Generalization of Functional Dependencies in Total Neutrosophic Relation

Meena Arora[1] , **U.S.Pandey**[2]

[1] **Research Scholar**
Singhania University,
Rajasthan, India

[2] **Delhi University,**
Delhi, India

Abstract

Essentially the data and documents on the Web are heterogeneous; inconsistency is unavoidable in Web mining. Using the presentation and reasoning method of our method, it is easier to capture imperfect information on the Web which will provide more potentially valued-added information. We introduce the concept of total neutrosophic relation with a new type of Functional dependency for the searching techniques using the neutrosophic theory to meet the predicates posed in natural language in order to answer imprecise queries of the lay users. For this neutrosophic set needs to be specified from a technical point of view. To this effect we define the set theoretic operators on an instance of neutrosophic set. It may be claimed that the method could be well incorporated in the existing commercial query languages so that the users of any level of knowledge can get some results to his queries.

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Keywords: *Neutrosophic set, Total Neutrosophic relation, Total Neutrosophic functional dependencies (TNfd), Beta neutrosophic key.*

1. Introduction

In real-life problems, the data associated are often imprecise, or non-deterministic. All real data cannot be precise because of their fuzzy nature. Imprecision can be of many types: non-matching data values, imprecise queries, inconsistent data, misaligned schemas, etc. A good relational database system should be capable of maintaining a good relationship among the data's and generate new relations among the existing data's in the database system.

In most cases of judgments', evaluation is done by human beings (or by n intelligent agent) where there certainly is a limitation of knowledge or intellectual functionaries. It is common feature of any human being. To decide "whether 3+2 = 5 or not", the hesitation is nil. But to judge whether a patient has cancer or not, a doctor, (decision maker), will hesitate because of the fact that a fraction of evaluation may remain indeterministic to him.

Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief system. For example, when we ask the opinion of an expert about certain statement, he or she may say that the possibility that the statement is true is 0.5 and the statement is false is 0.6 and the degree that he or she is not sure is 0.2.

Neutrosophic logic was created by Florentin Smarandache (1995) [6] and is an extension/combination of the fuzzy logic, intuitionistic logic, paraconsistent logic, and the three-valued logics that use an indeterminate value. A logic in which each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, is called *Neutrosophic Logic*. T, I, F are standard or non-standard subsets of the nonstandard interval $]^-0, 1^+[$, where $n_{inf} = \inf T + \inf I + \inf F \geq {}^-0$, and $n_{sup} = \sup T + \sup I + \sup F \leq 3^+$.

An important concept in relational schema design is that of a functional dependency of one set of attributes upon another. Normalization theory is based on the functional notion of functional dependency and moreover it helps in simplifying the structure of tables.

A functional dependency is a property of the semantics or meaning of the attributes. The database designers will use their understanding of the semantics of the attributes, to know how they relate each to one another. Certain FD's can be specified without referring specific relation, but as the property of those attributes. Consequently, a natural question that arises is that if two sets of attributes are not functionally dependent in the classical sense, are they really functionally independent? This problem was studied in ([1], [2], [3], [4], [5], [9]) using fuzzy logic, assuming that some or all data is fuzzy in nature.

Functional dependency plays a key role in establishing and maintaining a relationship among the data's that are functionally related to one another and they are separated from other non-related data's thus providing clear relationship among the set of data present. Normalization is basically used to eliminate data redundancy and provide data integrity. Axioms or rules of inference provide a simpler technique for reasoning about functional dependencies. We can also use other rules to find the logically implied functional dependencies.

The paper is organized as follows: In *section 2*, we present some preliminaries on Neutrosophic theory with neutrosophic logic and sets. In *section 3*, Neutrosophic sets are specified from technical point of view. To this effect, we define the set theoretic operators on an instance of neutrosophic set. The concept of Total Neutrosophic Relation is introduced in *section 4*. In *section 5* we introduce new terminology total neutrosophic functional dependencies with Armstrong's axioms and inference rules. In *section 6*, functional dependency using alpha, beta two parameters for nearness and equality is replaced by a single one to decrease the complexities is introduced. In *section 7*, a walkthrough is done with the help of real time examples. Finally we have concluded the paper in *section 8*.

2. Neutrosophic Sets

Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A.

$T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of$]^-0, 1^+[$. That is

$$T_A: X \rightarrow]^-0, 1^+[\quad (1)$$
$$I_A: X \rightarrow]^-0, 1^+[\quad (2)$$
$$F_A: X \rightarrow]^-0, 1^+[\quad (3)$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ so $^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

2.1 Operations with sets

Let S_1 and S_2 be two (unidimensional) real standard or non-standard subsets, then one defines [6]

Addition of sets:
$S_1 + S_2 = \{x|x=s_1+s_2$, where $s_1 \in S_1$and $s_2 \in S_2\}$,
with $\inf S_1+ S_2 = \inf S_1 + \inf S_2$, $\sup S_1 + S_2 = \sup S_1 + \sup S_2$;

Subtraction of sets:
$S1- S_2 = \{x|x= s_1- s_2$, where $s_1 \in S_1$and $s_2 \in S_2\}$.
For real positive subsets (most of the cases will fall in this range) one gets
$\inf S_1- S_2 = \inf S_1 - \sup S_2$, $\sup S_1- S_2 = \sup S_1 - \inf S_2$;

Multiplication of sets:
$S_1. S_2 = \{x|x= s_1. s_2$, where $s_1 \in S_1$ and $s_2 \in S_2\}$.
For real positive subsets (most of the cases will fall in this range) one gets
$\inf S_1. S_2 = \inf S_1. \inf S_2$, $\sup S_1. S_2 = \sup S_1. \sup S_2$;

Division of a set by a number:
Let $k \in R^*$, then $S_1 \oslash k = \{ x|x= s_1/k$, where $s_1 \in S_1\}$.

For all neutrosophic set operations: if, after calculations, one obtains numbers < 0 or > 1, one replaces them by $^-0$ or 1^+ respectively.

3. Neutrosophic Set Operators

In this section, we will now present the notion neutrosophic set operators as an instance of intutionistic set which can be used in real scientific and engineering applications.

In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors.

Consider parameters such as capability, trustworthiness and price of semantic Web services. These parameters are commonly used to define quality of service of semantic Web services. In this section, we will use the evaluation of quality of service of linguistic network service (LNS)[7] as running example to illustrate every set-theoretic operation on neutrosophic sets.

Example 1 :
Assume that $X = [x_1 ,x_2 , x_3]$. $x_1$is capability, x_2 is trustworthiness and x_3 is price. The values of x_1 ,x_2 and x_3 are in [0,1]. They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". A is a total valued neutrosophic set of X defined by

$A=\langle 0.3,0.4,0.5\rangle/x_1 + \langle 0.5,0.2,0.3\rangle/ x_2+ \langle 0.7,0.2,0.2\rangle / x_3$.

B is a total valued neutrosophic set of X defined by

$B = \langle 0.6,0.1,0.2\rangle/x_1 + \langle 0.3,0.2,0.6\rangle/x_2 + \langle 0.4,0.1,0.5\rangle/x_3$.

Definition 3.1. (Complement)
The complement of a total valued neutrosophic set A is denoted by c(A) and is defined by

$$T_{c(A)}(x) = F_A(x), \quad (4)$$
$$I_{c(A)}(x) = 1 - I_A(x), \quad (5)$$
$$F_{c(A)}(x) = T_A(x), \quad (6)$$

for all x in X.

Example 2
Let A be the total valued neutrosophic set defined in Example 1. Then, $c(A) = \langle 0.5,0.6,0.3\rangle/ x_1+ \langle 0.3,0.8,0.5\rangle/x_2 + \langle 0.2,0.8,0.7\rangle/x_3$.

Definition 3.2 (Union)
The union of two total valued neutrosophic sets A and B is a total valued neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = \max (T_A(x), T_B(x)), \quad (7)$$
$$I_C(x) = \max (I_A(x), I_B(x)), \quad (8)$$
$$F_C(x) = \min(F_A(x),F_B(x)), \quad (9)$$

for all x in X.

Example 3
Let A and B be the total valued neutrosophic sets defined in Example 1. Then,

$A \cup B = \langle 0.6, 0.4,0.2\rangle/x_1 + \langle 0.5, 0.2,0.3\rangle/x_2 + \langle 0.7, 0.2,0.2\rangle/x_3$.

Definition 3.3 (Intersection)
The intersection of two total valued neutrosophic sets A and B is a total valued neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = \min (T_A(x), T_B(x)), \quad (10)$$
$$I_C(x) = \min (I_A(x), I_B(x)), \quad (11)$$
$$F_C(x) = \max (F_A(x),F_B(x)), \quad (12)$$

for all x in X.

Example 4
Let A and B be the total valued neutrosophic sets defined in Example 1.
Then,
$A\cap B=\langle 0.3,0.1,0.5\rangle/x_1+\langle 0.3,0.2,0.6\rangle/x_2+ \langle 0.4,0.1,0.5\rangle/x_3$

Definition 3.4 (Difference)
The difference of two total valued neutrosophic set C, written as C = A-B, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = \min(T_A(x),F_B(x)), \quad (13)$$
$$I_C(x) = \min(I_A(x),1-I_B(x)), \quad (14)$$
$$F_C(x) = \max(F_A(x),T_B(x)), \quad (15)$$

for all x in X.

Example 5
Let A and B be the total valued neutrosophic sets defined in Example 1.
Then
$A-B=\langle 0.2,0.4,0.6\rangle/x_1+\langle 0.5,0.2,0.3\rangle/x_2 + \langle 0.5,0.2,0.4\rangle/x_3$.

Now we will define two operators: truth-favorite (Δ) and falsity-favorite (∇) to remove the indeterminacy in the single valued neutrosophic sets and transform it into intuitionistic fuzzy sets or paraconsistent sets. These two operators are unique on single valued neutrosophic sets.

Example 6 Assume that $X = [x_1 ,x_2 , x_3]$. x_1 is capability, x_2 is trustworthiness and x_3 is price. The values of x_1 ,x_2 and x_3 are in [0,1]. They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". A is a single valued neutrosophic set of X defined by

$A=<0.3,0.4,0.5>/ x_1 +<0.5,0.2,0.3>/ x_2 +<0.7,0.2,0.2> / x_3$.

B is a single valued neutrosophic set of X defined by

$B= <0.6,0.1,0.2>/x_1 + <0.3,0.2,0.6>/x_2 + <0.4,0.1,0.5>/x_3$.

Definition 3.5 (Truth-favorite) The truth-favorite of a single valued neutrosophic set A is a single valued neutrosophic set B, written as B = ΔA, whose truth-membership and falsity-membership functions are related to those of A by

$$T_B(x) = \min (T_A(x)+I_A(x),1) \quad (25)$$
$$I_B(x) = 0, \quad (26)$$
$$F_B(x) = F_A(x), \quad (27)$$

for all x in X.

Example 7 Let A be the single valued neutrosophic set defined in Example 1. Then
$\Delta A = <0.7, 0, 0.5>/x_1 + <0.7, 0, 0.3>/x_2 + <0.9, 0, 0.2>/x_3$.

Definition 3.6 (Falsity-favorite) The falsity-favorite of a single valued neutrosophic set B, written as B = A∇whose truth-membership and falsity-membership functions are related to those of A by

$$T_B(x)=T_A(x), \quad (28)$$
$$I_B(x)=0, \quad (29)$$
$$F_B(x)=\min(F_A(x)+I_A(x),1), \quad (30)$$

for all x in X.

Example 8 Let A be the single valued neutrosophic set defined in Example 1. Then

$\nabla A=<0.3, 0, 0.9>/x_1 + <0.5, 0, 0.5>/x_2 + <0.7, 0, 0.4>/x_3$.

4. Total Neutrosophic Relations

In this section, we generalize fuzzy relations in such a manner that we are now able to assign a measure of belief and a measure of doubt to each tuple. We shall refer to these generalized fuzzy relations as total neutrosophic relations.

A tuple in a neutrosophic relation is assigned a measure $\langle\alpha, \beta\rangle$, $0 \leq \alpha, \beta \leq 1$

Definition 4.1 Belief factor α :The interpretation of this measure is that we believe with confidence α that the tuple is in the relation.

In a neutrosophic relation R , R $(t)^+$ is the belief factor assigned to t by R.

Definition 4.2 Doubt factor**:** The interpretation of this measure is that we doubt with confidence β that the tuple is in the relation.

In a neutrosophic relation R , R $(t)^-$ is the doubt factor assigned to t by R. The belief and doubt confidence factors for a tuple need not add to exactly 1. This allows for incompleteness and inconsistency to be represented. If the belief and doubt factors add up to less than 1, we have incomplete information regarding the tuple's status in the relation and if the belief and doubt factors add up to more than 1, we have inconsistent information regarding the tuple's status in the relation.

Neutrosophic relations bound the grade of membership of a tuple to a subinterval [α , $1-\beta$] for the case $\alpha + \beta \leq 1$.

We now formalize the notion of a neutrosophic relation. Let a relation scheme (or just scheme) Σ be a finite set of attribute names, where for any attribute name A ∈Σ, dom(A) is a non-empty set of distinct values for A. A tuple on Σ is any total map t: $\Sigma \rightarrow \cup_{A\in\Sigma}$ dom (A), such that t(A) ∈ dom(A), for each A ∈Σ.

Let

$\tau(\sum)$ denotes the set of all tuples on any scheme$\sum$.
$T(\sum)$ be the set of all total neutrosophic relations on $\sum$
$C(\sum)$ be the set of all consistent neutrosophic relations on Σ.
$\mathcal{V}(\Sigma)$ be the set of all neutrosophic relations on$\sum$.

Definition 4.3 A neutrosophic relation R on scheme $\sum$ is any subset of $\tau(\sum) \times [0,1] \times [0,1]$.

For any t ∈ $\tau(\sum)$, we shall denote an element of R as $\langle t, R(t)^+, R(t)^-\rangle$ where $R(t)^+$ is the belief factor assigned to t by R and $R(t)^-$ is the doubt factor assigned to t by R. Let $\mathcal{V}(\Sigma)$ be the set of all neutrosophic relations on $\sum$.

Definition 4.4. A neutrosophic relation R on scheme $\sum$ is consistent if $R(t)^+ + R(t)^- \leq 1$, for all t ∈ $\tau(\Sigma)$.

Let $C(\Sigma)$ be the set of all consistent neutrosophic relations on Σ . R is said to be complete if $R(t)^+ + R(t)^- \geq 1$, for all t $\in \tau(\Sigma)$. If R is both consistent and complete, i.e. $R(t)^+ + R(t)^- = 1$, for all t $\in \tau(\Sigma)$, then it is a **total neutrosophic relation**, and let $T(\Sigma)$ be the set of all total neutrosophic relations on Σ.

For any t $\in \tau(\Sigma)$, we shall denote an element of R as <t, $R(t)^+$, $R(t)^-$>, where $R(t)^+$ is the belief factor assigned to t by R and $R(t)^-$ is the doubt factor assigned to t by R. Note that since contradictory beliefs are possible, so $R(t)^+ + R(t)^-$ could be greater than 1. Furthermore, $R(t)^+ + R(t)^-$ could be less than 1, giving rise to incompleteness.

As an *example*, suppose in the e-shopping environment, there are two items $Item_1$ and $Item_2$, which are evaluated by customers for some categories of quality - Capability, Trustworthiness and Price.

Let the evaluation results are captured by the following total neutrosophic relation EVAL_RESULT on scheme {Item_Name, Quality_Category} as shown in Table 1:

Table 1. Eval Result

ITEM_Name	Quality_Category	Evaluation
$Item_1$	Capability	⟨0.9,0.2⟩
$Item_1$	Trustworthiness	⟨1.0,0.0⟩
$Item_1$	Price	⟨0.1,0.8⟩
$Item_2$	Capability	⟨1.0,1.0⟩
$Item_2$	Price	⟨0.8,0.3⟩

The above relation contains the information that the confidence of $Item_1$ was evaluated "good" for category Capability is 0.9 and the doubt is 0.2. The confidence of $Item_1$ was evaluated "good" for category Trustworthiness is 1.0 and the doubt is 0.0. The confidence of $Item_1$ was evaluated "poor" for category Price is 0.1 and the doubt is 0.8. Also, the confidence of $Item_2$ was evaluated "good" for category Capability is 1.0 and the doubt is 1.0 (similarly, the confidence of $Item_2$ was evaluated "poor" for category Capability is 1.0 and the doubt is 1.0). The confidence of $Item_2$ was evaluated "good" for category Price is 0.8 and the doubt is 0.3. Note that the evaluation results of $Item_2$ for category Trustworthiness is unknown.

The above information contains results of fuzziness, incompleteness and inconsistency. Such information may be due to various reasons, such as evaluation not conducted, or evaluation results not yet available, the evaluation is not reliable, and different evaluation results for the same category producing different results, etc.

5. Functional Dependency for Total Neutrosophic Relation

Finally we introduce the concept of Functional Dependencies for total Neutrosophic relational databases. The need for introducing such dependencies is justified and explained. Then a detailed analysis of the new notion is conducted to verify the so-called Armstrong's axioms of Total Neutrosophic Relations.

We have considered a relational database (Codd's model, [8]) and indicate a new type of functional dependency, called functional dependency for total neutrosophic relation. In the Codd's model of a relational database, the real world of interest is expressed by means of relations. Implementation of this model is in terms of precise data only. The comparison of data of the same data types is done with classical logic. But, in real-life applications, the data associated are often imprecise, Neutrosophic or non-deterministic.

All real data cannot be precise because of their fuzzy or intuitionistic fuzzy nature. Consequently, for comparing such data, classical logic is not appropriate. The most important concept in relational databases is that of the functional dependency of one set of attributes upon another. We have observed that our approach is not a generalization of the type of fuzzy functional dependencies defined in ([1], [2], [3], [5])

Our main motivation is to capture the integrity arising out of neutrosophic constraints, and so we need to define a new type of functional dependency known as total neutrosophic functional dependency .

Let X is a set and R is a total neutrosophic tolerance relation on X. Consider two choice parameters $\alpha, \beta \in [0, 1]$. (Since these two parameters are to be predefined by the database designers and making indeterminacy zero for total neutrosophic relation for one of the instance, let us call them by terminology 'choice parameters')

Definition 5.1 : $(\alpha,\beta)_R$-equality of two elements.
Two elements $x_1, x_2 \in X$, are said to be (α,β)-equal if either $x_1 N_{(\alpha,\beta)R} x_2$
or $\exists$ $y_1, y_2, \ldots, y_{r-1}, y_r \in X$ such that $x_1 N_{(\alpha,\beta)R} y_1$ and $y_1 N_{(\alpha,\beta)R} y_2$ and $y_2 N_{(\alpha,\beta)R} y_3 \ldots\ldots$ and $y_{r-1} N_{(\alpha,\beta)R} y_r$ and $y_r N_{(\alpha,\beta)R} x_2$.
This equality is denoted by the notation $x_1 E_{(\alpha,\beta)} x_2$.

Definition 5.2 $(\alpha,\beta)_R$-equality of $t_1[x]$ and $t_2[x]$
Two tuples $t_1[x]$ and $t_2[x]$ are said to be $(\alpha,\beta)_R$-equal if $t_1[x_i]\ \varepsilon_{(\alpha,\beta)R} t_2[x_i]\ \forall\ i = 1,2,\ldots,n$.

Denote this equality by the notation $t_1[x]\ \varepsilon_{(\alpha,\beta)R}\ t_2[x]$.

5.1 Total Neutrosophic Functional Dependency

Let $X, Y \subset = \{ A1, A2, \ldots An \}$. . Choose two parameters $\alpha, \beta \in [0,1]$ and R as defined earlier. A total neutrosophic functional dependency $X\overrightarrow{(\alpha,\beta)R}\ Y$ is said to exist if, whenever t_1 [X] $\mathbf{E}_{(\alpha,\beta)R}$ t_2 [X] is true it is also the case that t_1 [Y] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [Y] is true.

We say that the set X of attributes if-functional defines the set Y of attributes at (α ,β)-level of choice. In another terminology, the set Y of attributes is if-functionally defined by the set X of attributes at (α ,β)-level of choice. Denote it by the notation $X\overrightarrow{(\alpha,\beta)R}\ Y$ or, simply by the notation $X\overrightarrow{(\alpha,\beta)R}\ Y$ because of the fact that R is already fixed and the choices α ,β may be left varied during analysis. Call it simply an (α ,β)− TN*fd*.

The following propositions are straightforward from the above definition of (α ,β)− TN*fd*.

Proposition 5.1

(i)For any subset X of R and for any $\alpha, \beta \in [0,1]$. Then

$$X\ \overrightarrow{(\alpha,\beta)R}\ X$$

(ii) Suppose that $0 \le \beta_1 \le \beta_2 \le 1$. Then.

$$X\ \overrightarrow{(\alpha,\beta 1)}\ Y \Rightarrow X\ \overrightarrow{(\alpha,\beta 2)}\ Y.$$

(iii) Suppose that $0 \le \alpha_1 \le \alpha_2 \le 1$. Then

$$X\ \overrightarrow{(\alpha 1,\beta)}\ Y \Rightarrow X\ \overrightarrow{(\alpha 2,\beta)}\ Y$$

5.2 Validation of Armstrong's Axioms with TNfds

In this section it is checked up whether Armstrong's axioms are also true with Rank Neutrosophic functional dependencies. Armstrong's axioms are the most important base of the theory of relational databases. The results which are true on crisp environment may not be true on neutrosophic environment. Consequently it is necessary now to study the Armstrong's axioms with Total neutrosophic functional dependencies and to explore the relevant results on the Armstrong's axioms of neutrosophic nature. Let us call them Total Neutrosophic Armstrong's Axioms (TN-Armstrong's-axioms).

Choose any pair of values of the choice parameters α, β in [0,1]. Then the following Propositions are true.

5.2.1 Propositions (TN-Armstrong's Axioms)

(A$_1$): If $Y \subset X$, then $X\ \overrightarrow{(\alpha,\beta)R}\ Y$
TN-Reflexive rule

Proof (A$_1$) :
Since $Y \subset X$, therefore whenever t_1 [X] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [X].is true, the result
t_1 [Y] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [Y] is also then obviously true. Hence

$$X \xrightarrow[(\alpha,\beta)R]{} Y$$

(A$_2$): If $X\ \overrightarrow{(\alpha,\beta)R}\ Y$, *then* $XZ\ \overrightarrow{(\alpha,\beta)R}\ YZ$
TN-Augmentation rule

Proof (A$_2$) :
Suppose that t_1 [X] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [X] ………..…………. (i)
Therefore we have t_1 [Y] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [Y] ……………... (ii)
Now suppose that t_1 [XZ] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [XZ] …………... (iii)
From (i) and (iii) we get t_1 [Z] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [Z] …………... (iv)
From (ii) and (iv) we get t_1 [YZ] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [YZ] ……….. (v)
From (iii) and (v) we deduce that $XZ\ \overrightarrow{(\alpha,\beta)R}\ YZ$. Hence Proved.

(A$_3$): If $X\ \overrightarrow{(\alpha,\beta)R}\ Y$, and $Y\overrightarrow{(\alpha,\beta)R}\ Z$, then $X\ \overrightarrow{(\alpha,\beta)R}Z$
TN-Transitive rule

Proof (A$_3$) :
Assume that both the TNfds $X\ \overrightarrow{(\alpha,\beta)R}Y$ and $Y\ \overrightarrow{(\alpha,\beta)R}Z$ hold in the relation r. Therefore whenever t_1 [X] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [X] is true, t_1 [Y] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [Y] is also true. But whenever t_1 [Y] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [Y] is true, t_1 [Z] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [Z] is also true. Combining these two we see that whenever t_1 [X] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [X] is true, the result t_1 [Z] $\boldsymbol{\varepsilon}_{(\alpha,\beta)R}$ t_2 [Z] is also true.
Therefore X $\overrightarrow{(\alpha,\beta)R}$ *Z*. Hence Proved.

Now inference rules for total neutrosophic relation can be derived from above armstrongs axioms as follows:

5.1.2 Propositions (TN-Inference Rules)

(A$_4$) If $X\ \overrightarrow{(\alpha,\beta)R}\ YZ$, then $X\ \overrightarrow{(\alpha,\beta)R}\ Y$
(T*N-Decomposition rule)*

Proof :
Given that $X\ \overrightarrow{(\alpha,\beta)R}\ YZ$ ……………………………… (i)
Now $Y \subset YZ$. Therefore by (A$_1$), we have
$YZ\overrightarrow{(\alpha,\beta)R}\ Y$ ………………………….. (ii)
Applying (A$_3$) on (i) and (ii) we get
$X\ \overrightarrow{(\alpha,\beta)R}\ Y$
Hence Proved.

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(A_5) If $X\ \overrightarrow{(\alpha,\beta)R}\ Y$ and $X\ \overrightarrow{(\alpha,\beta)R}\ Z$, then $X\ \overrightarrow{(\alpha,\beta)R}\ YZ$.
(TN-Union rule)

Proof:
Given that $X\ \overrightarrow{(\alpha,\beta)R}\ Y$ ……………………………….. (i)
and $X\ \overrightarrow{(\alpha,\beta)R}\ Z$ …………………………………… (ii)
From (i) we can write $X\ \overrightarrow{(\alpha,\beta)R}\ XY$ …………………(iii)
From (ii) we can write $XY\ \overrightarrow{(\alpha,\beta)R}\ YZ$ ……………….. (iv)
From (iii) and (iv) we get $X\ \overrightarrow{(\alpha,\beta)R}\ YZ$
Hence Proved.

The following propositions can be proved similarly.

(A_6) If $X\ \overrightarrow{(\alpha,\beta)R}\ Y$ and $WY\ \overrightarrow{(\alpha,\beta)R}\ Z$, then
WX $\overrightarrow{(\alpha,\beta)R}$ Z.
(TN-pseudotransitive rule)

Till here we have mentioned the total neutrosophic integrity constraints in relational databases with the notion of total neutrosophic functional dependency (TNfd). In the logical design of a relational database, integrity constraints play a vital role.

6. Generalization of Functional Dependencies in Total Neutrosophic Relations

Since FDs are user/context-dependent, it is desirable that the extended forms are as simple and understandable as possible to users and domain experts. That is, using a two-parameter measure (α, β) for nearness and equality needs to be replaced by a single one, because more parameters increase the computational complexities, involve more thresholds, and treatments. In such a case, we could choose both α, β to be identical, replacing α by β (as a special case).

The following fd can be read as "*X* functionally determines *Y* at β -level of choice" or "*Y* functionally depends on *X* at β -level of choice" and is called as β -fd. Clearly, by definition of β –fd, it follows that for any subset *X* of *R* and for any β ∈ [0, 1], $X \overrightarrow{\beta} X$.
X functionally determines at β – level of choice and vice versa.
For any β ∈ [0,1],
$X \overrightarrow{\beta} X$
If $0 \le \beta 2 \le \beta 1 \le 1$ then $X \overrightarrow{\beta 1} Y \Rightarrow X \overrightarrow{\beta 2} Y$
The fd $X \overrightarrow{1} Y \Rightarrow$ Classical fd $X \rightarrow Y$
$X \overrightarrow{0} Y \Rightarrow$ X does not functionally determines Y

β-reflexive rule
If Y ⊆ X then $X\ \overrightarrow{(\beta)R}\ Y$

β-augmentation rule
If $X\ \overrightarrow{(\beta)R}\ Y$ then $XZ\ \overrightarrow{(\beta)R}\ YZ$

β -transitive rule
If $X\ \overrightarrow{(\beta)R}\ Y$ and $Y\ \overrightarrow{(\beta)R}\ Z$ then $X\ \overrightarrow{(\beta)R}\ Z$

Modified β- transitive rule
If $X \overrightarrow{\beta 1} Y$ and $Y \overrightarrow{\beta 2} Z$ then $X \overrightarrow{\min(\beta 1, \beta 2)} Z$

Modified β -union Rule
If $X \overrightarrow{\beta 1} Y$ and $X \overrightarrow{\beta 2} Z$ then $X \overrightarrow{\min(\beta 1, \beta 2)} YZ$

Modified Pseudotransitive Rule
If $X \overrightarrow{\beta 1} Y$ and $WY \overrightarrow{\beta 2} Z$ then $WX \overrightarrow{\min(\beta 1, \beta 2)} Z$

Modified β -decomposition Rule
If $X \overrightarrow{\beta} YZ$, then $X \overrightarrow{\beta} Y$ and $X \overrightarrow{\beta} Z$

6.1 Beta Key
It is well known that in classical relational database key is a special case of functional dependency. Let us now extend the idea of classical key in the neutrosophic environment to define key with β -level of choice where β∈ [0, 1], is a choice parameter defined by the database designer. A formal definition of neutrosophic key is as follows:

Definition 6.1.1 (Beta Neutrosophic Key)

Let K ⊆ R and F be a set of fds for R. Then, K is called a key of R at β *-level of choice where* β∈ [0, 1], *if* $K \overrightarrow{\beta} R \in F$ *and* $K \overrightarrow{\beta} R$ *is not a partial FD.*

It is widely recognized that many attributes in real life applications are neutrosophic in nature and thus study of neutrosophic functional dependency in neutrosophic relational databases play a vital rule. Above here is the new notion of β–fd, which is based on an equivalence relation, with certain modifications.

7. Evaluation with a walkthrough example

Example 9: Let us assume a relation schema R = (A,B,C,D) and a set of fds

$F = \{ A \underset{0.75}{\longrightarrow} B,\ A \underset{0.8}{\rightarrow} C,\ A \underset{0.7}{\rightarrow} D\}$ of R. Find beta neutrosophic key of R.

Solution :

$A \underset{0.75}{\longrightarrow} B$ ………………………………………….. (i)

$A \underset{0.8}{\rightarrow} C$ ………………………………………….. (ii)

$A \underset{0.7}{\rightarrow} D$ ………………………………………….. (iii)

Applying β -Union rule on (i) and (ii), we get

$A \underset{0.75}{\longrightarrow} BC$ …………………………………….. (iv)

Again applying β -Union rule on (iii) and (iv), we get

$A \underset{0.7}{\rightarrow} BCD$ ……………………………………… (v)

Also we know

$A \underset{1}{\rightarrow} A$ …………………………………………… (vi)

From (v) and (vi) using β union rule, we get

$A \underset{0.7}{\rightarrow} ABCD$

i.e $A \underset{0.7}{\rightarrow} R$

Therefore A is a beta neutrosophic key of relation R at 0.7 level of choice.

Example 10 : Consider a EMPLOYEE relation schema as EMPLOYEE (Name, Address, Age, Expereience, Salary) and set of functional dependencies and β neutrosophic functional dependencies on EMPLOYEE are:

F = {Name → Address,

Name → Age,

$Age \underset{0.8}{\rightarrow} Experience$,

$Experience \underset{0.7}{\rightarrow} Salary\}$

i) To find a key of the relation EMPLOYEE.

Solution :

Given that

Name → Address ⇒ $Name \underset{1}{\rightarrow} Address$ ………… (i)

Name → Age ⇒ $Name \underset{1}{\rightarrow} Age$ ……………….. (ii)

$Age \underset{0.8}{\rightarrow} Experience$ ……………………………… (iii)

$Experience \underset{0.7}{\rightarrow} Salary$ …………………………….. (iv)

From (ii) and (iii) using β-transitive rule, we get

$Name \underset{0.8}{\rightarrow} Experience$ …………………………… (v)

From (v) and (vi) using β-transitive rule, we get

$Name \underset{0.7}{\rightarrow} Salary$ ……………………………….. (vi)

Now applying β -Union rule on (i), (ii), (v), (vi), we get

$Name \underset{0.7}{\rightarrow}$ Address Age Experience Salary ……… (vii)

And we know

$Name \underset{1}{\rightarrow} Name$ ………………………………… (viii)

From (vii) and (viii) using β -Union rule, we get

$Name \underset{0.7}{\rightarrow}$ Address Age Experience Salary

i.e $Name \underset{0.7}{\rightarrow} EMPLOYEE$

Hence, Name is a beta neutrosophic key of relation EMPLOYEE at the 0.7 level of choice.

8.Conclusions

It is widely recognized that many attributes in real life applications are neutrosophic in nature (i.e. where there is a possibility of belief, confidence and indeterminacy) and thus study of integrity constraints plays a vital role in neutrosophic databases. Among the integrity constraints, data dependencies constitute an important class. For example, a neutrosophic integrity constraint like, "salary of two lecturers of almost same age will be approximately equal" is certainly a constraint to a database designer which he cannot accommodate in his design with the help of crisp type of constraint.

Since FDs are user/context-dependent, it is desirable that the extended forms are as simple and understandable as possible to users and domain experts. So that is why a two parameter measure (α, β) for nearness and equality is replaced by a single one (β) as a special case. When neutrosophic data is processed, its indeterminacies are processed as well and the consequent results are more meaningful. This will help decision makers to compile useful information from a combination of raw data, documents or business models to identify and solve problems and make decisions. This is a complete new Method of functional dependencies in Total neutrosophic relations.

As future work, we want to extend this paper to study lossless join decomposition, dependency preservation and multivalued dependency, which constitute an important part of a good database design. We will create inference system based on neutrosophic set reduction and apply the theory to solve practical applications in areas such as expert system, information fusion system, question-answering system, bioinformatics and medical informatics, etc.

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Mrs. Meena Arora is B.Tech, M.Tech holder and is currently Pursuing Ph.D. She is currently working as a Assistant Professor in Computrer Engg. Department at JSS Academy of technical education, MTU University, Noida, U.P, India. She has 14 years of experience in academic field. She has published a book on Database Management Systems and has more than fourteen (14) years of experience. She has been adjudged as the best faculty and was awarded with **academic excellence award** during her stay in Dronacharya College Of Engineering, Gurgaon. She has participated in International and National Conferences. She also attended the Conference held in IIT, Delhi with **BILL GATES** regarding launch of Microsoft's "DreamSparks". She has published and presented nearly 10 research papers in International journals and Conferences.

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Dr U S Pandey, Ph.D (Computer Science) is an Associate Professor in School of Open Learning, University of Delhi, Delhi. He has Twenty years of teaching experience in the field of Computer Science and supervising many research scholars in the field of Mobile Commerce, Artificial Intelligence, Expert System, Data Warehouse, Data Mining, e-learning and e-governance. Dr. Pandey has got published two books and diverse a range of research papers to his credit in the 'National and International Journals' of repute. Dr. Pandey has worked as Professor & Director of VIPS affiliated to GGSIP University Delhi.