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# Fixed Point Theorem for Neutrosophic Triplet Partial Metric Space

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**Abstract:** Neutrosophic triplet is a new theory in neutrosophy. In a neutrosophic triplet set, there is a neutral element and antielement for each element. In this study, the concept of neutrosophic triplet partial metric space (NTPMS) is given and the properties of NTPMS are studied. We show that both classical metric and neutrosophic triplet metric (NTM) are different from NTPM. Also, we show that NTPMS can be defined with each NTMS. Furthermore, we define a contraction for NTPMS and we give a fixed point theory (FPT) for NTPMS. The FPT has been revealed as a very powerful tool in the study of nonlinear phenomena. This study is also part of the “Algebraic Structures of Neutrosophic Triplets, Neutrosophic Duplets, or Neutrosophic Multisets” which is a special issue.

**Keywords:** neutrosophic triplet set (NTS); partial metric spaces (PMS); fixed point theory (FPT)

## 1. Introduction

Neutrosophy was first studied by Smarandache in [1]. Neutrosophy consists of neutrosophic logic, probability, and sets. Actually, neutrosophy is generalization of fuzzy set in [2] and intuitionistic fuzzy set in [3]. Also, researchers have introduced neutrosophic theory in [4–6]. Recently, Olgun and Bal introduced the neutrosophic module in [7], Şahin, Uluçay, Olgun, and Kılıçman introduced neutrosophic soft lattices in [8], and Uluçay, Şahin, and Olgun studied soft normed rings in [9]. Furthermore, Smarandache and Ali studied NT theory in [10] and NT groups (NTG) in [11,12]. The greatest difference between NTG and classical groups is that there can be more than one unit element. That is, each element in a neutrosophic triplet group can be a separate unit element. In addition, the unit elements in the NTG must be different from the unit elements in the classical group. Also, a lot of researchers have introduced NT theory in [13–16]. Recently, Smarandache, Şahin, and Kargin studied neutrosophic triplet G-module in [17], and Bal, Shalla, and Olgun introduced neutrosophic triplet cosets and quotient groups in [18].

Matthew introduced the concept of partial metric spaces (PMS) in [19]. It is a generalization of usual metric space since self-distance cannot be zero in PMS. The most important use of PMS is to transfer mathematical techniques to computer science. Also, Matthew introduced Banach contraction theorem for PMS and a lot of researchers introduced PMS and its topological properties and FPT for PMS in [20–23]. If  $f$  is a mapping from a set  $E$  into itself, any element  $x$  of  $E$ , such that  $f(x) = x$ , is called a fixed point of  $f$ . Many problems, including nonlinear partial differential equations problems, may be recast as problems of finding a fixed point of a mapping in a space. Recently, Shukla introduced FPT for ordered contractions in partial b-metric space in [24]. Kim, Okeke, and

Lim introduced common coupled FPT for  $w$ -compatible mappings in PMS in [25]. Pant, Shukla, and Panicker introduced new FPT in PMS in [26].

In this paper, we first introduced PMS and contraction in NT theory. So, we obtained a new structure for developing NT theory. Thus, researchers can arrive at nonlinear partial differential equations problem solutions in NT theory. In Section 2, we give some basic results and definitions for NTPM and NTM. In Section 3, NTPMS is defined and some properties of a NTPMS are given. It was shown that both the classical metric and NTM are different from the NTPM, and NTPMS can be defined with each NTMS. Furthermore, the convergent sequence and Cauchy sequence in NTPMS are defined. Also, complete NTPMS are defined. Later, we define contractions for NTPM and we give some properties of these contractions. Furthermore, we give a FPT for NTPMS. In Section 4, we give conclusions.

## 2. Preliminaries

We give some basic results and definitions for NTPM and NTM in this section.

**Definition 1** ([19]). Let  $A$  be nonempty set. If the function  $p_m: A \times A \rightarrow \mathbb{R}^+$  satisfies the conditions given below;  $p$  is called a PM.  $\forall a, b, c \in A$ ;

- (i)  $p_m(a, a) = p_m(b, b) = p_m(a, b) = p_m(b, a) \Leftrightarrow a = b$ ;
- (ii)  $p_m(a, a) \leq p_m(a, b)$ ;
- (iii)  $p_m(a, b) = p_m(b, a)$ ;
- (iv)  $p_m(a, c) \leq p_m(a, b) + p_m(b, c) - p_m(b, b)$ ;

Also,  $(A, p_m)$  is called a PMS.

**Definition 2** ([12]). Let  $N$  be a nonempty and  $\#$  be a binary operation. Then,  $N$  is called a NT if the given below conditions are satisfied.

- (i) There is neutral element ( $neut(x)$ ) for  $x \in N$  such that  $x * neut(x) = neut(x) * x = x$ .
- (ii) There is anti element ( $anti(x)$ ) for  $x \in N$  such that  $x * anti(x) = anti(x) * x = neut(x)$ .

NT is shown by  $(x, neut(x), anti(x))$ .

**Definition 3** ([15]). Let  $(M, \#)$  be a NTS and  $a \# b \in N$ ,  $\forall a, b \in M$ . NTM is a map  $d_T: M \times M \rightarrow \mathbb{R}^+ \cup \{0\}$  such that  $\forall a, b, c \in M$ ,

- (a)  $d_T(a, b) \geq 0$
- (b) If  $a = b$ , then  $d_T(a, b) = 0$
- (c)  $d_T(a, b) = d_T(b, a)$
- (d) If there exists any element  $c \in M$  such that  $d_T(a, c) \leq d_T(a, c * neut(b))$ , then  $d_T(a, c * neut(b)) \leq d_T(a, b) + d_T(b, c)$ .

Also,  $((M, *), d_T)$  space is called NTMS.

## 3. Neutrosophic Triplet Partial Metric Space

Partial metric is the generalization of usual metric space, since self-distance cannot be zero in partial metric space. The most important use of PMS is to transfer mathematical techniques to computer science. Also, If  $f$  is a mapping from a set  $E$  into itself, any element  $x$  of  $E$  such that  $f(x) = x$  is called a fixed point of  $f$ . Many problems, including nonlinear partial differential equations problems, may be recast as problems of finding a fixed point of a mapping in a space. In this section, we introduced firstly PMS and FPT in NT theory. So, we obtained a new structure for developing NT theory. Thus, researchers can arrive at nonlinear partial differential equations problem solutions in NT theory.

**Definition 4.** Let  $(A, \#)$  be a NTS and  $a\#b \in A$ ,  $\forall a, b \in A$ . NTPM is a map  $p_N: AxA \rightarrow \mathbb{R}^+ \cup \{0\}$  such that  $\forall a, b, c \in A$

- (i)  $0 \leq p_N(a, a) \leq p_N(a, b)$
- (ii) If  $p_N(a, a) = p_N(a, b) = p_N(b, b) = 0$ , then there exists any  $a, b$  such that  $a = b$ .
- (iii)  $p_N(a, b) = p_N(a, b)$
- (iv) If there exists any element  $b \in A$  such that  $p_N(a, c) \leq p_N(a, c\#\text{neut}(b))$ , then  $p_N(a, c\#\text{neut}(b)) \leq p_N(a, b) + p_N(b, c) - p_N(b, b)$

Additionally,  $((A, \#), p_N)$  is called NTPMS.

**Example 1.** Let  $A$  be a nonempty set and  $P(A)$  be power set of  $A$  and  $m(X)$  be cardinal of  $X \in P(A)$ . Where, it is clear that  $X \cup X = X$ . Thus; we give that  $\text{neut}(X) = X$  and  $\text{anti}(X) = X$  for  $X \in P(A)$ . So,  $(P(A), \cup)$  is a NTS. We give the function  $p_N: P(A) \times P(A) \rightarrow \mathbb{R}^+ \cup \{0\}$  such that  $p_N(X, Y) = \max\{m(X), m(Y)\}$ . From Definition 4,

(i), (ii) and (iii) are apparent.

(iv) Let  $\emptyset$  be empty element of  $P(X)$ . Then,  $p_N(X, Y) = p_N(X, Y \cup \emptyset)$  since for  $p_N(X, Y \cup \emptyset) = p_N(X, Y) = \max\{m(X), m(Y)\}$ . Also, it is clear that  $\max\{m(X), m(Y)\} \leq \max\{m(X), m(Z)\} + \max\{m(Z), m(Y)\} - \max\{m(\emptyset), m(\emptyset)\}$ .

Therefore,  $p_N(X, Y \cup \emptyset) \leq p_N(X, \emptyset) + p_N(\emptyset, Y) - p_N(\emptyset, \emptyset)$ . Thus,  $((P(A), \cup), p_N)$  is a NTPMS.

**Corollary 1.** NTPM is different from the partial metric. Because there isn't a "#" binary operation and neutral of  $x$  in PMS.

**Corollary 2.** Generally the NTPM is different from NT metric, since for  $p_N(x, x) \geq 0$ .

**Theorem 1.** Let  $A$  be a nonempty set and  $P(A)$  be power set of  $A$  and  $m(X)$  be cardinal of  $X \in P(A)$  and  $(P(A), \#, d)$  be a NT metric space (NTMS). If there exists any  $Z \in P(A)$  such that  $m(Y\#\text{neut}(Z)) = m(Y)$ ; then  $((P(A), \#), p_N)$  is a NTPMS such that

$$p_N(X, Y) = \frac{d(X, Y) + m(X) + m(Y)}{2}$$

**Proof.**

- (i)  $p_N(X, X) = \frac{d(X, X) + m(X) + m(X)}{2} = m(X) \leq \frac{d(X, Y) + m(X) + m(Y)}{2} = p_N(X, Y)$ , since for  $d(X, X) = 0$ . Thus;  $0 \leq p_N(X, X) \leq p_N(X, Y)$  for  $X, Y \in P(A)$ .
- (ii) If  $p_N(X, X) = p_N(X, Y) = p_N(Y, Y) = 0$ , then
- (iii)  $\frac{d(X, X) + m(X) + m(X)}{2} = \frac{d(X, Y) + m(X) + m(Y)}{2} = \frac{d(Y, Y) + m(Y) + m(Y)}{2} = 0$  and  $d(X, Y) + m(X) + m(Y) = 0$ . Where,  $m(X) = 0, m(Y) = 0$  and  $d(X, Y) = 0$ . Thus,  $X = Y = \emptyset$  (empty set).
- (iv)  $p_N(X, Y) = \frac{d(X, Y) + m(X) + m(Y)}{2} = \frac{d(Y, X) + m(Y) + m(X)}{2} = p_N(Y, X)$ , since for  $d(X, Y) = d(Y, X)$ .
- (v) We suppose that there exists any  $Z \in P(A)$  such that  $m(Y\#\text{neut}(Z)) = m(Y)$  and  $p_N(X, Y) \leq p_N(X, Y\#\text{neut}(Z))$ . Thus,

$$\frac{d(X, Y) + m(X) + m(Y)}{2} \leq \frac{d(X, Y\#\text{neut}(Z)) + m(X) + m(Y\#\text{neut}(Z))}{2} \quad (1)$$

From (1),  $d(X, Y) \leq d(X, Y\#\text{neut}(Z))$ . Since  $(P(A), \#, d)$  is a NTMS,

$$d(X, Y\#\text{neut}(Z)) \leq d(X, Z) + d(X, Z) \quad (2)$$

From (1), (2)

$$\frac{d(X, Y) + m(X) + m(Y)}{2} \leq \frac{d(X, Y\#\text{neut}(Z)) + m(X) + m(Y\#\text{neut}(Z))}{2} \leq \frac{d(X, Z) + d(Z, Y) + m(X) + m(Y) + m(Z)}{2} = \frac{d(X, Z) + m(X) + m(Z)}{2} + \frac{d(Z, Y) + m(Z) + m(Y)}{2} - m(Z). \text{ Where, } p_N(Z, Z) = m(Z).$$

Thus,  $p_N(X, Y\#\text{neut}(Z)) \leq p_N(X, Z) + p_N(Z, Y) - p_N(Z, Z)$ . Hence,  $((P(A), \#), p_N)$  is a NTPMS.  $\square$

**Theorem 2.** Let  $(A, \#)$  be a NT set,  $k \in \mathbb{R}^+$  and  $((A, \#), d_T)$  be a NTMS. Then;  $((A, \#), p_N)$  is a NTPMS such that

$$p_N(a, b) = d_T(a, b) + k, \forall a, b \in A.$$

**Proof.**

- (i) Since for  $d_T(a, a) = 0$ ,  $0 \leq p_N(a, a) = d_T(a, a) + k = k \leq p_N(a, b) = d_T(a, b) + k$ . Thus;
- (ii)  $0 \leq p_N(a, a) \leq p_N(a, b)$ .
- (iii) There do not exists  $a, b \in A$  such that  $p_N(a, a) = p_N(a, b) = p_N(b, b) = 0$  since for  $k \in \mathbb{R}^+$  and  $d_T(a, a) = 0$ .
- (iv)  $p_N(a, b) = d_T(a, b) + k = d_T(b, a) + k$ , since for  $d_T(a, b) = d_T(b, a)$ .
- (v) Suppose that there exists any element  $c \in A$  such that  $p_N(a, b) \leq p_N(a, b\#neut(c))$ . Then  $d_T(a, b) + k \leq d_T(a, b\#neut(c)) + k$ . Thus,

$$d_T(a, b) \leq d_T(a, b\#neut(c)) \quad (3)$$

Also,

$$d_T(a, b\#neut(c)) \leq d_T(a, c) + d_T(c, b) \quad (4)$$

since for  $((A, \#), d_T)$  is a NTMS.

From (3) and (4),

$$p_N(a, b) \leq p_N(a, b\#neut(c)) = d_T(a, b\#neut(c)) + k \leq d_T(a, c) + d_T(c, b) + k = p_N(a, c) + p_N(c, b) - k.$$

where,  $p_N(c, c) = k$ . Thus;

$p_N(a, b\#neut(c)) \leq p_N(a, c) + p_N(c, b) - p_N(c, c)$ . Hence,  $((A, \#), p_N)$  is a NTPMS.  $\square$

**Corollary 3.** From Theorem 2, we can define NTPMS with each NTMS.

**Definition 5.** Let  $((A, \#), p_N)$  be a NTPMS,  $\{x_n\}$  be a sequence in NTPMS and  $a \in A$ . If for  $\forall \epsilon > 0$  and  $\forall n \geq M$ , there exist a  $M \in \mathbb{N}$  such that  $p_N(a, \{x_n\}) < \epsilon + p_N(a, a)$ , then  $\{x_n\}$  converges to  $a \in A$ . It is shown by

$$\lim_{n \rightarrow \infty} x_n = a \text{ or } x_n \rightarrow a.$$

**Definition 6.** Let  $((A, \#), p_N)$  be a NTPMS,  $\{x_n\}$  be a sequence in NTPMS and  $a \in A$ . If for  $\forall \epsilon > 0$  and  $\forall n, m \geq M$ , there exist a  $M \in \mathbb{N}$  such that  $p_N(\{x_m\}, \{x_n\}) < \epsilon + p_N(a, a)$ ; then  $\{x_n\}$  is a Cauchy sequence in  $((A, \#), p_N)$ .

**Theorem 3.** Let  $((A, \#), p_N)$  be a NTPMS,  $\{x_n\}$  be a convergent sequence in NTPMS and  $p_N(\{x_m\}, \{x_n\}) \leq p_N(\{x_m\}, \{x_n\}) *neut(a)$  for any  $a \in A$ . Then  $\{x_n\}$  is a Cauchy sequence in NTPMS.

**Proof.**

It is clear that

$$p_N(a, \{x_n\}) < \epsilon/2 + p_N(a, a) \quad (5)$$

for each  $n \geq M$  or

$$p_N(a, \{x_m\}) < \epsilon/2 + p_N(a, a) \quad (6)$$

for each  $m \geq M$

Because  $\{x_n\}$  is a convergent. Then, we suppose that  $p_N(\{x_m\}, \{x_n\}) \leq p_N(\{x_m\}, \{x_n\}) *neut(a)$  for any  $a \in A$ . It is clear that for  $n, m \geq M$ ;

$$p_N(\{x_m\}, \{x_n\}) \leq p_N(\{x_m\}, \{x_n\}) *neut(a) \leq p_N(a, \{x_n\}) + p_N(a, \{x_m\}) - p_N(a, a) \quad (7)$$

Because  $((A, \#), p_N)$  is a NTPMS. From (5)–(7),

$p_N(\{x_m\}, \{x_n\}) < \varepsilon/2 + p_N(a, a) + \varepsilon/2 + p_N(a, a) - p_N(a, a) = \varepsilon + p_N(a, a)$ . Thus;  $\{x_n\}$  is a Cauchy sequence in  $((A, \#), p_N)$ .  $\square$

**Definition 7.** Let  $((A, \#), p_N)$  be a NTPMS and  $\{x_n\}$  be a Cauchy sequence in NTPMS. If every  $\{x_n\}$  is convergent in  $((A, \#), p_N)$ , then  $((A, \#), p_N)$  is called a complete NTPMS.

**Definition 8.** Let  $((A, \#), p_N)$  be a NTPMS and  $m: A \rightarrow A$  be a map. If the map  $m$  and the NTPM  $p_N$  satisfy the conditions given below, then  $m$  is called a contraction for  $((A, \#), p_N)$ .

- (i) There exists any element  $c \in A$  such that  $p_N(a, b) \leq p_N(a, b^* \text{neut}(c))$ ;  $\forall a, b \in A$ .
- (ii) There exists  $k$  in  $[0, 1)$  such that  $p_N(m(a), m(b)) \leq k \cdot p_N(a, b)$ ;  $\forall a, b \in A$ .

**Example 2.** Let  $A = \{\emptyset, \{x\}, \{x, y\}\}$  be a set and  $m(X)$  be cardinal of  $X \in A$ . Where, it is clear that  $X \cap X = X$ . Thus, we give that  $\text{neut}(X) = X$  and  $\text{anti}(X) = X$ . So,  $(A, \cap)$  is a NTS. We give the function  $p_N: A \times A \rightarrow \mathbb{R}^+ \cup \{0\}$  such that  $p_N(X, Y) = \max\{2^{2-m(X)} - 1, 2^{2-m(Y)} - 1\}$ . From Definition 4,

(i), (ii) and (iii) are apparent.

(iv)  $p_N(X, \{x, y\}) = p_N(X, Y \cap \{x, y\})$  since for  $X, Y \in A$ . Furthermore, it is clear that  $\max\{2^{2-m(X)} - 1, 2^{2-m(Y)} - 1\} \leq \max\{2^{2-m(X)} - 1, 2^{2-m(\{x,y\})} - 1\} + \max\{2^{2-m(Z)} - 1, 2^{2-m(\{x,y\})} - 1\} - \max\{2^{2-m(\{x,y\})} - 1, 2^{2-m(\{x,y\})} - 1\}$ . Thus,

$p_N(X, Y \cap \{x, y\}) \leq p_N(X, \{x, y\}) + p_N(\{x, y\}, B) - p_N(\{x, y\}, \{x, y\})$ . Furthermore,  $((A, \cap), p_N)$  is a NTPMS.

$$\text{Let } m: A \rightarrow A \text{ be a map such that } m(X) = \begin{cases} \{x, y\}, & X = \{x, y\} \\ \{x\}, & X = \emptyset \\ \{x, y\}, & X = \{x\} \end{cases}$$

For  $k = 0, 2$

$$\begin{aligned} p_N(m(\emptyset), m(\emptyset)) &= p_N(\{x\}, \{x\}) = 1 \leq 0, 2. \quad p_N(\emptyset, \emptyset) = 1, 5 \\ p_N(m(\emptyset), m(\{x\})) &= p_N(\{x\}, \{x, y\}) = 1 \leq 0, 2. \quad p_N(\emptyset, \{x\}) = 1, 5 \\ p_N(m(\emptyset), m(\{x, y\})) &= p_N(\{x\}, \{x, y\}) = 1 \leq 0, 2. \quad p_N(\emptyset, \{x, y\}) = 1, 5 \\ p_N(m(\{x\}), m(\{x\})) &= p_N(\{x, y\}, \{x, y\}) = 0 \leq 0, 2. \quad p_N(\{x\}, \{x\}) = 0, 5 \\ p_N(m(\{x\}), m(\{x, y\})) &= p_N(\{x, y\}, \{x, y\}) = 0 \leq 0, 2. \quad p_N(\{x\}, \{x, y\}) = 0, 5 \\ p_N(m(\{x, y\}), m(\{x, y\})) &= p_N(\{x, y\}, \{x, y\}) = 0 \leq 0, 2. \quad p_N(\{x, y\}, \{x, y\}) = 0, 5 \end{aligned}$$

Thus,  $m$  is a contraction for  $((A, \cap), p_N)$

**Theorem 4.** For each contraction  $m$  over a complete NTPMS  $((A, \#), p_N)$ , there exists a unique  $x$  in  $A$  such that  $x = m(x)$ . Also,  $p_N(x, x) = 0$ .

**Proof.**

Let  $m$  be a contraction for  $((A, \#), p_N)$  complete NTPMS and  $x_n = m(x_{n-1})$  and  $x_0 \in A$  be a unique element. Also, we can take

$$p_N(x_n, x_k) \leq p_N(x_n, x_k^* \text{neut}(x_{n-1})) \quad (8)$$

since for  $m$  is a contraction over  $((A, \#), p_N)$  complete NTPMS. Then,

$$\begin{aligned} p_N(x_2, x_1) &= p_N(m(x_1), m(x_0)) \leq c \cdot p_N(x_1, x_0) \text{ and} \\ p_N(x_3, x_2) &= p_N(m(x_2), m(x_1)) \leq c \cdot p_N(x_2, x_1) \leq c^2 \cdot p_N(x_1, x_0). \text{ From mathematical induction,} \\ n &\geq m; \end{aligned}$$

$p_N(x_{m+1}, x_m) = p_N(m(x_m), m(x_{m-1})) \leq c \cdot p_N(x_m, x_{m-1}) \leq c^m \cdot p_N(x_1, x_0)$ . Thus; from (8) and definition of NTPMS,

$$\begin{aligned} p_N(x_n, x_m) &\leq p_N(x_n, x_m^* \text{neut}(x_{n-1})) \leq p_N(x_n, x_{n-1}) + p_N(x_{n-1}, x_m) - p_N(x_{n-1}, x_{n-1}) \\ &\leq c^{n-1} \cdot p_N(x_1, x_0) + p_N(x_{n-1}, x_m) - p_N(x_{n-1}, x_{n-1}) \\ &\leq c^{n-1} \cdot p_N(x_1, x_0) + p_N(x_{n-1}, x_{n-2}) + \dots + p_N(x_m, \\ &x_{m-1}) \leq (c^{n-1} + c^{n-2} + \dots + c^{m-1} + c^m) \cdot p_N(x_1, x_0) - \end{aligned}$$

$$\begin{aligned}
& \sum_{i=m}^{n-1} p_N(x_i, x_i) \\
& \leq \sum_{i=m}^{n-1} c^i \cdot p_N(x_1, x_0) - \sum_{i=m}^{n-1} p_N(x_i, x_i) \\
& \leq \sum_{i=m}^{n-1} c^i \cdot p_N(x_1, x_0) + p_N(x_0, x_0) \\
& = \sum_{i=m}^{n-1} c^i \cdot p_N(x_1, x_0) + p_N(x_0, x_0) \text{ (For } n, m \rightarrow \infty) \\
& = \frac{c^m}{1-c} p_N(x_1, x_0) + p_N(x_0, x_0) \rightarrow p_N(x_0, x_0).
\end{aligned}$$

Thus  $\{x_n\}$  is a Cauchy sequence. Also  $\{x_n\}$  is convergent such that  $x_n \rightarrow x$ . Because  $((A, \#), p_N)$  is complete NTPMS. Thus;  $m(x_n) \rightarrow m(x)$  since for  $x_n = m(x_{n-1})$ ;  $m(x_n) = x_{n+1} \rightarrow x$ . Thus;  $m(x) = x$ . Suppose that  $m(x) = x$  or  $m(y) = y$  for  $x, y \in x_n$ . Where;

$p_N(x, y) = p_N(m(x), m(y)) \leq c \cdot p_N(x, y)$ .  $p_N(x, y) > 0$ ,  $c \geq 1$  and it is a contradiction. Thus;  $p_N(x, y) = p_N(x, x) = p_N(y, y) = 0$  and  $x = y$ . Therefore,  $p_N(x, x) = 0$ .  $\square$

#### 4. Conclusions

In this paper, we introduced NTPMS. We also show that both the classical metric and NTM are different from the NT partial metric. This NT notion has more features than the classical notion. We also introduced contraction for PMS and we give a fixed point theory for PMS in NT theory. So, we obtained a new structure for developing NT theory. Thus, researchers can arrive at nonlinear partial differential equations problem solutions in NT theory thanks to NTPMS and FPT for NTPMS.

**Author Contributions:** In this paper, each Author contributed equally. M.S. introduced NTPMS and provided examples. A.K. introduced contraction for NTPMS and provided examples. M.A.C. gave fixed point theory for NTPMS and organized the paper.

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