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Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses

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ABSTRACT

Objective: In pattern recognition and medical diagnosis, similarity measure is an important mathematical tool. To overcome some disadvantages of existing cosine similarity measures of simplified neutrosophic sets (SNSs) in vector space, this paper proposed improved cosine similarity measures of SNSs based on cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures. Then, weighted cosine similarity measures of SNSs were introduced by taking into account the importance of each element. Further, a medical diagnosis method using the improved cosine similarity measures was proposed to solve medical diagnosis problems with simplified neutrosophic information.
Materials and methods: The improved cosine similarity measures between SNSs were introduced based on cosine function. Then, we compared the improved cosine similarity measures of SNSs with existing cosine similarity measures of SNSs by numerical examples to demonstrate their effectiveness and rationality for overcoming some shortcomings of existing cosine similarity measures of SNSs in some cases. In the medical diagnosis method, we can find a proper diagnosis by the cosine similarity measures between the symptoms and considered diseases which are represented by SNSs. Then, the medical diagnosis method based on the improved cosine similarity measures was applied to two medical diagnosis problems to show the applications and effectiveness of the proposed method.
Results: Two numerical examples all demonstrated that the improved cosine similarity measures of SNSs based on the cosine function can overcome the shortcomings of the existing cosine similarity measures between two vectors in some cases. By two medical diagnoses problems, the medical diagnoses using various similarity measures of SNSs indicated the identical diagnosis results and demonstrated the effectiveness and rationality of the diagnosis method proposed in this paper.
Conclusions: The improved cosine measures of SNSs based on cosine function can overcome some drawbacks of existing cosine similarity measures of SNSs in vector space, and then their diagnosis method is very suitable for handling the medical diagnosis problems with simplified neutrosophic information and demonstrates the effectiveness and rationality of medical diagnoses.
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1. Introduction

Due to the increased volume of information available to physicians from modem medical technologies, medical diagnosis contains a lot of incomplete, uncertainty, and inconsistent information, which is essential information about medical diagnosis problems. A symptom usually implies a lot of incomplete, uncertainty, and inconsistent information for a disease, which characterizes a relation between symptoms and diseases. Thus we work with the uncertainties and inconsistencies to lead us to proper decision making in medicine. In most of the medical diagnosis problems, there exist some patterns, and then the experts make a decision based on the similarity between unknown sample and the basic diagnosis patterns. In some practical situations, there is the possibility of each element having different truth-membership, indeterminacy-membership, and falsity-membership functions. Therefore, Smarandache [1] originally proposed the concept of a neutrosophic set from philosophical point of view. A neutrosophic set *A* in a universal set *X* is characterized independently by

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a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. However, the domain of definition and range of the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ in a neutrosophic set A is the non-standard unit interval $]^-0, 1^+[$, it is only used for philosophical applications, especially when a distinction is required between absolute and relative truth/falsity/indeterminacy. To easily use in technical applications of the neutrosophic set, the domain of definition and range of $T_A(x)$, $I_A(x)$ and $F_A(x)$ can be restrained to the normal standard real unit interval [0, 1]. As a simplified form of the neutrosophic set, a simplified neutrosophic set (SNS) in [2] is an appropriate choice as it easily expresses and deals with incomplete, uncertainty, and inconsistent information in real science and engineering fields. SNSs include single valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs) and a generalization of classic sets, fuzzy sets (FSs) [3], intuitionistic fuzzy sets (IFSs) [4] and interval-valued intuitionistic fuzzy sets (IVIFSs) [5]. However, FSs, IFSs and IVIFSs cannot represent and handle uncertainty and inconsistent information [1]. Then, similarity measure is not only an important mathematical tool in pattern recognition, medicine diagnosis, and decision making but also an important research topic in the neutrosophic theory. Various similarity measures have been proposed by some researchers. Broumi and Smarandache [6] defined the Hausdorff distance between neutrosophic sets and some similarity measures based on the distance, set theoretic approach, and matching function to calculate the similarity degree between neutrosophic sets. Majumdar and Samanta [7] introduced several similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [8] also presented the Hamming and Euclidean distances between INSs and their similarity measures and applied them to multiple attribute decision-making problems with interval neutrosophic information. Ye [9] further proposed the distance-based similarity measure of SVNSs and applied it to group decision making problems with single valued neutrosophic information. Furthermore, Ye [2] proposed three vector similarity measures for SNSs, including the Jaccard, Dice, and cosine similarity measures for SVNSs and INSs, and applied them to multicriteria decision-making problems with simplified neutrosophic information. Till now, existing similarity measures for neutrosophic sets are scarcely applied to medical diagnosis problems. However, the cosine similarity measures defined in vector space [2,10] have some drawbacks in some situations. For instance, they may produce no defined (unmeaningful) phenomena or some results calculated by the cosine similarity measures are unreasonable in some real cases (details given in Section 3). Therefore, in the situations, it is difficult to apply them to pattern recognition and medicine diagnosis. To overcome some drawbacks of existing cosine measures in [2], this paper aims to propose improved cosine similarity measures for SNSs and apply them to medical diagnosis. To do so, the rest of the article is structured as follows. In Section 2, we briefly introduce some basic concepts of SNSs. Section 3 reviews existing cosine similarity measures of SNSs in vector space and their drawbacks. Section 4 proposes improved cosine similarity measures of SNSs based on cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures, and investigates their properties. In Section 5, by two numerical examples we give the comparative analysis between the improved cosine similarity measures and existing cosine similarity measures for SNSs to show the effectiveness and rationality of the improved cosine measures. In Section 6, a medical diagnosis method is proposed based on the improved cosine similarity measures and is applied to medical diagnosis problems. Conclusions and further research are given in Section 7.

2. Some basic concepts of SNSs

Smarandache [1] originally presented the concept of a neutrosophic set from philosophical point of view. In a neutrosophic set A in a universal set X, its characteristic functions are expressed by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, respectively. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ is no restriction, i.e., $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

To apply a neutrosophic set to science and engineering areas, Ye [2] introduced SNS, which is a subclass of the neutrosophic set, and gave the following definition of a SNS.

Definition 1 *([2])*. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0, 1], such that $T_A(x)$: $X \to [0, 1]$, $I_A(x)$: $X \to [0, 1]$, and $F_A(x)$: $X \to [0, 1]$. Then, a simplification of the neutrosophic set A is denoted by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\},$$

which is called a SNS. It is a subclass of the neutrosophic set and includes the concepts of INS and SVNS.

On the one hand, if we only consider that the values of $T_A(x)$, $I_A(x)$ and $F_A(x)$ in a SNS A are single points in the real standard [0, 1] instead of subintervals/subsets in the real standard [0, 1], the SNS A can be described by three real numbers in the real unit interval [0, 1]. Therefore, the sum of $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$. In this case, the SNS A reduces to the SVNS A.

For two SVNSs $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle | x \in X\}$, there are the following relations [11]:

(1) Complement: $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle | x \in X\}$;
(2) Inclusion: $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for any x in X;
(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

On the other hand, if we only consider three membership degrees in a SNS A as the subunit interval of the real unit interval [0, 1], the SNS can be described by three interval numbers in the real unit interval [0, 1]. For each point x in X, we have that $T_A(x) = [\inf T_A(x), \sup T_A(x)]$, $I_A(x) = [\inf I_A(x), \sup I_A(x)]$, $F_A(x) = [\inf F_A(x), \sup F_A(x)] \subseteq [0, 1]$, and $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$ for any $x \in X$. In this case, the SNS A reduces to the INS A.

For two INSs $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle \mid x \in X\}$, there are the following relations [12]:

(1) Complement: $A^c = \{\langle x, [\inf F_A(x), \sup F_A(x)], [1 - \sup I_A(x), 1 - \inf I_A(x)], [\inf T_A(x), \sup T_A(x)\rangle \mid x \in X\}$;
(2) Inclusion: $A \subseteq B$ if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$, and $\sup F_A(x) \ge \sup F_B(x)$, for any x in X;
(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Especially when the upper and lower ends of three interval numbers $T_A(x)$, $I_A(x)$, $F_A(x)$ in A are equal, the INS A degrades to the SVNS A. Therefore, the SVNS A is a special case of the INS A, and also both are the special cases of the SNS A.

3. Existing cosine similarity measures of SNSs and their drawbacks

In this section, we introduce existing cosine similarity measures for SNSs in the literature [2] and review their drawbacks.

Then, similarity measures should satisfy axiomatic requirements in the following definition.

Definition 2. A real-valued function S: $\mathrm{SNS}(X) \times \mathrm{SNS}(X) \to [0, 1]$ is called a similarity measure on SNS(X) if it satisfies the following axiomatic requirements for $A, B, C \in \mathrm{SNS}(X)$:

(S1) $0 \le S(A, B) \le 1$;
(S2) $S(A, B) = 1$ if and only if $A = B$;
(S3) $S(A, B) = S(B, A)$;
(S4) If $A \subseteq B \subseteq C$, then $S(A, C) \le S(A, B)$ and $S(A, C) \le S(B, C)$.

3.1. Existing cosine similarity measure for SVNSs and its drawbacks

In this section, we only use SVNSs in SNSs. Assume that there are two SVNSs $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle \mid x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle \mid x_j \in X\}$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, where $T_A(x_j), I_A(x_j), F_A(x_j) \in [0, 1]$ for any $x_j \in X$ in A and $T_B(x_j), I_B(x_j), F_B(x_j) \in [0, 1]$ for any $x_j \in X$ in B. Then, Ye [2] presented the cosine similarity measure of SVNSs in vector space as follows:

$$C_1(A, B) = \frac{1}{n}\sum_{j=1}^{n} \frac{T_A(x_j)T_B(x_j) + I_A(x_j)I_B(x_j) + F_A(x_j)F_B(x_j)}{\sqrt{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)}\sqrt{T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)}}. \tag{1}$$

However, one can find some drawbacks of Eq. (1) as follows:

(1) For two SVNSs A and B, if $T_A(x_j) = I_A(x_j) = F_A(x_j) = 0$ and/or $T_B(x_j) = I_B(x_j) = F_B(x_j) = 0$ for any x_j in X ($j = 1, 2, \ldots, n$), Eq. (1) is undefined or unmeaningful. In this case, one cannot utilize it to calculate the cosine similarity measure between A and B.
(2) If $T_A(x_j) = 2T_B(x_j)$, $I_A(x_j) = 2I_B(x_j)$, and $F_A(x_j) = 2F_B(x_j)$ or $2T_A(x_j) = T_B(x_j)$, $2I_A(x_j) = I_B(x_j)$, and $2F_A(x_j) = F_B(x_j)$ for any x_j in X ($j = 1, 2, \ldots, n$). By applying Eq. (1), we have

$$C_1(A, B) = \frac{1}{n}\sum_{j=1}^{n} \frac{2T_A(x_j)T_B(x_j) + 2I_A(x_j)I_B(x_j) + 2F_A(x_j)F_B(x_j)}{2\sqrt{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)}\sqrt{T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)}} = \frac{1}{n}\sum_{j=1}^{n} \frac{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)}{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)} = 1.$$

Since $A \neq B$, the measure value of Eq. (1) is equal to 1. This means that it only satisfies the necessary condition of the property (S2) in Definition 2, but not the sufficient condition.

Therefore, in this case, it is unreasonable to apply it to pattern recognition and medical diagnosis.

3.2. Existing cosine similarity measure for INSs and its drawbacks

In this section, we only use INSs in SNSs. Assume that there are two INSs $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle \mid x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle \mid x_j \in X\}$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, where $T_A(x_j) = [\inf T_A(x_j), \sup T_A(x_j)]$, $I_A(x_j) = [\inf I_A(x_j), \sup I_A(x_j)]$, $F_A(x_j) = [\inf F_A(x_j), \sup F_A(x_j)] \subseteq [0, 1]$ for any $x_j \in X$ in A and $T_B(x_j) = [\inf T_B(x_j), \sup T_B(x_j)]$, $I_B(x_j) = [\inf I_B(x_j), \sup I_B(x_j)]$, $F_B(x_j) = [\inf F_B(x_j), \sup F_B(x_j)] \subseteq [0, 1]$ for any $x_j \in X$ in B. Then, Ye [2] presented the cosine similarity measure of INSs in vector space as follows:

$$C_2(A, B) = \frac{1}{n}\sum_{j=1}^{n} \frac{\begin{pmatrix}\inf T_A(x_j)\inf T_B(x_j) + \inf I_A(x_j)\inf I_B(x_j) + \inf F_A(x_j)\inf F_B(x_j) \\ + \sup T_A(x_j)\sup T_B(x_j) + \sup I_A(x_j)\sup I_B(x_j) + \sup F_A(x_j)\sup F_B(x_j)\end{pmatrix}}{\begin{pmatrix}\sqrt{[\inf T_A(x_j)]^2 + [\inf I_A(x_j)]^2 + [\inf F_A(x_j)]^2 + [\sup T_A(x_j)]^2 + [\sup I_A(x_j)]^2 + [\sup F_A(x_j)]^2} \\ \sqrt{[\inf T_B(x_j)]^2 + [\inf I_B(x_j)]^2 + [\inf F_B(x_j)]^2 + [\sup T_B(x_j)]^2 + [\sup I_B(x_j)]^2 + [\sup F_B(x_j)]^2}\end{pmatrix}}. \tag{2}$$

Similarly, one can find some drawbacks of Eq. (2) as follows:

(1) For two INSs A and B, if $T_A(x_j)=I_A(x_j)=F_A(x_j)=[0, 0]$ and/or $T_B(x_j)=I_B(x_j)=F_B(x_j)=[0, 0]$ for any x_j in X ($j=1, 2, \ldots, n$), Eq. (2) is undefined or unmeaningful. In this case, one cannot calculate the cosine similarity measure between A and B.
(2) If $T_A(x_j)=[2\inf T_B(x_j), 2\sup T_B(x_j)]$, $I_A(x_j)=[2\inf I_B(x_j), 2\sup I_B(x_j)]$, and $F_A(x_j)=[2\inf F_B(x_j), 2\sup F_B(x_j)]$ or $T_B(x_j)=[2\inf T_A(x_j), 2\sup T_A(x_j)]$, $I_B(x_j)=[2\inf I_A(x_j), 2\sup I_A(x_j)]$, and $F_B(x_j)=[2\inf F_A(x_j), 2\sup F_A(x_j)]$ for any x_j in X ($j=1, 2, \ldots, n$). By using Eq. (2), we have

$$C_2(A,B)=\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\begin{array}{l}\inf T_A(x_j)\inf T_B(x_j)+\inf I_A(x_j)\inf I_B(x_j)+\inf F_A(x_j)\inf F_B(x_j)+\sup T_A(x_j)\sup T_B(x_j)\\+\sup I_A(x_j)\sup I_B(x_j)+\sup F_A(x_j)\sup F_B(x_j)\end{array}\right)}{\left(\begin{array}{l}2\sqrt{[\inf T_A(x_j)]^2+[\inf I_A(x_j)]^2+[\inf F_A(x_j)]^2+[\sup T_A(x_j)]^2+[\sup I_A(x_j)]^2+[\sup F_A(x_j)]^2}\\\sqrt{[\inf T_B(x_j)]^2+[\inf I_B(x_j)]^2+[\inf F_B(x_j)]^2+[\sup T_B(x_j)]^2+[\sup I_B(x_j)]^2+[\sup F_B(x_j)]^2}\end{array}\right)}$$
$$=\frac{1}{n}\sum_{j=1}^{n}\frac{\left([\inf T_A(x_j)]^2+[\inf I_A(x_j)]^2+[\inf F_A(x_j)]^2+[\sup T_A(x_j)]^2+[\sup I_A(x_j)]^2+[\sup F_A(x_j)]^2\right)}{\left([\inf T_A(x_j)]^2+[\inf I_A(x_j)]^2+[\inf F_A(x_j)]^2+[\sup T_A(x_j)]^2+[\sup I_A(x_j)]^2+[\sup F_A(x_j)]^2\right)}=1.$$

Since $A \neq B$, the measure value of Eq. (2) is equal to 1. This means that it only satisfies the necessary condition of the property (S2) in Definition 2, but not the sufficient condition.

Therefore, in this case, the cosine similarity measure of INSs is unreasonable in the application of pattern recognition and medical diagnosis.

In order to overcome the above mentioned disadvantages, we shall improve cosine similarity measures of SNSs in the following section.

4. Improved cosine similarity measures for SNSs

4.1. Improved cosine similarity measures for SVNSs

Based on cosine function, we propose two improved cosine similarity measures between SVNSs and investigate their properties.

Let $A=\{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle | x_j \in X\}$ and $B=\{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle | x_j \in X\}$ be any two SVNSs in $X=\{x_1, x_2, \ldots, x_n\}$, where $T_A(x_j), I_A(x_j), F_A(x_j) \in [0, 1]$ for any $x_j \in X$ in A and $T_B(x_j), I_B(x_j), F_B(x_j) \in [0, 1]$ for any $x_j \in X$ in B. Then, based on cosine function, we propose two improved cosine similarity measures between A and B, respectively, as follows:

$$SC_1(A,B)=\frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi(|T_A(x_j)-T_B(x_j)| \vee |I_A(x_j)-I_B(x_j)| \vee |F_A(x_j)-F_B(x_j)|)}{2}\right], \tag{3}$$

$$SC_2(A,B)=\frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi(|T_A(x_j)-T_B(x_j)| + |I_A(x_j)-I_B(x_j)| + |F_A(x_j)-F_B(x_j)|)}{6}\right], \tag{4}$$

where the symbol "$\vee$" is the maximum operation. Then, the two improved cosine similarity measures satisfy the axiomatic requirements of similarity measures.

Proposition 1. For two SVNSs A and B in $X=\{x_1, x_2, \ldots, x_n\}$, the cosine similarity measure $SC_k(A, B)$ ($k=1, 2$) should satisfy the following properties (S1–S4):

(S1) $0 \le SC_k(A, B) \le 1$;
(S2) $SC_k(A, B)=1$ if and only if $A=B$;
(S3) $SC_k(A, B)=SC_k(B, A)$;
(S4) If C is a SVNS in X and $A \subseteq B \subseteq C$, then $SC_k(A, C) \le SC_k(A, B)$ and $SC_k(A, C) \le SC_k(B, C)$.

Proof.

(S1) Since the truth-membership degree, indeterminacy-membership degree, and falsity-membership degree in SVNS and the value of the cosine function are within [0, 1], the similarity measure based on the cosine function also is within [0, 1]. Hence $0 \le SC_k(A, B) \le 1$ for $k=1, 2$.
(S2) For any two SVNSs A and B, if $A=B$, this implies $T_A(x_j)=T_B(x_j)$, $I_A(x_j)=I_B(x_j)$, $F_A(x_j)=F_B(x_j)$ for $j=1, 2, \ldots, n$ and $x_j \in X$. Hence $\left|T_A(x_j)-T_B(x_j)\right|=0$, $\left|I_A(x_j)-I_B(x_j)\right|=0$, and $\left|F_A(x_j)-F_B(x_j)\right|=0$. Thus $SC_k(A, B)=1$ for $k=1, 2$.If $SC_k(A, B)=1$ for $k=1, 2$, this implies $\left|T_A(x_j)-T_B(x_j)\right|=0$, $\left|I_A(x_j)-I_B(x_j)\right|=0$, and $\left|F_A(x_j)-F_B(x_j)\right|=0$ since $\cos(0)=1$. Then, these equalities indicate $T_A(x_j)=T_B(x_j)$, $I_A(x_j)=I_B(x_j)$, $F_A(x_j)=F_B(x_j)$ for $j=1, 2, \ldots, n$ and $x_j \in X$. Hence $A=B$.
(S3) Proof is straightforward.

(S4) If $A \subseteq B \subseteq C$, then there are $T_A(x_j) \le T_B(x_j) \le T_C(x_j)$, $I_A(x_j) \ge I_B(x_j) \ge I_C(x_j)$, and $F_A(x_j) \ge F_B(x_j) \ge F_C(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Then, we have the following inequalities:

$$|T_A(x_j) - T_B(x_j)| \le |T_A(x_j) - T_C(x_j)|, \quad |T_B(x_j) - T_C(x_j)| \le |T_A(x_j) - T_C(x_j)|,$$
$$|I_A(x_j) - I_B(x_j)| \le |I_A(x_j) - I_C(x_j)|, \quad |I_B(x_j) - I_C(x_j)| \le |I_A(x_j) - I_C(x_j)|,$$
$$|F_A(x_j) - F_B(x_j)| \le |F_A(x_j) - F_C(x_j)|, \quad |F_B(x_j) - F_C(x_j)| \le |F_A(x_j) - F_C(x_j)|$$

Hence, $SC_k(A, C) \le SC_k(A, B)$ and $SC_k(A, C) \le SC_k(B, C)$ for $k = 1, 2$ since the cosine function is a decreasing function within the interval [0, π/2].

Therefore, we complete the proofs of these properties. □

Usually, one takes the weight of each element x_j for $x_j \in X$ into account and assumes that the weight of an element x_j is w_j ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Thus we can introduce the following weighted cosine similarity measures between SVNSs:

$$WSC_1(A, B) = \sum_{j=1}^{n} w_j \cos\left[\frac{\pi(|T_A(x_j) - T_B(x_j)| \vee |I_A(x_j) - I_B(x_j)| \vee |F_A(x_j) - F_B(x_j)|)}{2}\right], \quad (5)$$

$$WSC_2(A, B) = \sum_{j=1}^{n} w_j \cos\left[\frac{\pi(|T_A(x_j) - T_B(x_j)| + |I_A(x_j) - I_B(x_j)| + |F_A(x_j) - F_B(x_j)|)}{6}\right], \quad (6)$$

Especially when $w_j = 1/n$ for $j = 1, 2, \ldots, n$, Eqs. (5) and (6) reduce to Eqs. (3) and (4).

4.2. Improved cosine similarity measures for INSs

Similarly, we propose two improved cosine similarity measures between INSs and investigate their properties.

Let $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle |\ x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle |\ x_j \in X\}$ be any two INSs in $X = \{x_1, x_2, \ldots, x_n\}$, where $T_A(x_j) = [\inf T_A(x_j), \sup T_A(x_j)]$, $I_A(x_j) = [\inf I_A(x_j), \sup I_A(x_j)]$, $F_A(x_j) = [\inf F_A(x_j), \sup F_A(x_j)] \subseteq [0,1]$ for any $x_j \in X$ in A and $T_B(x_j) = [\inf T_B(x_j), \sup T_B(x_j)]$, $I_A(x_j) = [\inf I_B(x_j), \sup I_B(x_j)]$, $F_B(x_j) = [\inf F_B(x_j), \sup F_B(x_j)] \subseteq [0, 1]$ for any $x_j \in X$ in B. Then, based on the cosine function, we propose two improved cosine similarity measures between A and B, respectively, as follows:

$$SC_3(A, B) = \frac{1}{n}\sum_{j=1}^{n} \cos\left[\frac{\pi}{4}\Big(|\inf T_A(x_j) - \inf T_B(x_j)| \vee |\inf I_A(x_j) - \inf I_B(x_j)| \vee |\inf F_A(x_j) - \inf F_B(x_j)| \right.$$
$$\left. + |\sup T_A(x_j) - \sup T_B(x_j)| \vee |\sup I_A(x_j) - \sup I_B(x_j)| \vee |\sup F_A(x_j) - \sup F_B(x_j)|\Big)\right], \quad (7)$$

$$SC_4(A, B) = \frac{1}{n}\sum_{j=1}^{n} \cos\left[\frac{\pi}{12}\Big(\left|\inf T_A(x_j) - \inf T_B(x_j)\right| + \left|\inf I_A(x_j) - \inf I_B(x_j)\right| + \left|\inf F_A(x_j) - \inf F_B(x_j)\right| \right.$$
$$\left. + \left|\sup T_A(x_j) - \sup T_B(x_j)\right| + \left|\sup I_A(x_j) - \sup I_B(x_j)\right| + \left|\sup F_A(x_j) - \sup F_B(x_j)\right|\Big)\right], \quad (8)$$

where the symbol "∨" is the maximum operation. Then, the two improved cosine similarity measures of INSs satisfy the axiomatic requirements in Definition 2.

Proposition 2. For two INSs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the cosine similarity measure $SC_k(A, B)$ ($k = 3, 4$) should satisfy the following properties (S1–S4):

(S1) $0 \le SC_k(A, B) \le 1$;
(S2) $SC_k(A, B) = 1$ if and only if $A = B$;
(S3) $SC_k(A, B) = SC_k(B, A)$;
(S4) If C is an INS in X and $A \subseteq B \subseteq C$, then $SC_k(A, C) \le SC_k(A, B)$ and $SC_k(A, C) \le SC_k(B, C)$.

Proof.

(S1) Since the truth-membership degree, indeterminacy-membership degree, and falsity-membership degree in an INS and the value of the cosine function are within [0, 1], the similarity measure value based on the cosine function also is within [0, 1]. Thus $0 \le SC_k(A, B) \le 1$ for $k = 3, 4$.

(S2) For any two INSs A and B, if $A = B$, this implies $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Hence $\left|\inf T_A(x_j) - \inf T_B(x_j)\right| = 0$, $\left|\inf I_A(x_j) - \inf I_B(x_j)\right| = 0$, $\left|\inf F_A(x_j) - \inf F_B(x_j)\right| = 0$, $\left|\sup T_A(x_j) - \sup T_B(x_j)\right| = 0$, $\left|\sup I_A(x_j) - \sup I_B(x_j)\right| = 0$, and $\left|\sup F_A(x_j) - \sup F_B(x_j)\right| = 0$. Thus $SC_k(A, B) = 1$ for $k = 3, 4$.If $SC_k(A, B) = 1$ for $k = 3, 4$, this implies $\left|\inf T_A(x_j) - \inf T_B(x_j)\right| = 0$, $\left|\inf I_A(x_j) - \inf I_B(x_j)\right| = 0$, $\left|\inf F_A(x_j) - \inf F_B(x_j)\right| = 0$, $\left|\sup T_A(x_j) - \sup T_B(x_j)\right| = 0$,

$\left|\sup I_A(x_j) - \sup I_B(x_j)\right| = 0$, and $\left|\sup F_A(x_j) - \sup F_B(x_j)\right| = 0$ since cos(0) = 1. Then, these equalities indicate $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Hence $A = B$.

(S3) Proof is straightforward.

(S4) If $A \subseteq B \subseteq C$, then there are $\inf T_A(x_j) \le \inf T_B(x_j) \le \inf T_C(x_j)$, $\sup T_A(x_j) \le \sup T_B(x_j) \le \sup T_C(x_j)$, $\inf I_A(x_j) \ge \inf I_B(x_j) \ge \inf I_C(x_j)$, $\sup I_A(x_j) \ge \sup I_B(x_j) \ge \sup I_C(x_j)$, $\inf F_A(x_j) \ge \inf F_B(x_j) \ge \inf F_C(x_j)$, and $\sup F_A(x_j) \ge \sup F_B(x_j) \ge \sup F_C(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Then, we have the following inequalities:

$$|\inf T_A(x_j) - \inf T_B(x_j)| \le |\inf T_A(x_j) - \inf T_C(x_j)|,$$
$$|\inf T_B(x_j) - \inf T_C(x_j)| \le |\inf T_A(x_j) - \inf T_C(x_j)|,$$
$$|\sup T_A(x_j) - \sup T_B(x_j)| \le |\sup T_A(x_j) - \sup T_C(x_j)|,$$
$$|\sup T_B(x_j) - \sup T_C(x_j)| \le |\sup T_A(x_j) - \sup T_C(x_j)|,$$
$$|\inf I_A(x_j) - \inf I_B(x_j)| \le |\inf I_A(x_j) - \inf I_C(x_j)|,$$
$$|\inf I_B(x_j) - \inf I_C(x_j)| \le |\inf I_A(x_j) - \inf I_C(x_j)|,$$
$$|\sup I_A(x_j) - \sup I_B(x_j)| \le |\sup I_A(x_j) - \sup I_C(x_j)|,$$
$$|\sup I_B(x_j) - \sup I_C(x_j)| \le |\sup I_A(x_j) - \sup I_C(x_j)|,$$
$$|\inf F_A(x_j) - \inf F_B(x_j)| \le |\inf F_A(x_j) - \inf F_C(x_j)|,$$
$$|\inf F_B(x_j) - \inf F_C(x_j)| \le |\inf F_A(x_j) - \inf F_C(x_j)|,$$
$$|\sup F_A(x_j) - \sup F_B(x_j)| \le |\sup F_A(x_j) - \sup F_C(x_j)|,$$
$$|\sup F_B(x_j) - \sup F_C(x_j)| \le |\sup F_A(x_j) - \sup F_C(x_j)|.$$

Since the cosine function is a decreasing function within the interval [0, π/2], hence $SC_k(A, C) \le SC_k(A, B)$ and $SC_k(A, C) \le SC_k(B, C)$ for $k = 3, 4$.

Thus, we complete the proofs of these properties. □

When one takes the weight of each element x_j for $x_j \in X$ into account and assumes that the weight of an element x_j is w_j ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, we can introduce the following weighted cosine similarity measures between INSs A and B:

$$WSC_3(A, B) = \sum_{j=1}^{n} w_j \cos\left[\frac{\pi}{4}\left(|\inf T_A(x_j) - \inf T_B(x_j)| \vee |\inf I_A(x_j) - \inf I_B(x_j)| \vee |\inf F_A(x_j) - \inf F_B(x_j)| + |\sup T_A(x_j) - \sup T_B(x_j)| \vee |\sup I_A(x_j) - \sup I_B(x_j)| \vee |\sup F_A(x_j) - \sup F_B(x_j)|\right)\right], \quad (9)$$

$$WSC_4(A, B) = \sum_{j=1}^{n} w_j \cos\left[\frac{\pi}{12}\left(|\inf T_A(x_j) - \inf T_B(x_j)| + |\inf I_A(x_j) - \inf I_B(x_j)| + |\inf F_A(x_j) - \inf F_B(x_j)| + |\sup T_A(x_j) - \sup T_B(x_j)| + |\sup I_A(x_j) - \sup I_B(x_j)| + |\sup F_A(x_j) - \sup F_B(x_j)|\right)\right]. \quad (10)$$

Especially when $w_j = 1/n$ for $j = 1, 2, \ldots, n$, Eqs. (9) and (10) reduce to Eqs. (7) and (8). Then, when $T_A(x_j) = \inf T_A(x_j) = \sup T_A(x_j$, $I_A(x_j) = \inf I_A(x_j) = \sup I_A(x_j)$, and $F_A(x_j) = \inf F_A(x_j) = \sup F_A(x_j)$ for any $x_j \in X$ in A and $T_B(x_j) = \inf T_B(x_j) = \sup T_B(x_j)$, $I_B(x_j) = \inf I_B(x_j) = \sup I_B(x_j)$, $F_B(x_j) = \inf F_B(x_j) = \sup F_B(x_j)$ for any $x_j \in X$ in B, the INSs A and B reduce to the SVNSs A and B, and then Eqs. (7)–(10) reduce to Eqs. (3)–(6), respectively.

5. Comparative analyses of various cosine similarity measures

To compare the improved cosine measures with existing cosine measures [2] in simplified neutrosophic setting, we provide two numerical examples to demonstrate the effectiveness and rationality of the improved cosine similarity measures of SNSs.

Example 1. We consider two SVNSs A and B in $X = \{x\}$ and compare the improved cosine similarity measures with existing cosine similarity measure in [2]. The comparison of pattern recognitions is indicated by the numerical example in Table 1. By applying Eqs. (1), (3) and (4), these similarity measure results are shown in Table 1.

Example 2. Let us consider two INSs A and B in $X = \{x\}$ and compare the improved cosine similarity measures with existing cosine similarity measure in [2]. The comparison of pattern recognitions is demonstrated by the numerical example in Table 2. By using Eqs. (2), (7) and (8), these similarity measure results are shown in Table 2.

The results of Tables 1 and 2 show that the existing cosine similarity measures in [2] not only cannot carry out the recognition between Case 1 and Case 5 but also produces unreasonable phenomena for Case 5 and undefined (unmeaningful) phenomena for Case 4. This will get the decision maker into trouble in practical applications. However, the improved cosine similarity measures SC_1 and SC_3 cannot also carry

Table 1
Similarity measure values of Eqs. (1), (3) and (4).

	Case 1	Case 2	Case 3	Case 4	Case 5
A	$\langle x, 0.2, 0.3, 0.4\rangle$	$\langle x, 0.3, 0.2, 0.4\rangle$	$\langle x, 1, 0, 0\rangle$	$\langle x, 1, 0, 0\rangle$	$\langle x, 0.4, 0.2, 0.6\rangle$
B	$\langle x, 0.2, 0.3, 0.4\rangle$	$\langle x, 0.4, 0.2, 0.3\rangle$	$\langle x, 0, 1, 1\rangle$	$\langle x, 0, 0, 0\rangle$	$\langle x, 0.2, 0.1, 0.3\rangle$
$C_1(A, B)$ [2]	**1**	0.9655	0	**null**	**1**
$SC_1(A, B)$	1	0.9877	**0**	**0**	0.8910
$SC_2(A, B)$	1	0.9945	0	0.8660	0.9511

Table 2
Similarity measure values of Eqs. (2), (7) and (8).

	Case 1	Case 2	Case 3	Case 4	Case 5
A	$\langle x, [0.3, 0.5], [0.2, 0.4], [0, 0.1]\rangle$	$\langle x, [0.3, 0.5], [0.2, 0.4], [0.4, 0.5]\rangle$	$\langle x, [1, 1], [0, 0], [0, 0]\rangle$	$\langle x, [1, 1], [0, 0], [0, 0]\rangle$	$\langle x, [0.3, 0.4], [0.2, 0.3], [0.4, 0.5]\rangle$
B	$\langle x, [0.3, 0.5], [0.2, 0.4], [0, 0.1]\rangle$	$\langle x, [0.4, 0.5], [0.2, 0.4], [0.3, 0.5]\rangle$	$\langle x, [0, 0], [1, 1], [1, 1]\rangle$	$\langle x, [0, 0], [0, 0], [0, 0]\rangle$	$\langle x, [0.6, 0.8], [0.4, 0.6], [0.8, 1]\rangle$
$C_2(A, B)$ [2]	**1**	0.9895	0	**null**	**1**
$SC_3(A, B)$	1	0.9969	**0**	**0**	0.7604
$SC_4(A, B)$	1	0.9986	0	0.8660	0.8526

out the recognition between Case 3 and Case 4, but do not produce the undefined (unmeaningful) phenomena. Then, the improved cosine similarity measures SC_2 and SC_4 demonstrate stronger discrimination among them. Obviously, the improved cosine similarity measures are superior to the existing cosine similarity measures in [2]. Furthermore, the cosine similarity measures SC_2 and SC_4 are superior to the cosine similarity measures SC_1 and SC_3, respectively.

The two examples all demonstrate that in some cases the improved cosine similarity measures of SNSs based on the cosine function can overcome the disadvantages of the existing cosine similarity measures between two vectors.

6. Medical diagnoses using improved cosine similarity measures

Due to the increased volume of information available to physicians from modem medical technologies, medical diagnosis contains a lot of incomplete, uncertainty, and inconsistent information. In some practical situations, there is the possibility of each element having different truth-membership, indeterminacy-membership, and falsity-membership degrees, by which an SNS is expressed. Hence, similarity measures for SNSs are a suitable tool to deal with medical diagnosis problems with simplified neutrosophic information. Hereby, we apply the improved cosine similarity measures of SNSs to medical diagnosis.

In medical diagnosis problems, we propose a medical diagnosis method below.

Let us consider a set of diagnoses $Q = \{Q_1, Q_2, \ldots, Q_n\}$, and a set of symptoms $S = \{s_1, s_2, s_3, \ldots, s_m\}$. Assume that we take a sample from a patient P with all the symptoms. Then, the characteristic information of Q, S and P is represented by the form of SNSs. To find a proper diagnosis, we can calculate the cosine measure $SC_k(P, Q_i)$ ($k = 1, 2, 3, 4$; $i = 1, 2, \ldots, n$). The proper diagnosis Q_{i^*} for the patient P is derived by

$$i^* = arg \max_{1 \le i \le n} \{SC_k(P, Q_i)\}.$$

To illustrate the medical diagnostic process, we provide two medical diagnosis examples to demonstrate the applications and effectiveness of the medical diagnosis method using the improved cosine similarity measures.

6.1. Medical diagnosis under the single valued neutrosophic environment

Example 3. Let us consider the medical diagnosis problem adapted from [13]. Assume that a set of diagnoses is $Q = \{Q_1$ (viral fever), Q_2 (malaria), Q_3 (typhoid), Q_4 (gastritis), Q_5 (stenocardia)$\}$ and a set of symptoms is $S = \{s_1$ (fever), s_2 (headache), s_3 (stomach pain), s_4 (cough), s_5 (chest pain)$\}$. Then characteristic values of the considered diseases are represented by the form of SVNSs, which are shown in Table 3.

In the medical diagnosis, assume that we take a sample from a patient P_1 with all the symptoms, which is represented by the following SVNS information:

P_1 (patient) = $\{\langle s_1, 0.8, 0.2, 0.1\rangle, \langle s_2, 0.6, 0.3, 0.1\rangle, \langle s_3, 0.2, 0.1, 0.8\rangle, \langle s_4, 0.6, 0.5, 0.1\rangle, \langle s_5, 0.1, 0.4, 0.6\rangle\}$.

For convenient comparison, we utilize the existing cosine measure [2] and the two improved cosine measures to handle the diagnosis problem. By applying Eqs. (1), (3) and (4), we can obtain the results of the three similarity measures between the patient P_1 and the considered disease Q_i ($i = 1, 2, 3, 4, 5$), as shown in Table 4.

Table 3
Characteristic values of the considered diseases represented by the form of SVNSs.

	s_1 (fever)	s_2 (headache)	s_3 (stomach pain)	s_4 (cough)	s_5 (chest pain)
Q_1 (viral fever)	$\langle s_1, 0.4, 0.6, 0.0\rangle$	$\langle s_2, 0.3, 0.2, 0.5\rangle$	$\langle s_3, 0.1, 0.3, 0.7\rangle$	$\langle s_4, 0.4, 0.3, 0.3\rangle$	$\langle s_5, 0.1, 0.2, 0.7\rangle$
Q_2 (malaria)	$\langle s_1, 0.7, 0.3, 0.0\rangle$	$\langle s_2, 0.2, 0.2, 0.6\rangle$	$\langle s_3, 0.0, 0.1, 0.9\rangle$	$\langle s_4, 0.7, 0.3, 0.0\rangle$	$\langle s_5, 0.1, 0.1, 0.8\rangle$
Q_3 (typhoid)	$\langle s_1, 0.3, 0.4, 0.3\rangle$	$\langle s_2, 0.6, 0.3, 0.1\rangle$	$\langle s_3, 0.2, 0.1, 0.7\rangle$	$\langle s_4, 0.2, 0.2, 0.6\rangle$	$\langle s_5, 0.1, 0.0, 0.9\rangle$
Q_4 (gastritis)	$\langle s_1, 0.1, 0.2, 0.7\rangle$	$\langle s_2, 0.2, 0.4, 0.4\rangle$	$\langle s_3, 0.8, 0.2, 0.0\rangle$	$\langle s_4, 0.2, 0.1, 0.7\rangle$	$\langle s_5, 0.2, 0.1, 0.7\rangle$
Q_5 (stenocardia)	$\langle s_1, 0.1, 0.1, 0.8\rangle$	$\langle s_2, 0.0, 0.2, 0.8\rangle$	$\langle s_3, 0.2, 0.0, 0.8\rangle$	$\langle s_4, 0.2, 0.0, 0.8\rangle$	$\langle s_5, 0.8, 0.1, 0.1\rangle$

Table 4
Various similarity measure values for SVNS information.

	Viral fever (Q_1)	Malaria (Q_2)	Typhoid (Q_3)	Gastritis (Q_4)	Stenocardia (Q_5)
$C_1(P_1, Q_i)$ [2]	0.8505	**0.8661**	0.8185	0.5148	0.4244
$SC_1(P_1, Q_i)$	0.8942	**0.8976**	0.8422	0.6102	0.5607
$SC_2(P_1, Q_i)$	0.9443	**0.9571**	0.9264	0.8214	0.7650

Table 5
Various similarity measure values for INS information.

	Viral fever (Q_1)	Malaria (Q_2)	Typhoid (Q_3)	Gastritis (Q_4)	Stenocardia (Q_5)
$C_2(P_2, Q_i)$ [2]	0.6775	0.5613	**0.7741**	0.7198	0.6872
$SC_3(P_2, Q_i)$	0.7283	0.6079	**0.7915**	0.7380	0.7157
$SC_4(P_2, Q_i)$	0.8941	0.8459	**0.9086**	0.9056	0.8797

Since the largest similarity measure indicates the proper diagnosis, we can see from Table 4 that the patient P_1 suffers from malaria. Obviously, the medical diagnoses using various similarity measures of SVNSs indicate the same diagnosis results and demonstrate the effectiveness of these diagnoses. However, as mentioned above, since the improved cosine measures of SVNSs in this paper cannot produce undefined (unmeaningful) or unreasonable phenomena in some real cases, they can avoid some drawbacks of the existing cosine measure in [2] in some cases. Hence, the improved cosine measures of SVNSs are superior to the existing cosine measure of SVNSs.

To compare the diagnosis results of cosine similarity measures between SVNSs and between IFSs, here we quote the results of the cosine similarity measure of IFSs in [13] as follows:

$C_{IFS}(P_1, Q_1) = \mathbf{0.9046}$, $C_{IFS}(P_1, Q_2) = 0.8602$, $C_{IFS}(P_1, Q_3) = 0.8510$, $C_{IFS}(P_1, Q_4) = 0.5033$, and $C_{IFS}(P_1, Q_5) = 0.4542$.

Obviously, the diagnosis result of the patient P_1 is viral fever under the intuitionistic fuzzy environment, while the diagnosis result of the patient P_1 given in this paper is malaria under the single valued neutrosophic environment. Therefore, there are different diagnosis results under different environments. The reason is that the diagnosis method in [13] is on the basis of the cosine measure of IFSs, which only considers the truth-membership and falsity-membership degrees but not the indeterminate-membership degree; while the diagnosis method in this paper is based on the improved cosine measures of SVNSs and contains more information (the truth-membership, falsity-membership, and indeterminate-membership degrees). Therefore, different measure methods with different kinds of information represented by IFSs and SVNSs may give different diagnosis results. Furthermore, the diagnosis method in [13] cannot handle the diagnosis problem with single valued neutrosophic information and may produce an undefined (unmeaningful) or unreasonable phenomenon based on the cosine similarity measure of IFSs in some real cases, while the diagnosis method in this paper can deal with the diagnosis problems with intuitionistic fuzzy information and simplified neutrosophic information and overcome the indicated drawbacks. Hence, the diagnosis method based on the improved cosine measures of SVNSs is superior to the one of the cosine measure of IFSs [13].

6.2. Medical diagnosis under the interval neutrosophic environment

However, by only taking one time inspection, we wonder whether one can obtain a conclusion from a particular person with a particular decease or not. Hence, we have to examine the patient at different time intervals (e.g., two or three times a day) and can obtain that data collected from multiple time inspections for the patient are interval values rather than single values. In this case, the improved cosine measures of INSs are a better tool to find a proper disease diagnosis.

Example 4. Let us consider Example 3 again. Then, a patient P_2 with all the symptoms can be represented by the following INS information:

P_2 (patient) = {$\langle s_1$, [0.3, 0.5], [0.2, 0.3], [0.4, 0.5]$\rangle$, $\langle s_2$, [0.7, 0.9], [0.1, 0.2], [0.1, 0.2]$\rangle$, $\langle s_3$, [0.4, 0.6], [0.2, 0.3], [0.3, 0.4]$\rangle$, $\langle s_4$, [0.3, 0.6], [0.1, 0.3], [0.4, 0.7]$\rangle$, $\langle s_5$, [0.5, 0.8], [0.1, 0.4], [0.1, 0.3]$\rangle$}.

Similarly, we utilize the existing cosine measure [2] and the two improved cosine measures of INSs to handle the diagnosis problem. By applying Eqs. (2), (7) and (8), we can obtain the results of various similarity measures between the patient P_2 and the considered disease Q_i (i = 1, 2, 3, 4, 5), as shown in Table 5.

Therefore, we can see from Table 5 that the patient P_2 suffers from typhoid. Then, the medical diagnoses using various similarity measures of INSs indicate the same diagnosis results and demonstrate the effectiveness of these diagnoses. However, as aforementioned advantages of the improved cosine measures, the improved cosine measures between INSs can also overcome some drawbacks of the existing cosine measure of INSs in [2] in some cases and are superior to the cosine similarity measures of SVNSs because INSs are the extension of SVNSs and more suitable for expressing and handling medical diagnosis problems than SVNSs.

7. Conclusions

This paper proposed the improved cosine similarity measures for SNSs based on cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures. Then, the weighted cosine similarity measures of SNSs are introduced by considering the importance of each element. Compared with existing cosine similarity measures under simplified neutrosophic environment, the improved cosine measures of SNSs demonstrate their effectiveness and rationality and can overcome some drawbacks of existing cosine similarity measures of SNSs. Finally, two medical diagnosis problems with simplified neutrosophic information are provided to demonstrate the applications and effectiveness of the medical diagnosis method using the improved cosine similarity measures of SNSs.

In further work, it is necessary to apply the cosine similarity measures of SNSs to other areas such as decision making, image processing, and clustering analysis.

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