Group decision making using neutrosophic soft matrix: An algorithmic approach

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A B S T R A C T
This article proposes an algorithmic approach for group decision making (GDM) problems using neutrosophic soft matrix (NSM) and relative weights of experts. NSM is the matrix representation of neutrosophic soft sets (NSSs), where NSS is the combination of neutrosophic set and soft set. We propose a new idea for assigning relative weights to the experts based on cardinalities of NSSs. The relative weight is assigned to each of the experts based on their preferred attributes and opinions, which reduces the chance of unfairness in the decision making process. Firstly we introduce choice matrix and combined choice matrix using neutrosophic sets. Multiplying combined choice matrices with the individual NSMs, this study develops product NSMs, which are aggregated to find out the collective NSM. Then neutrosophic cross-entropy measure is used to rank the alternatives and for selecting the most desirable one (s). This study also provides a comparative analysis of the proposed weight based approach with the normal procedure, where weight is not considered. Finally, a case study illustrates the applicability of the proposed approach.

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1. Introduction

Fuzzy set (Zadeh, 1965a) uses membership degree \(\mu_A(x) \in [0, 1]\) to find the belongingness of an element to a set. When \(\mu_A(x)\) itself becomes uncertain, then it is hard to define by a crisp value for it. This was solved by using interval-valued fuzzy sets (IVFSs) in Turksen (1986). In some real life applications, one has to consider not only the truth membership supported by the evidence but also the falsity membership against the evidence, which is beyond the scope of fuzzy sets and IVFSs. Intuitionistic fuzzy set (IFS) (Atanassov, 1986) was introduced as a generalization of fuzzy sets to consider both truth membership and falsity membership. Later IFS was extended to the interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov, 1989) for generalization purpose. A bibliomet-
Some significant contributions in neutrosophic set. Peng et al. (2014) presented a new outranking approach for multi criteria decision making (MCDM) problems in the context of simplified neutrosophic environment based on ELECTRE method. Combining neutrosophic set with other mathematical models, a number of research works have been published. Maji (2012) introduced neutrosophic soft set (NSS) as a combination of NS and soft set (Molodtsov, 1999) and presented a neutrosophic soft set theoretic approach for a multi-observer object recognition problem. Combining generalized neutrosophic set (Salama, 2012) with soft set (Molodtsov, 1999), Broumi (2013) introduced the concept of generalized interval neutrosophic soft set. Broumi and Smarandache (2013a) developed intuitionistic neutrosophic soft set by combining intuitionistic neutrosophic set and soft set. Deli (2014) developed interval-valued neutrosophic soft set which is a combination of interval-valued neutrosophic set and soft set. Broumi et al. (2014a) defined neutrosophic parameterized soft set and studied some of its properties. They introduced neutrosophic parameterized aggregation operator and applied it in decision making problem. Broumi et al. (2014b), in 2014, extended generalized neutrosophic soft set and proposed the idea of generalized interval neutrosophic soft set. Deli and Broumi (2014) introduced the concept of neutrosophic soft matrix and redefined some operations of neutrosophic soft set given by Maji (2012). Deli et al. (2014) introduced the concept of neutrosophic soft multi-set theory and studied their properties and operations. Rivieccio (2008) presented a critical introduction to neutrosophic logics. The author defined suitable neutrosophic propositional connectives and discussed the relationship between neutrosophic logics and other well-known frameworks. Deli et al. (2015) introduced bipolar neutrosophic set and studied some of its operations. To compare the bipolar neutrosophic sets, they studied score functions and accuracy functions. Deli et al. (2016) proposed interval valued bipolar fuzzy weighted neutrosophic set (JVBFWN-set) as a generalization of fuzzy set, bipolar fuzzy set, neutrosophic set and bipolar neutrosophic set. Liu and Luo (2017) developed a series of power aggregation operators on simplified neutrosophic set (SNS) called simplified neutrosophic number power weighted averaging (SNNPWA) operator, simplified neutrosophic number power weighted geometric (SNNPGW) operator, simplified neutrosophic number power ordered weighted averaging (SNNPOWA) operator and simplified neutrosophic number power ordered weighted geometric (SNNPOWG) operator. Additionally, using the developed aggregation operators, they presented a multi attribute group decision making (MAGDM) approach within the framework of SNSs. In Ye (2015), Ye proposed a neutrosophic number tool for group decision making problems with indeterminate information under a
neutrosophic number environment. Some significant contributions in neutrosophic sets are given below in Table 1.

As per our knowledge, there are no articles published in neutrosophic sets and its hybridizations, where experts’ relative weights have been considered. Case I shows the final outcome without study is related to investment in business sectors, where two cases the combined NCMs are multiplied with the normalized NSMs to information, which reduces the chances of biasness. The proposed approach for GDM using NSMs and relative weights of experts. Initially experts provide their opinions using NSMs, which are normalized by the relative weights of the corresponding experts. The defined experts provide their opinions using NSMs, which are normalized by the relative weights of the corresponding experts. The proposed approach focuses on the parameter choices/attributes of various experts to find out the neutrosophic choice matrix (NCM) and combined NCM for individual decision maker/expert. In the process, the combined NCMS are multiplied with the normalized NSMs to obtain the product NSMs, which are aggregated to produce the resultant NSM. Then cross-entropy measure is applied on the alternatives of the resultant NSM to rank the alternatives. The case study is related to investment in business sectors, where two cases have been considered. Case I shows the final outcome without assigning any weight and Case II shows the result by assigning weights.

The rest of the article is organized as follows. Section 2 provides the basic notions and backgrounds of neutrosophic sets, NSSs, NSMs, and cross-entropy measure of neutrosophic sets. Section 3 proposes NCM, combined NCM, and some operations on NSMs. Proposed algorithmic approach is presented in section 4 followed by case study in Section 5. Then a brief discussion on the results is given in Section 6. Finally, we have concluded in Section 7.

2. Preliminaries

This section briefly describes neutrosophic set (NS), neutrosophic soft set (NSS), neutrosophic soft matrix (NSM), and cross-entropy measure of neutrosophic sets.

2.1. Neutrosophic set, NSS, and NSM

Definition 1 Smarandache (1999). Let U be an universe of discourse. The neutrosophic set A in U is expressed by A = ({x : T_A(x), I_A(x), F_A(x), x ∈ U), where the characteristic functions T, I, F : U → [−0, 1] respectively define the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ U to the set A with the condition: 

\[ 0 ≤ T_A(x) + I_A(x) + F_A(x) ≤ 3 \] 

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of [−0, 1] for convenient application in real life problems, we take the interval [0, 1] instead of [−0, 1]. For a given element x ∈ U, the triplet \((T_A(x), I_A(x), F_A(x))\) is usually called neutrosophic value (NV) or neutrosophic number (NN).

Soft set was introduced by Molodtsov (1999) as a generic mathematical tool for dealing with uncertain problems which cannot be handled using traditional mathematical tools. There are many theories, such as theory of probability, theory of fuzzy sets (Zadeh, 1965a), theory of neutrosophic sets (Smarandache, 1999, 2003), etc., which can be considered to deal with uncertainties. But all these theories have their inherent difficulties due to the inadequacy of the parameterization tool. Soft set is free from such difficulties which can be used for approximate description of objects without any restriction. As a definite outcome, soft set theory has emerged as a convenient and easily applicable tool in practice. Neutrosophic set is combined with soft set to get the advantages of both the neutrosophic set and soft set.

Definition 2 Maji (2012). Let U be a universe of discourse and E be a set of parameters. Let NS (U) denotes the set of all neutrosophic subsets of U and A ⊆ E. A pair \((N_A, E)\) is called a NSS over U, where \(N_A\) is a mapping given by \(N_A : E → NS(U)\).

Example 1. Let \(U = \{c_1, c_2, c_3\} = \{\text{celerio, xcent, eon}\}\) be the set of 3 models of cars, \(E = \{e_1, e_2, e_3, e_4, e_5\} = \{\text{speed, comfort, durability, costly, branded}\}\) be the set of parameters considered for a car, and \(A = \{e_1, e_2, e_3, e_4\} \subseteq E\). Let

\[ N_A(e_1) = \{c_1/(0.6, 0.4, 0.8), c_2/(0.4, 0.6, 0.8)\} \]
\[ N_A(e_2) = \{c_2/(0.4, 0.9, 0.7), c_3/(0.2, 0.3, 0.6)\} \]
\[ N_A(e_3) = \{c_1/(0.5, 0.3, 0.8), c_2/(0.1, 0.6, 0.3), c_3/(0.4, 0.1, 0.7)\} \]
\[ N_A(e_4) = \{c_2/(0.7, 0.1, 0.3), c_3/(0.5, 0.4, 0.7)\} \]

Here \(N_A(e_1) = \{c_1/(0.6, 0.4, 0.8), c_2/(0.4, 0.6, 0.8)\}\) implies the object of objects (cars) \(c_1, c_2, c_3\) (celerio, xcent) with the parameter \(e_1\) (speed). Parameter \(e_2\) is associated with the object \(c_1\) using the degree of membership 0.6, degree of indeterminacy 0.4, and degree of non-membership 0.8. This example shows that parameter \(e_1\) (branded) is not associated with any other cars, i.e., when the cars are being considered, the parameter branded has no significance particularly in this example.

Then the NSS \((N_A, E)\) is given by

\[ (N_A, E) = \left\{ \left( e_1, \{c_1/(0.6, 0.4, 0.8), c_2/(0.4, 0.6, 0.8)\} \right), \left( e_2, \{c_2/(0.4, 0.9, 0.7), c_3/(0.2, 0.3, 0.6)\} \right), \left( e_3, \{c_1/(0.5, 0.3, 0.8), c_2/(0.1, 0.6, 0.3), c_3/(0.4, 0.1, 0.7)\} \right), \left( e_4, \{\emptyset\} \right), \left( e_5, \{c_2/(0.7, 0.1, 0.3), c_3/(0.5, 0.4, 0.7)\} \right) \right\} \]

Definition 3 Deli and Brouni (2014). Let \((N_A, E)\) be a NSS over the initial universe U. Let E be a set of parameters and \(A ⊆ E\). Then a subset of \(U \times E\) is uniquely defined by the relation \((e, x) : e ∈ A, x ∈ N_A(e)\) and denoted by \(R_A = (N_A, E)\). Now the relation \(R_A\) is characterized by the truth function \(T_{A} : U \times E → [0, 1]\), indeterminacy \(I_A : U \times E → [0, 1]\), and the falsity function \(F_A : U \times E → [0, 1]\). Then \(T_{A}(x, e)\) is the truth value, \(I_{A}(x, e)\) is the indeterminacy value, and \(F_{A}(x, e)\) is the falsity value of the object x associated with the parameter e. \(R_A\) is represented as \(R_A = \{(T_{A}(x, e), I_{A}(x, e), F_{A}(x, e)) : 0 ≤ T_{A} + I_{A} + F_{A} ≤ 3, (x, e) ∈ U \times E\}\).
Now if the set of universe $U = \{x_1, x_2, \ldots, x_n\}$ and the set of parameters $E = \{e_1, e_2, \ldots, e_n\}$, then $R_A$ can be represented by a matrix as follows:

$$R_A = (a_{ij})_{m \times n} = \left( \begin{array}{cccc} a_{i1} & a_{i2} & \ldots & a_{in} \\ a_{i2} & a_{i3} & \ldots & \vdots \\ \vdots & \vdots & \ddots & \ddots \\ a_{im} & a_{i2} & \ldots & a_{in} \end{array} \right)$$

where $a_{ij} = (T_A(x_i, e_j), I_A(x_i, e_j), F_A(x_i, e_j))$.

The above matrix is called a neutrosophic soft matrix (NSM) of order $m \times n$ corresponding to the neutrosophic set $(N_A, E)$ over $U$.

**Example 2.** The matrix representation of the NSS, described above in Example 1, is shown in Table 2.

### 2.2. Cross-entropy of neutrosophic sets

Entropy is used to measure the degree of fuzziness or uncertain information in fuzzy set theory. The fuzzy entropy was first introduced by Zadeh (1965b, 1968) to quantify the amount of fuzziness. De Luca and Termini (1972) introduced some axioms to describe the degree of fuzziness of a fuzzy set based on Shannon's function. Shannon (1949) defined an information theory in 1949 which introduced cross-entropy. A measure of fuzzy cross-entropy between fuzzy sets was proposed by Shang and Jiang (1997) which is used to measure the discrimination information between two fuzzy sets. Zhang and Jiang (2008) defined vague cross-entropy measure between vague sets. Ye (2011) extended the idea of fuzzy cross-entropy to interval-valued intuitionistic fuzzy sets. Hesitant fuzzy linguistic entropy and cross-entropy measures were proposed by Gou (2017). Ye also proposed cross-entropy measure in Ye (2014a) for single valued neutrosophic set as an extension of the fuzzy cross-entropy.

Suppose $\bar{A} = (\bar{A}(x_1), \bar{A}(x_2), \ldots, \bar{A}(x_n))$ and $\bar{B} = (\bar{B}(x_1), \bar{B}(x_2), \ldots, \bar{B}(x_n))$ be two fuzzy sets in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. Then the fuzzy cross-entropy between $\bar{A}$ and $\bar{B}$ is defined as

$$H(\bar{A}, \bar{B}) = \sum_{i=1}^{n} \left( \bar{A}(x_i) \log_2 \left( \frac{\bar{A}(x_i)}{\bar{A}(x_i) + \bar{B}(x_i)} \right) + (1 - \bar{A}(x_i)) \log_2 \left( \frac{1 - \bar{A}(x_i)}{1 - \bar{A}(x_i) + \bar{B}(x_i)} \right) \right),$$

(1)

which describes the degree of discrimination of $\bar{A}$ from $\bar{B}$. Since $H(\bar{A}, \bar{B})$ is not symmetric with respect to its arguments, a symmetric discrimination information measure was proposed in Shang and Jiang (1997) as $I(\bar{A}, \bar{B}) = H(\bar{A}, \bar{B}) + H(\bar{B}, \bar{A})$, where $I(\bar{A}, \bar{B}) \geq 0$ and $I(\bar{A}, \bar{B}) = 0$ if and only of $\bar{A} = \bar{B}$. The cross-entropy and symmetric discrimination information measures between two fuzzy sets have been extended to the context of SVNSs. Let $A = \{X_T(x_i), I_A(x_i), F_A(x_i) ; x_i \in X\}$ and $B = \{X_T(x_i), I_B(x_i), F_B(x_i) ; x_i \in X\}$ be two SVNSs and $X = \{x_1, x_2, \ldots, x_n\}$ be the universe of discourse, where $X_T(x_i), I_A(x_i), F_A(x_i), X_T(x_i), I_B(x_i), F_B(x_i) \in [0, 1]$. Now considering truth-membership, indeterminacy-membership, and falsity-membership, the amount of information discrimination of $T_A(x_i)$ from $T_B(x_i)$, $I_A(x_i)$ from $I_B(x_i)$, and $F_A(x_i)$ from $F_B(x_i)$ are respectively computed using (2), (3), and (4) given below.

### Table 2

<table>
<thead>
<tr>
<th>U/E</th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$e_3$</th>
<th>$e_4$</th>
<th>$e_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>(0.6, 0.4, 0.8)</td>
<td>0</td>
<td>(0.5, 0.3, 0.8)</td>
<td>0</td>
<td>(0.7, 0.1, 0.3)</td>
</tr>
<tr>
<td>$c_2$</td>
<td>(0.4, 0.6, 0.8)</td>
<td>(0.4, 0.3, 0.7)</td>
<td>(0.1, 0.5, 0.3)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0</td>
<td>(0.2, 0.3, 0.6)</td>
<td>0</td>
<td>(0.4, 0.3, 0.7)</td>
<td>(0.6, 0.5, 0.7)</td>
</tr>
</tbody>
</table>
Using (5) and (6), Ye (2014a) defined weighted neutrosophic cross-entropy \( D_i(A', A_i) \) between an alternative \( A_i \) and the ideal alternative \( A' \) as given in (7). Here the smaller the value of \( D_i(A', A_i) \), the better will be the alternative \( A_i \) and the alternative \( A_i \) will be close to the ideal alternative \( A' \). The concept of ideal alternative is used in decision making environment for the purpose of identifying the best alternative in the decision set. Normally the ideal alternative does not exist in real world, but it provides theoretical framework for evaluating the alternatives.

The ideal alternative \( A' \) in neutrosophic concept is defined as \( A' = (T', I', F') = (1, 0, 0) \).

3. NCM, combined NCM, and basic operations on NSMs

This section presents NCM, combined NCM, and operations on NSMs, such as addition, complement, and product of NSMs with the combined NCMs.

3.1. NCM and combined NCM

This sub-section defines NCM and combined NCM with necessary examples.

Definition 4. NCM is a square matrix whose rows and columns both indicates parameters. If \( \xi \) is a NCM, then its element \( \xi(i,j) \) is defined as

\[
\xi(i,j) = (0, 0, 1) \text{ when } i \text{th row and } j \text{th column both are not indeterminate.}
\]

Example 3. Let U and E are same as in Example 1. Suppose Mr. X is interested for buying a car based on the attributes

\[
\mathcal{A}_X = \{e_1, e_2, e_3\} \subset E.
\]

Then the NCM for Mr. X can be represented as

\[
\xi_X(i,j) = e_X = \left\{ \begin{array}{l}
(0.0, 0.5, 1), (0.0, 5, 1), (0.0, 5, 1), (0.0, 5, 1) \\
(0.0, 5, 1), (0.0, 5, 1), (0.0, 1), (0.0, 1) \\
(0.0, 5, 1), (0.0, 1), (0.0, 5, 1), (0.0, 1) \\
(0.5, 1), (0.5, 1), (0.5, 1), (0.5, 1) \\
\end{array} \right\}.
\]

Example 4. Suppose Mr. X and Mr. Y want to buy a car as per their combined opinion. Attributes preferred by Mr. X are mentioned in Example 3. Let Mr. Y shows his/her preference in \( \mathcal{A}_Y = \{e_1, e_2\} \subset E \). Combined NCM \( \xi_X^Y \), for Mr. X is given below.

\[
\xi_X^Y(i,j) = e_X = \left\{ \begin{array}{l}
(0.0, 0.5, 1), (0.0, 5, 1), (0.0, 5, 1), (0.0, 5, 1) \\
(0.0, 5, 1), (0.0, 5, 1), (0.0, 1), (0.0, 1) \\
(0.0, 5, 1), (0.0, 1), (0.0, 5, 1), (0.0, 1) \\
(0.5, 1), (0.5, 1), (0.5, 1), (0.5, 1) \\
\end{array} \right\}.
\]

3.2. Addition of NSMs

Two NSMs, \((N, A)_m \times n = [a_{ij}]_{m \times n}\) and \((N, B)_m \times n = [b_{ij}]_{m \times n}\) are said to be conformable for addition, if they have the same order. The addition of two NSMs matrices \([a_{ij}]_{m \times n}\) and \([b_{ij}]_{m \times n}\) is defined by \([c_{ij}]_{m \times n} = [a_{ij}]_{m \times n} \oplus [b_{ij}]_{m \times n}\) where \([c_{ij}]_{m \times n}\) is also the NSM of order \(m \times n\) and \([c_j] = (\max(T_{ij}, T_{kj}), \arg(T_{ij}, T_{kj}), \min(F_{ij}, F_{kj})) \forall i, j\).

Example 5. Suppose NSMs \((a_{ij})_{m \times n}\) and \((b_{ij})_{m \times n}\) are given below.

\[
(a_{ij})_{m \times n} = \left\{ \begin{array}{l}
(0.6, 0.5, 0.3), (0.0, 5, 1), (0.0, 1), (0.0, 1), (0.0, 5, 1) \\
(0.8, 0.9, 0.3), (0.0, 6, 1), (0.8, 6, 0.3), (0.0, 2, 1), (0.1, 7, 1) \\
(0.8, 0.5, 0.4), (0.0, 5, 1), (0.0, 5, 4), (0.0, 5, 1), (0.2, 1, 2) \\
\end{array} \right\}.
\]

\[
(b_{ij})_{m \times n} = \left\{ \begin{array}{l}
(0.4, 0.5, 0.7), (1.0, 5, 1), (0.4, 0.2, 0.7), (0.1, 2, 0), (0.1, 0, 5) \\
(0.0, 2, 1, 0), (1.0, 4, 0), (0.2, 0, 4, 0), (1.0, 8, 0), (1.0, 3, 0) \\
(0.2, 0, 5, 0, 6), (1.0, 5, 0), (0.2, 0, 5, 0, 6), (1.0, 8, 0), (1.0, 8, 0) \\
\end{array} \right\}.
\]

Then the addition of these two NSMs gives \((c_j)_{m \times n}\) where

\[
(c_{ij})_{m \times n} = (a_{ij})_{m \times n} \oplus (b_{ij})_{m \times n} = \left\{ \begin{array}{l}
(0.6, 0.5, 0.3), (1.0, 5, 1), (0.0, 5, 0.3), (1.0, 5, 0), (1.0, 5) \\
(0.8, 0.5, 0.3), (1.0, 5, 0), (0.8, 0.5, 0.3), (1.0, 5, 0), (1.0, 5) \\
(0.8, 0.5, 0.4), (1.0, 5, 0), (0.8, 0.5, 0.4), (1.0, 5, 0), (1.0, 5) \\
\end{array} \right\}.
\]

3.3. Product of NSM and combined NCM

The number of columns of NSM \((N, A)\) is equal to the number of rows of the combined NSM \(\xi^X\), then \((N, A)\) and \(\xi^X\) are said to be conformable for the product \((N, A) \otimes \xi^X\) and the product \((N, A) \otimes \xi^X\) becomes an NSM, which is denoted by \((N, P)\). If \((N, A) = (a_{ij})_{m \times n}\) and \(\xi^X = (b_{ik})_{m \times p}\), then \((N, P) = (N, A) \otimes \xi^X = (c_{ij})_{m \times p}\) where

\[
c_{ik} = (\min(T_{ij}, T_{kj}), \min(I_{ij}, I_{kj}), \min(F_{ij}, F_{kj})).
\]

Example 6. Let NCM and combined NCM of Mr. X are respectively defined as given below.

\[
(N, A)_X = \left\{ \begin{array}{l}
(0.0, 0, 0), (0.0, 0), (0.3, 0.5, 0.6), (0.6, 0.7, 0.3), (0.3, 0.6, 0.7) \\
(0.0, 0, 0), (0.7, 0.6, 0.3), (0.8, 0.2, 0.7), (0.7, 0.2, 0.3) \\
(0.0, 0, 0), (0.7, 0.8, 0.4), (0.6, 0, 1, 0.7), (0.8, 0.1, 0.4) \\
\end{array} \right\},
\]

\[
(N, P)_X = (N, A)_X \otimes \xi_X^Y = \left\{ \begin{array}{l}
(0.0, 5, 1), (0.0, 5, 1), (0.0, 5, 1), (0.0, 5, 1) \\
(0.0, 5, 1), (0.0, 5, 1), (0.0, 5, 1), (0.0, 5, 1) \\
(0.0, 5, 1), (0.0, 5, 1), (1.0, 5, 0), (0.5, 1) \\
(0.0, 5, 1), (0.0, 5, 1), (1.0, 5, 0), (0.5, 1) \\
\end{array} \right\}.
\]

3.4. Complement of NSM

Complement of an NSM \((N, A) = (a_{ij})_{m \times n}\) is denoted by \((N^c, A) = (a^c_{ij})_{m \times n}\) where \(a^c_{ij}\) is the matrix representation of the NSS \((N, A)\). \(a^c_{ij}\) is the matrix representation of the NSS\((N^c, A)\) and defined as \(a^c_{ij} = (T_{ij}, F_{ij}, E_{ij}) = (1 - T_{ij}, 1 - F_{ij}, 1 - E_{ij})\).
Example 7. Let NSM \( (d_i)_{m,n} \) is given below.
\[
(N,A) = \begin{pmatrix}
(0.6,0.5,0.3), (0.5,0.1), (0.6,0.8,0.3), (0.8,0.1), (0.5,0.1) \\
(0.8,0.5,0.4), (0.5,0.1), (0.8,0.5,0.4), (0.5,0.1), (0.2,0.1) \\
(0.6,0.5,0.4), (0.5,0.1), (0.8,0.5,0.4), (0.5,0.1), (0.2,0.1)
\end{pmatrix}
\]

Then the complement \((N^{\complement},A)\) is given by
\[
(N^{\complement},A) = \begin{pmatrix}
(0.4,0.5,0.7), (1.0,0.1), (0.4,0.2,0.7), (1.0,0.2,0.0), (1.0,0.5) \\
(0.2,0.1,0.7), (1.0,0.4,0), (0.2,0.4,0.7), (1.0,0.8,0), (1.0,0.3) \\
(0.2,0.5,0.6), (1.0,0.5,0), (0.2,0.5,0.6), (1.0,0.5,0), (1.0,0.8,0)
\end{pmatrix}
\]

3.5. Cardinality of NSM

More cardinality is assigned to the NSM, which gives more emphasis to the given opinion on the set of attributes irrespective of the number of attributes focused by the NSM. The cardinal set of neutrosophic soft set \((N(A),E)\), denoted by \((cN(A),E)\) and defined by \((cN(A),E) = \{ (T_{cN(A)}(x),I_{cN(A)}(x),F_{cN(A)}(x)), x \in E \}\), is a neutrosophic soft set over \(E\). The membership \(T_{cN(A)}(x)\), indeterminacy \(I_{cN(A)}(x)\) and falsity membership value \(F_{cN(A)}(x)\) are respectively defined by
\[
T_{cN(A)}(x) = \sum_{a \in U} T_{N(A)}(x)/|E_a|, I_{cN(A)}(x) = \sum_{a \in U} (0.5 - I_{N(A)}(x))/|E_a|,
\]
and
\[
F_{cN(A)}(x) = \sum_{a \in U} F_{N(A)}(x)/|E_a|,
\]
where
\[
E_a = \{ i \mid T_{N(A)}(x) \leq I_{N(A)}(x), F_{N(A)}(x) \neq 0 \forall x \in E \}.
\]

Cardinal score of a neutrosophic soft set corresponding to the cardinal set \((cN(A),E)\) is defined as
\[
S(cN(A)) = (T_{cN(A)}(x) + I_{cN(A)}(x) - F_{cN(A)}(x))/|U|.
\]

Example 8. Let two experts Mr. X and Mr. Y provide their opinions using their NSMs as given below (see Tables 3 and 4).

Cardinality score of Mr. X is given below.
\[
S(cN(A)) = (T_{cN(A)}(x) + I_{cN(A)}(x) - F_{cN(A)}(x))/|U| = \frac{0.3 + 0.4 + 0.8}{3} = 0.5
\]

Cardinal score \(S(cN(A))\) for NSM \(R'_A\) is computed, where \(l = 1, 2, \ldots, k\).

Step 1: NCM \(\xi_l(i,j)\) and combined NCM \(\xi_l'(i,j)\) of each of the decision makers \(d_i, l = 1, 2, \ldots, k\) are computed in the context of NSS based on their choice parameters or attributes.

Step 2: Cardinal score \(S(cN(A))\) is multiplied with the corresponding NSM \(R'_A\), \(l = 1, 2, \ldots, k\) to produce the normalized NSM. Let \(\tilde{A}_{ij}^{(l)}\) be an NSM and \(h\) is the cardinal score, then the normalized NSM, denoted by \(\tilde{N}_{NSM}^{(l)}\), is defined by \(\tilde{N}_{NSM}^{(l)}(A_{ij}) = |h \cdot \tilde{A}_{ij}^{(l)}|_{m \times n} \forall i,j,k\).

Step 3: Product of normalized NSM \(\tilde{N}_{NSM}^{(l)}\) and combined choice \(\xi_l'(i,j)\) for each decision maker \(d_i\) is calculated as given in Section 3.3 and denoted by \(P'_{NSM}^{l}\).

Step 4: Aggregation of the product NSMs \(P'_{NSM}^{l} \forall l\) is done as defined in Section 3.2, which produces the resultant NSM denoted by \(N_{NSM}\).

Step 5: Neutrosophic cross-entropy, discussed above in Section 2.2, between the ideal alternative and the \(i\)th alternative \(o_i\), is computed to rank the alternatives.

Step 7: Alternative(s) having lowest cross-entropy value is selected as the most desirable one(s).
In this approach, NCM shows the impact of choice parameters of individual experts in decision making process, whereas combined NCM is used to provide the impact of choice parameters of individual experts with respect to other experts. Basically NCM and combined NCM have been used to provide more importance to the choice parameters of different experts. Here cardinal score is used to assign relative weights to the experts. When an expert is confident about his/her opinion, this approach assigns more weight, i.e., more cardinal score to the corresponding NSM. By normalizing the NSMs, we provide more importance to the NSMs of confident experts and less importance to the NSMs of less confident experts. Then normalized NSMs are associated with the combined NCMS with the hope that the resultant NSM will focus on both of the expert’s confident and choice parameters. Finally, we compute neutrosophic cross-entropy between our alternatives and the ideal alternative to find the ranking.

5. Case study

In order to demonstrate the application of the proposed method, we cite an example about the investment for three possible business sectors. Let three experts Mr. John, Mr. Smith, and Mr. Peter, the members of a set $D = \{d_1, d_2, d_3\}$ jointly want to select a business sector for investment. Their proposed business sectors are travel agencies, hotel, and restaurant, given by $U = \{o_1, o_2, o_3\}$. These business sectors have a set of common attributes: first time investment, risk factor, profit, place of running business, and quality of services, given by $E = \{e_1, e_2, e_3, e_4, e_5\}$. Among three experts, Mr. John is interested in profit, place of running business, and quality of services, i.e., $(e_1, e_2, e_3, e_4)$. Mr. Smith shows his interest in first time investment and profit, i.e., $(e_1, e_5)$ and Mr. Peter is interested in first time investment, profit, and quality of services, i.e., $(e_1, e_2, e_3)$. Opinions of Mr. John, Mr. Smith, and Mr. Peter are represented in the three different NSMs, $R^d_1$, $R^d_2$, and $R^d_3$ respectively, which are given below.

$R^d_1 = \left\{ \begin{array}{c}
(0.0,0.0), (0.0,0.0), (0.3,0.5,0.6), (0.6,0.7,0.3), (0.3,0.6,0.7) \\
(0.0,0.0), (0.0,0.0), (0.7,0.6,0.3), (0.8,0.2,0.7), (0.7,0.2,0.3) \\
(0.0,0.0), (0.0,0.0), (0.7,0.8,0.4), (0.8,0.1,0.4) \\
\end{array} \right\}$

$R^d_2 = \left\{ \begin{array}{c}
(0.6,0.4,0.8), (0.0,0.0), (0.3,0.5,0.6), (0.0,0.0), (0.0,0.0) \\
(0.7,0.5,0.6), (0.0,0.0), (0.7,0.6,0.3), (0.0,0.0), (0.0) \\
(0.8,0.3,0.4), (0.0,0.0), (0.7,0.8,0.4), (0.0,0.0), (0.0), \\
\end{array} \right\}$

$R^d_3 = \left\{ \begin{array}{c}
(0.6,0.4,0.8), (0.0,0.0), (0.3,0.5,0.6), (0.0,0.0), (0.3,0.6,0.7) \\
(0.7,0.5,0.6), (0.0,0.0), (0.7,0.6,0.3), (0.0,0.0), (0.7,0.2,0.3) \\
(0.8,0.3,0.4), (0.0,0.0), (0.7,0.8,0.4), (0.0,0.0), (0.8,0.1,0.4) \\
\end{array} \right\}$

In this case study, we consider two cases. In the first case, we use non-normalized NSM and normalized NSM in the second case.

Case 1. In this case, we use non-normalized NSM as input.

[Step 1]: Neutrosophic choice matrices for the experts and their corresponding combined choice matrices are given below.

$\tilde{z}_i(i,j) = \left\{ \begin{array}{c}
(0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1) \\
(0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1) \\
(0.0, 0.5, 1), (0.0, 0.5, 1), (1.0, 0.5, 0), (1.0, 0.5, 0) \\
(0.0, 0.5, 1), (0.0, 0.5, 1), (1.0, 0.5, 0), (1.0, 0.5, 0) \\
(0.0, 0.5, 1), (0.0, 0.5, 1), (1.0, 0.5, 0), (1.0, 0.5, 0) \\
\end{array} \right\}$

$\tilde{\xi}_i(i,j) = \left\{ \begin{array}{c}
(1.0, 0.5, 0), (0.0, 0.5, 1), (1.0, 0.5, 0), (0.0, 0.5, 1) \\
(0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1) \\
(1.0, 0.5, 0), (0.0, 0.5, 1), (1.0, 0.5, 0), (0.0, 0.5, 1) \\
(0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1) \\
(0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1) \\
\end{array} \right\}$

[Step 2 & Step 3]: These steps are not applied in Case 1.

[Step 4]: Product of NSM $R^d_1$ and combined choice matrix $\tilde{z}_i(i,j)$ for each decision maker $d_i, i = 1, 2, 3$ are

$p^d_{NSM} = R^d_1 \odot \tilde{z}_i(i,j) = \left\{ \begin{array}{c}
(0.0, 0.0), (0.0, 0.0), (0.3, 0.5, 0.6), (0.6, 0.7, 0.3), (0.3, 0.6, 0.7) \\
(0.0, 0.0), (0.0, 0.0), (0.7, 0.6, 0.3), (0.8, 0.2, 0.7), (0.7, 0.2, 0.3) \\
(0.0, 0.0), (0.0, 0.0), (0.7, 0.8, 0.4), (0.8, 0.1, 0.4) \\
\end{array} \right\}$

$\tilde{z}_i(i,j) = \left\{ \begin{array}{c}
(0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1) \\
(0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1), (0.0, 0.5, 1) \\
(1.0, 0.5, 0), (0.0, 0.5, 1), (1.0, 0.5, 0), (0.0, 0.5, 1) \\
(1.0, 0.5, 0), (0.0, 0.5, 1), (1.0, 0.5, 0), (0.0, 0.5, 1) \\
(0.0, 0.5, 1), (0.0, 0.5, 1), (1.0, 0.5, 0), (1.0, 0.5, 0) \\
\end{array} \right\}$

$p^d_{NSM} = R^d_1 \odot \tilde{z}_i(i,j) = \left\{ \begin{array}{c}
(0.0, 0.0), (0.0, 0.0), (0.3, 0.5, 0.6), (0.6, 0.7, 0.3), (0.3, 0.6, 0.7) \\
(0.0, 0.0), (0.0, 0.0), (0.7, 0.6, 0.3), (0.8, 0.2, 0.7), (0.7, 0.2, 0.3) \\
(0.0, 0.0), (0.0, 0.0), (0.7, 0.8, 0.4), (0.8, 0.1, 0.4) \\
\end{array} \right\}$
\[ p_{R_{NM}}^m = R_{A_{m}}^N \odot \zeta(i, j) = \left[ \begin{array}{cccc}
0.605.03, & 0.051, & 0.0605.03, & 0.051, \\
0.0605.03, & 0.051, & 0.0805.03, & 0.051, \\
0.0805.02, & 0.051, & 0.0805.02, & 0.051, \\
0.051, & 0.051, & 0.0605.06, & 0.051, \\
0.051, & 0.051, & 0.0705.03, & 0.051, \\
0.051, & 0.051, & 0.0805.04, & 0.051, \\
0.051, & 0.051, & 0.0605.06, & 0.051, \\
0.051, & 0.051, & 0.0705.03, & 0.051, \\
0.051, & 0.051, & 0.0805.04, & 0.051, 
\end{array} \right] \]

\[ R_{NM}^m = \begin{cases}
(0.60.0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.7.0.5.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0.8.0.3.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0.0.0.5.1, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0.0.0.5.1, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0.0.0.5.1, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0.0.0.5.1, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0.0.0.5.1, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0.0.0.5.1, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
\end{cases} \]

\[ I_{NNM}^m(x) = \sum_{i=1}^{n} 0.5 - I_{NNM}^m(x)/|E| = \frac{0.5 - 0.5 + 0.5 - 0.5 + 0.5 - 0.5 + 0.5 - 0.5}{3} = \frac{0 + 2 + 1 + 1 + 3 + 3 + 4 + 4}{3} = 2.1/3 \]

\[ F_{CN(A)}(x) = \sum_{i=1}^{n} F_{N(A)}(x)/|E| = 0.6 + 0.3 + 0.7 + 0.3 + 0.7 + 0.4 + 0.2(-0.7) + 0.4)/3 = 0.7/3 \]

\[
S'(cN(A)) = S'(cN(A)) = (5.5 + (2.1 - 3.9))/3 = 0.411
\]

Similarly, \[ S'(cN(A)) = 0.23, S'(cN(A)) = 0.28 \]

**Step 3:** Normalized NSMs for the decision makers, John, Smith and Peter are respectively

\[
N_{NM}[a, b] = \frac{h + w_{ij}}{1.5} = \begin{cases}
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
\end{cases}
\]

\[
N_{NM}[a, b] = \frac{h + w_{ij}}{1.5} = \begin{cases}
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
\end{cases}
\]

\[
N_{NM}[a, b] = \frac{h + w_{ij}}{1.5} = \begin{cases}
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
\end{cases}
\]

**Step 4:** Product of normalized NSM \[ N_{NM} \] and combined choice matrix \[ \zeta(i, j) \] for each decision maker \[ d_i \] is computed as follows.

\[
p_{NM} = N_{NM} \odot \zeta(i, j) = \begin{cases}
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
\end{cases}
\]

\[
p_{NM} = N_{NM} \odot \zeta(i, j) = \begin{cases}
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
\end{cases}
\]

\[
p_{NM} = N_{NM} \odot \zeta(i, j) = \begin{cases}
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
\end{cases}
\]

\[
p_{NM} = N_{NM} \odot \zeta(i, j) = \begin{cases}
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0) \\
(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
\end{cases}
\]

**Case II:** It uses normalized NSM as input.

**Step 1:** This step is similar as in case 1.

**Step 2:** It calculates the cardinal scores \[ S'(cN(A)), S'(cN(A)), \] and \[ S'(cN(A)) \] respectively for the NSMs \[ R_{A_{m}}^N, R_{A_{m}}^R, \] and \[ R_{A}^N \].

\[
S'(cN(A)) = (T_{cN(A)}(x) + I_{cN(A)}(x) - F_{cN(A)}(x))/|E| = \frac{(0.3 + 0.6 + 0.3 + 0.7 + 0.8 + 0.7 + 0.6 + 0.8)/3 = 5.5/3}
\]
The hotel business sector will be selected as the collective decision of the experts. In (Das and Kar, 2014; Das et al., 2014), authors considered the experts’ prescribed opinion and is computed by deriving the cardinal score of the corresponding NSM. The more the cardinal score, the more the relative weight assigned to an expert.

When experts’ opinions about the selected set of attributes are used here in case study, Case I does not use the relative weight, while Case II uses it. As per collective opinion of a group of experts, the order of selection of the business sectors is given in Table 5. The result shows different ordering in Case II as we have considered the experts’ relative weights.

Since our proposed algorithmic approach for solving GDM problems using NSM and relative weight of experts. Firstly, we have presented NSM and discussed some of its relevant operations. Next we have proposed the relative weight assigning procedure of experts using cardinal score in the context of neutrosophic environments. The proposed algorithm is based on combined choice matrix, product NSM, cardinal score, and cross entropy measure of NSMs, which yields the collective opinion of a group of decision makers. The case study is related to the selection of a business sector for investment purpose, where a set of three experts suggest their opinions about a common set of attributes. Future scope of this research work could be to investigate the application of robustness in GDM in the framework of neutrosophic set. Also researchers might focus on various properties of NSMs and then apply them to suitable uncertain decision making problems.

**7. Conclusion**

This article has proposed an algorithmic approach for solving GDM problems using NSM and relative weight of experts. Firstly, we have presented NSM and discussed some of its relevant operations. Next we have proposed the relative weight assigning procedure of experts using cardinal score in the context of neutrosophic environments. The proposed algorithm is based on combined choice matrix, product NSM, cardinal score, and cross entropy measure of NSMs, which yields the collective opinion of a group of decision makers. The case study is related to the selection of a business sector for investment purpose, where a set of three experts suggest their opinions about a common set of attributes. Future scope of this research work could be to investigate the application of robustness in GDM in the framework of neutrosophic set. Also researchers might focus on various properties of NSMs and then apply them to suitable uncertain decision making problems.

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**References**


**Table 5**

<table>
<thead>
<tr>
<th>Case I</th>
<th>$o_1 &gt; o_2 &gt; o_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case II</td>
<td>$o_2 &gt; o_3 &gt; o_1$</td>
</tr>
</tbody>
</table>

**Step 5:** Aggregation of the product NSMs, $P^i_{R_{NSM}}$, $VI$ is denoted by $R_{NSM}$.

**Step 6:** Cross-entropy values of $(A^+, o_1)$, $(A^+, o_2)$, and $(A^+, o_3)$ are

$$ D_1(A^+, o_1) = 15.235, \quad D_2(A^+, o_2) = 14.075, \quad D_3(A^+, o_3) = 14.366 $$

Since $D_2(A^+, o_2)$ has least cross-entropy value, alternative $o_2$, i.e., hotel business sector will be selected as the collective decision of John, Smith, and Peter.

**6. Discussion on the results**

Expert’s opinion exhibit a vital role in GDM (Das et al., 2015; 2017). This study proposes a decision making methodology using NSM, which is used to represent the opinion of individual decision maker. The relative weight assigned to an expert is based on the expert’s prescribed opinion and is computed by deriving the cardinal score of the corresponding NSM. The more the cardinal score, the more important is the opinion. When an expert is more confident about her opinion, more cardinal score is assigned to that expert. Due to lack of information or limited domain knowledge, experts often prefer to express their opinions only for a subset of attributes instead of the entire attribute set. Often it is also found that experts are confident about a few attributes among the subset of attributes. In that case, cardinal score will be more where an expert is confident about her opinion irrespective of the number of attributes provided. This is similar to our real life situations. It also removes the biasness which might be imposed by different experts and as a result adds more credibility to the final decision.

In (Das and Kar, 2014; Das et al., 2014), authors considered to assign more cardinal score when an expert provides his/her opinion about more number of attributes, which is also practical in real life environment. More specifically, which case should be considered for assigning higher cardinal score, may be case dependent. When experts’ opinions about the selected set of attributes are quite significant and no attributes can be ignored, then one can consider the relative weight assigning procedure as proposed in Das and Kar (2014); Das et al. (2014). But when opinion about some of the selected attributes are not significant and can be ignored, then the approach proposed here can be used. Moreover, the key aspect of the proposed algorithm is that it does not use any knowledgebase for finding the distances and similarity measures of the individual experts. Rather more importance is given on the parameter selection of experts by finding initial choice matrices and then combined choice matrices. Among the two cases used here in case study, Case I does not use the relative weight, while Case II uses it. As per collective opinion of a group of experts, the order of selection of the business sectors is given in Table 5. The result shows different ordering in Case II as we have considered the experts’ relative weights. Case I produces the final outcome as $o_1$, i.e., restaurant business sector while Case II produces $o_2$, i.e., hotel business sector as per the collective opinion.


