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Forecasting model based on neutrosophic logical relationship and Jaccard similarity

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Abstract: Daily fluctuation trends of a stock market consist of three statuses like up, equal and down. It can be represented by a neutrosophic set which consists of three terms like the truth-membership, indeterminacy-membership and falsity-membership functions. In this paper, we propose a novel forecasting model based on neutrosophic set theory and fuzzy logical relationships between the status of history and current. Firstly, the original time series of the stock market are converted to fluctuation time series by comparing each piece of data with that of the previous day. Then, fuzzify the fluctuation time series into fuzzy-fluctuation time series in terms of the pre-defined up, equal and down intervals. Next, the fuzzy logical relationships can be expressed by two neutrosophic sets according to the probabilities for different statuses of each current and a certain range of corresponding histories. Finally, based on the neutrosophic logical relationships and the status of history, Jaccard similarity measure is employed to find the most proper logical rule to forecast its future. The authentic Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) time series datasets are used as an example to illustrate the forecasting procedure and performance comparison. The experimental results show that the proposed method can successfully forecast the stock market and other similar kinds of time series. We also apply the proposed method to forecast Shanghai Stock Exchange Composite Index (SHSECI) to verify its effectiveness and universality.

Keywords: Fuzzy time series; forecasting; fuzzy logical relationship; neutrosophic set; Jaccard similarity

1. Introduction

It is well known that there is a statistical long range dependency between the current values and historical values in different times of certain time series [1]. Therefore, many researchers have developed various models to predict the future of such time series based on the historical data sets, e.g., the regression analysis model [2], the autoregressive moving average (ARIMA) model [3], the autoregressive conditional heteroscedasticity (ARCH) model [4], the generalized ARCH (GARCH) model [5], and so on. However, crisp data used in those models are sometimes unavailable as such time series contains lots of uncertainties. In fact, models satisfy the constraints precisely can miss the true optimum design within the confines of practical and realistic approximations. Therefore, Song and Chissom proposed the fuzzy time series (FTS) forecasting model [6–8] to predict the future of such nonlinear and complicated problems. In a financial context, FTS approaches have been widely applied in stock index forecasting [9–13]. In order to improve the accurate forecasts for stock market indices, some researchers combine fuzzy and non-fuzzy time series with heuristic optimization

methods in forecasting strategies [14]. Other approaches even introduce neural networks and machine learning procedures to find forecasting rules from historical time series [15-17].

The major points in FTS models are related to the fuzzifying of original time series, the establishment of fuzzy logical relationships from historical training datasets and the forecasting and defuzzification of the outputs. Concerning the basic steps of fuzzifying method, various proposals have been considered to determine the effective length of intervals, e.g., the optimal interval length based on averages and distribution methods [18], the optimal interval length using statistical theory [18-24], unequal interval length method based on ratios of data [25], length determination method based on PSO techniques [10], etc. To state appropriate fuzzy logical relationships, Yu [26] proposed a weight assignment model based on the recurrent fuzzy relationships for each individual relationship. Aladag et al. [27] considered artificial neural networks as a basic high-order method for the establishment of logical relationships. Fuzzy auto regressive (AR) models and fuzzy auto regressive and moving average (ARMA) models are also widely used to reflect the recurrence and the weights of different fuzzy logical relationships [9-10, 28-37]. These obtained logical relationships will be used as rules during the forecasting process. However, the proportions of the lagged variables in AR or ARMA models only represent the general best fitness for certain training datasets, without taking into account the differences between individual relationships. Although weight assignment model considers the differences between individual relationships, it has to deal with special relationships which appear in testing dataset but never happen in training dataset. These FTS methods look for point forecasts, without taking into account the implicit uncertainty in the ex-post forecasts.

The main question concerning to forecast stock market indices is related to the trend changes. In fact, daily fluctuation trends of a stock market consist of three statuses like up, equal and down. Therefore, the relationships between historical and current statuses can be regarded as the relationships between two weighted fuzzy-fluctuation statuses. In order to deal with the indeterminate and inconsistent logical relationships, we use neutrosophic sets to represent the trends and weights of the relationships and utilize the similarity measures to look for the most consistent rules in the forecasting and defuzzification steps. The concept of neutrosophic set was originally proposed from a philosophical point of view by Smarandache [38]. A neutrosophic set is characterized independently by a truth-membership function, an indeterminacy-membership function and a falsity-membership function. Its similarity measure is a key role for decision making problems in uncertainty environments. Researchers have proposed various similarity measures and mainly applied them to decision making, e.g., Jaccard, Dice and cosine similarity measures [39], the distance-based similarity measures [40], the entropy measures [41], etc. Although neutrosophic sets have been successfully applied to decision making [39-44], they are scarcely applied to forecasting problems.

In this paper, we introduce neutrosophic sets to stock market forecasting. We propose a novel forecasting model based on neutrosophic set theory and fuzzy logical relationships between statuses of history and current. Firstly, the original time series of the stock market are converted to fluctuation time series by comparing each piece of data with that of the previous day. Then, fuzzify the fluctuation time series into fuzzy-fluctuation time series in terms of the pre-defined up, equal and down intervals. Next, the fuzzy logical relationships can be expressed by two neutrosophic sets according to the probabilities for different statuses of each current and a certain range of corresponding histories. Finally, based on the neutrosophic logical relationships and statuses of nearby history, Jaccard similarity measure is employed to find the most proper logical rule to forecast its future.

The remaining content of this paper is organized as follows: Section 2 introduces some preliminaries of fuzzy-fluctuation time series and concepts and similarity measures of neutrosophic sets. Section 3 describes a novel approach for forecasting based on fuzzy-fluctuation trends and logical relationships. In Section 4, the proposed model is used to forecast the stock market using TAIEX datasets from 1997 to 2005 and SHSECI from 2007 to 2015. Conclusions and potential issues for future research are summarized in Section 5.

2. Preliminaries

2.1. Definition of fuzzy-fluctuation time series (FFTS)

Song and Chissom [6-8] combined fuzzy set theory with time series and presented the definitions of fuzzy time series. In this section, we extend fuzzy time series to fuzzy-fluctuation time series (FFTS) and propose the related concepts.

Definition 1. Let $L = \{l_1, l_2, \dots, l_g\}$ be a fuzzy set in the universe of discourse U ; it can be defined by its membership function, $\mu_L : U \rightarrow [0, 1]$, where $\mu_L(u_i)$ denotes the grade of membership of u_i , $U = \{u_1, u_2, \dots, u_i, \dots, u_l\}$.

The fluctuation trends of a stock market can be expressed by a linguistic set $L = \{l_1, l_2, l_3\} = \{\text{down, equal, up}\}$. The element l_i and its subscript i is strictly monotonically increasing [45], so the function can be defined as follows: $f : l_i = f(i)$.

Definition 2. Let $F(t) (t = 1, 2, \dots, T)$ be a time series of real numbers, where T is the number of the time series. $G(t)$ is defined as a fluctuation time series, where $G(t) = F(t) - F(t-1), (t = 2, 3, \dots, T)$. Each element of $G(t)$ can be represented by a fuzzy set $S(t) (t = 2, 3, \dots, T)$ as defined in Definition 1. Then we call time series $G(t)$ to be fuzzified into a fuzzy-fluctuation time series (FFTS) $S(t)$.

Definition 3. Let $S(t) (t = n+1, n+2, \dots, T, n \geq 1)$ be a FFTS. If $S(t)$ is determined by $S(t-1), S(t-2), \dots, S(t-n)$, then the fuzzy-fluctuation logical relationship is represented by:

$$S(t-1), S(t-2), \dots, S(t-n) \rightarrow S(t) \quad (1)$$

and it is called the n th-order fuzzy-fluctuation logical relationship (FFLR) of the fuzzy-fluctuation time series, where $S(t-n), \dots, S(t-2), S(t-1)$ is called the left-hand side (LHS) and $S(t)$ is called the right-hand side (RHS) of the FFLR, and $S(k) (k = t, t-1, t-2, \dots, t-n) \in L$.

2.2. Basic Concept of Neutrosophic Logical Relationship (NLR)

Smarandache [38] originally presented the neutrosophic set theory. Based on neutrosophic set theory, we propose the concept of fuzzy-neutrosophic logical relationship which employs the three terms of a neutrosophic set to reflect the fuzzy-fluctuation trends and weights of a n th-order FFLR.

Definition 4. Let $P_{A(t)}^i$ be the probabilities of each element $l_i (l_i \in L)$ in the LHS of a n th-order FFLR $S(t-1), S(t-2), \dots, S(t-n) \rightarrow S(t)$, it can be generated by:

$$P_{A(t)}^i = \frac{\sum_{j=1}^n w_{i,j}}{n} \quad i = 1, 2, 3 \quad (2)$$

where $w_{i,j} = 1$ if $S(t-j) = l_i$ and 0 otherwise. Let X be a universal set, the left-hand side of a neutrosophic logical relationship is defined by:

$$A(t) = \left\{ \left\langle x, P_{A(t)}^1, P_{A(t)}^2, P_{A(t)}^3 \right\rangle \mid x \in X \right\} \quad (3)$$

Definition 5. For $S(t) (t = n+1, n+2, \dots, T)$ be a FFTS and $A(t)$ be the LHS of a neutrosophic logical relationship. The FFLRs with the same $A(t)$ can be grouped into a FFLRG by putting all their RHSs together as on the RHS of the FFLRG. The RHSs of the FFLRG for $A(t)$ can be represented by a neutrosophic set as Definition 4:

$$B_{A(t)} = \left\{ \left\langle x, P_{B_{A(t)}}^1, P_{B_{A(t)}}^2, P_{B_{A(t)}}^3 \right\rangle \mid x \in X \right\} \quad (4)$$

where $P_{B_{A(t)}}^i$ ($i=1,2,3$) represent the down, equal or up probability of the RHSs of the FFRLG for $A(t)$. $P_{B_{A(t)}}^i$ ($i=1,2,3$) is called the right-hand side of a neutrosophic logical relationship.

In this way, the FFLR $S(t-1), S(t-2), \dots, S(t-n) \rightarrow S(t)$ can be represented by a neutrosophic logical relationship (NLR) $A(t) \rightarrow B_{A(t)}$.

Definition 6 [39]. Let $A(t_1)$ and $A(t_2)$ be two neutrosophic sets. The Jaccard similarity measure between $A(t_1)$ and $A(t_2)$ in vector space is defined as follows:

$$J(A(t_1), A(t_2)) = \frac{\sum_{i=1}^3 P_{A(t_1)}^i P_{A(t_2)}^i}{\sum_{i=1}^3 (P_{A(t_1)}^i)^2 + \sum_{i=1}^3 (P_{A(t_2)}^i)^2 - \sum_{i=1}^3 P_{A(t_1)}^i P_{A(t_2)}^i} \quad (5)$$

3. A Novel Forecasting Model Based on NLRs

In this paper, we propose a novel forecasting model based on high-order neutrosophic logical relationships and Jaccard similarity measures. In order to compare the forecasting results with other researchers' work [9,17,24,33,46–50], the authentic TAIEX (Taiwan Stock Exchange Capitalization Weighted Stock Index) is employed to illustrate the forecasting process. The data from January 1999 to October 1999 are used as training time series and the data from November 1999 to December 1999 are used as testing dataset. The basic steps of the proposed model are shown in Figure 1.

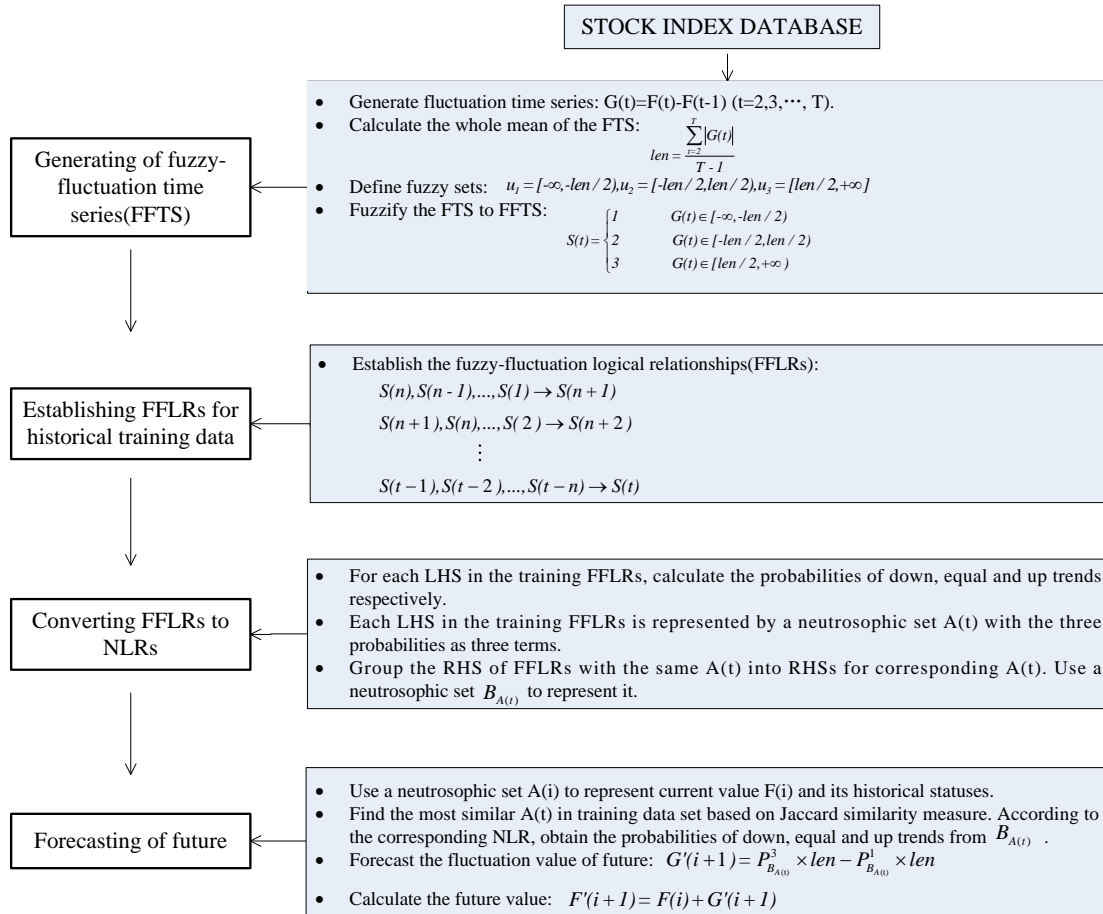


Figure 1. Flowchart of our proposed forecasting model.

Step 1: Construct FFTS for historical training data

For each element $F(t) (t = 1, 2, \dots, T)$ in the historical training time series, its fluctuation trend is determined by $G(t) = F(t) - F(t-1), (t = 2, 3, \dots, T)$. According to the range and orientation of the fluctuations, $G(t) (t = 2, 3, \dots, T)$ can be fuzzified into a linguistic set {down, equal, up}. Let len be the whole mean of all elements in the fluctuation time series $G(t) (t = 2, 3, \dots, T)$, define $u_1 = [-\infty, -len/2)$, $u_2 = [-len/2, len/2)$, $u_3 = [len/2, +\infty]$, then $G(t) (t = 2, 3, \dots, T)$ can be fuzzified into a fuzzy-fluctuation time series $S(t) (t = 2, 3, \dots, T)$.

Step 2: Establish n th-order FFLRs for training data set

According to Definition 3, each $S(t) (t > n)$ in the historical training data set can be represented by its previous n days' fuzzy-fluctuation numbers to establish the training FFLRs.

Step 3: Convert the FFLRs to NLRs

According to Definition 4, the LHS of each FFLR can be expressed by a neutrosophic set $A(t)$. Then, we can generate the RHSs $B_{A(t)}$ for different LHSs respectively as described in Definition 5. Thus, the FFLRs for the historical training dataset are converted into NLRs.

Step 4: Forecast test time series

For each observed point $F(i)$ in the test time series, we can use a neutrosophic set $A(i)$ to represent its n th-order fuzzy-fluctuation trends. Then, for each $A(t)$ obtained in step 3, compare $A(i)$ with $A(t)$ respectively and find the most similar one based on the Jaccard similarity measure method described in Definition 6. Next, use the corresponding $B_{A(t)}$ as forecasting rule to predict the fluctuation value $G'(i+1)$ of next point. Finally, get the forecasting value by $F'(i+1) = F(i) + G'(i+1)$.

4. Empirical Analysis

4.1. Forecasting TAIEX

Many studies use TAIEX1999 as an example to illustrate their proposed forecasting methods [9,17,24,33,46–50]. In order to compare the accuracy with their models, we also use TAIEX1999 to illustrate the proposed method.

Step 1: Calculate the fluctuation trend for each element in the historical training dataset of TAIEX1999. Then, we use the whole mean of the fluctuation numbers of the training dataset to fuzzify the fluctuation trends into FFTS. For example, the whole mean of the historical dataset of TAIEX1999 from January to October is 85. That is to say, $len = 85$. For $F(1) = 6152.43$ and $F(2) = 6199.91$, $G(2) = 47.48$, $S(2) = 3$. In this way, the historical training dataset can be represented by a fuzzified fluctuation dataset as shown in Appendix Table A1.

Step 2: Based on the FFTS from 5 January 1999 to 30 October shown in Table A1, the n th-order FFLRs for the forecasting model are established as shown in Table A2. The subscript i is used to represent element l_i in the FFLRs for convenience.

Step 3: In order to convert the FFLRs to NLRs, first of all, the LHSs of the FFLRs in Table A2 are represented by a neutrosophic set, respectively (shown in Table A2). Then, group the RHSs of the FFLRs with the same LHS neutrosophic set value into RHSs group. Use a neutrosophic set to represent the RHSs group. For example, the LHS of FFLR $2,3,1,1,1,2,2,3,3 \rightarrow 1$ can be represented by the neutrosophic set $(0.33,0.33,0.33)$. The detailed grouping and converting processes are shown in Figure 2.

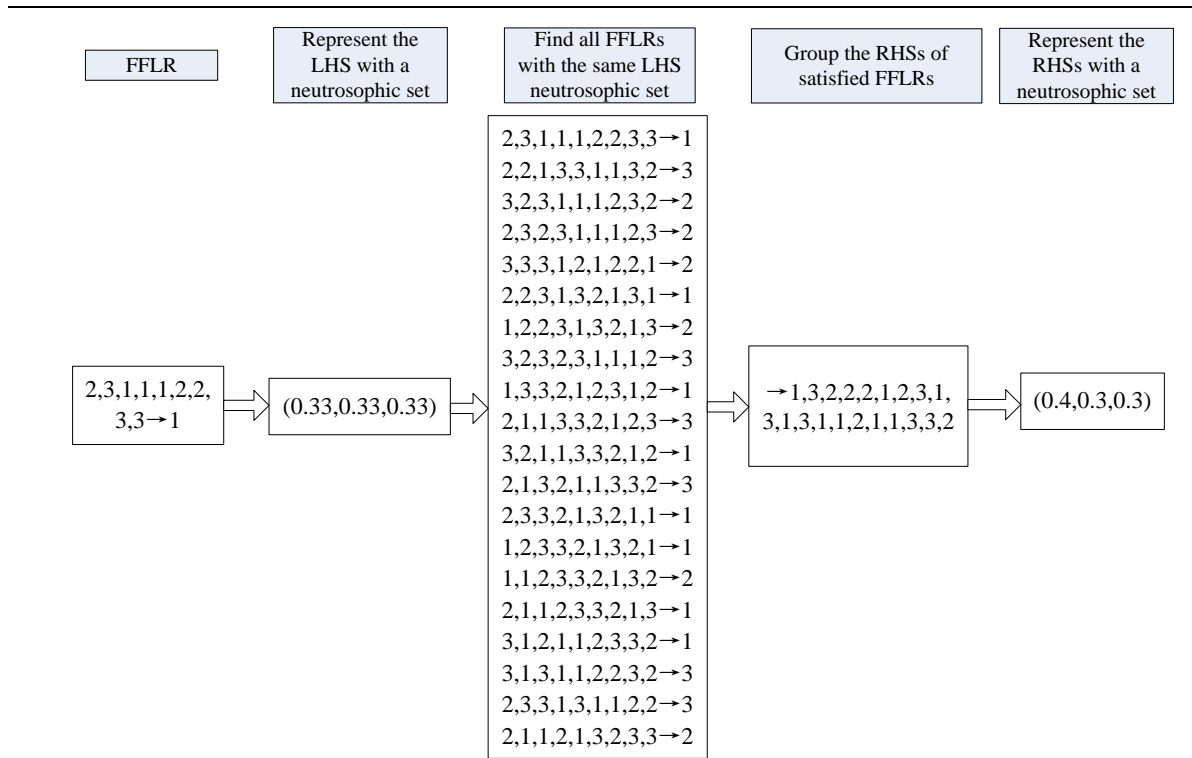


Figure 2. Group and converting processes for FFLR 2,3,1,1,1,2,2,3,3→2.

In this way, the FFLR 2,3,1,1,1,2,2,3,3→1 and other members of the same group is converted into a NLR (0.33,0.33,0.33)→(0.4,0.3,0.3). Therefore, the FFLRs in Table A2 can be converted into NLRs as shown in Table 1.

Table 1. Neutrosophic logical relationships (NLRs) for historical training data of TAIEX1999.

NLRs	NLRs	NLRs
(0.33,0.33,0.33)→(0.4,0.3,0.3)	(0.22,0.33,0.44)→(0,0.6,0.4)	(0.22,0.78,0)→(0.5,0.5,0)
(0.44,0.33,0.22)→(0.23,0.46,0.31)	(0.22,0.44,0.33)→(0.33,0.33,0.33)	(0.33,0.67,0)→(0,0,1)
(0.44,0.44,0.11)→(0.4,0.33,0.27)	(0.11,0.56,0.33)→(0.17,0.5,0.33)	(0.11,0.11,0.78)→(0,1,0)
(0.33,0.44,0.22)→(0.54,0.23,0.23)	(0.11,0.67,0.22)→(0.17,0.33,0.5)	(0,0.22,0.78)→(0,1,0)
(0.33,0.56,0.11)→(0.25,0.5,0.25)	(0.22,0.56,0.22)→(0.25,0.5,0.25)	(0,0.33,0.67)→(0,1,0)
(0.56,0.33,0.11)→(0.36,0.27,0.36)	(0.11,0.44,0.44)→(0,0.38,0.63)	(0.56,0.22,0.22)→(0.25,0.25,0.5)
(0.67,0.22,0.11)→(0,1,0)	(0.11,0.33,0.56)→(0.33,0.17,0.5)	(0.44,0.11,0.44)→(0.5,0.5,0)
(0.56,0.44,0)→(0,0,1)	(0.11,0.22,0.67)→(0.43,0.43,0.14)	(0.56,0.11,0.33)→(1,0,0)
(0.44,0.22,0.33)→(0.29,0.43,0.29)	(0,0.56,0.44)→(0.33,0,0.67)	(0.67,0,0.33)→(0,1,0)
(0.33,0.22,0.44)→(0.31,0.38,0.31)	(0,0.44,0.56)→(0.14,0.43,0.43)	(0.67,0.11,0.22)→(0.5,0.25,0.25)
(0.22,0.22,0.56)→(0.25,0.25,0.5)	(0.11,0.78,0.11)→(0,0.8,0.2)	(0.22,0.67,0.11)→(0,0,1)
(0.33,0.11,0.56)→(0,0.5,0.5)	(0,0.89,0.11)→(0.25,0.75,0)	
(0.22,0.11,0.67)→(0.29,0.29,0.43)	(0.11,0.89,0)→(0.5,0.5,0)	

Step 4: Use the NLRs obtained from historical training data to forecast the test dataset from 1 November 1999 to 30 December. For example, the forecasting value of the TAIEX on 1 November 1999 is calculated as follows:

Firstly, the 9th-order historical fuzzy-fluctuation trends 3,2,2,2,2,3,1,2,2 on 1 November 1999 can be represented by a neutrosophic set (0.11,0.67,0.22). Then, use Jaccard similarity measure method as Definition 6 to choose the most optimal NLR from the NLRs listed in Table 1. Obviously, the NLR (0.11,0.67,0.22)→(0.17,0.33,0.5) is the best rule for further forecasting. Therefore, the forecasted fuzzy-fluctuation number is:

$$S'(i+1) = (-0.17) + 0.5 = 0.33$$

The forecasted fluctuation from current value to next value can be obtained by defuzzifying the fluctuation fuzzy number:

$$G'(i + 1) = S'(i + 1) \times len = 0.33 \times 85 = 28.05$$

Finally, the forecasted value can be obtained by current value and the fluctuation value:

$$F(i+1) = F(i) + G'(i+1) = 7854.85 + 28.05 = 7882.9$$

The other forecasting results are shown in Table 2 and Figure 3.



Figure 3. Forecasting results from 1 November 1999 to 30 December 1999.

Table 2. Forecasting results from 1 November1999 to 30 December 1999.

Date (MM/DD/YYYY)	Actual	Forecast	(Forecast-Actual) ²	Date (MM/DD/YYYY)	Actual	Forecast	(Forecast-Actual) ²
11/1/1999	7,814.89	7,882.90	4,625.36	12/1/1999	7,766.20	7,720.87	2,054.81
11/2/1999	7,721.59	7,842.94	14,725.82	12/2/1999	7,806.26	7,766.20	1,604.80
11/3/1999	7,580.09	7,721.59	20,022.25	12/3/1999	7,933.17	7,797.76	18,335.87
11/4/1999	7,469.23	7,580.09	12,289.94	12/4/1999	7,964.49	7,924.67	1,585.63
11/5/1999	7,488.26	7,469.23	362.14	12/6/1999	7,894.46	7,955.99	3,785.94
11/6/1999	7,376.56	7,488.26	12,476.89	12/7/1999	7,827.05	7,885.96	3,470.39
11/8/1999	7,401.49	7,365.51	1,294.56	12/8/1999	7,811.02	7,827.05	256.96
11/9/1999	7,362.69	7,390.44	770.06	12/9/1999	7,738.84	7,802.52	4,055.14
11/10/1999	7,401.81	7,351.64	2,517.03	12/10/1999	7,733.77	7,745.64	140.90
11/11/1999	7,532.22	7,486.82	2,061.16	12/13/1999	7,883.61	7,707.42	31,042.92
11/15/1999	7,545.03	7,521.17	569.30	12/14/1999	7,850.14	7,857.26	50.69
11/16/1999	7,606.20	7,545.03	3,741.77	12/15/1999	7,859.89	7,823.79	1,303.21
11/17/1999	7,645.78	7,606.20	1,566.58	12/16/1999	7,739.76	7,859.89	14,431.22
11/18/1999	7,718.06	7,673.83	1,956.29	12/17/1999	7,723.22	7,728.71	30.14
11/19/1999	7,770.81	7,731.66	1,532.72	12/18/1999	7,797.87	7,723.22	5,572.62
11/20/1999	7,900.34	7,799.71	10,126.40	12/20/1999	7,782.94	7,797.87	222.90
11/22/1999	8,052.31	7,924.99	16,210.38	12/21/1999	7,934.26	7,782.94	22,897.74
11/23/1999	8,046.19	8,052.31	37.45	12/22/1999	8,002.76	7,947.86	3,014.01
11/24/1999	7,921.85	8,046.19	15,460.44	12/23/1999	8,083.49	8,056.32	738.21
11/25/1999	7,904.53	7,936.30	1,009.33	12/24/1999	8,219.45	8,137.05	6,789.76
11/26/1999	7,595.44	7,918.98	104,678.13	12/27/1999	8,415.07	8,233.90	32,822.57
11/29/1999	7,823.90	7,629.44	37,814.69	12/28/1999	8,448.84	8,390.42	3,412.90
11/30/1999	7,720.87	7,845.15	15,445.52	Root Mean Square Error(RMSE)			98.76

The forecasting performance can be assessed by comparing the difference between the forecasted values and the actual values. The widely used indicators in time series models comparisons are the mean squared error (MSE), root of the mean squared error (RMSE), mean absolute error (MAE), mean percentage error (MPE), etc. These indicators are defined by Equations (6)–(9):

$$MSE = \frac{\sum_{t=1}^n (forecast(t) - actual(t))^2}{n} \quad (6)$$

$$RMSE = \sqrt{\frac{\sum_{t=1}^n (forecast(t) - actual(t))^2}{n}} \quad (7)$$

$$MAE = \frac{\sum_{t=1}^n |(forecast(t) - actual(t))|}{n} \quad (8)$$

$$MPE = \frac{\sum_{t=1}^n |(forecast(t) - actual(t))| / actual(t)}{n} \quad (9)$$

where n denotes the number of values forecasted, $forecast(t)$ and $actual(t)$ denote the predicted value and actual value at time t , respectively. With respect to the proposed method for the 9th-order, the MSE, RMSE, MAE, and MPE are 9753.63, 98.76, 76.32, and 0.01, respectively.

Let the order number n vary from 2 to 10, the RMSEs for different n th-order forecasting models are listed in Table 4. The item "Average" refers to the RMSE for the average forecasting results of these different n th-order ($n=2,3,\dots,10$) models.

Table 4. Comparison of forecasting errors for different n th-orders .

n	2	3	4	5	6	7	8	9	10	Average
RMSE	100.22	100.9	100.66	99.81	102.83	103.48	100.36	98.76	108.99	99.03

In practical forecasting, the average of results for different n th-order ($n=2,3,\dots,9$) forecasting models is adopted to avoid the uncertainty. The proposed method is employed to forecast the TAIEX from 1997 to 2005. The forecasting results and errors are shown in Figure 3 and Table 5.

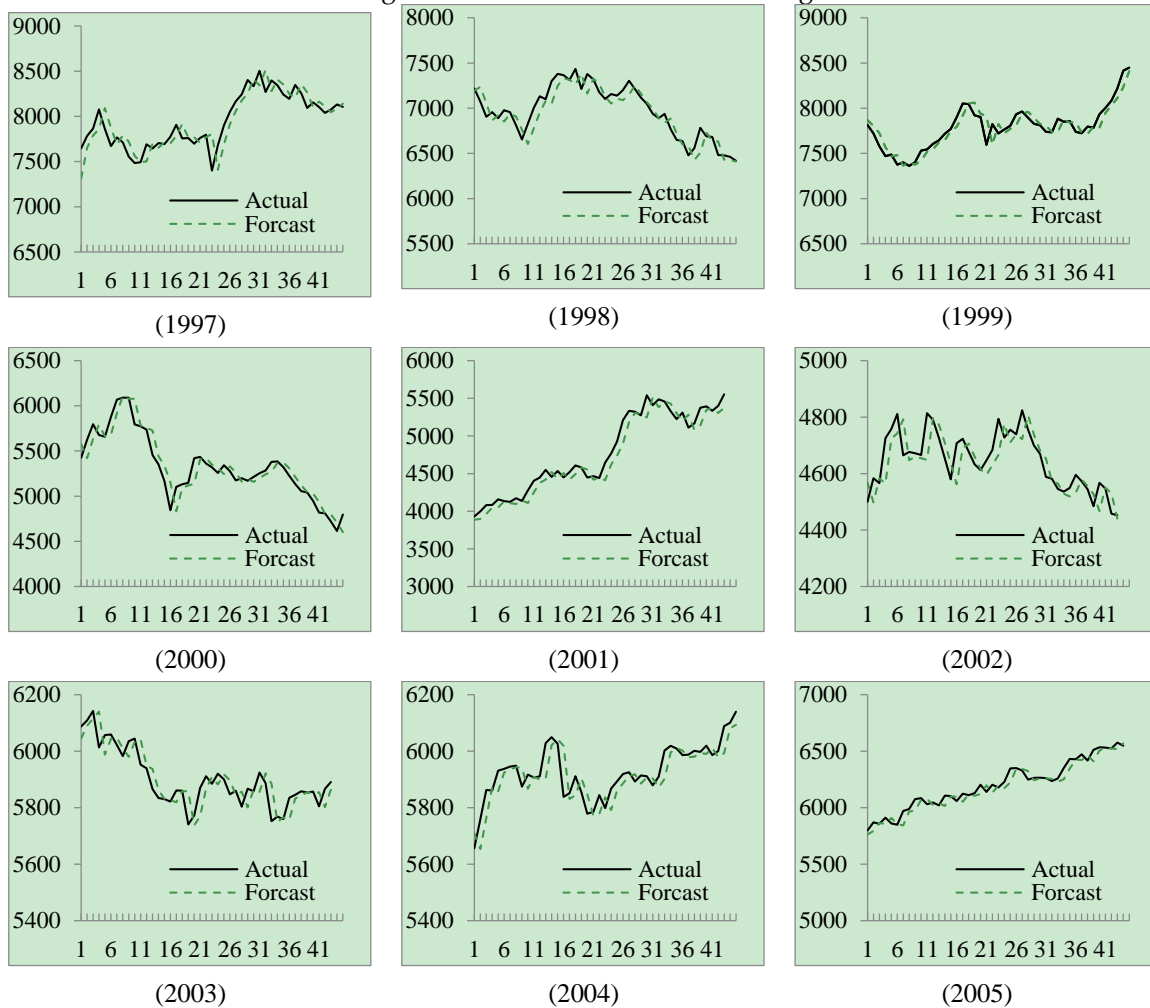


Figure 3. The stock market fluctuation for TAIEX test dataset (1997–2005).

Table 5. RMSEs of forecast errors for TAIEX 1997 to 2005.

Year	1997	1998	1999	2000	2001	2002	2003	2004	2005
RMSE	141.89	119.85	99.03	128.62	125.64	66.29	53.2	56.11	55.83

Table 6 shows a comparison of RMSEs for different methods for forecasting the TAIEX1999. From this table, we can see that the performance of the proposed method is acceptable. The greatest advantage of the proposed method is that it doesn't need to determine the boundary of discourse and the intervals for number fuzzifying. Though RMSEs of some of the other methods outperform the proposed method, they often need to determine complex discretization partitioning rules or use

adaptive expectation models to justify the final forecasting results. The method proposed in this paper is simpler and easily realized by a computer program.

Table 6. A comparison of RMSEs for different methods for forecasting the TAIEX1999.

Methods	RMSE
Yu's Method(2005) [33]	145
Hsieh et al.'s Method(2011) [50]	94
Chang et al.'s Method(2011) [47]	100
Cheng et al.'s Method(2013) [49]	103
Chen et al.'s Method(2013) [48]	102.11
Chen and Chen's Method(2015) [9]	103.9
Chen and Chen's Method(2015) [46]	92
Zhao et al.'s Method(2016) [24]	110.85
Jia et al.'s Method(2017) [17]	99.31
The Proposed Method	99.03

4.2. Forecasting SHSECI

The SHSECI (Shanghai Stock Exchange Composite Index) is the most famous stock market index in China. In the following, we apply the proposed method to forecast the SHSECI from 2007 to 2015. For each year, the authentic datasets of historical daily SHSECI closing prices from January to October are used as the training data, and the datasets from November to December are used as the testing data. The RMSEs of forecast errors are shown in Table 7.

Table 7. RMSEs of forecast errors for SHSECI from 2007 to 2015.

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015
RMSE	113.47	71.6	49.14	45.35	27.74	25.83	19.95	41.42	64.6

From Table 7, we can see that the proposed method can successfully predict the SHSECI stock market.

5. Conclusions

In this paper, a novel forecasting model is proposed based on neutrosophic logical relationships and Jaccard similarity measure. The proposed method is based on the fluctuations of the time series. The high-order fuzzy-fluctuation logical relationships are represented by neutrosophic logical relationships. Therefore, we can use Jaccard similarity measure method to find the most optimal forecasting rules. The most advantage of this method is that it can deal with the problem of lack of rules. Experiments show that the parameters generated from the training dataset can be successfully used for future datasets as well. In order to compare the performance with that of other methods, we take the TAIEX1999 as an example. We also forecasted TAIEX1997–2005 and SHSECI 2007–2015 to verify its effectiveness and universality. In the future, we will consider other factors which might affect the fluctuation of the stock market, such as the trade volume, the beginning value, the end value, etc. We will also consider the influence of other stock markets, such as the Dow Jones, the NASDAQ, the M1b, and so on.

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Appendix A

Table A1. Historical training data and fuzzified fluctuation data of TAIEX1999.

Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified
1/5/1999	6152.43	-	-	4/17/1999	7581.5	114.68	3	7/26/1999	7595.71	-128.81	1
1/6/1999	6199.91	47.48	3	4/19/1999	7623.18	41.68	2	7/27/1999	7367.97	-227.74	1
1/7/1999	6404.31	204.4	3	4/20/1999	7627.74	4.56	2	7/28/1999	7484.5	116.53	3
1/8/1999	6421.75	17.44	2	4/21/1999	7474.16	-153.58	1	7/29/1999	7359.37	-125.13	1
1/11/1999	6406.99	-14.76	2	4/22/1999	7494.6	20.44	2	7/30/1999	7413.11	53.74	3
1/12/1999	6363.89	-43.1	1	4/23/1999	7612.8	118.2	3	7/31/1999	7326.75	-86.36	1
1/13/1999	6319.34	-44.55	1	4/26/1999	7629.09	16.29	2	8/2/1999	7195.94	-130.81	1
1/14/1999	6241.32	-78.02	1	4/27/1999	7550.13	-78.96	1	8/3/1999	7175.19	-20.75	2
1/15/1999	6454.6	213.28	3	4/28/1999	7496.61	-53.52	1	8/4/1999	7110.8	-64.39	1
1/16/1999	6483.3	28.7	2	4/29/1999	7289.62	-206.99	1	8/5/1999	6959.73	-151.07	1
1/18/1999	6377.25	-106.05	1	4/30/1999	7371.17	81.55	3	8/6/1999	6823.52	-136.21	1
1/19/1999	6343.36	-33.89	2	5/3/1999	7383.26	12.09	2	8/7/1999	7049.74	226.22	3
1/20/1999	6310.71	-32.65	2	5/4/1999	7588.04	204.78	3	8/9/1999	7028.01	-21.73	2
1/21/1999	6332.2	21.49	2	5/5/1999	7572.16	-15.88	2	8/10/1999	7269.6	241.59	3
1/22/1999	6228.95	-103.25	1	5/6/1999	7560.05	-12.11	2	8/11/1999	7228.68	-40.92	2
1/25/1999	6033.21	-195.74	1	5/7/1999	7469.33	-90.72	1	8/12/1999	7330.24	101.56	3
1/26/1999	6115.64	82.43	3	5/10/1999	7484.37	15.04	2	8/13/1999	7626.05	295.81	3
1/27/1999	6138.87	23.23	2	5/11/1999	7474.45	-9.92	2	8/16/1999	8018.47	392.42	3
1/28/1999	6063.41	-75.46	1	5/12/1999	7448.41	-26.04	2	8/17/1999	8083.43	64.96	3
1/29/1999	5984	-79.41	1	5/13/1999	7416.2	-32.21	2	8/18/1999	7993.71	-89.72	1
1/30/1999	5998.32	14.32	2	5/14/1999	7592.53	176.33	3	8/19/1999	7964.67	-29.04	2
2/1/1999	5862.79	-135.53	1	5/15/1999	7576.64	-15.89	2	8/20/1999	8117.42	152.75	3
2/2/1999	5749.64	-113.15	1	5/17/1999	7599.76	23.12	2	8/21/1999	8153.57	36.15	2
2/3/1999	5743.86	-5.78	2	5/18/1999	7585.51	-14.25	2	8/23/1999	8119.98	-33.59	2
2/4/1999	5514.89	-228.97	1	5/19/1999	7614.6	29.09	2	8/24/1999	7984.39	-135.59	1
2/5/1999	5474.79	-40.1	2	5/20/1999	7608.88	-5.72	2	8/25/1999	8127.09	142.7	3
2/6/1999	5710.18	235.39	3	5/21/1999	7606.69	-2.19	2	8/26/1999	8097.57	-29.52	2
2/8/1999	5822.98	112.8	3	5/24/1999	7588.23	-18.46	2	8/27/1999	8053.97	-43.6	1
2/9/1999	5723.73	-99.25	1	5/25/1999	7417.03	-171.2	1	8/30/1999	8071.36	17.39	2
2/10/1999	5798	74.27	3	5/26/1999	7426.63	9.6	2	8/31/1999	8157.73	86.37	3
2/20/1999	6072.33	274.33	3	5/27/1999	7469.01	42.38	2	9/1/1999	8273.33	115.6	3
2/22/1999	6313.63	241.3	3	5/28/1999	7387.37	-81.64	1	9/2/1999	8226.15	-47.18	1
2/23/1999	6180.94	-132.69	1	5/29/1999	7419.7	32.33	2	9/3/1999	8073.97	-152.18	1
2/24/1999	6238.87	57.93	3	5/31/1999	7316.57	-103.13	1	9/4/1999	8065.11	-8.86	2

2/25/1999	6275.53	36.66	2	6/1/1999	7397.62	81.05	3	9/6/1999	8130.28	65.17	3
2/26/1999	6318.52	42.99	3	6/2/1999	7488.03	90.41	3	9/7/1999	7945.76	-184.52	1
3/1/1999	6312.25	-6.27	2	6/3/1999	7572.91	84.88	3	9/8/1999	7973.3	27.54	2
3/2/1999	6263.54	-48.71	1	6/4/1999	7590.44	17.53	2	9/9/1999	8025.02	51.72	3
3/3/1999	6403.14	139.6	3	6/5/1999	7639.3	48.86	3	9/10/1999	8161.46	136.44	3
3/4/1999	6393.74	-9.4	2	6/7/1999	7802.69	163.39	3	9/13/1999	8178.69	17.23	2
3/5/1999	6383.09	-10.65	2	6/8/1999	7892.13	89.44	3	9/14/1999	8092.02	-86.67	1
3/6/1999	6421.73	38.64	2	6/9/1999	7957.71	65.58	3	9/15/1999	7971.04	-120.98	1
3/8/1999	6431.96	10.23	2	6/10/1999	7996.76	39.05	2	9/16/1999	7968.9	-2.14	2
3/9/1999	6493.43	61.47	3	6/11/1999	7979.4	-17.36	2	9/17/1999	7916.92	-51.98	1
3/10/1999	6486.61	-6.82	2	6/14/1999	7973.58	-5.82	2	9/18/1999	8016.93	100.01	3
3/11/1999	6436.8	-49.81	1	6/15/1999	7960	-13.58	2	9/20/1999	7972.14	-44.79	1
3/12/1999	6462.73	25.93	2	6/16/1999	8059.02	99.02	3	9/27/1999	7759.93	-212.21	1
3/15/1999	6598.32	135.59	3	6/17/1999	8274.36	215.34	3	9/28/1999	7577.85	-182.08	1
3/16/1999	6672.23	73.91	3	6/21/1999	8413.48	139.12	3	9/29/1999	7615.45	37.6	2
3/17/1999	6757.07	84.84	3	6/22/1999	8608.91	195.43	3	9/30/1999	7598.79	-16.66	2
3/18/1999	6895.01	137.94	3	6/23/1999	8492.32	-116.59	1	10/1/1999	7694.99	96.2	3
3/19/1999	6997.29	102.28	3	6/24/1999	8589.31	96.99	3	10/2/1999	7659.55	-35.44	2
3/20/1999	6993.38	-3.91	2	6/25/1999	8265.96	-323.35	1	10/4/1999	7685.48	25.93	2
3/22/1999	7043.23	49.85	3	6/28/1999	8281.45	15.49	2	10/5/1999	7557.01	-128.47	1
3/23/1999	6945.48	-97.75	1	6/29/1999	8514.27	232.82	3	10/6/1999	7501.63	-55.38	1
3/24/1999	6889.42	-56.06	1	6/30/1999	8467.37	-46.9	1	10/7/1999	7612	110.37	3
3/25/1999	6941.38	51.96	3	7/2/1999	8572.09	104.72	3	10/8/1999	7552.98	-59.02	1
3/26/1999	7033.25	91.87	3	7/3/1999	8563.55	-8.54	2	10/11/1999	7607.11	54.13	3
3/29/1999	6901.68	-131.57	1	7/5/1999	8593.35	29.8	2	10/12/1999	7835.37	228.26	3
3/30/1999	6898.66	-3.02	2	7/6/1999	8454.49	-138.86	1	10/13/1999	7836.94	1.57	2
3/31/1999	6881.72	-16.94	2	7/7/1999	8470.07	15.58	2	10/14/1999	7879.91	42.97	3
4/1/1999	7018.68	136.96	3	7/8/1999	8592.43	122.36	3	10/15/1999	7819.09	-60.82	1
4/2/1999	7232.51	213.83	3	7/9/1999	8550.27	-42.16	2	10/16/1999	7829.39	10.3	2
4/3/1999	7182.2	-50.31	1	7/12/1999	8463.9	-86.37	1	10/18/1999	7745.26	-84.13	1
4/6/1999	7163.99	-18.21	2	7/13/1999	8204.5	-259.4	1	10/19/1999	7692.96	-52.3	1
4/7/1999	7135.89	-28.1	2	7/14/1999	7888.66	-315.84	1	10/20/1999	7666.64	-26.32	2
4/8/1999	7273.41	137.52	3	7/15/1999	7918.04	29.38	2	10/21/1999	7654.9	-11.74	2
4/9/1999	7265.7	-7.71	2	7/16/1999	7411.58	-506.46	1	10/22/1999	7559.63	-95.27	1
4/12/1999	7242.4	-23.3	2	7/17/1999	7366.23	-45.35	1	10/25/1999	7680.87	121.24	3
4/13/1999	7337.85	95.45	3	7/19/1999	7386.89	20.66	2	10/26/1999	7700.29	19.42	2
4/14/1999	7398.65	60.8	3	7/20/1999	7806.85	419.96	3	10/27/1999	7701.22	0.93	2
4/15/1999	7498.17	99.52	3	7/21/1999	7786.65	-20.2	2	10/28/1999	7681.85	-19.37	2
4/16/1999	7466.82	-31.35	2	7/22/1999	7678.67	-107.98	1	10/29/1999	7706.67	24.82	2
4/17/1999	7581.5	114.68	3	7/23/1999	7724.52	45.85	3	10/30/1999	7854.85	148.18	3

Table A2. The FFLRs and the converted left hand side of NLRs for Historical training data of TAIEX1999.

Date (MM/DD/ YYYY)	FFLR	LHS of NLR	Date (MM/DD /YYYY)	FFLR	LHS of NLR	Date (MM/DD /YYYY)	FFLR	LHS of NLR	Date (MM/DD/ YYYY)	FFLR	LHS of NLR
1999/1/18	2,3,1,1,1,2,2,3,3→2	(0.33,0.33,0.33)	1999/4/3	3,3,2,2,1,3,3,1,1→3	(0.33,0.22,0.44)	1999/6/11	2,3,3,3,3,2,3,3,3→2	(0,0.22,0.78)	1999/8/21	3,2,1,3,3,3,3,2,3→3	(0.11,0.22,0.67)
1999/1/19	1,2,3,1,1,1,2,2,3→1	(0.44,0.33,0.22)	1999/4/6	1,3,3,2,2,1,3,3,1→1	(0.33,0.22,0.44)	1999/6/14	2,2,3,3,3,3,2,3,3→2	(0,0.33,0.67)	1999/8/23	2,3,2,1,3,3,3,3,2→2	(0.11,0.33,0.56)
1999/1/20	2,1,2,3,1,1,1,2,2→2	(0.44,0.44,0.11)	1999/4/7	2,1,3,3,2,2,1,3,3→2	(0.22,0.33,0.44)	1999/6/15	2,2,2,3,3,3,3,2,3→2	(0,0.44,0.56)	1999/8/24	2,2,3,2,1,3,3,3,3→2	(0.11,0.33,0.56)
1999/1/21	2,2,1,2,3,1,1,1,2→2	(0.44,0.44,0.11)	1999/4/8	2,2,1,3,3,2,2,1,3→2	(0.22,0.44,0.33)	1999/6/16	2,2,2,2,3,3,3,3,2→2	(0,0.56,0.44)	1999/8/25	1,2,2,3,2,1,3,3,3→1	(0.22,0.33,0.44)
1999/1/22	2,2,2,1,2,3,1,1,1→2	(0.44,0.44,0.11)	1999/4/9	3,2,2,1,3,3,2,2,1→3	(0.22,0.44,0.33)	1999/6/17	3,2,2,2,2,3,3,3,3→3	(0,0.44,0.56)	1999/8/26	3,1,2,2,3,2,1,3,3→3	(0.22,0.33,0.44)
1999/1/25	1,2,2,2,1,2,3,1,1→1	(0.44,0.44,0.11)	1999/4/12	2,3,2,2,1,3,3,2,2→2	(0.11,0.56,0.33)	1999/6/21	3,3,2,2,2,2,3,3,3→3	(0,0.44,0.56)	1999/8/27	2,3,1,2,2,3,2,1,3→2	(0.22,0.44,0.33)
1999/1/26	1,1,2,2,2,1,2,3,1→1	(0.44,0.44,0.11)	1999/4/13	2,2,3,2,2,1,3,3,2→2	(0.11,0.56,0.33)	1999/6/22	3,3,3,2,2,2,3,3→3	(0,0.44,0.56)	1999/8/30	1,2,3,1,2,2,3,2,1→1	(0.33,0.44,0.22)
1999/1/27	3,1,1,2,2,2,1,2,3→3	(0.33,0.44,0.22)	1999/4/14	3,2,2,3,2,2,1,3,3→3	(0.11,0.44,0.44)	1999/6/23	3,3,3,3,2,2,2,2,3→3	(0,0.44,0.56)	1999/8/31	2,1,2,3,1,2,2,3,2→2	(0.22,0.56,0.22)
1999/1/28	2,3,1,1,2,2,2,1,2→2	(0.33,0.56,0.11)	1999/4/15	3,3,2,2,3,2,2,1,3→3	(0.11,0.44,0.44)	1999/6/24	1,3,3,3,3,2,2,2,2→1	(0.11,0.44,0.44)	1999/9/1	3,2,1,2,3,1,2,2,3→3	(0.22,0.44,0.33)
1999/1/29	1,2,3,1,1,2,2,2,1→1	(0.44,0.44,0.11)	1999/4/16	3,3,3,2,2,3,2,2,1→3	(0.11,0.44,0.44)	1999/6/25	3,1,3,3,3,3,2,2,2→3	(0.11,0.33,0.56)	1999/9/2	3,3,2,1,2,3,1,2,2→3	(0.22,0.44,0.33)
1999/1/30	1,1,2,3,1,1,2,2,2→1	(0.44,0.44,0.11)	1999/4/17	2,3,3,3,2,2,3,2,2→2	(0,0.56,0.44)	1999/6/28	1,3,1,3,3,3,3,2,2→1	(0.22,0.22,0.56)	1999/9/3	1,3,3,2,1,2,3,1,2→1	(0.33,0.33,0.33)
1999/2/1	2,1,1,2,3,1,1,2,2→2	(0.44,0.44,0.11)	1999/4/19	3,2,3,3,3,2,2,3,2→3	(0,0.44,0.56)	1999/6/29	2,1,3,1,3,3,3,3,2→2	(0.22,0.22,0.56)	1999/9/4	1,1,3,3,2,1,2,3,1→1	(0.44,0.22,0.33)
1999/2/2	1,2,1,1,2,3,1,1,2→1	(0.56,0.33,0.11)	1999/4/20	2,3,2,3,3,3,2,2,3→2	(0,0.44,0.56)	1999/6/30	3,2,1,3,1,3,3,3,3→3	(0.22,0.11,0.67)	1999/9/6	2,1,1,3,3,2,1,2,3→2	(0.33,0.33,0.33)
1999/2/3	1,1,2,1,1,2,3,1,1→1	(0.67,0.22,0.11)	1999/4/21	2,2,3,2,3,3,3,2,2→2	(0,0.56,0.44)	1999/7/2	1,3,2,1,3,1,3,3,3→1	(0.33,0.11,0.56)	1999/9/7	3,2,1,1,3,3,2,1,2→3	(0.33,0.33,0.33)
1999/2/4	2,1,1,2,1,1,2,3,1→2	(0.56,0.33,0.11)	1999/4/22	1,2,2,3,2,3,3,3,2→1	(0.11,0.44,0.44)	1999/7/3	3,1,3,2,1,3,1,3,3→3	(0.33,0.11,0.56)	1999/9/8	1,3,2,1,1,3,3,2,1→1	(0.44,0.22,0.33)
1999/2/5	1,2,1,1,2,1,1,2,3→1	(0.56,0.33,0.11)	1999/4/23	2,1,2,2,3,2,3,3,3→2	(0.11,0.44,0.44)	1999/7/5	2,3,1,3,2,1,3,1,3→2	(0.33,0.22,0.44)	1999/9/9	2,1,3,2,1,1,3,3,2→2	(0.33,0.33,0.33)
1999/2/6	2,1,2,1,1,2,1,1,2→2	(0.56,0.44,0)	1999/4/26	3,2,1,2,2,3,2,3,3→3	(0.11,0.44,0.44)	1999/7/6	2,2,3,1,3,2,1,3,1→2	(0.33,0.33,0.33)	1999/9/10	3,2,1,3,2,1,1,3,3→3	(0.33,0.22,0.44)
1999/2/8	3,2,1,2,1,1,2,1,1→3	(0.56,0.33,0.11)	1999/4/27	2,3,2,1,2,2,3,2,3→2	(0.11,0.56,0.33)	1999/7/7	1,2,2,3,1,3,2,1,3→1	(0.33,0.33,0.33)	1999/9/13	3,3,2,1,3,2,1,1,3→3	(0.33,0.22,0.44)
1999/2/9	3,3,2,1,2,1,1,2,1→3	(0.44,0.33,0.22)	1999/4/28	1,2,3,2,1,2,2,3,2→1	(0.22,0.56,0.22)	1999/7/8	2,1,2,2,3,1,3,2,1→2	(0.33,0.44,0.22)	1999/9/14	2,3,3,2,1,3,2,1,1→2	(0.33,0.33,0.33)
1999/2/10	1,3,3,2,1,2,1,1,2→1	(0.44,0.33,0.22)	1999/4/29	1,1,2,3,2,1,2,2,3→1	(0.33,0.44,0.22)	1999/7/9	3,2,1,2,2,3,1,3,2→3	(0.22,0.44,0.33)	1999/9/15	1,2,3,3,2,1,3,2,1→1	(0.33,0.33,0.33)
1999/2/20	3,1,3,3,2,1,2,1,1→3	(0.44,0.22,0.33)	1999/4/30	1,1,1,2,3,2,1,2,2→1	(0.44,0.44,0.11)	1999/7/12	2,3,2,1,2,2,3,1,3→2	(0.22,0.44,0.33)	1999/9/16	1,1,2,3,3,2,1,3,2→1	(0.33,0.33,0.33)
1999/2/22	3,3,1,3,3,2,1,2,1→3	(0.33,0.22,0.44)	1999/5/3	3,1,1,1,2,3,2,1,2→3	(0.44,0.33,0.22)	1999/7/13	1,2,3,2,1,2,2,3,1→1	(0.33,0.44,0.22)	1999/9/17	2,1,1,2,3,3,2,1,3→2	(0.33,0.33,0.33)
1999/2/23	3,3,3,1,3,3,2,1,2→3	(0.22,0.22,0.56)	1999/5/4	2,3,1,1,1,2,3,2,1→2	(0.44,0.33,0.22)	1999/7/14	1,1,2,3,2,1,2,2,3→1	(0.33,0.44,0.22)	1999/9/18	1,2,1,1,2,3,3,2,1→1	(0.44,0.33,0.22)
1999/2/24	1,3,3,3,1,3,3,2,1→1	(0.33,0.11,0.56)	1999/5/5	3,2,3,1,1,1,2,3,2→3	(0.33,0.33,0.33)	1999/7/15	1,1,1,2,3,2,1,2,2→1	(0.44,0.44,0.11)	1999/9/20	3,1,2,1,1,2,3,3,2→3	(0.33,0.33,0.33)
1999/2/25	3,1,3,3,3,1,3,3,2→3	(0.22,0.11,0.67)	1999/5/6	2,3,2,3,1,1,1,2,3→2	(0.33,0.33,0.33)	1999/7/16	2,1,1,1,2,3,2,1,2→2	(0.44,0.44,0.11)	1999/9/27	1,3,1,2,1,1,2,3,3→1	(0.44,0.22,0.33)
1999/2/26	2,3,1,3,3,3,1,3,3→2	(0.22,0.11,0.67)	1999/5/7	2,2,3,2,3,1,1,1,2→2	(0.33,0.44,0.22)	1999/7/17	1,2,1,1,1,2,3,2,1→1	(0.56,0.33,0.11)	1999/9/28	1,1,3,1,2,1,1,2,3→1	(0.56,0.22,0.22)
1999/3/1	3,2,3,1,3,3,3,1,3→3	(0.22,0.11,0.67)	1999/5/10	1,2,2,3,2,3,1,1,1→1	(0.44,0.33,0.22)	1999/7/19	1,1,2,1,1,1,2,3,2→1	(0.56,0.33,0.11)	1999/9/29	1,1,1,3,1,2,1,1,2→1	(0.67,0.22,0.11)
1999/3/2	2,3,2,3,1,3,3,3,1→2	(0.22,0.22,0.56)	1999/5/11	2,1,2,2,3,2,3,1,1→2	(0.33,0.44,0.22)	1999/7/20	2,1,1,2,1,1,1,2,3→2	(0.56,0.33,0.11)	1999/9/30	2,1,1,1,3,1,2,1,1→2	(0.67,0.22,0.11)
1999/3/3	1,2,3,2,3,1,3,3,3→1	(0.22,0.22,0.56)	1999/5/12	2,2,1,2,2,3,2,3,1→2	(0.22,0.56,0.22)	1999/7/21	3,2,1,1,2,1,1,1,2→3	(0.56,0.33,0.11)	1999/10/1	2,2,1,1,1,3,1,2,1→2	(0.56,0.33,0.11)
1999/3/4	3,1,2,3,2,3,1,3,3→3	(0.22,0.22,0.56)	1999/5/13	2,2,2,1,2,2,3,2,3→2	(0.11,0.67,0.22)	1999/7/22	2,3,2,1,2,1,1,1,1→2	(0.56,0.33,0.11)	1999/10/2	3,2,2,1,1,1,3,1,2→3	(0.44,0.33,0.22)
1999/3/5	2,3,1,2,3,2,3,1,3→2	(0.22,0.33,0.44)	1999/5/14	2,2,2,2,1,2,2,3,2→2	(0.11,0.78,0.11)	1999/7/23	1,2,3,2,1,1,2,1,1→1	(0.56,0.33,0.11)	1999/10/4	2,3,2,2,1,1,1,3,1→2	(0.44,0.33,0.22)
1999/3/6	2,2,3,1,2,3,2,3,1→2	(0.22,0.44,0.33)	1999/5/15	3,2,2,2,2,1,2,2,3→3	(0.11,0.67,0.22)	1999/7/26	3,1,2,3,2,1,1,2,1→3	(0.44,0.33,0.22)	1999/10/5	2,2,3,2,2,1,1,1,3→2	(0.33,0.44,0.22)
1999/3/8	2,2,2,3,1,2,3,2,3→2	(0.11,0.56,0.33)	1999/5/17	2,3,2,2,2,2,1,2,2→2	(0.11,0.78,0.11)	1999/7/27	1,3,1,2,3,2,1,1,2→1	(0.44,0.33,0.22)	1999/10/6	1,2,2,3,2,2,1,1,1→1	(0.44,0.44,0.11)
1999/3/9	2,2,2,2,3,1,2,3,2→2	(0.11,0.67,0.22)	1999/5/18	2,2,3,2,2,2,2,1,2→2	(0.11,0.78,0.11)	1999/7/28	1,1,3,1,2,3,2,1,1→1	(0.56,0.22,0.22)	1999/10/7	1,1,2,2,3,2,2,1,1→1	(0.44,0.44,0.11)

1999/3/10	3,2,2,2,2,3,1,2,3→3	(0.11,0.56,0.33)	1999/5/19	2,2,2,3,2,2,2,1→2	(0.11,0.78,0.11)	1999/7/29	3,1,1,3,1,2,3,2,1→3	(0.44,0.22,0.33)	1999/10/8	3,1,1,2,2,3,2,2,1→3	(0.33,0.44,0.22)
1999/3/11	2,3,2,2,2,3,1,2→2	(0.11,0.67,0.22)	1999/5/20	2,2,2,2,3,2,2,2→2	(0,0.89,0.11)	1999/7/30	1,3,1,1,3,1,2,3,2→1	(0.44,0.22,0.33)	1999/10/11	1,3,1,1,2,2,3,2,2→1	(0.33,0.44,0.22)
1999/3/12	1,2,3,2,2,2,3,1→1	(0.22,0.56,0.22)	1999/5/21	2,2,2,2,2,3,2,2→2	(0,0.89,0.11)	1999/7/31	3,1,3,1,1,3,1,2,3→3	(0.44,0.11,0.44)	1999/10/12	3,1,3,1,1,2,2,3,2→3	(0.33,0.33,0.33)
1999/3/15	2,1,2,3,2,2,2,3→2	(0.11,0.67,0.22)	1999/5/24	2,2,2,2,2,3,2,2→2	(0,0.89,0.11)	1999/8/2	1,3,1,3,1,1,3,1,2→1	(0.56,0.11,0.33)	1999/10/13	3,3,1,3,1,1,2,2,3→3	(0.33,0.22,0.44)
1999/3/16	3,2,1,2,3,2,2,2→3	(0.11,0.67,0.22)	1999/5/25	2,2,2,2,2,2,3,2→2	(0,0.89,0.11)	1999/8/3	1,1,3,1,3,1,1,3,1→1	(0.67,0,0.33)	1999/10/14	2,3,3,1,3,1,1,2,2→2	(0.33,0.33,0.33)
1999/3/17	3,3,2,1,2,3,2,2,2→3	(0.11,0.56,0.33)	1999/5/26	1,2,2,2,2,2,2,3→1	(0.11,0.78,0.11)	1999/8/4	2,1,1,3,1,3,1,1,3→2	(0.56,0.11,0.33)	1999/10/15	3,2,3,3,1,3,1,1,2→3	(0.33,0.22,0.44)
1999/3/18	3,3,3,2,1,2,3,2,2→3	(0.11,0.44,0.44)	1999/5/27	2,1,2,2,2,2,2,2→2	(0.11,0.89,0)	1999/8/5	1,2,1,1,3,1,3,1,1→1	(0.67,0.11,0.22)	1999/10/16	1,3,2,3,3,1,3,1,1→1	(0.44,0.11,0.44)
1999/3/19	3,3,3,3,2,1,2,3,2→3	(0.11,0.33,0.56)	1999/5/28	2,2,1,2,2,2,2,2→2	(0.11,0.89,0)	1999/8/6	1,1,2,1,1,3,1,3,1→1	(0.67,0.11,0.22)	1999/10/18	2,1,3,2,3,3,1,3,1→2	(0.33,0.22,0.44)
1999/3/20	3,3,3,3,3,2,1,2,3→3	(0.11,0.22,0.67)	1999/5/29	1,2,2,1,2,2,2,2→1	(0.22,0.78,0)	1999/8/7	1,1,1,2,1,1,3,1,3→1	(0.67,0.11,0.22)	1999/10/19	1,2,1,3,2,3,3,1,3→1	(0.33,0.22,0.44)
1999/3/22	2,3,3,3,3,3,2,1,2→2	(0.11,0.33,0.56)	1999/5/31	2,1,2,2,1,2,2,2→2	(0.22,0.78,0)	1999/8/9	3,1,1,1,2,1,1,3,1→3	(0.67,0.11,0.22)	1999/10/20	1,1,2,1,3,2,3,3,1→1	(0.44,0.22,0.33)
1999/3/23	3,2,3,3,3,3,3,2,1→3	(0.11,0.22,0.67)	1999/6/1	1,2,1,2,2,1,2,2,2→1	(0.33,0.67,0)	1999/8/10	2,3,1,1,1,2,1,1,3→2	(0.56,0.22,0.22)	1999/10/21	2,1,1,2,1,3,2,3,3→2	(0.33,0.33,0.33)
1999/3/24	1,3,2,3,3,3,3,3,2→1	(0.11,0.22,0.67)	1999/6/2	3,1,2,1,2,2,1,2,2→3	(0.33,0.56,0.11)	1999/8/11	3,2,3,1,1,1,2,1,1→3	(0.56,0.22,0.22)	1999/10/22	2,2,1,1,2,1,3,2,3→2	(0.33,0.44,0.22)
1999/3/25	1,1,3,2,3,3,3,3,3→1	(0.22,0.11,0.67)	1999/6/3	3,3,1,2,1,2,2,1,2→3	(0.33,0.44,0.22)	1999/8/12	2,3,2,3,1,1,1,2,1→2	(0.44,0.33,0.22)	1999/10/25	1,2,2,1,1,2,1,3,2→1	(0.44,0.44,0.11)
1999/3/26	3,1,1,3,2,3,3,3,3→3	(0.22,0.11,0.67)	1999/6/4	3,3,3,1,2,1,2,2,1→3	(0.33,0.33,0.33)	1999/8/13	3,2,3,2,3,1,1,1,2→3	(0.33,0.33,0.33)	1999/10/26	3,1,2,2,1,1,2,1,3→3	(0.44,0.33,0.22)
1999/3/29	3,3,1,1,3,2,3,3,3→3	(0.22,0.11,0.67)	1999/6/5	2,3,3,3,1,2,1,2,2→2	(0.22,0.44,0.33)	1999/8/16	3,3,2,3,2,3,1,1,1→3	(0.33,0.22,0.44)	1999/10/27	2,3,1,2,2,1,1,2,1→2	(0.44,0.44,0.11)
1999/3/30	1,3,3,1,1,3,2,3,3→1	(0.33,0.11,0.56)	1999/6/7	3,2,3,3,3,1,2,1,2→3	(0.22,0.33,0.44)	1999/8/17	3,3,3,2,3,2,3,1,1→3	(0.22,0.22,0.56)	1999/10/28	2,2,3,1,2,2,1,1,2→2	(0.33,0.56,0.11)
1999/3/31	2,1,3,3,1,1,3,2,3→2	(0.33,0.22,0.44)	1999/6/8	3,3,2,3,3,3,1,2,1→3	(0.22,0.22,0.56)	1999/8/18	3,3,3,3,2,3,2,3,1→3	(0.11,0.22,0.67)	1999/10/29	2,2,2,3,1,2,2,1,1→2	(0.33,0.56,0.11)
1999/4/1	2,2,1,3,3,1,1,3,2→2	(0.33,0.33,0.33)	1999/6/9	3,3,3,2,3,3,3,1,2→3	(0.11,0.22,0.67)	1999/8/19	1,3,3,3,3,2,3,2,3→1	(0.11,0.22,0.67)	1999/10/30	2,2,2,2,3,1,2,2,1→2	(0.22,0.67,0.11)
1999/4/2	3,2,2,1,3,3,1,1,3→3	(0.33,0.22,0.44)	1999/6/10	3,3,3,3,2,3,3,3,1→3	(0.11,0.11,0.78)	1999/8/20	2,1,3,3,3,3,2,3,2→2	(0.11,0.33,0.56)			

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