

Generalized Alpha Closed sets in Neutrosophic topological spaces

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ABSTRACT

This paper committed to the investigation of Neutrosophic topological spaces. In this paper Generalized Alpha Closed Sets and Generalized Alpha Open Sets are presented. Some of its properties are contemplated.

Keywords: Neutrosophic Set, Generalized Neutrosophic Set, Neutrosophic Topology

Introduction and Preliminaries

The idea of neutrosophic sets was first presented by Smarandache [7, 8] as a generalization of intuitionistic fuzzy sets where we have the degree of membership, the degree of indeterminacy and the degree of non – membership of each component in X.

Definition 1 [7]

A Neutrosophic set A on the universe of discourse X is defined as

$$A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$$

Where $\mu_A, \sigma_A, \gamma_A: X \rightarrow]-0, 1+ [$ [and $-0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3+$ From a philosophical point of view, the Neutrosophic set takes the value from real standard or non-standard subsets of $] -0, 1+ [$. But in real life application in science and engineering problems it is difficult to use Neutrosophic set with values from real standard or non-standard subset of $] -0, 1+ [$. Hence we consider the Neutrosophic set which takes the value from the subsets of $[0,1]$. Set of all Neutrosophic set over X is denoted by $\mathcal{N}(X)$.

Definition 2 [12]

Let $A, B \in \mathcal{N}(X)$, Then

i. (Inclusion) If $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \geq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$, then A is Neutrosophic subset of B and denoted by $A \sqsubseteq B$.

ii. (Equality) If $A \sqsubseteq B$ and $B \sqsubseteq A$, then $A=B$.

iii. (Intersection) Neutrosophic intersection of A and B, denoted by $A \sqcap B$. And defined by

$$A \sqcap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle : x \in X \}.$$

iv. (Union) Neutrosophic union of A and B, denoted by $A \sqcup B$. and defined by

$$A \sqcup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle : x \in X \}.$$

v. (Complement) Neutrosophic complement of A denoted by A^c and defined by

$$A^c = \{ \langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$$

vi. (Universal Set) If $\mu_A(x) = 1$, $\sigma_A(x) = 0$ and $\gamma_A(x) = 0$ for all $x \in X$, A is said to be Neutrosophic universal set, denoted by \tilde{X} .

vii. (Empty Set) If $\mu_A(x) = 0$, $\sigma_A(x) = 1$ and $\gamma_A(x) = 1$ for all $x \in X$, A is said to be Neutrosophic empty set, denoted by $\tilde{\emptyset}$.

Definition 3 [12]

Let $\tau \subseteq \mathcal{N}(X)$, then τ is called a Neutrosophic topology on X if

- i. \tilde{X} and $\tilde{\emptyset}$ belongs to τ .
- ii. The union of any number of Neutrosophic sets in τ belongs to τ .
- iii. The intersection of any number of Neutrosophic sets in τ belongs to τ .

The pair (X, τ) is called a Neutrosophic topological space over X . Moreover the members of τ are said to be Neutrosophic open sets in X . If $A^c \in \tau$ then $A \in \mathcal{N}(X)$ is said to be Neutrosophic closed set in X .

Definition 4 [12]

Let (X, τ) be a neutrosophic topological space over X and $A \in \mathcal{N}(X)$. Then, the neutrosophic interior of A , denoted by $\text{int}(A)$ is the union of all neutrosophic open subsets of A , Clearly $\text{int}(A)$ is the biggest neutrosophic open sets over X which containing A .

Definition 5 [12]

Let (X, τ) be a neutrosophic topological space over X and $A, B \in \mathcal{N}(X)$. Then,

- i. $\text{int}(\tilde{\emptyset}) = \tilde{\emptyset}$ and $\text{int}(\tilde{X}) = \tilde{X}$.
- ii. $\text{int}(A) \sqsubseteq A$.
- iii. A is neutrosophic open set if and only if $A = \text{int}(A)$
- iv. $\text{int}(\text{int}(A)) = \text{int}(A)$
- v. $A \sqsubseteq B$ implies $\text{int}(A) \sqsubseteq \text{int}(B)$.
- vi. $\text{int}(A) \sqcup \text{int}(B) \sqsubseteq \text{int}(A \sqcup B)$
- vii. $\text{int}(A \sqcap B) = \text{int}(A) \sqcap \text{int}(B)$

Definition 6 [12]

Let (X, τ) be a neutrosophic topological space over X and $A \in \mathcal{N}(X)$. Then, the neutrosophic closure of A , denoted by $\text{cl}(A)$ is the intersection of all neutrosophic closed super sets of A , Clearly $\text{cl}(A)$ is the smallest neutrosophic closed sets over X which contains A

Definition 7 [12]

Let (X, τ) be a neutrosophic topological space over X and $A, B \in \mathcal{N}(X)$. Then,

- i. $\text{cl}(\tilde{\emptyset}) = \tilde{\emptyset}$ and $\text{cl}(\tilde{X}) = \tilde{X}$.
- ii. $A \sqsubseteq \text{cl}(A)$.
- iii. A is neutrosophic closed set if and only if $A = \text{cl}(A)$
- iv. $\text{cl}(\text{cl}(A)) = \text{cl}(A)$
- v. $A \sqsubseteq B$ implies $\text{cl}(A) \sqsubseteq \text{cl}(B)$.
- vi. $\text{cl}(A \sqcup B) = \text{cl}(A) \sqcup \text{cl}(B)$
- vii. $\text{cl}(A \sqcap B) \sqsubseteq \text{cl}(A) \sqcap \text{cl}(B)$

Definition 8 [12]

Let (X, τ) be neutrosophic topological space over X and $A, B \in \mathcal{N}(X)$. Then

- i. $\text{int}(A^c) = (\text{cl}(A))^c$
- ii. $\text{cl}(A^c) = (\text{int}(A))^c$

Definition 9 [1]

A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic semi open set (briefly NSOS) if $A \subseteq \text{Ncl}(\text{Nint}(A))$

Definition 10 [1]

A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic semi closed set (briefly NSCS) if $\text{Nint}(\text{Ncl}(A)) \subseteq A$

Definition 11 [1]

A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic α -open set (briefly $N\alpha$ OS) if $A \subseteq \text{Nint}(\text{Ncl}(\text{Nint}(A)))$.

Definition 12 [1]

A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic α -closed set (briefly $N\alpha$ CS) if $\text{Ncl}(\text{Nint}(\text{Ncl}(A))) \subseteq A$.

Definition 13 [1]

A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic pre open (briefly NPOS) if $A \subseteq \text{Nint}(\text{Ncl}(A))$.

Definition 14 [1]

A neutrosophic set A in a topological space (X, τ) is called neutrosophic pre closed (briefly NPCS) if $\text{Ncl}(\text{Nint}(A)) \subseteq A$.

Definition 15 [1]

A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic regular open (briefly NROS) if $A = \text{Nint}(\text{Ncl}(A))$.

Definition 16 [1]

A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic regular closed (briefly NRCS) if $A = \text{Ncl}(\text{Nint}(A))$.

Definition 17 [1]

A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic semi pre open or β -open (briefly $N\beta$ OS) if $A \subseteq \text{Ncl}(\text{Nint}(\text{Ncl}(A)))$

Definition 18 [11]

A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic ω closed (N_ω -closed set for short) if $\text{Ncl}(A) \subseteq G$ whenever $A \subseteq G$ and G is N_ω -closed.

Definition 19 [11]

A neutrosophic set A in X is called N_ω -open in X if A^c is an N_ω -closed in X .

1. Generalized Alpha Closed Sets in Neutrosophic

Definition 1.1.

Neutrosophic set A in (X, τ) is said to be Neutrosophic generalized alpha closed sets ($NG\alpha$ CS in short) if $\text{Ntr}_{\alpha\text{cl}}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\alpha$ OS in (X, τ) .

The family of all $NG\alpha$ CS of a Neutrosophic topological space (X, τ) is denoted by $NG\alpha\text{CS}(X)$.

Note 1.1

In this paper we denote neutrosophic closure as Ntr_{cl} neutrosophic interior as Ntr_{int} and neutrosophic alpha closure as $\text{Ntr}_{\alpha\text{cl}}$.

Example 1.2:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Take $G = \{\langle x, (0.4, 0.6, 0.7), (0.8, 0.7, 0.8) \rangle\}$. Here the only α -open set are \emptyset, X, G then the neutrosophic set $A = \{\langle x, (0.6, 0.6, 0.7), (0.8, 0.7, 0.8) \rangle\}$ is $NG\alpha CS$ in (X, τ) .

Theorem 1.3: Every Neutrosophic Closed Set (NCS in short) in (X, τ) is $NG\alpha CS$, but not conversely.

Proof:

Let A be any set which contained in U and U is $N\alpha OS$ in (X, τ) . Since $Ntr_{\alpha cl}(A) \subseteq Ntr_{cl}(A)$ and A is NCS, $Ntr_{\alpha cl}(A) \subseteq Ntr_{cl}(A) = A \subseteq U$. Therefore A is an $NG\alpha CS$ in X .

Example 1.4:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology and (X, τ) on X . Take $G = \{\langle x, (0.3, 0.5, 0.3), (0.3, 0.5, 0.4) \rangle\}$. Let $A = \{\langle x, (0.6, 0.5, 0.3), (0.5, 0.5, 0.4) \rangle\}$ be any NS in X , here $Ntr_{\alpha cl}(A) \subseteq X$ whenever $A \subseteq X$ for all $N\alpha OS$ G in X . A is $NG\alpha CS$. But not a NCS in X . since $Ntr_{cl}(A) = G^c \neq A$

Theorem 1.5: Every Neutrosophic α -closed Set ($N\alpha CS$ in short) is $NG\alpha CS$ but not conversely.

Proof:

Let A be any set which contained in U and U is $N\alpha OS$ in (X, τ) . By the hypothesis $Ntr_{\alpha cl}(A) = A$. Hence $Ntr_{\alpha cl}(A) \subseteq U$. Therefore A is $NG\alpha CS$ in X .

Example 1.6:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology and (X, τ) on X . Take $G = \{\langle x, (0.3, 0.5, 0.3), (0.3, 0.5, 0.4) \rangle\}$. Let $A = \{\langle x, (0.6, 0.5, 0.3), (0.5, 0.5, 0.4) \rangle\}$ be any NS in X , here $Ntr_{\alpha cl}(A) \subseteq X$ whenever $A \subseteq X$ for all $N\alpha OS$ G in X . therefore A is $NG\alpha CS$. But not a $NG\alpha CS$ in X . since $Ntr_{cl}(A) = G^c \not\subseteq A$.

Theorem 1.7: Every Neutrosophic Regular Closed Set (NRCS in short) is an $NG\alpha CS$ but not conversely.

Proof:

Let A is a NRCS in (X, τ) . By Definition, $A = Ntr_{cl}(Ntr_{int}(A))$, This implies $Ntr_{cl}(A) = Ntr_{cl}(Ntr_{int}(A))$. Therefore $Ntr_{cl}(A) = A$. That is A is an NCS in X . By theorem 1.3 A is an $NG\alpha CS$ in X .

Example 1.8:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Take $G = \{\langle x, (0.3, 0.5, 0.3), (0.3, 0.5, 0.4) \rangle\}$. Let $A = \{\langle x, (0.8, 0.5, 0.3), (0.5, 0.5, 0.5) \rangle\}$ be any NS in X , here A is a $NG\alpha CS$. But not a NRCS in (X, τ) , since $Ntr_{cl}(Ntr_{int}(A)) = G^c \neq A$

Example 1.9:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology and (X, τ) on X . Take $G = \{\langle x, (0.4, 0.4, 0.4), (0.6, 0.6, 0.6) \rangle\}$. Let $A = \{\langle x, (0.5, 0.5, 0.5), (0.3, 0.7, 0.7) \rangle\}$ be any NS in X , here A is a $NGCS$ in X . consider the $N\alpha OS$ $G_1 = \{\langle x, (0.2, 0.2, 0.3), (0.8, 0.6, 0.7) \rangle\}$. Here $A \subseteq G_1$ but $Ntr_{\alpha cl}(A) \not\subseteq G_1$. Hence A is not a $NG\alpha CS$ in (X, τ) .

Theorem 1.10: Every Neutrosophic Generalized α –closed Set (NG α CS in short) is an N α GCS in X. But converse is not true in general.

Proof:

Let A be any set which contained in U and U is N α OS in (X, τ). Since every open set is α open set we have $Ntr_{\alpha cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an NOS in (X, τ). Hence A is an N α GCS in X.

Example 1.11:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X. Where $G = \{\langle x, (0.2, 0.2, 0.3), (0.8, 0.6, 0.7) \rangle\}$. Let $A = \{\langle x, (0.6, 0.2, 0.4), (0.9, 0.6, 0.8) \rangle\}$ be any NS in X, here A is a N α GCS in X consider the N α OS $G_1 = \{\langle x, (0.3, 0.2, 0.8), (0.7, 0.7, 0.3) \rangle\}$. Here $A \subseteq G_1$ but $Ntr_{\alpha cl}(A) \not\subseteq G_1$. Hence A is not a NG α CS in (X, τ)

Theorem 1.12: Every Neutrosophic Generalized α –closed Set (NG α CS in short) is NSGCS but its converse may not be true.

Proof:

Let A be any set which contained in U and U is an NSOS in (X, τ). By hypothesis $Ntr_{\alpha cl}(A) \subseteq A$, which implies $Ntr_{cl}(Ntr_{int}(Ntr_{cl}(A))) \subseteq U$. That is $Ntr_{int}(Ntr_{cl}(A)) \subseteq U$, which implies $A \cup Ntr_{int}(Ntr_{cl}(A)) \subseteq U$. Then $Ntr_{scl}(A) \subseteq U$, U is NSOS. Therefore A is NSGCS in (X, τ)

Example 1.13:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X. Take $G = \{\langle x, (0.3, 0.4, 0.6), (0.6, 0.6, 0.4) \rangle\}$. Here only α –open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.3, 0.2, 0.5), (0.6, 0.6, 0.8) \rangle\}$ be any NS in X. Then $Ntr_{scl}(A) = X$. Clearly $Ntr_{scl}(A) \subseteq X$ whenever $A \subseteq X$ for all NSOS G in X. A is NSGCS in (X, τ). But not a NG α CS in X since $Ntr_{\alpha cl}(A) = G^c \not\subseteq G$

Theorem 1.14: Every Neutrosophic Generalized α –closed Set (NG α CS in short) is NGSCS but its converse may not be true.

Proof:

Let A be any set which contained in U and U is an N α OS in (X, τ). By hypothesis $Ntr_{\alpha cl}(A) \subseteq U$, which implies $Ntr_{cl}(Ntr_{int}(Ntr_{cl}(A))) \subseteq U$. That is $Ntr_{int}(Ntr_{cl}(A)) \subseteq U$, which implies $A \cup Ntr_{int}(Ntr_{cl}(A)) \subseteq U$. Therefore $Ntr_{scl}(A) \subseteq U$, U is NOS. Therefore A is NGSCS of (X, τ).

Example 1.15:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology (X, τ) on X. where $G = \{\langle x, (0.3, 0.2, 0.3), (0.8, 0.6, 0.7) \rangle\}$. Here only α –open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.3, 0.2, 0.6), (0.8, 0.9, 0.8) \rangle\}$ be any NS in X, here $Ntr_{scl}(A) = X$. Clearly $Ntr_{scl}(A) \subseteq X$ whenever $A \subseteq X$ for all NOS G in X. A is NGSCS in (X, τ). But not a NG α CS since $Ntr_{\alpha cl}(A) = G^c \neq A$

Theorem 1.16: Every Neutrosophic Generalized α –closed Set (NG α CS in short) is Neutrosophic Generalized Pre Closed Set (NGPCS in short) but its converse may not be true.

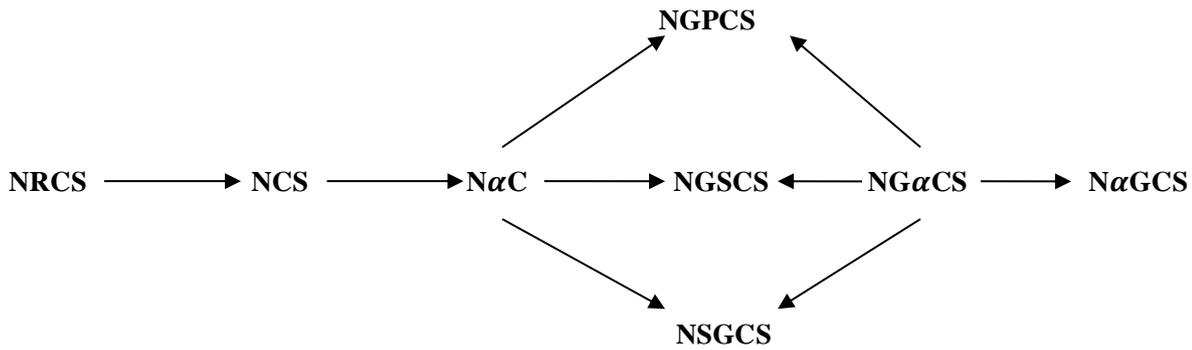
Proof:

Let A be any set which contained in U and U is an N α OS in (X, τ). By hypothesis $Ntr_{\alpha cl}(A) \subseteq U$, which implies $Ntr_{cl}(Ntr_{int}(Ntr_{cl}(A))) \subseteq U$. That is $Ntr_{cl}(Ntr_{int}(A)) \subseteq U$, which implies $A \cup Ntr_{cl}(Ntr_{int}(A)) \subseteq U$. Therefore $Ntr_{pcl}(A) \subseteq U$, U is NOS. Therefore A is NGPCS in (X, τ).

Example 1.17:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X. Where $G = \{\langle x, (0.6, 0.2, 0.8), (0.2, 0.6, 0.1) \rangle\}$. Here only α –open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.3, 0.4, 0.4), (0.4, 0.6, 0.5) \rangle\}$ be any NS in X, here $Ntr_{pcl}(A) \subseteq X$. Therefore A is a NGPCS in (X, τ) but not a NG α CS since $Ntr_{\alpha cl}(A) = G^c \not\subseteq G$

The following diagram implications are true:



Remark 1.18:

A NP closedness is independent of an $NG\alpha$ closedness.

Example 1.19:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.3, 0.4, 0.5), (0.8, 0.6, 0.9) \rangle\}$. Here only α -open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.5, 0.6, 0.4), (0.8, 0.3, 0.2) \rangle\}$ be any $NG\alpha CS(x)$, But not a $NPCS(X)$ since $Ntr_{cl}(Ntr_{int}(A)) = G^c \not\subseteq A$

Example 1.20:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.2, 0.1, 0.3), (0.8, 0.4, 0.7) \rangle\}$. $A = \{\langle x, (0.4, 0.2, 0.4), (0.7, 0.6, 0.8) \rangle\}$ be any $NPCS(x)$, But not a $NG\alpha CS(x)$ since $Ntr_{acl}(A) = G^c \not\subseteq G$

Remark 1.21:

A neutrosophic closed set and neutrosophic generalized α -closed set are independent to each other.

Example 1.22:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.2, 0.2, 0.3), (0.8, 0.6, 0.7) \rangle\}$. Let $A = \{\langle x, (0.5, 0.8, 0.4), (0.5, 0.5, 0.8) \rangle\}$ be any $NG\alpha CS(x)$, here $Ntr_{acl}(A) \subseteq G$ But not a $NSCS(X)$ since $Ntr_{int}(A) = X \not\subseteq A$.

Example 1.23:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.3, 0.2, 0.3), (0.5, 0.6, 0.5) \rangle\}$. Let $A = \{\langle x, (0.1, 0.2, 0.4), (0.3, 0.6, 0.8) \rangle\}$ be any $NSCS(X)$ but not a $NG\alpha CS(x)$ since $Ntr_{acl}(A) = G^c \not\subseteq G$.

Remark 1.24:

The intersection of any two $NG\alpha CS$ is not a $NG\alpha CS$ in general as seen from the following example.

Example 1.25:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.1, 0.2, 0.1), (0.8, 0.1, 0.7) \rangle\}$. Let $A = \{\langle x, (0.6, 0.7, 0.4), (0.7, 0.6, 0.8) \rangle\}$, $B = \{\langle x, (0.3, 0.2, 0.4), (0.4, 0.6, 0.8) \rangle\}$ are $NG\alpha CS$ but $A \cap B$ is not a $NG\alpha CS$ in X .

Theorem 1.26: If A is an NOS and $NG\alpha CS$ in (X, τ) , then A is a $N\alpha CS$ in X .

Proof:

Let A is a NOS in X . Since $A \subseteq A$, by hypothesis $Ntr_{\alpha cl}(A) \subseteq A$. But from the definition $A \subseteq Ntr_{\alpha cl}(A)$. Therefore $Ntr_{\alpha cl}(A) = A$. Hence A is a $N\alpha CS(X)$.

2. Generalized Alpha Open Sets in Neutrosophic

In this section we introduce neutrosophic generalized alpha open set and studied some of its properties.

Definition 2.1:

A neutrosophic set A is said to be neutrosophic generalized alpha open set ($NG\alpha OP$ in short) in (X, τ) if the complement of A^c is a $NG\alpha CS$ in X .

The family of all $NG\alpha OS$ of a NTS (X, τ) is denoted by $NG\alpha O(X)$

Note

In this paper we denote neutrosophic closure as Ntr_{cl} neutrosophic interior as Ntr_{int} and neutrosophic alpha closure as $Ntr_{\alpha cl}$.

Theorem 2.2:

For any NTS (X, τ) , we have the following

- Every NOS is a $NG\alpha OS$
- Every $N\alpha OS$ is a $NG\alpha OS$,
- Every NROS is a $NG\alpha OS$. But the converses are not true in general.

Proof:

Straight forward

Example 2.3:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.2, 0.3, 0.8), (0.7, 0.6, 0.7) \rangle\}$. Here only α -open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.6, 0.7, 0.4), (0.3, 0.4, 0.3) \rangle\}$, be any NS in X . A is an $NG\alpha OS$, but not an NOS in X .

Example 2.4

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.3, 0.2, 0.7), (0.6, 0.6, 0.7) \rangle\}$. Here only α -open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.6, 0.6, 0.4), (0.3, 0.2, 0.2) \rangle\}$, be any NS in X . A is an $NG\alpha OS$, but not an $N\alpha OS$ in X

Example 2.5

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.1, 0.2, 0.3), (0.9, 0.6, 0.7) \rangle\}$. Here only α -open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.7, 0.2, 0.4), (0.8, 0.6, 0.7) \rangle\}$, be any NS in X . A is an $NG\alpha OS$, but not an NROS in X

Theorem 2.6

For any NTS (X, τ) , we have the following:

- Every $NG\alpha OS$ is a $NGSOS$
- Every $NG\alpha OS$ is a $NSGOS$
- Every $NG\alpha OS$ is a $NGPOS$. But the converses are not to be true in general.

Proof:

Straight forward

Example 2.7:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.2, 0.4, 0.3), (0.7, 0.7, 0.7) \rangle\}$. Here only α -open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.5, 0.2, 0.5), (0.4, 0.6, 0.2) \rangle\}$, be any NS in X . A is an NGSOS, but not an $NG\alpha OS$ in X

Example 2.8:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.4, 0.2, 0.3), (0.3, 0.2, 0.1) \rangle\}$. Here only α -open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.3, 0.2, 0.3), (0.3, 0.6, 0.1) \rangle\}$, be any NS in X . A is an NSGOS, but not an $NG\alpha OS$ in X

Example 2.9

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.2, 0.1, 0.3), (0.2, 0.6, 0.7) \rangle\}$. Here only α -open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.6, 0.7, 0.4), (0.3, 0.6, 0.8) \rangle\}$, be any NS in X . A is an NGPOS, but not an $NG\alpha OS$ in X

Remark 2.10

The union of any two any $NG\alpha OS$ is not an $NG\alpha OS$ in general as seen from following example.

Example 2.11:

Let $X = \{a, b\}$ and $\tau = \{\emptyset, X, G\}$ is a neutrosophic topology on X . Where $G = \{\langle x, (0.2, 0.2, 0.3), (0.8, 0.6, 0.7) \rangle\}$. Here only α -open sets are $\{\emptyset, X\}$. Let $A = \{\langle x, (0.5, 0.2, 0.4), (0.6, 0.7, 0.8) \rangle\}$, and $B = \{\langle x, (0.7, 0.2, 0.4), (0.8, 0.6, 0.7) \rangle\}$ are $NG\alpha OS$ but $A \cup B$ is not an $NG\alpha OS$.

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