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Implicative Neutrosophic Quadruple BCK-Algebras and Ideals

G. Muhiuddin [1,*], Ahmad N. Al-Kenani [2], Eun Hwan Roh [3] and Young Bae Jun [4]

1 Department of Mathematics, University of Tabuk, Tabuk 71491, Saudi Arabia
2 Department of Mathematics, King Abdulaziz University, P.O. Box 80219, Jeddah 21589, Saudi Arabia; aalkenani10@hotmail.com
3 Department of Mathematics Education, Chinju National University of Education, Jinju 52673, Korea; ehroh9988@gmail.com
4 Department of Mathematics Education, Gyeongsang National University, Jinju 52828, Korea; skywine@gmail.com
* Correspondence: chishtygm@gmail.com

Received: 15 November 2018; Accepted: 13 February 2019; Published: 21 February 2019

Abstract: A neutrosophic set is initiated by Smarandache, and it is a novel tool to deal with vagueness considering the truth, indeterminacy and falsity memberships satisfying the condition that their sum is less than 3. The concept of neutrosophic quadruple numbers was introduced by Florentin Smarandache. Using this idea, Jun et al. introduced the notion of neutrosophic quadruple BCK/BCI-numbers, and studied neutrosophic quadruple BCK/BCI-algebras. As a continuation of Jun et al.'s paper, the notion of implicative neutrosophic quadruple BCK-algebras is introduced, and several properties are investigated. Given a set Y, conditions for the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ to be a neutrosophic quadruple BCI-algebra are provided. Conditions for the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ to be an implicative neutrosophic quadruple BCK-algebra are provided. Given subsets I and J of a BCK-algebra Y, conditions for the neutrosophic quadruple (I, J)-set $\mathcal{N}_q(I, J)$ to be an implicative ideal of the neutrosophic quadruple BCK-algebra $\mathcal{N}_q(Y)$ are discussed.

Keywords: (commutative, implicative) neutrosophic quadruple BCK-algebra; (commutative, implicative) neutrosophic quadruple ideal; neutrosophic quadruple (I, J)-set

1. Introduction

BCK/BCI-algebras are an algebraic structure, which was introduced by Imai, Iséki and Tanaka in 1966, that describes fragments of the propositional calculus involving implication known as BCK and BCI logics. The notion of neutrosophic set, which is developed by Smarandache (see [1–3]), is a more general platform which extends the notions of (intuitionistic) fuzzy set, interval valued (intuitionistic) fuzzy set and classic set. Neutrosophic set theory has useful applications in several branches. Decision-making problems are some of the most widely used phenomena in our real-life applications or in various fields like science, engineering, operation research, and management sciences. Garg and Nancy [4] developed a nonlinear programming model based on the technique for order preference by similarity to ideal solution (TOPSIS), in order to solve decision-making problems in which criterion values and their importance are given in the form of interval neutrosophic numbers (INNs). Garg and Nancy [5] presented some new operational laws called logarithm operational laws with real number base for the single-valued neutrosophic (SVN) numbers, and applied it to multiattribute decision making. In algebraic structures of BCK/BCI-algebras and semigroup, neutrosophic set theory is discussed in the papers [6–15]. Smarandache [16] introduced the notion of neutrosophic quadruple numbers. Akinleye et al. [17] introduced the concept of neutrosophic quadruple algebraic structures.

Jun et al. [18] studied the neutrosophic quadruple algebraic structures in BCK/BCI-algebras, and they introduced the notion of neutrosophic quadruple BCK/BCI-algebras.

In this article, we introduce the concept of implicative neutrosophic quadruple BCK-algebras, and investigate several properties. In the first and second sections, introduction and basic notions/results are displayed. In the third section, we discuss several properties and (implicative) neutrosophic quadruple ideals in (implicative) neutrosophic quadruple BCK-algebras. Given a set Y, we discuss conditions for the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ to be a neutrosophic quadruple BCI-algebra. We provide conditions for the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ to be an implicative neutrosophic quadruple BCK-algebra. Given subsets I and J of a BCK-algebra Y, we find conditions for the neutrosophic quadruple (I, J)-set $\mathcal{N}_q(I, J)$ to be an implicative ideal of the neutrosophic quadruple BCK-algebra $\mathcal{N}_q(Y)$.

2. Preliminaries

A *BCI-algebra* (see [19]) is defined to be a set Y with a binary operation $*$ and a special element 0 which satisfies the following conditions:

(I) $(\forall a, b, c \in Y)\ (((a * b) * (a * c)) * (c * b) = 0)$,
(II) $(\forall a \in Y)\ (a * 0 = a)$,
(III) $(\forall u, v \in Y)\ (u * v = 0,\ v * u = 0\ \Rightarrow\ u = v)$.

If a BCI-algebra Y satisfies the following identity:

(IV) $(\forall a \in Y)\ (0 * a = 0)$,

then Y is called a *BCK-algebra*. Any BCK/BCI-algebra Y satisfies the following conditions:

$$a * 0 = a, \tag{1}$$
$$a \le b\ \Rightarrow\ a * c \le b * c,\ c * b \le c * a, \tag{2}$$
$$(a * b) * c = (a * c) * b, \tag{3}$$
$$(a * c) * (b * c) \le a * b \tag{4}$$

for all $a, b, c \in Y$.

A BCK-algebra Y is said to be

- *commutative* if the following assertion is valid:

$$(\forall a, b \in Y)\ (a * (a * b) = b * (b * a)). \tag{5}$$

- *implicative* if the following assertion is valid:

$$(\forall a, b \in Y)\ (a * (b * a) = a). \tag{6}$$

A subset I of a BCK-algebra Y is called

- an *ideal* of Y if it satisfies:

$$0 \in I, \tag{7}$$
$$(\forall a \in Y)\ (\forall b \in I)\ (a * b \in I\ \Rightarrow\ a \in I), \tag{8}$$

- a *commutative ideal* of Y if it satisfies Label (7) and

$$(\forall a, b \in Y)(\forall c \in I)\ ((a * b) * c \in I\ \Rightarrow\ a * (b * (b * a)) \in I). \tag{9}$$

- an *implicative ideal* (see [20]) of Y if it satisfies (7) and

$$(\forall a,b,c \in Y)(((a * (b * a)) * c \in I,\ c \in I \ \Rightarrow\ a \in I). \tag{10}$$

We refer the reader to the books [19,20] for further information regarding BCK/BCI-algebras, and to [2,3] for further information regarding neutrosophic set theory.

Definition 1 ([18])**.** *Let Y be a nonempty set. A neutrosophic quadruple Y-number is an ordered quadruple (a, xT, yI, zF) where $a, x, y, z \in Y$ and T, I, F have their usual neutrosophic logic meanings.*

The set of all neutrosophic quadruple Y-numbers is denoted by $\mathcal{N}_q(Y)$, that is,

$$\mathcal{N}_q(Y) := \{(a, xT, yI, zF) \mid a, x, y, z \in Y\},$$

and it is called the *neutrosophic quadruple set* based on Y or *neutrosophic quadruple Y-set*.

Let Y be a set with a binary operation $*$ and a special number 0. We define a binary operation $\tilde{*}$ on $\mathcal{N}_q(Y)$ by

$$(a, xT, yI, zF) \,\tilde{*}\, (b, uT, vI, wF) = (a * b, (x * u)T, (y * v)I, (z * w)F)$$

for all (a, xT, yI, zF), $(b, uT, vI, wF) \in \mathcal{N}_q(Y)$. Given $x_1, x_2, x_3, x_4 \in Y$, the neutrosophic quadruple Y-number (x_1, x_2T, x_3I, x_4F) is denoted by $\tilde{x}$, that is,

$$\tilde{x} = (x_1, x_2T, x_3I, x_4F),$$

and the zero neutrosophic quadruple Y-number $(0, 0T, 0I, 0F)$ is denoted by $\tilde{0}$, that is,

$$\tilde{0} = (0, 0T, 0I, 0F).$$

If Y has an order relation "$\leq$", then we define an order relation "$\ll$" and the equality "$=$" on $\mathcal{N}_q(Y)$ as follows:

$$\tilde{a} \ll \tilde{b} \Leftrightarrow a_i \leq b_i \text{ for } i = 1,2,3,4,$$
$$\tilde{a} = \tilde{b} \Leftrightarrow a_i = b_i \text{ for } i = 1,2,3,4$$

for all $\tilde{a}, \tilde{b} \in \mathcal{N}_q(Y)$. It is easy to verify that, if "$\leq$" is a partial order on Y, then "$\ll$" is a partial order on $\mathcal{N}_q(Y)$.

Definition 2 ([18])**.** *Given a set Y with a binary operation $*$ and a special number 0, the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ is called a neutrosophic quadruple BCK/BCI-algebra if $(\mathcal{N}_q(Y); \tilde{*}, \tilde{0})$ is a BCK/BCI-algebra.*

3. Implicative Neutrosophic Quadruple Ideals

In this section, we first consider conditions for a the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ to be a neutrosophic quadruple BCI-algebra. We define the notion of (commutative, implicative) neutrosophic quadruple BCK-algebra and investigate related properties.

Theorem 1. *Given a set Y with a binary operation $*$ and a special number 0, if the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ has a binary operation "$\tilde{*}$" and a partial ordering "$\ll$" such that*

(1) $(\tilde{x} \,\tilde{*}\, \tilde{y}) \,\tilde{*}\, (\tilde{x} \,\tilde{*}\, \tilde{z}) \ll \tilde{z} \,\tilde{*}\, \tilde{y}$,
(2) $\tilde{x} \,\tilde{*}\, (\tilde{x} \,\tilde{*}\, \tilde{y}) \ll \tilde{y}$,
(3) $\tilde{x} \,\tilde{*}\, \tilde{y} = \tilde{0} \ \Leftrightarrow\ \tilde{x} \ll \tilde{y}$

for all $\tilde{x}, \tilde{y}, \tilde{z} \in \mathcal{N}_q(Y)$, then $\mathcal{N}_q(Y)$ is a neutrosophic quadruple BCI-algebra.

Proof. Let $\tilde{x}, \tilde{y}, \tilde{z} \in \mathcal{N}_q(Y)$. Using conditions (1) and (3) of this theorem, we have

$$((\tilde{x} \tilde{*} \tilde{y}) \tilde{*} (\tilde{x} \tilde{*} \tilde{z})) \tilde{*} (\tilde{z} \tilde{*} \tilde{y}) = \tilde{0}. \tag{11}$$

Assume that $\tilde{x} \tilde{*} \tilde{y} = \tilde{0}$ and $\tilde{y} \tilde{*} \tilde{x} = \tilde{0}$. Then, $\tilde{x} \ll \tilde{y}$ and $\tilde{y} \ll \tilde{x}$ by (3), which implies that $\tilde{x} = \tilde{y}$ by the anti-symmetry of $\ll$. By the condition (3) of this theorem and the reflexivity of $\ll$, we get $\tilde{x} \tilde{*} \tilde{x} = \tilde{0}$. Using conditions (2) and (3) of this theorem, we have

$$(\tilde{x} \tilde{*} (\tilde{x} \tilde{*} \tilde{y})) \tilde{*} \tilde{y} = \tilde{0}. \tag{12}$$

Putting $\tilde{y} = \tilde{0}$ in (12) implies that

$$(\tilde{x} \tilde{*} (\tilde{x} \tilde{*} \tilde{0})) \tilde{*} \tilde{0} = \tilde{0}. \tag{13}$$

If we substitute $\tilde{x} \tilde{*} \tilde{0}$ and $\tilde{x}$ for $\tilde{y}$ and $\tilde{z}$, respectively, in (11), then

$$\begin{aligned} \tilde{0} &= ((\tilde{x} \tilde{*} (\tilde{x} \tilde{*} \tilde{0})) \tilde{*} (\tilde{x} \tilde{*} \tilde{x})) \tilde{*} (\tilde{x} \tilde{*} (\tilde{x} \tilde{*} \tilde{0})) \\ &= ((\tilde{x} \tilde{*} (\tilde{x} \tilde{*} \tilde{0})) \tilde{*} \tilde{0}) \tilde{*} (\tilde{x} \tilde{*} (\tilde{x} \tilde{*} \tilde{0})) \\ &= \tilde{0} \tilde{*} (\tilde{x} \tilde{*} (\tilde{x} \tilde{*} \tilde{0})). \end{aligned}$$

Hence, $\tilde{x} \tilde{*} (\tilde{x} \tilde{*} \tilde{0}) = \tilde{0}$. On the other hand, we get $(\tilde{x} \tilde{*} \tilde{0}) \tilde{*} \tilde{x} = (\tilde{x} \tilde{*} (\tilde{x} \tilde{*} \tilde{x})) \tilde{*} \tilde{x} = \tilde{0}$. It follows that $\tilde{x} \tilde{*} \tilde{0} = \tilde{x}$. Hence, $(\mathcal{N}_q(Y); \tilde{*}, \tilde{0})$ is a BCI-algebra, and therefore $\mathcal{N}_q(Y)$ is a neutrosophic quadruple BCI-algebra. □

Definition 3. *Given a set Y with a binary operation $*$ and a special number 0, the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ is called a (commutative, implicative) neutrosophic quadruple BCK-algebra if $(\mathcal{N}_q(Y); \tilde{*}, \tilde{0})$ is a (commutative, implicative) BCK-algebra.*

Example 1. *Given a set $Y = \{0, a\}$, consider the neutrosophic quadruple Y-set as follows:*

$$\mathcal{N}_q(Y) = \{\tilde{\alpha}_0, \tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4, \tilde{\alpha}_5, \tilde{\alpha}_6, \tilde{\alpha}_7, \tilde{\alpha}_8, \tilde{\alpha}_9, \tilde{\alpha}_{10}, \tilde{\alpha}_{11}, \tilde{\alpha}_{12}, \tilde{\alpha}_{13}, \tilde{\alpha}_{14}, \tilde{\alpha}_{15}\},$$

where

$$\tilde{\alpha}_0 = (0, 0T, 0I, 0F), \tilde{\alpha}_1 = (0, 0T, 0I, aF), \tilde{\alpha}_2 = (0, 0T, aI, 0F), \tilde{\alpha}_3 = (0, 0T, aI, aF),$$

$$\tilde{\alpha}_4 = (0, aT, 0I, 0F), \tilde{\alpha}_5 = (0, aT, 0I, aF), \tilde{\alpha}_6 = (0, aT, aI, 0F), \tilde{\alpha}_7 = (0, aT, aI, aF),$$

$$\tilde{\alpha}_8 = (a, 0T, 0I, 0F), \tilde{\alpha}_9 = (a, 0T, 0I, aF), \tilde{\alpha}_{10} = (a, 0T, aI, 0F), \tilde{\alpha}_{11} = (a, 0T, aI, aF),$$

$$\tilde{\alpha}_{12} = (a, aT, 0I, 0F), \tilde{\alpha}_{13} = (a, aT, 0I, aF), \tilde{\alpha}_{14} = (a, aT, aI, 0F), \tilde{\alpha}_{15} = (a, aT, aI, aF).$$

Define a binary operation "$\tilde{*}$" on $\mathcal{N}_q(Y)$ by Table 1.

Table 1. Binary operation " $\tilde{*}$ ".

$\tilde{*}$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_3$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_6$	$\tilde{\alpha}_7$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_{11}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_{13}$	$\tilde{\alpha}_{14}$	$\tilde{\alpha}_{15}$
$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$
$\tilde{\alpha}_1$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$
$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$
$\tilde{\alpha}_3$	$\tilde{\alpha}_3$	$\tilde{\alpha}_2$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_3$	$\tilde{\alpha}_2$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_3$	$\tilde{\alpha}_2$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_3$	$\tilde{\alpha}_2$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$
$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$
$\tilde{\alpha}_5$	$\tilde{\alpha}_5$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_4$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_5$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_4$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$
$\tilde{\alpha}_6$	$\tilde{\alpha}_6$	$\tilde{\alpha}_6$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_6$	$\tilde{\alpha}_6$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$
$\tilde{\alpha}_7$	$\tilde{\alpha}_7$	$\tilde{\alpha}_6$	$\tilde{\alpha}_5$	$\tilde{\alpha}_4$	$\tilde{\alpha}_3$	$\tilde{\alpha}_2$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_7$	$\tilde{\alpha}_6$	$\tilde{\alpha}_5$	$\tilde{\alpha}_4$	$\tilde{\alpha}_3$	$\tilde{\alpha}_2$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$
$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$
$\tilde{\alpha}_9$	$\tilde{\alpha}_9$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$
$\tilde{\alpha}_{10}$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_0$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$
$\tilde{\alpha}_{11}$	$\tilde{\alpha}_{11}$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_9$	$\tilde{\alpha}_8$	$\tilde{\alpha}_{11}$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_9$	$\tilde{\alpha}_8$	$\tilde{\alpha}_3$	$\tilde{\alpha}_2$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_3$	$\tilde{\alpha}_2$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$
$\tilde{\alpha}_{12}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$
$\tilde{\alpha}_{13}$	$\tilde{\alpha}_{13}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_{13}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_9$	$\tilde{\alpha}_8$	$\tilde{\alpha}_9$	$\tilde{\alpha}_8$	$\tilde{\alpha}_5$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$	$\tilde{\alpha}_4$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$
$\tilde{\alpha}_{14}$	$\tilde{\alpha}_{14}$	$\tilde{\alpha}_{14}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_8$	$\tilde{\alpha}_8$	$\tilde{\alpha}_6$	$\tilde{\alpha}_6$	$\tilde{\alpha}_4$	$\tilde{\alpha}_4$	$\tilde{\alpha}_2$	$\tilde{\alpha}_2$	$\tilde{\alpha}_0$	$\tilde{\alpha}_0$
$\tilde{\alpha}_{15}$	$\tilde{\alpha}_{15}$	$\tilde{\alpha}_{14}$	$\tilde{\alpha}_{13}$	$\tilde{\alpha}_{12}$	$\tilde{\alpha}_{11}$	$\tilde{\alpha}_{10}$	$\tilde{\alpha}_9$	$\tilde{\alpha}_8$	$\tilde{\alpha}_7$	$\tilde{\alpha}_6$	$\tilde{\alpha}_5$	$\tilde{\alpha}_4$	$\tilde{\alpha}_3$	$\tilde{\alpha}_2$	$\tilde{\alpha}_1$	$\tilde{\alpha}_0$

Then, $\mathcal{N}_q(Y)$ is a (commutative, implicative) neutrosophic quadruple BCK-algebra.

Lemma 1 ([18]). *If Y is a BCK/BCI-algebra, then $\mathcal{N}_q(Y)$ is a neutrosophic quadruple BCK/BCI-algebra.*

Theorem 2. *If Y is an implicative BCK-algebra, then the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ is an implicative neutrosophic quadruple BCK-algebra.*

Proof. Let Y be an implicative BCK-algebra. Then, Y is a BCK-algebra, and so $\mathcal{N}_q(Y)$ is a neutrosophic quadruple BCK-algebra by Lemma 1. Let $\tilde{x}$, $\tilde{y} \in \mathcal{N}_q(Y)$. Then, $x_i * (y_i * x_i) = x_i$ for all $i = 1, 2, 3, 4$ since $x_i, y_i \in Y$ and Y is an implicative BCK-algebra. Hence,

$$\begin{aligned} \tilde{x} \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x}) &= (x_1 * (y_1 * x_1), (x_2 * (y_2 * x_2))T, (x_3 * (y_3 * x_3))I, (x_4 * (y_4 * x_4))F) \\ &= (x_1, x_2T, x_3I, x_4F) = \tilde{x}, \end{aligned}$$

and therefore $\mathcal{N}_q(Y)$ is an implicative neutrosophic quadruple BCK-algebra. □

Lemma 2 ([21]). *If Y is a commutative BCK-algebra, then the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ is a commutative neutrosophic quadruple BCK-algebra.*

Since every implicative BCK-algebra is a commutative BCK-algebra, we have the following corollary.

Corollary 1. *Every neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ based on an implicative BCK-algebra Y is a commutative neutrosophic quadruple BCK-algebra.*

Proposition 1. *The neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ based on an implicative BCK-algebra Y satisfies the following assertions:*

(1) $(\tilde{x} \,\tilde{*}\, (\tilde{x} \,\tilde{*}\, \tilde{y})) \,\tilde{*}\, (\tilde{x} \,\tilde{*}\, \tilde{y}) = \tilde{y} \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x})$,
(2) $(\tilde{x} \,\tilde{*}\, (\tilde{x} \,\tilde{*}\, \tilde{y})) \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x}) = \tilde{y} \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x})$,
(3) $(\tilde{x} \,\tilde{*}\, (\tilde{x} \,\tilde{*}\, \tilde{y})) \,\tilde{*}\, (\tilde{x} \,\tilde{*}\, \tilde{y}) = (\tilde{y} \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x})) \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x})$

for all $\tilde{x}, \tilde{y} \in \mathcal{N}_q(Y)$.

Proof. Let Y be an implicative BCK-algebra. Then,

$$(x_i * (x_i * y_i)) * (x_i * y_i) = y_i * (y_i * x_i)$$

for all $x_i, y_i \in Y$ with $i = 1, 2, 3, 4$. Thus,

$$\begin{aligned}(\tilde{x} \circledast (\tilde{x} \circledast \tilde{y})) \circledast (\tilde{x} \circledast \tilde{y}) &= \big((x_1, x_2T, x_3I, x_4F) \circledast ((x_1, x_2T, x_3I, x_4F) \circledast (y_1, y_2T, y_3I, y_4F))\big) \\ &\quad \circledast \big((x_1, x_2T, x_3I, x_4F) \circledast (y_1, y_2T, y_3I, y_4F)\big) \\ &= \big((x_1 * (x_1 * y_1)) * (x_1 * y_1), ((x_2 * (x_2 * y_2)) * (x_2 * y_2))T, \\ &\quad ((x_3 * (x_3 * y_3)) * (x_3 * y_3))I, ((x_4 * (x_4 * y_4)) * (x_4 * y_4))F\big) \\ &= \big(y_1 * (y_1 * x_1), (y_2 * (y_2 * x_2))T, (y_3 * (y_3 * x_3))I, (y_4 * (y_4 * x_4))F\big) \\ &= \tilde{y} \circledast (\tilde{y} \circledast \tilde{x}).\end{aligned}$$

This proves (1). Similarly, we can prove (2) and (3). □

Theorem 3. *If the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ based on a BCK-algebra Y satisfies the condition (3) in Proposition 1, then it is an implicative neutrosophic quadruple BCK-algebra.*

Proof. By Lemma 1, we know that $\mathcal{N}_q(Y)$ is a neutrosophic quadruple BCK-algebra. Let $\tilde{x}, \tilde{y} \in \mathcal{N}_q(Y)$. Then,

$$(x_i * (x_i * y_i)) * (x_i * y_i) = (y_i * (y_i * x_i)) * (y_i * x_i) \tag{14}$$

for all $i = 1, 2, 3, 4$. If we substitute $x_i * y_i$ for y_i in (14), then

$$\begin{aligned} x_i * y_i &= ((x_i * y_i) * 0) * 0 = ((x_i * y_i) * ((x_i * y_i) * x_i)) * ((x_i * y_i) * x_i) \\ &= (x_i * (x_i * (x_i * y_i))) * (x_i * (x_i * y_i)) \\ &= (x_i * y_i) * (x_i * (x_i * y_i)) \\ &= (x_i * (x_i * (x_i * y_i)) * y_i \\ &= (x_i * y_i) * y_i. \end{aligned} \tag{15}$$

It follows from (15) and (3) in Proposition 1 that

$$x_i * (x_i * y_i) = (x_i * (x_i * y_i)) * (x_i * y_i) = (y_i * (y_i * x_i)) * (y_i * x_i) = y_i * (y_i * x_i). \tag{16}$$

Using (15) and (16), we have

$$x_i * (x_i * (y_i * x_i)) = (y_i * x_i) * ((y_i * x_i) * x_i) = ((y_i * x_i) * x_i) * ((y_i * x_i) * x_i) = 0.$$

Obviously, $(x_i * (y_i * x_i)) * x_i = 0$. Hence, $x_i * (y_i * x_i) = x_i$, and thus

$$\begin{aligned}\tilde{x} \circledast (\tilde{y} \circledast \tilde{x}) &= \big(x_1 * (y_1 * x_1), (x_2 * (y_2 * x_2))T, (x_3 * (y_3 * x_3))I, (x_4 * (y_4 * x_4))F\big) \\ &= (x_1, x_2T, x_3I, x_4F) = \tilde{x}.\end{aligned}$$

Hence, $(\mathcal{N}_q(Y); \circledast, \tilde{0})$ is an implicative BCK-algebra, and therefore $\mathcal{N}_q(Y)$ is an implicative neutrosophic quadruple BCK-algebra. □

Given subsets I and J of a BCK-algebra Y, consider the set

$$\mathcal{N}_q(I, J) := \{(a, xT, yI, zF) \in \mathcal{N}_q(Y) \mid a, x \in I; y, z \in J\},$$

which is called the *neutrosophic quadruple* (I, J)*-set*. It is clear that the neutrosophic quadruple (I, J)-set is a subset of the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$.

Theorem 4. *If I and J are implicative ideals of a BCK-algebra Y, then the neutrosophic quadruple (I, J)-set $\mathcal{N}_q(I, J)$ is an implicative ideal of the neutrosophic quadruple BCK-algebra $\mathcal{N}_q(Y)$.*

Proof. Assume that I and J are implicative ideals of a BCK-algebra Y. Obviously, $\tilde{0} \in \mathcal{N}_q(I, J)$. Let $\tilde{x} = (x_1, x_2T, x_3I, x_4F)$, $\tilde{y} = (y_1, y_2T, y_3I, y_4F)$ and $\tilde{z} = (z_1, z_2T, z_3I, z_4F)$ be elements of $\mathcal{N}_q(Y)$ such that $(\tilde{x} \tilde{*} (\tilde{y} \tilde{*} \tilde{x})) \tilde{*} \tilde{z} \in \mathcal{N}_q(I, J)$ and $\tilde{z} \in \mathcal{N}_q(I, J)$. Then,

$$(\tilde{x} \tilde{*} (\tilde{y} \tilde{*} \tilde{x})) \tilde{*} \tilde{z} = \big((x_1 * (y_1 * x_1)) * z_1, ((x_2 * (y_2 * x_2)) * z_2)T,$$
$$((x_3 * (y_3 * x_3)) * z_3)I, ((x_4 * (y_4 * x_4)) * z_4)F\big) \in \mathcal{N}_q(I, J),$$

and so $(x_1 * (y_1 * x_1)) * z_1 \in I$, $(x_2 * (y_2 * x_2)) * z_2 \in I$, $(x_3 * (y_3 * x_3)) * z_3 \in J$ and $(x_4 * (y_4 * x_4)) * z_4 \in J$. Since $\tilde{z} \in \mathcal{N}_q(I, J)$, we have $z_1, z_2 \in I$ and $z_3, z_4 \in J$. Since I and I are implicative ideals of Y, it follows that $x_1, x_2 \in I$ and $x_3, x_4 \in J$. Hence, $\tilde{x} = (x_1, x_2T, x_3I, x_4F) \in \mathcal{N}_q(I, J)$, and therefore $\mathcal{N}_q(I, J)$ is an implicative ideal of $\mathcal{N}(Y)$. □

Lemma 3 ([21]). *If I and J are commutative ideals of a BCK-algebra Y, then the neutrosophic quadruple (I, J)-set $\mathcal{N}_q(I, J)$ is a commutative ideal of the neutrosophic quadruple BCK-algebra $\mathcal{N}_q(Y)$.*

Since every implicative ideal is a commutative ideal, we have the following corollary.

Corollary 2. *If I and J are implicative ideals of a BCK-algebra Y, then the neutrosophic quadruple (I, J)-set $\mathcal{N}_q(I, J)$ is a commutative ideal of the neutrosophic quadruple BCK-algebra $\mathcal{N}_q(Y)$.*

The following example illustrates Theorem 4.

Example 2. *Consider a BCK-algebra $Y = \{0, a, b, c\}$ in which the binary operation $*$ is given by Table 2,*

Table 2. Binary operation "$*$".

$*$	**0**	***a***	***b***	***c***
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

Then, the neutrosophic quadruple BCK-algebra $\mathcal{N}_q(Y)$ has 256 elements. Note that $I := \{0, a\}$ and $J := \{0, b\}$ are implicative ideals of Y. Hence, the neutrosophic quadruple (I, J)-set $\mathcal{N}_q(I, J)$ is given as follows:

$$\mathcal{N}_q(I, J) = \{\tilde{0}, \tilde{1}, \tilde{2}, \tilde{3}, \tilde{4}, \tilde{5}, \tilde{6}, \tilde{7}, \tilde{8}, \tilde{9}, \tilde{10}, \tilde{11}, \tilde{12}, \tilde{13}, \tilde{14}, \tilde{15}\}$$

and it is an implicative ideal of the neutrosophic quadruple BCK-algebra $\mathcal{N}_q(Y)$ where

$$\tilde{0} = (0, 0T, 0I, 0F), \tilde{1} = (0, 0T, 0I, bF), \tilde{2} = (0, 0T, bI, 0F), \tilde{3} = (0, 0T, bI, bF),$$

$$\tilde{4} = (0, aT, 0I, 0F), \tilde{5} = (0, aT, 0I, bF), \tilde{6} = (0, aT, bI, 0F), \tilde{7} = (0, aT, bI, bF),$$

$$\tilde{8} = (a, 0T, 0I, 0F), \tilde{9} = (a, 0T, 0I, bF), \tilde{10} = (a, 0T, bI, 0F), \tilde{11} = (a, 0T, bI, bF),$$

$$\tilde{12} = (a, aT, 0I, 0F), \tilde{13} = (a, aT, 0I, bF), \tilde{14} = (a, aT, bI, 0F), \tilde{15} = (a, aT, bI, bF).$$

Proposition 2. *If I and J are implicative ideals of a BCK-algebra Y, then the neutrosophic quadruple (I,J)-set $\mathcal{N}_q(I,J)$ satisfies the following assertion:*

$$(\forall \tilde{x}, \tilde{y} \in \mathcal{N}_q(Y)) \left(\tilde{y} \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x}) \in \mathcal{N}_q(I,J) \;\Rightarrow \tilde{x} \,\tilde{*}\, (\tilde{x} \,\tilde{*}\, \tilde{y}) \in \mathcal{N}_q(I,J)\right). \tag{17}$$

Proof. Assume that I and J are implicative ideals of a BCK-algebra Y and $\tilde{y} \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x}) \in \mathcal{N}_q(I,J)$ for all $\tilde{x}, \tilde{y} \in \mathcal{N}_q(Y)$. Then,

$$\tilde{y} \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x}) = (y_1 * (y_1 * x_1), (y_2 * (y_2 * x_2))T, (y_3 * (y_3 * x_3))I, (y_4 * (y_4 * x_4))F) \in \mathcal{N}_q(I,J),$$

and so $y_1 * (y_1 * x_1) \in I$, $y_2 * (y_2 * x_2) \in I$, $y_3 * (y_3 * x_3) \in J$ and $y_4 * (y_4 * x_4) \in J$. Since $x_i * (x_i * y_i) \leq x_i$ for $i = 1,2,3,4$, we have

$$y_i * x_i \leq y_i * (x_i * (x_i * y_i)),$$

which implies that

$$\begin{aligned}(x_i * (x_i * y_i)) * (y_i * (x_i * (x_i * y_i))) &\leq (x_i * (x_i * y_i)) * (y_i * x_i) \\ &= (x_i * (y_i * x_i)) * (x_i * y_i) \leq y_i * (y_i * x_i),\end{aligned}$$

that is, $((x_i * (x_i * y_i)) * (y_i * (x_i * (x_i * y_i)))) * (y_i * (y_i * x_i)) = 0 \in I \cap J$ for $i = 1,2,3,4$. Since $y_i * (y_i * x_i) \in I$ for $i = 1,2$, $y_j * (y_j * x_j) \in J$ for $j = 3,4$, and I and J are implicative ideals of Y, it follows from (10) that $x_i * (x_i * y_i) \in I$ for $i = 1,2$, and $x_j * (x_j * y_j) \in J$ for $j = 3,4$. Hence,

$$\tilde{x} \,\tilde{*}\, (\tilde{x} \,\tilde{*}\, \tilde{y}) = (x_1 * (x_1 * y_1), (x_2 * (x_2 * y_2))T, (x_3 * (x_3 * y_3))I, (x_4 * (x_4 * y_4))F) \in \mathcal{N}_q(I,J).$$

This completes the proof. □

Lemma 4 ([18]). *If I and J are ideals of a BCK-algebra Y, then the neutrosophic quadruple (I,J)-set $\mathcal{N}_q(I,J)$ is an ideal of $\mathcal{N}_q(Y)$.*

Theorem 5. *Let I and J be ideals of a BCK-algebra Y such that*

$$(x * y) * y \in I \text{ (resp., } J) \;\Rightarrow\; x * y \in I \text{ (resp., } J), \tag{18}$$

$$y * (y * x) \in I \text{ (resp., } J) \;\Rightarrow\; x * (x * y) \in I \text{ (resp., } J) \tag{19}$$

for all $x, y \in Y$. Then, the neutrosophic quadruple (I,J)-set $\mathcal{N}_q(I,J)$ is an implicative ideal of $\mathcal{N}_q(Y)$.

Proof. If I and J are ideals of a BCK-algebra Y, then $\mathcal{N}_q(I,J)$ is an ideal of $\mathcal{N}_q(Y)$ by Lemma 4. Suppose $(\tilde{x} \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x})) \,\tilde{*}\, \tilde{z} \in \mathcal{N}_q(I,J)$ and $\tilde{z} \in \mathcal{N}_q(I,J)$ for all $\tilde{x}, \tilde{y}, \tilde{z} \in \mathcal{N}_q(Y)$. Then,

$$\begin{aligned}(\tilde{x} \,\tilde{*}\, (\tilde{y} \,\tilde{*}\, \tilde{x})) \,\tilde{*}\, \tilde{z} = \big(&(x_1 * (y_1 * x_1)) * z_1, ((x_2 * (y_2 * x_2)) * z_2)T, \\ &((x_3 * (y_3 * x_3)) * z_3)I, ((x_4 * (y_4 * x_4)) * z_4)F\big) \in \mathcal{N}_q(I,J)\end{aligned}$$

and $\tilde{z} = (z_1, z_2T, z_3I, z_4F) \in \mathcal{N}_q(I,J)$. It follows that $z_1, z_2 \in I$, $z_3, z_4 \in J$, $(x_1 * (y_1 * x_1)) * z_1 \in I$, $(x_2 * (y_2 * x_2)) * z_2 \in I$, $(x_3 * (y_3 * x_3)) * z_3 \in J$ and $(x_4 * (y_4 * x_4)) * z_4 \in J$. Since I and J are ideals of Y, we have $x_1 * (y_1 * x_1) \in I$, $x_2 * (y_2 * x_2) \in I$, $x_3 * (y_3 * x_3) \in J$ and $x_4 * (y_4 * x_4) \in J$. Since $(y_i * (y_i * x_i)) * (y_i * x_i) \leq x_i * (y_i * x_i)$ for $i = 1,2,3,4$, it follows that $(y_i * (y_i * x_i)) * (y_i * x_i) \in I$ for $i = 1,2$, and $(y_j * (y_j * x_j)) * (y_j * x_j) \in J$ for $j = 3,4$. Using (18), we obtain $y_i * (y_i * x_i) \in I$ for $i = 1,2$, and $y_j * (y_j * x_j) \in J$ for $j = 3,4$. It follows from (19) that

$$x_i * (x_i * y_i) \in I \text{ for } i = 1,2, \text{ and } x_j * (x_j * y_j) \in J \text{ for } j = 3,4. \tag{20}$$

Note that $(x_i * y_i) * z_i \leq x_i * y_i \leq x_i * (y_i * x_i)$ for $i = 1,2,3,4$. Thus, $(x_i * y_i) * z_i \in I$ for $i = 1,2$, and $(x_i * y_i) * z_i \in J$ for $j = 3,4$. Since $z_1, z_2 \in I$ and $z_3, z_4 \in J$, we obtain $x_i * y_i \in I$ for $i = 1,2$, and $x_j * y_j \in J$ for $j = 3,4$, which imply from (20) that $x_1, x_2 \in I$ and $x_3, x_4 \in J$. Hence, $\tilde{x} = (x_1, x_2T, x_3I, x_4F) \in \mathcal{N}_q(I,J)$, and therefore $\mathcal{N}_q(I,J)$ is an implicative ideal of $\mathcal{N}_q(Y)$. □

Theorem 6. *Let I and J be ideals of a BCK-algebra Y such that*

$$x * (y * x) \in I \text{ (resp., } J) \Rightarrow x \in I \text{ (resp., } J) \tag{21}$$

for all $x, y \in Y$. Then, the neutrosophic quadruple (I,J)-set $\mathcal{N}_q(I,J)$ is an implicative ideal of $\mathcal{N}_q(Y)$.

Proof. If I and J are ideals of a BCK-algebra Y, then $\mathcal{N}_q(I,J)$ is an ideal of $\mathcal{N}_q(Y)$ by Lemma 4. Let $\tilde{x}, \tilde{y}, \tilde{z} \in \mathcal{N}_q(Y)$ be such that $(\tilde{x} \tilde{*} (\tilde{y} \tilde{*} \tilde{x})) \tilde{*} \tilde{z} \in \mathcal{N}_q(I,J)$ and $\tilde{z} \in \mathcal{N}_q(I,J)$. Then,

$$\begin{aligned}(\tilde{x} \tilde{*} (\tilde{y} \tilde{*} \tilde{x})) \tilde{*} \tilde{z} = \big(&(x_1 * (y_1 * x_1)) * z_1, ((x_2 * (y_2 * x_2)) * z_2)T, \\ &((x_3 * (y_3 * x_3)) * z_3)I, ((x_4 * (y_4 * x_4)) * z_4)F\big) \in \mathcal{N}_q(I,J)\end{aligned}$$

and $\tilde{z} = (z_1, z_2T, z_3I, z_F) \in \mathcal{N}_q(I,J)$. It follows that $z_i \in I$, $(x_i * (y_i * x_i)) * z_i \in I$ for $i = 1,2$ and $z_j \in J$, $(x_j * (y_j * x_j)) * z_j \in J$ for $j = 3,4$. Since I and J are ideals of Y, we have $x_i * (y_i * x_i) \in I$ for $i = 1,2$ and $x_j * (y_j * x_j) \in J$ for $j = 3,4$. Using (21), we get $x_1, x_2 \in I$ and $x_3, x_4 \in J$. Hence $\tilde{x} = (x_1, x_2T, x_3I, x_4F) \in \mathcal{N}_q(I,J)$, and therefore $\mathcal{N}_q(I,J)$ is an implicative ideal of $\mathcal{N}_q(Y)$. □

Lemma 5 ([20]). *If I is an implicative ideal of a BCK-algebra Y, then every ideal A containing I is implicative.*

Theorem 7. *Let A, B, I and J be ideals of a BCK-algebra Y such that $A \subseteq I$ and $B \subseteq J$. If A and B are implicative ideals of Y, then the neutrosophic quadruple (I,J)-set $\mathcal{N}_q(I,J)$ is an implicative ideal of $\mathcal{N}_q(Y)$.*

Proof. If A, B, I and J are ideals of Y, then $\mathcal{N}_q(A,B)$ and $\mathcal{N}_q(I,J)$ are ideals of $\mathcal{N}_q(Y)$ by Lemma 4 and $\mathcal{N}_q(A,B) \subseteq \mathcal{N}_q(I,J)$. Since A and B are implicative ideals of Y, it follows from Theorem 4 that $\mathcal{N}_q(A,B)$ is an implicative ideal of $\mathcal{N}_q(Y)$. Therefore, the neutrosophic quadruple (I,J)-set $\mathcal{N}_q(I,J)$ is an implicative ideal of $\mathcal{N}_q(Y)$ by Lemma 5. □

4. Conclusions

Based on the concept of neutrosophic quadruple numbers which is introduced by Florentin Smarandache, Jun et al. have introduced the notion of neutrosophic quadruple BCK/BCI-numbers, and have studied neutrosophic quadruple BCK/BCI-algebras. As a continuation of Jun et al.'s paper which has been published in Axioms, we have introduced the notion of implicative neutrosophic quadruple BCK-algebras and have investigated several properties. Given a set Y, we have provided conditions for the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ to be a neutrosophic quadruple BCI-algebra, and have considered conditions for the neutrosophic quadruple Y-set $\mathcal{N}_q(Y)$ to be an implicative neutrosophic quadruple BCK-algebra. Given subsets I and J of a BCK-algebra Y, we have discussed conditions for the neutrosophic quadruple (I,J)-set $\mathcal{N}_q(I,J)$ to be an implicative ideal of the neutrosophic quadruple BCK-algebra $\mathcal{N}_q(Y)$. In the forthcoming research and papers, we will continue these ideas and will define new notions. We will study several kinds of neutrosophic quadruple ideals in neutrosophic quadruple BCK/BCI-algebras.

Author Contributions: Conceiving the idea, G.M. and Y.B.J.; literature review, A.N.A., E.H.R.; writing—original draft preparation, G.M.; review and editing, G.M., A.N.A., E.H.R.; supervision, Y.B.J.

Funding: The first author is partially supported by the research grant S-0064-1439, Deanship of Scientific Research, University of Tabuk, Tabuk-71491, Saudi Arabia.

Acknowledgments: The authors are grateful to the learned referees for careful detailed reading and helpful comments/suggestions that improved the overall presentation of this paper.

Conflicts of Interest: The authors declare no conflict of interest.

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