{n_N}}\sqrt{\left[1 + \frac{k_N^2}{4}(\alpha_{N4} - 1) + k_N\alpha_{N3}\right]}\right)\right); \tag{2}$$
$$\mu_{Y_N} \in \{\mu_{Y_L}, \mu_{Y_U}\},\ \sigma_{Y_N} \in \{\sigma_{Y_L}, \sigma_{Y_U}\},\ k_N \in \{k_{aL}, k_{aU}\}$$

Let $Z_{Np_U} = U - \mu_{Ny}/\sigma_{Ny}$ be the neutrosophic standard normal variable ([28]), the NOC given in Equation (2) can be written as follows

$$L_{Na}(p) = \Phi_N\left\{\left((Z_{Np_U} - k_N)\sqrt{\frac{n_N}{\sqrt{Q_N}}}\right)\right\} \tag{3}$$

where $Q_N = \left[1 + (k_N^2/4)(\alpha_{N4} - 1) + k_N\alpha_{N3}\right]$ and $\Phi_N(x)$ denote neutrosophic cumulative standard normal distribution.

Neutrosophic Non-Linear Optimization

The plan parameters $n_N \in \{n_L, n_U\}$, $k_N \in \{k_{aL}, k_{aU}\}$ will be obtained such that both the producer's risk α and consumer's risk β are satisfied. To obtain this target, the plan parameters will be determined such that $L_{Na}(p_1) \ge 1 - \alpha$ and $L_{Na}(p_2) \le \beta$, where p_1 and p_2 are the acceptable quality level (AQL) and limiting quality level (LQL), respectively. The plan parameters $n_N \in \{n_L, n_U\}$ *and* $k_N \in \{k_{aL}, k_{aU}\}$ can be obtained through following the neutrosophic non-linear optimization solution

$$\text{minimize } n_N \in \{n_L, n_U\} \tag{4a}$$

subject to

$$L_N(p_1) = \Phi_N\left\{\left((Z_{Np_{U1}} - k_N)\sqrt{\frac{n_N}{\sqrt{Q_N}}}\right)\right\} \ge 1 - \alpha;\ k_N \in \{k_{aL}, k_{aU}\};\ n_N \in \{n_L, n_U\} \tag{4b}$$

and

$$L_N(p_2) = \Phi_N\left\{\left(\left(Z_{Np_{U2}} - k_N\right)\sqrt{\frac{n_N}{\sqrt{Q_N}}}\right)\right\} \leq \beta;\ k_N \in \{k_{aL}, k_{aU}\};\ n_N \in \{n_L, n_U\} \quad \text{(4c)}$$

The plan parameters $n_N \in \{n_L, n_U\}$, $k_N \in \{k_{aL}, k_{aU}\}$ will be determined through Equations (4a)–(4c) using a grid search method. It has been noted that several combination of $n_N \in \{n_L, n_U\}$, $k_N \in \{k_{aL}, k_{aU}\}$ exist that satisfy Equation (4a)–(4c). The plan parameter having the smaller range (R $= n_U - n_L$) in indeterminacy interval will be chosen and reported in Tables 1–3 for various values of α, β, a_N, AQL and LQL.

Table 1. The neutrosophic plan parameters when α = 0.95 and β = 0.10.

p_1	p_2	a_N	α_{N3}	α_{N4}	n_N	k_N	$L_N(p_1)$	$L_N(p_2)$
0.01	0.025	[11,22]	[2.714,2.308]	[175.6,254.9]	[1706,1918]	[2.368,2.372]	[0.9845,0.09503]	[0.9793,0.094397]
0.01	0.05	[10,21]	[2.811,2.325]	[172.1,246.6]	[484,525]	[2.197,2.245]	[0.9854,0.08649]	[0.9641,0.060927]
0.01	0.1	[11,18]	[2.714,2.388]	[175.6,222.4]	[192,257]	[2.088,2.14]	[0.9644,0.05059]	[0.9591,0.023991]
0.01	0.25	[17,28]	[2.415,2.236]	[214.6,305.9]	[122,149]	[1.574,1.842]	[0.9994,0.08587]	[0.9869,0.01808]
0.01	0.5	[9,27]	[2.94,2.245]	[171,297.3]	[38,89]	[1.339,1.768]	[0.9945,0.08611]	[0.974,0.004256]
0.02	0.05	[6,14]	[3.81,2.525]	[218.7,192.8]	[1103,1288]	[2.114,2.148]	[0.9616,0.09953]	[0.9507,0.041457]
0.02	0.1	[21,23]	[2.325,2.293]	[246.6,263.3]	[368,419]	[1.939,1.951]	[0.971,0.07512]	[0.9727,0.058404]
0.02	0.2	[26,29]	[2.256,2.227]	[288.7,314.5]	[190,192]	[1.648,1.725]	[0.9936,0.08987]	[0.983,0.058857]
0.02	0.25	[10,11]	[2.811,2.714]	[172.1,175.6]	[154,160]	[1.65,1.817]	[0.9942,0.03101]	[0.9672,0.008]
0.02	0.5	[22,29]	[2.308,2.227]	[254.9,314.5]	[85,86]	[1.313,1.352]	[0.9979,0.03562]	[0.9952,0.035612]
0.05	0.1	[9,23]	[2.94,2.293]	[171,263.3]	[1898,2046]	[1.755,1.772]	[0.9955,0.08073]	[0.9871,0.065411]
0.05	0.15	[6,10]	[3.81,2.811]	[218.7,172.1]	[515,544]	[1.678,1.698]	[0.964,0.06447]	[0.9648,0.036618]
0.05	0.2	[11,24]	[2.714,2.28]	[175.6,271.7]	[326,521]	[1.619,1.686]	[0.9685,0.03263]	[0.9523,0.006865]
0.05	0.25	[13,19]	[2.576,2.365]	[186.3,230.4]	[159,234]	[1.497,1.502]	[0.9646,0.08775]	[0.9802,0.05685]
0.05	0.5	[11,12]	[2.714,2.637]	[175.6,180.5]	[100,152]	[1.352,1.376]	[0.9776,0.01274]	[0.9904,0.002472]

Table 2. The neutrosophic plan parameters when α = 0.95 and β = 0.05.

p_1	p_2	a_N	α_{N3}	α_{N4}	n_N	k_N	$L_N(p_1)$	$L_N(p_2)$
0.01	0.025	[17,27]	[2.415,2.245]	[214.6,297.3]	[2038,2297]	[2.404,2.411]	[0.9669,0.04046]	[0.9583,0.0379624]
0.01	0.05	[6,28]	[3.81,2.236]	[218.7,305.9]	[753,812]	[2.284,2.311]	[0.9736,0.01591]	[0.9526,0.0131754]
0.01	0.1	[12,13]	[2.637,2.576]	[180.5,186.3]	[270,364]	[2.153,2.187]	[0.9651,0.01454]	[0.9718,0.00389]
0.01	0.25	[14,15]	[2.525,2.483]	[192.8,199.8]	[90,136]	[1.779,1.812]	[0.9836,0.04614]	[0.9933,0.0161476]
0.01	0.5	[24,29]	[2.28,2.227]	[271.7,314.5]	[56,85]	[1.734,1.768]	[0.9512,0.01843]	[0.9694,0.0056095]
0.02	0.05	[9,11]	[2.94,2.714]	[171,175.6]	[1731,2067]	[2.108,2.161]	[0.9923,0.04951]	[0.9759,0.0082816]
0.02	0.1	[11,23]	[2.714,2.293]	[175.6,263.3]	[403,648]	[2.015,2.026]	[0.9554,0.02186]	[0.9698,0.0085848]
0.02	0.2	[6,11]	[3.81,2.714]	[218.7,175.6]	[153,288]	[1.776,1.972]	[0.9684,0.04723]	[0.9509,0.0006528]
0.02	0.25	[7,30]	[3.381,2.218]	[185.4,323.2]	[180,197]	[1.645,1.721]	[0.9965,0.02489]	[0.9843,0.0213151]
0.02	0.5	[15,16]	[2.483,2.446]	[199.8,207.1]	[81,103]	[1.368,1.44]	[0.997,0.02343]	[0.9972,0.0083661]
0.05	0.1	[13,27]	[2.576,2.245]	[186.3,297.3]	[1702,2210]	[1.818,1.818]	[0.9511,0.02239]	[0.9538,0.020496]
0.05	0.15	[6,17]	[3.81,2.415]	[218.7,214.6]	[731,781]	[1.668,1.696]	[0.9871,0.04121]	[0.9813,0.0218219]
0.05	0.2	[13,21]	[2.576,2.325]	[186.3,246.6]	[285,569]	[1.614,1.7]	[0.9594,0.04693]	[0.9541,0.0032972]
0.05	0.25	[10,25]	[2.811,2.267]	[172.1,280.2]	[329,339]	[1.468,1.551]	[0.9978,0.03336]	[0.9812,0.0209033]
0.05	0.5	[24,28]	[2.28,2.236]	[271.7,305.9]	[99,124]	[1.23,1.267]	[0.9882,0.0426]	[0.9894,0.0245211]

Table 3. The neutrosophic plan parameters when α = 0.90 and β = 0.10.

p_1	p_2	a_N	α_{N3}	α_{N4}	n_N	k_N	$L_N(p_1)$	$L_N(p_2)$
0.01	0.025	[20,30]	[2.344,2.218]	[238.5,323.2]	[1337,1825]	[2.399,2.429]	[0.9329,0.09126]	[0.9094,0.043424]
0.01	0.05	[25,27]	[2.267,2.245]	[280.2,297.3]	[447,462]	[2.25,2.306]	[0.9429,0.07925]	[0.9029,0.048256]
0.01	0.1	[10,23]	[2.811,2.293]	[172.1,263.3]	[143,223]	[2.052,2.062]	[0.955,0.09413]	[0.9691,0.064611]
0.01	0.25	[5,19]	[4.648,2.365]	[343.8,230.4]	[73,99]	[1.891,1.993]	[0.9172,0.06665]	[0.9308,0.015943]
0.01	0.5	[13,27]	[2.576,2.245]	[186.3,297.3]	[41,78]	[1.584,1.965]	[0.9721,0.03959]	[0.9043,0.002908]
0.02	0.05	[13,30]	[2.576,2.218]	[186.3,323.2]	[1120,1244]	[2.139,2.16]	[0.9484,0.0594]	[0.9078,0.055194]
0.02	0.1	[10,19]	[2.811,2.365]	[172.1,230.4]	[303,392]	[1.955,1.995]	[0.9635,0.06768]	[0.953,0.037956]
0.02	0.2	[8,24]	[3.118,2.28]	[173.9,271.7]	[159,191]	[1.756,1.939]	[0.9818,0.04094]	[0.9088,0.011826]
0.02	0.25	[11,25]	[2.714,2.267]	[175.6,280.2]	[101,126]	[1.652,1.793]	[0.9788,0.06578]	[0.938,0.031912]
0.02	0.5	[10,12]	[2.811,2.637]	[172.1,180.5]	[55,58]	[1.451,1.53]	[0.9812,0.03261]	[0.9694,0.022133]
0.05	0.1	[14,26]	[2.525,2.256]	[192.8,288.7]	[1206,1512]	[1.804,1.818]	[0.9354,0.06039]	[0.9184,0.04372]
0.05	0.15	[16,22]	[2.446,2.308]	[207.1,254.9]	[493,530]	[1.645,1.683]	[0.9781,0.094]	[0.9581,0.064101]
0.05	0.2	[13,18]	[2.576,2.388]	[186.3,222.4]	[221,308]	[1.634,1.637]	[0.9246,0.05986]	[0.9465,0.038161]
0.05	0.25	[22,23]	[2.308,2.293]	[254.9,263.3]	[276,280]	[1.458,1.504]	[0.9924,0.06826]	[0.9851,0.046336]
0.05	0.5	[10,19]	[2.811,2.365]	[172.1,230.4]	[62,100]	[1.35,1.41]	[0.9445,0.03897]	[0.9523,0.012875]

From Tables 1–3, we note that for fixed α, $n_N \in \{n_L, n_U\}$ increase as β decrease from 0.10 to 0.05. Similarly, for the fixed value of β, $n_N \in \{n_L, n_U\}$ decrease as α changes from 0.95 to 0.90. From Tables 1–3, it is clearly noted that decreasing the consumer's risk is compensated with a large sample for the inspection purpose.

3. Comparative Study

In this section, we will compare the proposed variable sampling plan for Pareto Distribution using neutrosophic statistics with Sathya Narayanan and Rajarathinam [37]'s sampling plan based on the classical statistics. For the fair comparison, we will consider the same values of AQL, LQL, α and β. To save the space, we will consider $\alpha = 0.90$ and $\beta = 0.10$. The sample size n of Sathya Narayanan and Rajarathinam [37]'s plan and $n_N \in \{n_L, n_U\}$ of the proposed plan are placed in Table 4. From Table 4, it can be noted that the proposed variable sampling plan for Pareto Distribution using neutrosophic statistics has a smaller indeterminacy interval than Sathya Narayanan and Rajarathinam [37]'s plan. For example, when AQL = 0.01 and LQL = 0.025, the proposed plan has $n_N \in \{1337, 1825\}$ while the Sathya Narayanan and Rajarathinam [37]'s plan is required n = 2733. It means that under the uncertainty environment, when AQL = 0.01 and LQL = 0.025, the industrial engineers should select a sample from 1337 to 1825. The existing sampling plan provides only the single determined. From this analysis, it can be concluded that the proposed plan based on neutrosophic statistics needs a much smaller sample size for the inspection of the proposed as compared to an existing plan based on the classical statistics. Therefore, the proposed plan is more adequate to be applied in uncertainty than the existing plan. Therefore, the proposed plan is more economical than Sathya Narayanan and Rajarathinam [37]'s plan in terms of inspection cost which is directly related to sample size.

Table 4. The Comparison of Proposed Plan and [37] Plan when α = 0.90 and β = 0.10.

p1	p2	Proposed Plan	Single Plan
		n_N	n
0.01	0.025	[1337,1825]	[2733,2733]
0.01	0.05	[447,462]	[657,657]
0.01	0.1	[143,223]	[522,522]
0.01	0.25	[73,99]	[418,418]
0.01	0.5	[41,78]	[115,115]
0.02	0.05	[1120,1244]	[3621,3621]
0.02	0.1	[303,392]	[1260,1260]
0.02	0.2	[159,191]	[715,715]
0.02	0.25	[101,126]	[321,321]
0.02	0.5	[55,58]	[297,297]
0.05	0.1	[1206,1512]	[2078,2078]
0.05	0.15	[493,530]	[2499,2499]
0.05	0.2	[221,308]	[1518,1518]
0.05	0.25	[276,280]	[1085,1085]
0.05	0.5	[62,100]	[165,165]

4. Case Study

In this section, we discuss the application of the proposed variable sampling plan for Pareto Distribution using neutrosophic statistics with the help of ball bearing data. The ball bearing has been widely used to minimize the rotational fraction in the verity of industries including, for example, the food industry, printing machines and high speed machines. The diameter of the ball bearing is an important variable of interest which should be designed to satisfy the given specification limits. A slight difference in diameter from the speciation limit U = 14 mm may make it unfit to use in machines. The diameter data is obtained from the measurement process so the possibility of some neutrosophic observations cannot be ignored. Suppose that the quality assurance department wants to apply the proposed for the inspection of the ball bearing diameter with $\alpha = 0.95$, $\beta = 0.10$, AQL = 0.01 and LQL = 0.5. At this stage, the experimenter is uncertain about the selection of sample size for the inspection of the ball bearing product. From Table 1, the neutrosophic plan parameters are noted as: $n_N \in \{38, 89\}$ and $k_N \in \{1.339, 1.768\}$. Therefore, the experimenter can select a random sample between 38 to 89. Suppose he decided to select a random sample of 55 from the ball bearing product. The ball bearing diameter data follows the Pareto distribution with $a_N \in \{10.22, 10.34\}$, $b_N \in \{0.1000275,\ 0.1004329\}$. The diameter data having some neutrosophic observations is shown in Table 5.

Table 5. The ball bearing data.

Observations				
[0.1154,0.1154]	[0.1171,0.1171]	[0.1055,0.1053]	[0.1157,0.1157]	[0.1044,0.10154]
[0.1415,0.1088]	[0.1152,0.1234]	[0.11421,0.1142	[0.1018,0.1095]	[0.1102,0.1010]
[0.1032,0.1581]	[0.1065,0.1023]	[0.1072,0.1072]	[0.1043,0.1092]	[0.1018,0.11269]
[0.1099,0.1152]	[0.1014,0.1016]	[0.1051,0.1087]	[0.1122,0.1043]	[0.1013,0.1013]
[0.1031,0.1159]	[0.1070,0.1149]	[0.1017,0.1006]	[0.1308,0.11553]	[0.1019,0.1019]
[0.1144,0.1144]	[0.1143,0.1022]	[0.1648,0.1197]	[0.1038,0.1428]	[0.1234,0.1234]
[0.1095,0.1070]	[0.1184,0.1100]	[0.1231,0.1437]	[0.1166,0.1023]	[0.1015,0.1102]
[0.1396,0.1071]	[0.1231,0.1143]	[0.1067,0.1067]	[0.1025,0.1208]	[0.1061,0.1112]
[0.1089,0.1044]	[0.1015,0.1015]	[0.1012,0.1012]	[0.1007,0.1007]	[0.1040,0.1040]
[0.1129,0.1012]	[0.1015,0.1012]	[0.1025,0.1025]	[0.1140,0.1126]	[0.1081,0.1081]
[0.1251,0.15]	[0.1000,0.1223]	[0.1040,0.1040]	[0.10043,0.1004]	[0.1050,0.1050]

The statistic $Y_N = \overline{X}_N + k_N s_N$ is computed as follows

The neutrosophic mean is computed as follows

$$\overline{X}_L = \sum_{i=1}^{n} x_i^L / n_L = 0.1108, \overline{X}_U = \sum_{i=1}^{n} x_i^U / n_U = 0.1115$$

which yields

$$\overline{X}_N \in \{0.1108, 0.1115\}$$

The neutrosophic standard deviation is calculated as

$s_L = \sqrt{\sum_{i=1}^{n} \left(x_i^L - \overline{X}_L\right)^2 / n_L} = 0.0120$ and $s_U = \sqrt{\sum_{i=1}^{n} \left(x_i^U - \overline{X}_U\right)^2 / n_U}$ = 0.0124 which yield $S_N \in \{0.0120, 0.0124\}$. Based on $\overline{X}_N \in \{0.1108, 0.1115\}$ and $S_N \in \{0.0120, 0.0124\}$, we have $Y_N \in \{0.1269, 0.1321\}$.

The proposed plan is implemented as follows

Step-1: Select a neutrosophic random sample of size $n_N \in \{38, 89\}$ from the lot of the product and record the observations for $X_{Ni} \in \{X_L, X_U\} = i = 1, 2, 3, \ldots, 55$.

Step-2: Accept the lot of the ball bearing product as $Y_N \in \{0.1269, 0.1321\} \leq U = 14$.

From this example, it is concluded that the proposed sampling plan can be applied effectively and adequately for the inspection of ball bearing under the indeterminate conditions.

5. Concluding Remarks

In this paper, the variable sampling plan for Pareto Distribution using neutrosophic statistics is proposed. The necessary measures of the proposed plan are given for the implementation of the plan in the industry. The proposed plan is an extension of the plan under the classical statistics. The proposed plan provides the plan parameters in an interval while the existing plan provides only the determined values of the plan parameter. The proposed plan can be applied in the industry when there is uncertainty in observations or parameters or both. From the comparison, it is concluded that the proposed plan is more economical, adequate, flexible and informative than the existing sampling plan. An application of the proposed plan in the ball bearing industry is given. The proposed plan can be applied in the food industry, automobile industry and aerospace industry. The proposed plan for other distributions is proposed for future research.

Author Contributions: Conceived and designed the experiments, M.A.; N.K. and A.H.A.-M.; Performed the experiments, M.A. and N.K.; Analyzed the data, M.A. and N.K.; Contributed reagents/materials/analysis tools, M.A.; Wrote the paper, M.A.

Funding: This article was funded by the Deanship of Scientific Research (DSR) at King Abdulaziz University, Jeddah. The authors, therefore, acknowledge with thanks DSR technical and financial support.

Acknowledgments: The authors are deeply thankful to the editor and reviewers for their valuable suggestions to improve the quality of this manuscript.

Conflicts of Interest: The authors declare no conflict of interest regarding this paper.

References

1. Schilling, E.G.; Neubauer, D.V. *Acceptance Sampling in Quality Control*; CRC Press: Boca Raton, FL, USA, 2017.
2. Montgomery, D.C. *Introduction to Statistical Quality Control*; John Wiley & Sons: Hoboken, NJ, USA, 2007.
3. Gharaibeh, N.G.; Liu, L.; Wani, S.S. Skip-lot acceptance sampling plans for highway construction and materials. *J. Constr. Eng. Manag.* **2011**, *138*, 791–796. [CrossRef]
4. Collani, E. A note on acceptance sampling for variables. *Metrika* **1991**, *38*, 19–36. [CrossRef]
5. Seidel, W. Is sampling by variables worse than sampling by attributes? A decision theoretic analysis and a new mixed strategy for inspecting individual lots. *Sankhyā Indian J. Stat. Ser. B* **1997**, *59*, 96–107.
6. Itay, N.; Yisrael, P.; Edna, S. Developing a sampling plan based on Cpk. *Qual. Eng.* **2009**, *21*, 306–318. [CrossRef]
7. Klufa, J. Exact calculation of the Dodge-Romig LTPD single sampling plans for inspection by variables. *Stat. Pap.* **2010**, *51*, 297–305. [CrossRef]
8. Aslam, M.; Wu, C.W.; Azam, M.; Jun, C.H. Variable sampling inspection for resubmitted lots based on process capability index Cpk for normally distributed items. *Appl. Math. Model.* **2013**, *37*, 667–675. [CrossRef]
9. Aslam, M.; Yen, C.H.; Chang, C.H.; Jun, C.H. Multiple dependent state variable sampling plans with process loss consideration. *Int. J. Adv. Manuf. Technol.* **2014**, *71*, 1337–1343. [CrossRef]
10. Kurniati, N.; Yeh, R.-H.; Wu, C.-W. Designing a variables two-plan sampling system of type TNTVSS-(n T, n N; k) for controlling process fraction nonconforming with unilateral specification limit. *Int. J. Prod. Res.* **2015**, *53*, 2011–2025. [CrossRef]
11. Seifi, S.; Nezhad, M.S.F. Variable sampling plan for resubmitted lots based on process capability index and Bayesian approach. *Int. J. Adv. Manuf. Technol.* **2017**, *88*, 2547–2555. [CrossRef]
12. Khan, N.; Aslam, M.; Ahmad, L.; Jun, C.H. Multiple dependent state repetitive sampling plans with or without auxiliary variable. *Commun. Stat.-Simul. Comput.* **2017**, 1–15. [CrossRef]
13. Wang, F.K.; Tamirat, Y.; Lo, S.C.; Aslam, M. Dependent Mixed and Mixed Repetitive Sampling Plans for Linear Profiles. *Qual. Reliab. Eng. Int.* **2017**, *33*, 1669–1683. [CrossRef]
14. Wu, C.-W.; Shu, M.-H.; Chang, Y.-N. Variable-sampling plans based on lifetime-performance index under exponential distribution with censoring and its extensions. *Appl. Math. Model.* **2018**, *55*, 81–93. [CrossRef]
15. Cheng, S.-R.; Hsu, B.-M.; Shu, M.-H. Fuzzy testing and selecting better processes performance. *Ind. Manag. Data Syst.* **2007**, *107*, 862–881. [CrossRef]
16. Jamkhaneh, E.B.; Gildeh, B.S. Acceptance Double Sampling Plan using Fuzzy Poisson Distribution 1. 2012. Available online: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.390.1273 (accessed on 1 December 2018).
17. Jamkhaneh, E.B.; Gildeh, B.S. Sequential sampling plan using fuzzy SPRT. *J. Intell. Fuzzy Syst.* **2013**, *25*, 785–791.
18. Afshari, R.; Gildeh, B.S. Construction of fuzzy multiple deferred state sampling plan. In Proceedings of the 2017 Joint 17th World Congress of International Fuzzy Systems Association and 9th International Conference on Soft Computing and Intelligent Systems (IFSA-SCIS), Otsu, Japan, 27–30 June 2017.
19. Afshari, R.; Gildeh, B.S.; Sarmad, M. Multiple Deferred State Sampling Plan with Fuzzy Parameter. *Int. J. Fuzzy Syst.* **2018**, *20*, 549–557. [CrossRef]
20. Afshari, R.; Sadeghpour Gildeh, B.; Sarmad, M. Fuzzy multiple deferred state attribute sampling plan in the presence of inspection errors. *J. Intell. Fuzzy Syst.* **2017**, *33*, 503–514. [CrossRef]
21. Elango, S.; Venkatesh, A.; Sivakumar, G. A Fuzzy Mathematical Analysis For The Effect Of Trh Using Acceptance Sampling Plans. *Int. J. Pure Appl. Math.* **2017**, *117*, 1–7.
22. Kanagawa, A.; Ohta, H. A design for single sampling attribute plan based on fuzzy sets theory. *Fuzzy Sets Syst.* **1990**, *37*, 173–181. [CrossRef]

23. Jamkhaneh, E.B.; Sadeghpour-Gildeh, B.; Yari, G. Inspection error and its effects on single sampling plans with fuzzy parameters. *Struct. Multidiscip. Optim.* **2011**, *43*, 555–560. [CrossRef]
24. Turanoğlu, E.; Kaya, İ.; Kahraman, C. Fuzzy acceptance sampling and characteristic curves. *Int. J. Comput. Intell. Syst.* **2012**, *5*, 13–29. [CrossRef]
25. Venkateh, A.; Elango, S. Acceptance sampling for the influence of TRH using crisp and fuzzy gamma distribution. *Aryabhatta J. Math. Inform.* **2014**, *6*, 119–124.
26. Viertl, R. On reliability estimation based on fuzzy lifetime data. *J. Stat. Plan. Inference* **2009**, *139*, 1750–1755. [CrossRef]
27. Smarandache, F. Neutrosophic logic-generalization of the intuitionistic fuzzy logic. *arXiv* **2003**, arXiv:math/0303009.
28. Smarandache, F. *Introduction to Neutrosophic Statistics*; Infinite Study: Conshohocken, PA, USA, 2014.
29. Chen, J.; Ye, J.; Du, S. Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. *Symmetry* **2017**, *9*, 208. [CrossRef]
30. Chen, J.; Ye, J.; Du, S.; Yong, R. Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. *Symmetry* **2017**, *9*, 123. [CrossRef]
31. Aslam, M. A New Sampling Plan Using Neutrosophic Process Loss Consideration. *Symmetry* **2018**, *10*, 132. [CrossRef]
32. Aslam, M. Design of Sampling Plan for Exponential Distribution under Neutrosophic Statistical Interval Method. *IEEE Access* **2018**, *6*, 64153–64158. [CrossRef]
33. Aslam, M.; Arif, O. Testing of Grouped Product for the Weibull Distribution Using Neutrosophic Statistics. *Symmetry* **2018**, *10*, 403. [CrossRef]
34. Aslam, M.; Bantan, R.A.; Khan, N. Design of a New Attribute Control Chart Under Neutrosophic Statistics. *Int. J. Fuzzy Syst.* **2018**. [CrossRef]
35. Aslam, M.; Khan, N.; Khan, M. Monitoring the Variability in the Process Using Neutrosophic Statistical Interval Method. *Symmetry* **2018**, *10*, 562. [CrossRef]
36. Aslam, M.; Raza, M.A. Design of New Sampling Plans for Multiple Manufacturing Lines Under Uncertainty. *Int. J. Fuzzy Syst.* **2018**, 1–15. [CrossRef]
37. Sathya Narayanan, G.; Rajarathinam, V. A procedure for the selection of single sampling plans by variables based on Pareto distribution. *J. Qual. Reliab. Eng.* **2013**, *2013*. [CrossRef]