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# Detour Interior and Boundary vertices of BSV Neutrosophic Graphs 

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#### Abstract

:

In the present article, we deduce a characterization of BSVN detour interior and boundary vertices. We established the relations between BSVN cut node and BSVN detour boundary nodes. Further, we studied properties of BSVN boundary nodes and BSVN interior nodes. Application of detour boundary node, detour interior is given on modeling wireless sensor network in terms of BSVN graphs.


Keywords: Detour distance, BSVN detour boundary nodes, BSVN detour interior nodes.

## 1. Introduction:

The Neutrosophic sets launch by Smarandache $[15,16]$ are a great exact implement for the situation uncertainty in the real world. These uncertainty idea comes from the theories of fuzzy sets [9], intuitionistic fuzzy sets [6, 8] and interval valued intuitionistic fuzzy sets [7]. The representation of the neutrosophic sets are truth, indeterminacy and falsity value. These T, I, F values belongs to standard or nonstandard unit interval denoted by $]-0,1+[[10,14]$.

The idea of subclass of the NS and SVNS introduced by Wang et al. [17]. The idea of SVNS initiation by intuitionistic fuzzy sets [5,11], in this the functions Truth, Indeterminacy, Falsity are not dependent and these values are present within [0, 1] [12].

Neutrosophic theory is widely expands in all fields especially authors discoursed about topology with respect to neutrosophic [18].

Graph theory has at this time turn into a most important branch of mathematics. It is the division of combinatory. The Graph is a extensively important to analyze combinatorial complication in dissimilar areas in mathematics, optimization and computer science. Mainly significant object is well-known. The uncertainty on the subject of vertice and edges or both representation to be a fuzzy graph.

In a graph theory the new graph model was invites by using BSVN set is known as BSVN Graph(BSVNG). In [3, 4, 13] Broumi et al. explained BSVN graphs from the recall of fuzzy, bipolar fuzzy and single valued neutrosophic graphs.

In this manuscript, discus about BSVN graphs and neutrosophic detour distance between two vertices of the graph based on this define BSVN eccentricity, radius, diameter, center and periphery with respect to detour distance. Also find some important results on these topics.

## 2. Preliminaries:

## Explanation 2.1 BSVN sets:-

A BSVN set is explained as the membership functions represented as an object in $W$ is denoted by $\left\{<w, T^{P}, I^{P}, F^{P}, T^{N}, I^{N}, F^{N}>: w \in W\right\}$, the functions $T^{P}, I^{P}, F^{P}$ are mapping from $W$ to $[0,1]$ and $T^{N}, I^{N}, F^{N}$ are mapping from $W$ to $[-1,0]$.

## Explanation 2.2 BSVN relation on $W$

Let $W$ be a non-empty set. Then we call mapping $Z=\left(W, T^{P}, I^{P}, F^{P}, T^{N}, I^{N}, F^{N}\right)$, $F^{N}\left(w_{1}, w_{2}\right): W \times W \rightarrow[-1,0] \times[0,1]$ is a BSVN relation on $W$ such that $T_{Z}^{P}\left(w_{1}, w_{2}\right) \in[0,1], I_{Z}^{P}\left(w_{1}, w_{2}\right) \in[0,1], F_{Z}^{P}\left(w_{1}, w_{2}\right) \in[0,1]$ $T_{Z}^{N}\left(w_{1}, w_{2}\right) \in[-1,0], I_{Z}^{N}\left(w_{1}, w_{2}\right) \in[-1,0], F_{Z}^{N}\left(w_{1}, w_{2}\right) \in[-1,0]$.

Explanation 2.3 Let $Z_{1}=\left(T_{Z_{1}}^{P}, I_{Z_{1}}^{P}, F_{Z_{1}}^{P}, T_{Z_{1}}^{N}, I_{Z_{1}}^{N}, F_{Z_{1}}^{N}\right)$ and $Z_{2}=\left(T_{Z_{2}}^{P}, I_{Z_{2}}^{P}, F_{Z_{2}}^{P}, T_{Z_{2}}^{N}, I_{Z_{2}}^{N}, F_{Z_{2}}^{N}\right)$ be a BSVN graphs on a set $W$. If $Z_{2}$ is a BSVN relation on $Z_{1}$, then $T_{Z_{2}}^{P}\left(w_{1}, w_{2}\right) \leq \min \left(T_{Z_{2}}^{P}\left(w_{1}\right), T_{Z_{2}}^{P}\left(w_{2}\right)\right) \geq \max \left(T_{Z_{2}}^{N}\left(w_{1}\right), T_{Z_{2}}^{N}\left(w_{2}\right)\right)$
$I_{Z_{2}}^{P}\left(w_{1}, w_{2}\right) \geq \max \left(I_{Z_{1}}^{P}\left(w_{1}\right), I_{Z_{1}}^{P}\left(w_{2}\right)\right) \leq \min \left(I_{Z_{1}}^{N}\left(w_{1}\right), I_{Z_{1}}^{N}\left(w_{2}\right)\right)$
$F_{Z_{2}}^{P}\left(w_{1}, w_{2}\right) \geq \max \left(F_{z_{1}}^{P}\left(w_{1}\right), F_{z_{1}}^{P}\left(w_{2}\right)\right) \leq \min \left(F_{z_{1}}^{N}\left(w_{1}\right), F_{z_{1}}^{N}\left(w_{2}\right)\right)$ for all $w_{1}, w_{2} \in W$

Explanation 2.4. The symmetric property defined on BSVN relation $Z$ on $W$ is explained by
$T_{Z}^{P}\left(w_{1}, w_{2}\right)=T_{Z}^{P}\left(w_{2}, w_{1}\right), \quad I_{Z}^{P}\left(w_{1}, w_{2}\right)=I_{Z}^{P}\left(w_{2}, w_{1}\right), F_{Z}^{P}\left(w_{1}, w_{2}\right)=F_{Z}^{P}\left(w_{2}, w_{1}\right)$
$T_{Z}^{N}\left(w_{1}, w_{2}\right)=T_{Z}^{N}\left(w_{2}, w_{1}\right), I_{Z}^{N}\left(w_{1}, w_{2}\right)=I_{Z}^{N}\left(w_{2}, w_{1}\right), F_{Z}^{N}\left(w_{1}, w_{2}\right)=F_{Z}^{N}\left(w_{2}, w_{1}\right)$
for all $w_{1}, w_{2} \in W$

## Explanation 2.5 BSVN graph

The new graph in SVN is denoted by $G^{*}=(V, E)$ is a pair $G=\left(Z_{1}, Z_{2}\right)$, where $Z_{1}=\left(T_{Z_{1}}^{P}, I_{Z_{1}}^{P}, F_{Z_{1}}^{P}, T_{Z_{1}}^{N}, I_{Z_{1}}^{N}, F_{Z_{1}}^{N}\right)$ is a BSVNS in $V$ and $Z_{2}=\left(T_{Z_{2}}^{P}, I_{Z_{2}}^{P}, F_{Z_{2}}^{P}, T_{Z_{2}}^{N}, I_{Z_{2}}^{N}, F_{Z_{2}}^{N}\right)$ is BSVNS in $V^{2}$ defined as
$T_{Z_{2}}^{P}\left(w_{1}, w_{2}\right) \leq \min \left(T_{Z_{1}}^{P}\left(w_{1}\right), T_{Z_{1}}^{P}\left(w_{2}\right)\right)$
$I_{Z_{2}}^{P}\left(w_{1}, w_{2}\right) \geq \max \left(I_{Z_{1}}^{P}\left(w_{1}\right), I_{Z_{1}}^{P}\left(w_{2}\right)\right)$
$F_{Z_{2}}^{P}\left(w_{1}, w_{2}\right) \geq \max \left(F_{Z_{1}}^{P}\left(w_{1}\right), F_{Z_{1}}^{P}\left(w_{2}\right)\right)$
$T_{Z_{2}}^{N}\left(w_{1}, w_{2}\right) \geq \max \left(T_{Z_{1}}^{N}\left(w_{1}\right), T_{Z_{1}}^{N}\left(w_{2}\right)\right)$
$I_{Z_{2}}^{N}\left(w_{1}, w_{2}\right) \leq \min \left(I_{z_{1}}^{N}\left(w_{1}\right), I_{Z_{1}}^{N}\left(w_{2}\right)\right)$
$F_{Z_{2}}^{N}\left(w_{1}, w_{2}\right) \leq \min \left(F_{Z_{1}}^{N}\left(w_{1}\right), F_{Z_{1}}^{N}\left(w_{2}\right)\right)$ for all $w_{1}, w_{2} \in V$

The BSVNSG of an edge denoted by $w_{1} w_{2} \in V^{2}$

Explanation 2.6 Let $G=\left(Z_{1}, Z_{2}\right)$ be a BSVNSG and $a_{1}, c_{1} \in V$

A path $P: a_{1}=w_{0}, w_{1}, w_{2}, \ldots \ldots . . . w_{k-1}, w_{k}=c_{1}$ in $G$ is sequence of distinct vertices such that $\binom{T_{Z_{2}}^{P}\left(w_{m-1}, w_{m}\right)>0, I_{Z_{2}}^{P}\left(w_{m-1}, w_{m}\right)>0, F_{Z_{2}}^{P}\left(w_{m-1}, w_{m}\right)>0}{,T_{Z_{2}}^{N}\left(w_{m-1}, w_{m}\right)>0, I_{Z_{2}}^{N}\left(w_{m-1}, w_{m}\right)>0, F_{Z_{2}}^{N}\left(w_{m-1}, w_{m}\right)>0}, \quad m=1,2, \ldots \ldots \ldots ., k$ and length of the path is $k$, where is $a_{1}$ called initial vertex and $c_{1}$ is called terminal vertex in the path.

Explanation 2.7 A BSVN graph $G=\left(Z_{1}, Z_{2}\right)$ of $G^{*}=(V, E)$ is called strong BSVN graph if
$T_{Z_{2}}^{P}\left(w_{1}, w_{2}\right)=\min \left(T_{z_{1}}^{P}\left(w_{1}\right), T_{Z_{1}}^{P}\left(w_{2}\right)\right)$
$I_{Z_{2}}^{P}\left(w_{1}, w_{2}\right)=\max \left(I_{Z_{1}}^{P}\left(w_{1}\right), I_{Z_{1}}^{P}\left(w_{2}\right)\right)$
$F_{Z_{2}}^{P}\left(w_{1}, w_{2}\right)=\max \left(F_{Z_{1}}^{P}\left(w_{1}\right), F_{Z_{1}}^{P}\left(w_{2}\right)\right)$
$T_{Z_{2}}^{N}\left(w_{1}, w_{2}\right)=\max \left(T_{Z_{1}}^{N}\left(w_{1}\right), T_{Z_{1}}^{N}\left(w_{2}\right)\right)$
$I_{Z_{2}}^{N}\left(w_{1}, w_{2}\right)=\min \left(I_{Z_{1}}^{N}\left(w_{1}\right), I_{Z_{1}}^{N}\left(w_{2}\right)\right)$
$F_{Z_{2}}^{N}\left(w_{1}, w_{2}\right)=\min \left(F_{Z_{1}}^{N}\left(w_{1}\right), F_{Z_{1}}^{N}\left(w_{2}\right)\right)$ for all $\left(w_{1}, w_{2}\right) \in E$

If $P: a_{1}=w_{0}, w_{1}, w_{2}, \ldots \ldots . . w_{k-1}, w_{k}=c_{1}$ be a path of length $k$ between $a_{1}$ and $c_{1}$ then
$\left(T_{Z_{2}}^{P}\left(a_{1}, c_{1}\right), I_{Z_{2}}^{P}\left(a_{1}, c_{1}\right), F_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right)^{k}$ and $\left(T_{Z_{2}}^{N}\left(a_{1}, c_{1}\right), I_{Z_{2}}^{N}\left(a_{1}, c_{1}\right), F_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right)^{k}$ is defined as
$\left(T_{Z_{2}}^{P}\left(a_{1}, c_{1}\right), I_{Z_{2}}^{P}\left(a_{1}, c_{1}\right), F_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right)^{k}=\left\{\begin{array}{l}\sup \left\{T_{Z_{2}}^{P}\left(a_{1}, w_{1}\right) \wedge T_{Z_{2}}^{P}\left(w_{1}, w_{2}\right) \wedge \ldots . \wedge T_{Z_{2}}^{P}\left(w_{k-1}, c_{1}\right)\right\}, \\ \inf \left\{I_{Z_{2}}^{P}\left(a_{1}, w_{1}\right) \vee I_{Z_{2}}^{P}\left(w_{1}, w_{2}\right) \vee \ldots . . \vee I_{Z_{2}}^{P}\left(w_{k-1}, c_{1}\right)\right\}, \\ \inf \left\{F_{Z_{2}}^{P}\left(a_{1}, w_{1}\right) \vee F_{Z_{2}}^{P}\left(w_{1}, w_{2}\right) \vee \ldots . . \vee F_{Z_{2}}^{P}\left(w_{k-1}, c_{1}\right)\right\}\end{array}\right.$
$\left(T_{Z_{2}}^{N}\left(a_{1}, c_{1}\right), I_{Z_{2}}^{N}\left(a_{1}, c_{1}\right), F_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right)^{k}=\left\{\begin{array}{l}\sup \left\{T_{Z_{2}}^{N}\left(a_{1}, w_{1}\right) \vee T_{Z_{2}}^{N}\left(w_{1}, w_{2}\right) \vee \ldots . . \vee T_{Z_{2}}^{N}\left(w_{k-1}, c_{1}\right)\right\}, \\ \inf \left\{I_{Z_{2}}^{N}\left(a_{1}, w_{1}\right) \wedge I_{Z_{2}}^{N}\left(w_{1}, w_{2}\right) \wedge \ldots . . \wedge I_{Z_{2}}^{N}\left(w_{k-1}, c_{1}\right)\right\}, \\ \inf \left\{F_{Z_{2}}^{N}\left(a_{1}, w_{1}\right) \wedge F_{Z_{2}}^{N}\left(w_{1}, w_{2}\right) \wedge \ldots . \wedge F_{Z_{2}}^{N}\left(w_{k-1}, c_{1}\right)\right\}\end{array}\right.$
$\left(T_{Z_{2}}^{P}\left(a_{1}, c_{1}\right), I_{Z_{2}}^{P}\left(a_{1}, c_{1}\right), F_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right)^{\infty}$ and $\left(T_{Z_{2}}^{N}\left(a_{1}, c_{1}\right), I_{Z_{2}}^{N}\left(a_{1}, c_{1}\right), F_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right)^{\infty}$ is said to be the strength of connectedness between two vertices $a_{1}$ and $c_{1}$ in $G$, where
$\left(T_{Z_{2}}^{P}\left(a_{1}, c_{1}\right), I_{Z_{2}}^{P}\left(a_{1}, c_{1}\right), F_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right)^{\infty}=\left(\sup _{k \in N}\left\{T_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right\}, \inf _{k \in N}\left\{I_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right\}, \inf _{k \in N}\left\{F_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right\}\right)$
$\left(T_{Z_{2}}^{N}\left(a_{1}, c_{1}\right), I_{Z_{2}}^{N}\left(a_{1}, c_{1}\right), F_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right)^{\infty}=\left(\inf _{k \in N}\left\{T_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right\}, \sup _{k \in N}\left\{I_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right\}, \sup _{k \in N}\left\{F_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right\}\right)$

If $\quad\left(T_{Z_{2}}^{P}\left(a_{1}, c_{1}\right) \geq\left(T_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right)^{\infty}, I_{Z_{2}}^{P}\left(a_{1}, c_{1}\right) \leq\left(I_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right)^{\infty}, F_{Z_{2}}^{P}\left(a_{1}, c_{1}\right) \leq\left(F_{Z_{2}}^{P}\left(a_{1}, c_{1}\right)\right)^{\infty}\right) \quad$ and $\left(T_{Z_{2}}^{N}\left(a_{1}, c_{1}\right) \leq\left(T_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right)^{\infty}, I_{Z_{2}}^{N}\left(a_{1}, c_{1}\right) \geq\left(I_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right)^{\infty}, F_{Z_{2}}^{N}\left(a_{1}, c_{1}\right) \geq\left(F_{Z_{2}}^{N}\left(a_{1}, c_{1}\right)\right)^{\infty}\right)$ then the arc $a_{1} c_{1}$ in $G$ is called a strong arc. A path $a_{1}-c_{1}$ is strong path if all arcs on the path are strong.

## 3. BSVN detour distance

## Explanation 3.1

BSVN detour distance is defined as the length of , $a-c$ strong path between $a$ and $c$ if there is no other strong path longer than $P$ between $a$ and $c$ and we denote this by B.S.N.D $(a, c)$. Any $a-c$ strong path whose length is $B \cdot \operatorname{S.N} \cdot D(a, c)$ is called a $a-c B S V N$ detour path.

## 4. BN detour boundary node of a BN graph

Explanation 4.1 In a connected BN graph $G$, a node $n_{2}$ is said to be a BN detour boundary node of a node $n_{1}$ if $\operatorname{B.N.D}\left(n_{1}, n_{2}\right) \geq \operatorname{B.N.D}\left(n_{1}, n_{3}\right)$ for each $n_{3}$ in $G$, where $n_{3}$ is a neighbor of $n_{2}$. The set of all BN detour boundary nodes of $n_{l}$ denoted by $n_{1}^{\prime}$ B.N.D.

Explanation 4.2 If the BN sub graph formed by strong neighbor of a node $n_{2}$ in a BN graph $G$, form a complete BN graph then the node $n_{2}$ is said to be a complete node of $G$.

Theorem 4.3 A node in a complete BN graph is BN detour boundary node of every other nodes iff the node is complete.

Proof. Let a node $n_{2}$ be a complete node in a connected BN graph $G$. Let $n_{l}$ be an another node of $G$. Each arc in $G$ is strong, because of completeness of $G$ [1]. So B.N.D $\left(n_{1}, n_{2}\right)=|V|$ $-1=B . N . D\left(n_{1}, n_{3}\right), \forall n_{3} \in N\left(n_{2}\right)$, where $|V|=$ numbers of nodes in $G$.Therefore $n_{2}$ is a BN detour boundary node of $n_{l}$.

Conversely, let $n_{2}$ be a BN detour boundary node of every other node. Then each arc in $G$ is strong, because of completeness of $G[1]$. Then B.N.D $\left(n_{1}, n_{2}\right)=|V|-1, \forall n_{1} \in G$. So all neighbor of $n_{2}$ are strong neighbor. Hence by Explanation 4.2, the node $n_{2}$ is complete.

Theorem 4.4 If a node in a connected BN graph $G$ is a complete node of $G$, then the node is a BN detour boundary node of all other node.

Proof. Let a node $n_{2}$ be a complete node in a connected BN graph $G$ and let $n_{l}$ be another node of $G$. Assume that $n_{l}=w_{0}, w_{1}, \ldots, w_{k-1}, w_{k}=n_{2}$ be a $n_{1}-n_{2} \mathrm{BN}$ detour and $n_{3}$ be a strong neighbor of $n_{2}$. There arise two cases

Case 1: If $n_{3}=w_{k-1}$, then $B \cdot N \cdot D\left(n_{1}, n_{3}\right) \leq B \cdot N \cdot D\left(n_{1}, n_{2}\right)$. Hence $n_{2}$ be a BN detour boundary node of $n_{l}$.

Case 2: If $n_{3} \neq w_{k-1}$, since $n_{3}$ is a strong neighbor of $n_{2}$, so the $\operatorname{arc}\left(n_{3}, w_{k-1}\right)$ is a strong arc and also $n_{3} \neq w_{k-1}$. So the length of the path $n_{1}=w_{0}, w_{1}, \ldots, w_{k-1}, n_{3}, w_{k}=n_{2}$ is greater than than the length of the path $n_{l}=w_{0}, w_{1}, \ldots, w_{k-1}, w_{k}=n_{2}$. Hence B.N.D $\left(n_{1}, n_{3}\right) \leq B \cdot N . D\left(n_{1}\right.$, $\left.n_{2}\right)$. Therefore $n_{2}$ is a BN detour boundary node of $n_{1}$.

Theorem 4.5 A connected $B N$ graph $G$ is a $B N$ tree iff $G$ is $B N$ detour graph.

Proof. Let $G$ be a BN tree. Then between any two nodes in $G$, there is exactly one BN strong path. So B.N.D $\left(n_{1}, n_{2}\right)=B . N . D\left(n_{1}, n_{2}\right)$ for any two nodes $n_{1}, n_{2}$ in $G$. Hence $G$ is BN detour graph.

Conversely, let $G$ be a BN detour graph, which has $|V|$ nodes. Then B.N.D $\left(n_{1}, n_{2}\right)=B \cdot N \cdot D\left(n_{1}\right.$, $n_{2}$ )for any two nodes $n_{1}, n_{2}$ in $G$. If $|V|=2$ then $G$ is a BN tree.

Let $|V| \geq 3$. If possible, let $G$ be not a BN tree. So $\exists$ two nodes $p, q$ in $G$ for which there is at least two strong path between $p$ and $q$. Let $Q_{1}$ and $Q_{2}$ be two $p-q$ bipolar neutrosophic strong paths. So $Q_{1} \cup Q_{2}$ has a cycle $C($ say $)$ in $G$. If node $n_{1}$ and $n_{2}$ are adjacent nodes in $G$, then we have $B . N . D\left(n_{1}, n_{2}\right)=1$ and $B \cdot N . D\left(n_{1}, n_{2}\right)>1$. This contradicts the fact that B.N.D $\left(n_{1}, n_{2}\right)$ $=B . N . D\left(n_{1}, n_{2}\right)$. Hence $G$ is a BN tree.

Theorem 4.6 In a BN tree $G$, a node $n_{2}$ is a BN detour boundary node of $G$ iff $n_{2}$ cannot be a BN cut node of $G$.

Proof. Let $G$ be a BN tree and a node $n_{2}$ in $G$ be a BN detour boundary node of a node $n_{3}$ in $G$. If possible, let $n_{2}$ be a BN cut node of $G$.

Let $E$ be a BN maximum spanning tree in $G$, which is unique in $G$. Since $n_{2}$ is a BN cut node, so $n_{2}$ cannot be an internal node of $E$. Let $x \in N_{B . N . S}\left(n_{2}\right)$ such that $x$ does not lie on the BN
detour in $E$. Therefore B.N. $D(p, q)$ is same when $p, q$ be any two nodes of $E$ and $G$ both. But B.N.D $\left(n_{3}, x\right)=B . N . D\left(n_{3}, n_{2}\right)+B . N . D\left(n_{2}, x\right)>B . N . D\left(n_{3}, n_{2}\right)$. This contradicts the fact that $n_{2}$ is a BN detour boundary node of a node $n_{3}$ in $G$. Therefore the node $n_{2}$ cannot be a BN cut node of $G$.

Conversely, let $n_{2}$ be not a BN cut node of the BN graph $G$. So $n_{2}$ is end node of maximum bipolar spanning tree, which is unique. Then $n_{2}$ has a strong neighbor which is also unique [2]. So there does not exist any extension of any BN detour for a node $x$ to $n_{2}$. Hence $n_{2}$ is a BN detour boundary node of $G$.

Explanation 4.7 A node $n_{l}$ in a BN graph $G$ is said to be a BN end node of $G$ if $n_{2}$ is only strong neighbor of $n_{1}$, where $n_{2} \in G$.

Theorem 4.8 A node $n_{2}$ in a $B N$ tree $G$ is a $B N$ detour boundary node iff $n_{2}$ is a $B N$ end node.

Proof. Let a node $n_{2}$ be a BN detour boundary node for a node $n_{l}$ in a BN tree $G$. Let $E$ be a maximum bipolar spanning tree in $G$, which is unique in $G$ [2]. By Explanation 4.7, each node of $G$ is a BN cut node of $G$ or a BN end node of $G$ [2]. So by Explanation 4.7, $n_{2}$ must be a BN end node of $G$.

Conversely, let $n_{2}$ be a BN end node of a BN tree $G$. Let $E$ be the maximum bipolar spanning tree of $G$. Then $n_{2}$ is a BN end node of $E$. Hence $n_{2}$ is not a BN cut node of $G$. Therefore by Explanation 4.7, $n_{2}$ is a BN detour boundary node of $G$.

## 5. BN detour interior node of a BN graph

In a connected BN graph $G$, a node $n_{2}$ lie between the nodes $n_{1}$ and $n_{3}$ in the sense of BN detour distance if B.N.D $\left(n_{1}, n_{3}\right)=\operatorname{B.N.D}\left(n_{1}, n_{2}\right)+B \cdot N . D\left(n_{2}, n_{3}\right)$.

Explanation 5.1 In a connected BN graph $G$, a node $n_{2}$ is said to be a BN detour interior node if for each node $n_{1}$ in $G$ different from $n_{2}$, there is a node $n_{3}$ in $G$ for which B.N.D $\left(n_{1}\right.$, $\left.n_{3}\right)=B \cdot N \cdot D\left(n_{1}, n_{2}\right)+B \cdot N \cdot D\left(n_{2}, n_{3}\right)$.

Explanation 5.2 The set of all BN detour interior node of $G$, denoted by $\operatorname{Int}_{\text {B.N.D }}(G)$, form a BN sub graph of $G$.

Theorem 5.3 A node in a connected $B N$ graph $G$ is a $B N$ detour boundary node of $G$ iff the node cannot be a $B N$ detour interior node of $G$.

Proof. Let $n_{2}$ be a BN detour boundary node of a node $n_{I}$ in a connected BN graph $G$. If possible, let $n_{2}$ be a BN detour interior node of $G$. So there exist a node $n_{3}$ different from $n_{1}$ and $n_{2}$ such that $n_{2}$ lies between $n_{1}$ and $n_{3}$.

Let $U: n_{l}=\mathrm{w}_{1}, w_{2}, \ldots, n_{2}=\mathrm{w}_{k}, w_{k+1}, \ldots, w_{l}=n_{3}$ be a $n_{l}-n_{3} \mathrm{BN}$ detour and $1<k<l$. Then $\mathrm{w}_{k+1} \in N_{B . N . S}\left(n_{2}\right)$, and this implies B.N.D $\left(n_{1}, w_{k+1}\right)>\operatorname{B.N.D}\left(n_{1}, n_{2}\right)$, this is a contradiction. Hence $n_{2}$ cannot be a BN detour interior node of $G$.

Conversely, let a node $n_{2}$ in $G$, which is not a BN detour interior node of $G$. Then there exist a node $n_{1}$ in $G$ for which any node $n_{3}$ different from $n_{1}$ and $n_{2}, \operatorname{B.N.D}\left(n_{1}, n_{3}\right) \neq B \cdot \operatorname{N.D}\left(n_{1}\right.$, $\left.n_{2}\right)+B . N . D\left(n_{2}, n_{3}\right)$. Therefore B.N.D $\left(n_{1}, q\right) \leq B . N . D\left(n_{1}, n_{2}\right)$ where $q \in N_{B . N . S}\left(n_{2}\right)$. This implies that $n_{2}$ is a BN detour boundary node of $n_{l}$.

Theorem 5.4 A BN end node of a connected BN graph $G$ cannot be a $B N$ detour interior node.

Proof. Let $q$ be a BN end node of a BN graph $G$. Then there is only one BN strong neighbor of $q$. So there is no strong BN detour for which $n_{2}$ lies between $n_{1}$ and $n_{3}$, where $n_{1}$ and $n_{3}$ be two node of $G$ and also different from $n_{2}$. Hence $n_{2}$ is not a BN detour interior node of $G$.

## 6. Application:

## Modeling of wireless sensor network in terms of BSVN graph and determination of its

 boundary and interior stationsIn a wireless sensor network, if the sensor failure or sensor give expansive errors or disconnection of network, then the capability of each station to capture the sense of occurrence and communication between them are uncertain. Here we present a BN graph $G$ (see Figure 1) which is applied on a wireless sensor network (W.S.N) to determine its boundary and interior station which is shown in Figure 1. The nodes are a, b, c, d, e, f of $G$ represents the stations and each edge represents the communication between corresponding stations.

The positive membership value of each nodes of $G$ represents the capability of the station to capture the sense of occurrence. Its value is 0 if the capability is $\leq 5 \%$ and its value is 1 if the capability is $\geq 80 \%$. So the positive membership value of each node lies in $(0,1)$ if the capability lies between $>5 \%$ and $<80 \%$. The negative membership value of each node of $G$ represents the disability of the station to capture the sense of occurrence (disability means it gives expansive error, change in sensor position or disconnection of network). Its value is 0 if the disability is $\leq 10 \%$ and its value is -1 if the disability is $\geq 75 \%$. So the negative membership value of each node lies in $(-1,0)$ if the disability lies between > $10 \%$ and < $75 \%$. The positive membership value of each edge represents the ability to communicate of two corresponding stations. Its value is 0 if the ability is $\leq 25 \%$ and its value is 1 if the ability is $\geq 70 \%$. So the positive membership value of each edge lies in $(0,1)$ if the ability lies between $>25 \%$ and $<70 \%$. The negative membership value of each edge represents the disability to communicate of two corresponding stations. Its value is 0 if the disability is $\leq$ $30 \%$ and its value is -1 if the disability is $\geq 80 \%$. So the negative membership value of each edge lies in $(-1,0)$ if the disability lies between $>30 \%$ and $<80 \%$. Here the middle value is called indeterminacy value of neutrosophic theory. This value is based upon the confusion between Truth value and falsity.

In W.S.N connecting and covering the whole area are very essential. If the sensor failure or sensor give expansive errors or disconnection of network to coverage the whole area, then we have to find out the boundary stations and interior stations of the W.S.N, which is equivalent to determine the BN detour boundary and interior nodes of $G$.


Figure 1: Modeling of wireless sensor network in terms of BSVN graph

For the BN graph in Figure 1,
B.N. $D(\mathrm{a}, c)=1$, B.N. $D(a, b)=3$, B.N. $D(a, e)=4$, B.N. $D(a, d)=1$, B.N. $D(a, f)=3$,
$B \cdot N \cdot D(\mathrm{~b}, e)=1, B \cdot N \cdot D(\mathrm{~b}, d)=2, \quad B \cdot N \cdot D(\mathrm{~b}, f)=2$,
B.N. $D(\mathrm{c}, b)=4, B \cdot N . D(\mathrm{c}, e)=5, \quad B \cdot N . D(\mathrm{c}, d)=2, B \cdot N \cdot D(\mathrm{c}, f)=4$,
$B \cdot N . D(\mathrm{e}, d)=3$, B.N.D $(\mathrm{e}, f)=3$,
$B . N . D(\mathrm{~d}, f)=2$.

So $a^{\prime} B . F . D=\{\mathrm{e}\}, c^{\prime} B . F . D=\{\mathrm{e}\}, b^{\prime} B . F . D=\{\mathrm{c}\}, e^{\prime} B . F . D=\{\mathrm{c}\}, d^{\prime} B . F . D=\{\mathrm{e}\}$, $f^{\prime} B . F . D=\{\mathrm{c}\}$.

Therefore $\mathrm{c}, e$ are BN detour boundary nodes and $\mathrm{a}, b, e, d$ are BN detour interior nodes of $G$.
Hence the stations at c, e are boundary stations and the stations $\mathrm{a}, b, d, f$ are the interior stations of the W.S.N.

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