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## Review Article

Innovative Approaches for N -valued Interval Neutrosophic Sets and their Execution in Medical Diagnosis

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#### Abstract

The objective of the study was to find out the relationship between the disease and the symptoms seen within patients and diagnose the disease that impacted the patient using $n$-valued interval neutrosophic sets. Neoteric methods were devised in $n$-valued interval neutrosophic sets. Utilization of medical diagnosis was commenced with using prescribed procedures to identify a person suffering from the disease for a considerable period. The result showed that the proposed methods were free from shortcomings that affect the existing methods and found to be more accurate in diagnosing the diseases. It was concluded that the techniques adopted in this study were more reliable and easier to handle medical diagnosis problems.


Key words: N -valued interval neutrosophic set, grade function, logarithmic distance, exponential measure, medical diagnosis
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## INTRODUCTION

Kumbakonam is a thickly populated town. Although underground drainage system is available here, it is yet to cover all the houses in the town. So, open drainage system continues to be in practice in different places of the town. Further this town is racing fast towards total sanitation in all spheres. As a result, Kumbakonam continues to be a repository of all new kinds of diseases. This created an urge to carry out research in the medical field. By introducing innovative methods in the research, the diseases can be diagnosed instantly and infallibly.

A number of real life problems in engineering, medical sciences, social sciences, economics etc., involve imprecise data and their solution involves the use of mathematical principles based on uncertainty and imprecision. Such uncertainties are being dealt with the help of topics like probability theory, fuzzy set theory ${ }^{1}$, rough set theory ${ }^{2}$ etc., Healthcare industry has been trying to complement the services offered by conventional clinical decision making systems with the integration of fuzzy logic techniques in them. As it is not an easy task for a clinician to derive a fool proof diagnosis, it is advantageous to automate few initial steps of diagnosis which would not require intervention from an expert doctor. Neutrosophic set which is a generalized set possesses all attributes necessary to encode medical knowledge base and capture medical inputs.

As medical diagnosis demands large amount of information processing, large portion of which is quantifiable, also intuitive thought process involve rapid unconscious data processing and combines available information by law of average, the whole process offers low intra and inter personal consistency. So contradictions, inconsistency, indeterminacy and fuzziness should be accepted as unavoidable as they are integrated in the behavior of biological systems as well as in their characterization. To model an expert doctor it is imperative that it should not disallow uncertainty as it would be then inapt to capture fuzzy or incomplete knowledge that might lead to the danger of fallacies due to misplaced precision.

As medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different sets of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having different truth membership, indeterminate and false membership functions. The unique feature of $n$-valued interval neutrosophic set is that it contains multi truth membership, indeterminate and false membership.

By taking one time inspection, there may be error in diagnosis. Hence, multi time inspection, by taking the samples of the same patient at different times gives the best diagnosis. So, n-valued interval neutrosophic sets and their applications play a vital role in medical diagnosis.

In 1965, fuzzy set theory was firstly given by Zadeh ${ }^{1}$ which is applied in many real applications to handle uncertainty. Sometimes membership function itself is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy sets was proposed to capture the uncertainty of grade of membership. Atanassov ${ }^{3}$ introduced the intuitionistic fuzzy sets which consider both truth-membership and falsity-membership. De et al. ${ }^{4}$ presented an application of intuitionistic fuzzy set in medical diagnosis. $\mathrm{Ye}^{5}$ introduced the concept of cosine similarity measures for intuitionistic fuzzy sets. Miaoying ${ }^{6}$ presented the cotangent similarity function for intuitionistic fuzzy sets. Later on, intuitionistic fuzzy sets were extended to the interval valued intuitionistic fuzzy sets. Intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief systems. So, neutrosophic set (generalization of fuzzy sets, intuitionistic fuzzy sets and so on) defined by Smarandache ${ }^{7}$ has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exists in real world from philosophical point of view. Wang et al. ${ }^{8}$ proposed the single valued neutrosophic set. Similarity and entropy between neutrosophic sets were proposed by Mamjumdar and Samanta ${ }^{9}$. Wang et al. ${ }^{10}$ proposed the set theoretic operations on an instance of neutrosophic set is called interval valued neutrosophic set which is more flexible and practical than neutrosophic set. Similarity measures between interval valued neutrosophic sets were proposed by $\mathrm{Ye}^{11}$. Interval valued neutrosophic soft sets were introduced by Deli ${ }^{12}$.

Sebastian and Ramakrishnan ${ }^{13}$ studied a new concept called fuzzy multi sets (FMS), which is the generalization of multi sets. Shinoj and Sunil ${ }^{14}$ extended the concept of fuzzy multi sets by introducing intuitionistic fuzzy multi sets (IFMS). Rajarajeswari and Uma ${ }^{15}$ proposed the normalized hamming similarity measure between them. However, the concepts of FMS and IFMS are not capable of dealing with indeterminacy. Ye and $\mathrm{Ye}^{16}$ introduced the concept of single valued neutrosophic multi sets. Distance based similarity measures between them were introduced by Ye et al. ${ }^{17}$. Smarandache ${ }^{18}$ extended the classical neutrosophic logic to $n$-valued refined neutrosophic logic, by refining each neutrosophic component $\mathrm{T}, \mathrm{I}, \mathrm{F}$ into respectively, $\mathrm{T}_{1}, \mathrm{~T}_{2}, \cdots, \mathrm{~T}_{\mathrm{m}}, \mathrm{I}_{1}, \mathrm{I}_{2}, \cdots, \mathrm{I}_{\mathrm{p}}$ and $\mathrm{F}_{1}, \mathrm{~F}_{2}, \cdots, \mathrm{~F}_{\mathrm{T}}$. Deli et al. ${ }^{19}$ studied a new concept called neutrosophic refined
sets. Broumi and Deli ${ }^{20}$ proposed the correlation measure between them. Broumi et al. ${ }^{21}$ generalize the concept of n-valued neutrosophic sets to the case of $n$-valued interval neutrosophic sets.

In this study, using the notion of $n$-valued interval neutrosophic set was provided an exemplary for medical diagnosis. In order to make this, various methods were implemented.

## PRELIMINARIES

Interval neutrosophic set ${ }^{\mathbf{1 0}}$ : Let X be a space of points (objects), with a generic element in $X$ denoted by $x$. An interval neutrosophic set $A$ in $X$ is characterized by the truth-membership function $T_{A}$, indeterminacy-membership function $I_{A}$ and falsity-membership function $F_{A}$. For each point x in $\mathrm{X}, \mathrm{T}_{\mathrm{A}}(\mathrm{x}), \mathrm{I}_{\mathrm{A}}(\mathrm{x}), \mathrm{F}_{\mathrm{A}}(\mathrm{x}) \subseteq[0,1]$ with the condition that $0 \leq T_{A}(x)+I_{A}(x)+F_{A}(x) \leq 3$.

Interval neutrosophic relation ${ }^{\mathbf{1 0}}$ : Let $X$ and $Y$ be two non-empty crisp sets. An interval neutrosophic relation $R(X, Y)$ is a subset of product space $X \times Y$ and is characterized by the truth membership function $T_{R}(x, y)$, the indeterminacy membership function $I_{R}(x, y)$ and the falsity membership function $F_{R}(x, y)$, where $x \in X$ and $y \in Y$ and $T_{R}(x, y), I_{R}(x, y)$, $F_{R}(x, y) \subseteq[0,1]$.

Sup-star composition ${ }^{\mathbf{1 0}}$ : Let $X$ and $Y$ be two non-empty crisp sets. An interval neutrosophic relation $R(X, Y)$ is a subset of product space $X \times Y$ and is characterized by the membership functions for the composition of interval neutrosophic relations $R(X, Y)$ and $S(Y, Z)$ are given by the interval neutrosophic sup-star composition of $R$ and $S$ :

$$
\begin{align*}
& T_{R o s}(x, z)=\sup _{y \in Y} \min \left(T_{R}(x, y), T_{S}(y, z)\right) \\
& I_{R o s}(x, z)=\sup _{y \in Y} \min \left(I_{R}(x, y), I_{s}(y, z)\right) \\
& F_{R o s}(x, z)=\inf _{y \in Y} \max \left(F_{R}(x, y), F_{S}(y, z)\right) \tag{1}
\end{align*}
$$

where, $x \in X$ and $y \in Y$ and $T_{R}(x, y), I_{R}(x, y), F_{R}(x, y) \subseteq[0,1]$.
$\mathbf{N}$-valued interval neutrosophic set ${ }^{21}$ : Let X be a universe, a n-valued interval neutrosophic set on $X$ can be defined as follows:

Where:

$$
\begin{aligned}
& \inf T_{A}^{1}(x), \inf _{A}^{2}(x), \ldots, \inf T_{A}^{p}(x), \inf _{A}^{1}(x), \inf _{A}^{2}(x), \ldots, \inf I_{A}^{p}(x) \\
& \inf F_{A}^{1}(x), \inf _{F_{A}^{2}}^{2}(x), \ldots, \inf _{F_{A}^{p}}^{p}(x), \sup _{T_{A}^{1}}^{1}(x), \sup _{A}^{2}(x), \ldots, \sup _{T_{A}^{p}}^{p}(x) \\
& \sup _{I_{A}^{1}}^{1}(x), \sup _{A}^{2}(x), \ldots, \sup _{I_{A}^{p}}^{p}(x), \sup _{F_{A}^{1}}(x), \sup _{F_{A}^{2}}^{2}(x), \ldots, \sup _{F_{A}^{p}}^{p}(x) \in[0,1]
\end{aligned}
$$

Such that:

$$
0 \leq \sup _{A}^{j}(x)+\sup _{A}^{j}(x)+\sup _{F_{A}}^{j}(x) \leq 3 \forall j=1,2,3, \ldots, p
$$

Inclusion ${ }^{21}$ : A n-valued interval neutrosophic set $A$ is contained in the other $n$-valued interval neutrosophicset $B$, denoted by $A \subseteq B$ if and only if:

$$
\begin{align*}
& \operatorname{supT}_{A}^{1}(x) \leq \sup _{B}^{1}(x), \sup _{A}^{2}(x) \leq \sup _{B}^{2}(x), \ldots, \sup _{A}^{p}(x) \leq \sup _{B}^{p}(x) \\
& \inf I_{A}^{1}(x) \geq \inf _{I_{B}^{1}}^{1}(x), \inf I_{A}^{2}(x) \geq \inf I_{B}^{2}(x), \ldots, \inf I_{A}^{p}(x) \geq \inf I_{B}^{p}(x) \\
& \sup _{A}^{1}(x) \geq \sup _{I_{B}^{1}}^{1}(x), \sup _{I_{A}^{2}}^{2}(x) \geq \sup _{I_{B}^{2}}^{2}(x), \ldots, \sup _{I_{A}^{p}}^{p}(x) \geq \sup _{B}^{p}(x) \\
& \inf F_{A}^{1}(x) \geq \inf F_{B}^{1}(x), \inf F_{A}^{2}(x) \geq \inf _{F_{B}^{2}}^{2}(x), \ldots, \inf F_{A}^{p}(x) \geq \inf F_{B}^{p}(x) \\
& \sup _{A}^{1}(x) \geq \sup _{B}^{1}(x), \sup _{A}^{2}(x) \geq \sup _{B}^{2}(x), \ldots, \\
& \sup _{\mathrm{A}}^{\mathrm{p}}(\mathrm{x}) \geq \sup _{\mathrm{B}}^{\mathrm{p}}(\mathrm{x}) \forall \mathrm{x} \in \mathrm{X} \tag{2}
\end{align*}
$$

## PROPOSED DEFINITIONS

The proposed definitions are as follows:

Grade function: Let $\mathrm{A}=([\mathrm{a}, \mathrm{b}],[\mathrm{c}, \mathrm{d}],[\mathrm{e}, \mathrm{f}])$ be an interval neutrosophic number, a grade function $E$ of an interval neutrosophic value, based on the truth-membership degree, indeterminacy-membership degree and falsity-membership degree is defined as:

$$
\begin{equation*}
E(A)=\frac{((1-a)+b+c+d+e-f)^{2}}{4} \tag{3}
\end{equation*}
$$

## Proposition 1:

$$
\mathrm{E}(\mathrm{~A}) \geq 0
$$

Proof: The proof is straightforward.

## Theorem 1: Let:

$$
\mathrm{A}=\left(\left[\mathrm{a}_{1}, \mathrm{~b}_{1}\right],\left[\mathrm{c}_{1}, \mathrm{~d}_{1}\right],\left[\mathrm{e}_{1}, \mathrm{f}_{1}\right]\right)
$$

and:

$$
\mathrm{B}=\left(\left[\mathrm{a}_{2}, \mathrm{~b}_{2}\right],\left[\mathrm{c}_{2}, \mathrm{~d}_{2}\right],\left[\mathrm{e}_{2}, \mathrm{f}_{2}\right]\right)
$$

be two interval neutrosophic numbers. If $A \subseteq B$ then $E(A) \geq E(B)$.

Proof: ${ }^{\prime \prime}{ }^{\prime}(3)^{\prime}$ :

$$
\mathrm{E}(\mathrm{~A})=\frac{\left(\left(1-\mathrm{a}_{1}\right)+\mathrm{b}_{1}+\mathrm{c}_{1}+\mathrm{d}_{1}+\mathrm{e}_{1}-\mathrm{f}_{1}\right)^{2}}{4}
$$

and:

$$
E(B)=\frac{\left(\left(1-a_{2}\right)+b_{2}+c_{2}+d_{2}+e_{2}-f_{2}\right)^{2}}{4}
$$

Since $A \subseteq B, a_{1} \leq a_{2}, b_{1} \leq b_{2}, c_{1} \geq c_{2}, d_{1} \geq d_{2}, e_{1} \geq e_{2}$ and $f_{1} \geq f_{2}$ :

$$
\begin{aligned}
& \therefore\left(\mathrm{a}_{2}-\mathrm{a}_{1}\right) \geq 0,\left(\mathrm{~b}_{2}-\mathrm{b}_{1}\right) \geq 0,\left(\mathrm{c}_{1}-\mathrm{c}_{2}\right) \geq 0,\left(\mathrm{~d}_{1}-\mathrm{d}_{2}\right) \geq 0,\left(\mathrm{e}_{1}-\mathrm{e}_{2}\right) \geq 0 \\
& \text { and }\left(\mathrm{f}_{1}-\mathrm{f}_{2}\right) \geq 0
\end{aligned}
$$

Hence $E(A)-E(B) \geq 0$.

Similarity grade function: Let $\mathrm{A}=([\mathrm{a}, \mathrm{b}],[\mathrm{c}, \mathrm{d}],[\mathrm{e}, \mathrm{f}])$ be an interval neutrosophic number, a similarity grade function N of an interval neutrosophic value, based on the truth-membership degree, indeterminacy-membership degree and falsity-membership degree is defined as:

$$
\begin{equation*}
N(A)=1-\frac{(1-a)+b+c+d+e-f}{6} \tag{4}
\end{equation*}
$$

## Proposition 2:

$$
\mathrm{N}(\mathrm{~A}) \in[0,1]
$$

Proof: The proof is straightforward.

Theorem 2: Let:

$$
\mathrm{A}=\left(\left[\mathrm{a}_{1}, \mathrm{~b}_{1}\right],\left[\mathrm{c}_{1}, \mathrm{~d}_{1}\right],\left[\mathrm{e}_{1}, \mathrm{f}_{1}\right]\right)
$$

and:

$$
\mathrm{B}=\left(\left[\mathrm{a}_{2}, \mathrm{~b}_{2}\right],\left[\mathrm{c}_{2}, \mathrm{~d}_{2}\right],\left[\mathrm{e}_{2}, \mathrm{f}_{2}\right]\right)
$$

be two interval neutrosophic numbers. If $A \subseteq B$ then $N(A) \leq N(B)$.

Proof: $\mathrm{By}^{\prime}(4)^{\prime}$ :

$$
\mathrm{N}(\mathrm{~A})=1-\frac{\left(1-\mathrm{a}_{1}\right)+\mathrm{b}_{1}+\mathrm{c}_{1}+\mathrm{d}_{1}+\mathrm{e}_{1}-\mathrm{f}_{1}}{6}
$$

and:

$$
\mathrm{N}(\mathrm{~B})=1-\frac{\left(1-\mathrm{a}_{2}\right)+\mathrm{b}_{2}+\mathrm{c}_{2}+\mathrm{d}_{2}+\mathrm{e}_{2}-\mathrm{f}_{2}}{6}
$$

Since $A \subseteq B, a_{1} \leq a_{2}, b_{1} \leq b_{2}, c_{1} \geq c_{2}, d_{1} \geq d_{2}, e_{1} \geq e_{2}$ and $f_{1} \geq f_{2}$ :

$$
\therefore\left(a_{2}-a_{1}\right) \geq 0,\left(b_{2}-b_{1}\right) \geq 0,\left(c_{1}-c_{2}\right) \geq 0,\left(d_{1}-d_{2}\right) \geq 0,\left(e_{1}-e_{2}\right) \geq 0
$$

$$
\text { and }\left(f_{1}-f_{2}\right) \geq 0
$$

Hence $N(A)-N(B) \leq 0$.
Logarithmic distance: Let:
and:

Be two $n$-valued interval neutrosophic sets then the logarithmic distance:

(5)

## Proposition 3:

- $\quad L D_{\text {nums }}(A, B)>0$
- $\quad L D_{\text {nvins }}(A, B)=\operatorname{LD}$ nvins $(B, A)$
- If $A \subseteq B \subseteq C$ then $L D_{\text {Nvins }}(A, C) \geq L D_{\text {NVINs }}(A, B)$ and $L D_{\text {NuINs }}(A, C)$ $\geq \mathrm{LD}_{\text {NuINs }}(\mathrm{B}, \mathrm{C})$


## Proof:

- The proof is straightforward
- It was well known that:

$$
\begin{aligned}
& \left|\inf _{T_{A}^{j}}^{d}\left(x_{i}\right)-\inf T_{B}^{j}\left(x_{i}\right)\right|=\inf T_{B}^{j}\left(x_{i}\right)-\inf T_{A}^{j}\left(x_{i}\right) \mid \\
& \left|\sup _{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{B}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|=\left|\sup T_{B}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\sin _{A}^{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
\end{aligned}
$$

$$
\begin{aligned}
& \left|\inf _{\mathrm{F}_{\mathrm{A}}^{\mathrm{j}}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{F}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|=\inf \mathrm{F}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{F}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \mid \\
& \left|\operatorname{supF}_{A}^{\prime}\left(\mathrm{x}_{\mathrm{i}}\right)-\operatorname{supF}_{\mathrm{B}}^{\mathrm{B}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|=\left|\sup \mathrm{F}_{\mathrm{B}}^{\mathrm{B}}\left(\mathrm{x}_{\mathrm{i}}\right)-\operatorname{supF}_{\mathrm{A}}^{\mathrm{A}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
\end{aligned}
$$




- By '(2)':

$$
\inf _{\mathrm{T}_{A}^{\mathrm{j}}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \inf \mathrm{T}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \inf _{\mathrm{T}_{C}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)}
$$

$$
\operatorname{supT}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \sup _{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \sup _{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)
$$

$$
\inf _{I_{A}^{j}}^{j}\left(x_{i}\right) \geq \inf _{I_{B}^{j}}^{j}\left(x_{i}\right) \geq \inf _{I_{C}^{j}}^{j}\left(x_{i}\right)
$$

$$
\sup _{A}^{j}\left(x_{i}\right) \geq \sup _{B}^{j}\left(x_{i}\right) \geq \sup _{C}^{j}\left(x_{i}\right)
$$

$\inf F_{A}^{j}\left(x_{i}\right) \geq \inf F_{B}^{j}\left(x_{i}\right) \geq \inf F_{C}^{j}\left(x_{i}\right)$
$\sup F_{A}^{j}\left(x_{i}\right) \geq \sup F_{B}^{j}\left(x_{i}\right) \geq \sup _{C}^{j}\left(x_{i}\right)$

Hence:

Here, the logarithmic distance is an increasing function:
$\therefore \operatorname{LD}_{\text {NVINS }}(A, C) \geq L D_{\text {NVINS }}(A, B)$ and $L D_{\text {NVINS }}(A, C) \geq L D_{\text {NVINS }}(B, C)$

$$
\begin{aligned}
& \left|\inf \mathrm{T}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{T}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\inf \mathrm{T}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf _{\mathrm{T}}^{\mathrm{C}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left.\left|\operatorname{supT}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{B}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq \mid \sup _{\mathrm{A}}^{\mathrm{A}}\left(\mathrm{x}_{\mathrm{i}}\right)-{\sup T_{C}^{j}}_{\mathrm{C}}^{\mathrm{x}} \mathrm{x}\right) \mid \\
& \left|\inf _{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{I}_{B}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq \inf \mathrm{I}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{I}_{\mathrm{C}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right) \mid \\
& \left|\sup _{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\operatorname{supI}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\sup _{I_{A}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup \mathrm{I}_{C}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\inf F_{A}^{j}\left(x_{i}\right)-\inf F_{B}^{j}\left(x_{i}\right)\right| \leq\left|\inf F_{A}^{j}\left(x_{i}\right)-\inf F_{C}^{j}\left(x_{i}\right)\right| \\
& \left|\operatorname{supF}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\operatorname{supF}_{B}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\operatorname{supF}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
\end{aligned}
$$

## Exponential measure: Let:

and:
be two $n$-valued interval neutrosophic sets then the exponential measure:

(6)

## Proposition 4:

- $E M_{\text {muns }}(A, B)>0$
- $E M_{\text {muns }}(A, B)=E M_{\text {nuns }}(B, A)$
- If $A \subseteq B \subseteq C$ then $E M_{\text {NVINS }}(A, C) \leq E M_{\text {wvins }}(A, B)$ and $E M_{\text {Nvins }}$ $(A, C) \leq E M_{\text {NUNS }}(B, C)$


## Proof:

- The proof is straightforward
- It was well known that:

$$
\begin{aligned}
& \left.\mid \inf _{T_{A}^{j}}^{\left(x_{i}\right)}\right)-\inf T_{B}^{j}\left(x_{i}\right)\left|=\inf T_{B}^{j}\left(x_{i}\right)-\inf T_{A}^{j}\left(x_{i}\right)\right|
\end{aligned}
$$



$$
=E M_{\text {NINS }}(B, A)
$$

- $\mathrm{By}^{\prime}(2)^{\prime}$ :

$$
\inf _{T_{A}^{j}}^{j}\left(x_{i}\right) \leq \inf T_{B}^{j}\left(x_{i}\right) \leq \inf T_{c}^{j}\left(x_{i}\right)
$$

$$
\sup _{T_{A}^{j}}\left(x_{i}\right) \leq \sup _{T_{B}^{j}}^{j}\left(x_{i}\right) \leq \sup _{C}^{d}\left(x_{i}\right)
$$

$$
\inf _{I_{A}^{j}}^{j}\left(x_{i}\right) \geq \inf I_{B}^{j}\left(x_{i}\right) \geq \operatorname{inff}_{I_{c}^{j}}^{j}\left(x_{i}\right)
$$

$$
\operatorname{supI}_{A}^{i}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \sup _{I_{B}^{\prime}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \operatorname{supI}_{\mathrm{C}}^{\mathrm{C}}\left(\mathrm{x}_{\mathrm{i}}\right)
$$

$\inf \mathrm{F}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \inf \mathrm{F}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \inf _{\mathrm{F}_{\mathrm{C}}^{\mathrm{j}}}\left(\mathrm{x}_{\mathrm{i}}\right)$

$$
\sup _{\mathrm{F}_{\mathrm{A}}^{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \geq{\sup \mathrm{F}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \sup \mathrm{F}_{\mathrm{c}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)}^{\text {and }}
$$

Hence:

$$
\begin{aligned}
& \left|\inf _{T_{A}^{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf _{T_{B}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\inf T_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf _{T_{C}^{j}}\left(\mathrm{X}_{\mathrm{i}}\right)\right| \\
& \left|\sup _{T_{A}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{B}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\operatorname{supT}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\operatorname{supt}_{\mathrm{c}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
\end{aligned}
$$

$$
\begin{aligned}
& \left|\inf _{I_{A}^{\prime}}^{j}\left(x_{i}\right)-\inf I_{B}^{j}\left(x_{i}\right)\right|=\left|\inf I_{B}^{j}\left(x_{i}\right)-\inf I_{A}^{j}\left(x_{i}\right)\right|
\end{aligned}
$$

$$
\begin{aligned}
& \left.\mid \inf _{F_{A}^{j}\left(x_{i}\right)}\right)-\inf _{F_{B}^{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\left|=\inf \mathrm{F}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf _{\mathrm{F}_{\mathrm{A}}^{\mathrm{j}}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left.\left|\sup _{\mathrm{F}}^{\mathrm{A}}\left(\mathrm{x}_{\mathrm{i}}\right)-\operatorname{supF}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|=\mid \sup _{\mathrm{F}}^{\mathrm{B}} \mathrm{( } \mathrm{x}_{\mathrm{i}}\right)-\operatorname{supF}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \mid
\end{aligned}
$$

$$
\begin{aligned}
& \left|\inf _{I_{A}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf _{\mathrm{I}_{B}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\inf \mathrm{I}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{I}_{\mathrm{C}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
\end{aligned}
$$

$$
\begin{aligned}
& \left|\inf F_{A}^{j}\left(x_{i}\right)-\inf F_{B}^{j}\left(x_{i}\right)\right| \leq\left|\inf F_{A}^{j}\left(x_{i}\right)-\inf F_{C}^{j}\left(x_{i}\right)\right| \\
& \left|\sup _{F_{A}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{B}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\sup _{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup \mathrm{F}_{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
\end{aligned}
$$

Here, the exponential measure is a decreasing function:
$\therefore E M_{\text {NVINS }}(A, C) \leq E M_{\text {NVINS }}(A, B)$ and $E M_{\text {NVINS }}(A, C) \leq E M_{\text {NVINS }}(B, C)$
Similarity measure: Let:
and:
be two $n$-valued interval neutrosophic sets then the similarity measure:


## Proposition 5:

- $S_{\text {NVINS }}(A, B) \leq 1$
- $\quad S M_{\text {NVINS }}(A, B)=S M_{\text {NVINS }}(B, A)$
- If $A \subseteq B \subseteq C$ then $S M_{\text {NVINS }}(A, C) \leq \operatorname{SM}_{\text {NVINS }}(A, B)$ and $S M_{\text {NVINS }}(A$, $C) \leq \mathrm{SM}_{\text {NVINS }}(\mathrm{B}, \mathrm{C})$


## Proof:

- The proof is straightforward
- It was well known that:

$$
\left|\inf T_{A}^{j}\left(x_{i}\right)-\inf T_{B}^{j}\left(x_{i}\right)\right|=\left|\inf T_{B}^{j}\left(x_{i}\right)-\inf _{T_{A}^{j}}^{j}\left(x_{i}\right)\right|
$$

$$
\left|\sup _{\mathrm{A}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|=\left|\sup _{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{T}_{\mathrm{A}}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
$$

$$
\left|\inf _{I_{A}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf I_{B}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right|=\left|\inf I_{B}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf _{I_{A}^{j}}^{\left(\mathrm{x}_{\mathrm{i}}\right)}\right|
$$

$$
\left|\sup _{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{I_{B}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right|=\left|\sup _{I_{B}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{I_{A}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
$$

$$
\left|\inf _{F_{A}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{F}_{B}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|=\left|\inf \mathrm{F}_{B}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{F}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
$$

$$
\left|\sup _{F_{A}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup \mathrm{F}_{B}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|=\left|\sup _{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
$$




$$
=S_{M}
$$

- $\quad \mathrm{By}{ }^{\prime}(2)$ ':

$$
\begin{aligned}
& \inf \mathrm{T}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \inf \mathrm{T}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \inf _{\mathrm{T}_{C}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)} \\
& \operatorname{supT}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \operatorname{supT}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \operatorname{supT}_{C}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)
\end{aligned}
$$

$$
\inf I_{A}^{j}\left(x_{i}\right) \geq \inf _{I_{B}^{j}}^{j}\left(x_{i}\right) \geq \inf I_{C}^{j}\left(x_{i}\right)
$$

$\operatorname{supI}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \operatorname{supI}_{B}^{j}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \sup \mathrm{I}_{C}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)$

$$
\begin{align*}
& \inf F_{A}^{j}\left(x_{i}\right) \geq \inf F_{B}^{j}\left(x_{i}\right) \geq \inf F_{C}^{j}\left(x_{i}\right)  \tag{8}\\
& \operatorname{supF}_{A}^{j}\left(x_{i}\right) \geq \operatorname{supF}_{B}^{j}\left(x_{i}\right) \geq \operatorname{supF}_{C}^{j}\left(x_{i}\right)
\end{align*}
$$

$$
\mathrm{l}_{\mathrm{NVINS}}(\mathrm{~A}, \mathrm{~B})=\frac{1}{2 \mathrm{p}} \log \left[\left|\frac{2+\mathrm{SM}_{\mathrm{NVINS}}(\mathrm{~A}, \mathrm{~B})}{2-\mathrm{SM}_{\mathrm{NVINS}}(\mathrm{~A}, \mathrm{~B})}\right|\right]
$$

## Proposition 6:

Hence:

$$
\begin{aligned}
& \left|\inf _{\mathrm{T}_{A}^{j}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{T}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\inf \mathrm{T}_{\mathrm{A}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{T}_{C}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\operatorname{supT}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\sup _{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\operatorname{supT}_{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\inf I_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{I}_{\mathrm{B}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\inf \mathrm{I}_{\mathrm{A}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{I}_{\mathrm{C}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\operatorname{supI}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\operatorname{supI}_{\mathrm{A}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup \mathrm{I}_{C}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\inf F_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{F}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\inf \mathrm{F}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{F}_{C}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\sup _{F_{A}}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\operatorname{supF}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\operatorname{supF}_{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
\end{aligned}
$$

Here, the similarity measure is a decreasing function:
$\therefore \mathrm{SM}_{\text {NVINS }}(\mathrm{A}, \mathrm{C}) \leq \mathrm{SM}_{\text {NVINS }}(\mathrm{A}, \mathrm{B})$ and $\mathrm{SM}_{\text {NVINS }}(\mathrm{A}, \mathrm{C}) \leq \mathrm{SM}_{\text {NVINS }}(\mathrm{B}, \mathrm{C})$

## Logarithmic function: Let:

and:
be two n-valued interval neutrosophic sets. Then, the logarithmic function based on similarity measure formula:

- $\quad I_{\text {NVINS }}(A, B) \leq 1$
- $\quad I_{\text {NVINS }}(A, B)=I_{\text {NVINS }}(B, A)$
- If $A \subseteq B \subseteq C$ then $I_{\text {NVINS }}(A, C)<I_{\text {NVINS }}(A, B)$ and $I_{\text {NVINS }}(A$,
$C)<l_{\text {NVINS }}(B, C)$


## Proof:

- The proof is straightforward
- Since $S M_{\text {NVINS }}(A, B)=S M_{\text {NVINS }}(B, A)$ :

$$
\begin{gathered}
l_{\mathrm{NVINS}}(\mathrm{~A}, \mathrm{~B})=\frac{1}{2 \mathrm{p}} \log \left[\left|\frac{2+\mathrm{SM}_{\mathrm{NVINS}}(\mathrm{~A}, \mathrm{~B})}{2-\mathrm{SM}_{\mathrm{NVINS}}(\mathrm{~A}, \mathrm{~B})}\right|\right] \\
=\frac{1}{2 \mathrm{p}} \log \left[\left|\frac{2+\mathrm{SM}_{\mathrm{NVINS}}(\mathrm{~B}, \mathrm{~A})}{2-\mathrm{SM}_{\mathrm{NVINS}}(\mathrm{~B}, \mathrm{~A})}\right|\right] \\
=\mathrm{I}_{\mathrm{NVINS}}(\mathrm{~B}, \mathrm{~A})
\end{gathered}
$$

- $\quad \mathrm{By}{ }^{\prime}(2)^{\prime}$ :

$$
\begin{aligned}
& \inf _{T_{A}^{j}}^{j}\left(x_{i}\right) \leq \inf _{T_{B}^{j}}^{j}\left(x_{i}\right) \leq \inf _{T_{C}^{j}}^{j}\left(x_{i}\right) \\
& \operatorname{supT}_{A}^{j}\left(x_{i}\right) \leq \sup _{B}^{j}\left(x_{i}\right) \leq \sup _{C}^{j}\left(x_{i}\right) \\
& \inf I_{A}^{j}\left(x_{i}\right) \geq \inf _{I_{B}^{j}}^{j}\left(x_{i}\right) \geq \inf _{C}^{j}\left(x_{i}\right) \\
& \operatorname{supI}_{A}^{j}\left(x_{i}\right) \geq \operatorname{supI}_{B}^{j}\left(x_{i}\right) \geq \operatorname{supI}_{C}^{j}\left(x_{i}\right) \\
& \inf _{F_{A}^{j}}^{j}\left(x_{i}\right) \geq \inf _{F_{B}^{j}}\left(x_{i}\right) \geq \inf _{F_{C}}^{j}\left(x_{i}\right)
\end{aligned}
$$

$$
\sup _{F_{A}}^{j}\left(x_{i}\right) \geq \sup _{F_{B}}^{j}\left(x_{i}\right) \geq \sup _{F_{C}}^{j}\left(x_{i}\right)
$$

Hence:

$$
\begin{aligned}
& \left|\inf T_{A}^{j}\left(x_{i}\right)-\inf T_{B}^{j}\left(x_{i}\right)\right| \leq\left|\inf T_{A}^{j}\left(x_{i}\right)-\inf T_{C}^{j}\left(x_{i}\right)\right| \\
& \left|\operatorname{supT}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{B}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\sup _{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{C}}^{\mathrm{C}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\inf \mathrm{I}_{A}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf _{\mathrm{I}_{B}^{j}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\inf _{\mathrm{I}}^{\mathrm{A}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf _{\mathrm{I}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\sup _{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{I}}^{\mathrm{B}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\sup \mathrm{I}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup \mathrm{I}_{C}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
\end{aligned}
$$

$$
\begin{aligned}
& \left|\inf F_{A}^{j}\left(x_{i}\right)-\inf F_{B}^{j}\left(x_{i}\right)\right| \leq\left|\inf F_{A}^{j}\left(x_{i}\right)-\inf F_{C}^{j}\left(x_{i}\right)\right| \\
& \left|\sup _{F_{A}}^{j}\left(x_{i}\right)-\sup _{B}^{j}\left(x_{i}\right)\right| \leq\left|\sup _{A}^{j}\left(x_{i}\right)-\sup _{C}^{j}\left(x_{i}\right)\right|
\end{aligned}
$$

Here, the logarithmic function is a decreasing function:

$$
\therefore I_{\text {NVINS }}(A, C) \leq \leq_{\text {NVINS }}(A, B) \text { and } I_{\text {NVINS }}(A, C) \leq I_{\text {NVINS }}(B, C)
$$

## Definition: Let:

and:
be two n-valued interval neutrosophic sets. Then the exponential function based on similarity measure formula:

$$
\begin{equation*}
\mathrm{e}_{\text {NVINS }}(\mathrm{A}, \mathrm{~B})=\frac{1}{2 \mathrm{n}}\left[\mathrm{e}^{\left[\frac{2+\mathrm{SM}_{\text {NvNs }}(\mathrm{A}, \mathrm{~B})}{\mathrm{p}\left(2-\mathrm{SM}_{\text {NvNs }}(\mathrm{A}, \mathrm{~B})\right)}\right]}\right] \tag{9}
\end{equation*}
$$

## Proposition 7:

- $e_{\text {NVINS }}(A, B)>0$
- $e_{\text {NVINS }}(A, B)=e_{\text {NVINS }}(B, A)$
- If $A \subseteq B \subseteq C$ then $e_{\text {NVINs }}(A, C) \leq e_{\text {NVINs }}(A, B)$ and $e_{\text {NVINs }}(A$, $C) \leq e_{\text {NVINs }}(B, C)$


## Proof:

- The proof is straightforward
- Since, SM $_{\text {NVINS }}(A, B)=\operatorname{SM}_{\text {NVINS }}(B, A)$ :

$$
\mathrm{e}_{\text {Nvins }}(\mathrm{A}, \mathrm{~B})=\frac{1}{2 \mathrm{n}}\left[\mathrm{e}^{\left[\frac{2+\mathrm{SM}_{\text {Nvus }}(\mathrm{P}(\mathrm{~A}, \mathrm{~B})}{\left.\mathrm{S} \mathrm{SM}_{\text {Nvss }}(\mathrm{A}, \mathrm{~B})\right)}\right]}\right]
$$

$$
=\frac{1}{2 \mathrm{n}}\left[\mathrm{e}^{\left[\frac{2+\mathrm{SM}_{\text {NvNs }}(\mathrm{B}, \mathrm{~A})}{\mathrm{p}\left(2-\mathrm{SM}_{\text {NvNs }}(\mathrm{B}, \mathrm{~A})\right)}\right]}\right]
$$

$$
=\mathrm{e}_{\mathrm{NVINS}}(\mathrm{~B}, \mathrm{~A})
$$

- $\quad \mathrm{By}{ }^{\prime}(2)^{\prime}$ :

$$
\begin{aligned}
& \inf _{\mathrm{T}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \inf _{\mathrm{T}_{\mathrm{B}}^{\mathrm{j}}}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \inf _{\mathrm{T}_{\mathrm{C}}^{\mathrm{j}}}\left(\mathrm{x}_{\mathrm{i}}\right)}^{\operatorname{supT}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \operatorname{supT}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \leq \operatorname{supT}_{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)} \\
& \inf \mathrm{I}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \inf _{\mathrm{I}_{B}^{j}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \inf _{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)
\end{aligned}
$$

$$
\sup _{A}^{j}\left(x_{i}\right) \geq \sup _{B}^{j}\left(x_{i}\right) \geq \operatorname{supI}_{C}^{j}\left(x_{i}\right)
$$

$$
\inf _{F_{A}^{j}}^{j}\left(x_{i}\right) \geq \inf F_{B}^{j}\left(x_{i}\right) \geq \inf F_{C}^{j}\left(x_{i}\right)
$$

$$
\sup _{A}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right) \geq \sup _{\mathrm{F}_{\mathrm{B}}^{\mathrm{j}}}\left(\mathrm{x}_{\mathrm{i}}\right) \geq{\sup \mathrm{F}_{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)}
$$

Hence:

Here, the exponential function is a decreasing function:

$$
\therefore \mathrm{e}_{\text {NVINS }}(\mathrm{A}, \mathrm{C}) \leq \mathrm{e}_{\text {NVINS }}(\mathrm{A}, \mathrm{~B}) \text { and } \mathrm{e}_{\text {NVINS }}(\mathrm{A}, \mathrm{C}) \leq \mathrm{e}_{\text {NVINS }}(\mathrm{B}, \mathrm{C})
$$

$$
\begin{aligned}
& \left|\operatorname{supT}_{A}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\sup _{A}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{C}}^{\mathrm{C}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\inf _{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{I}_{B}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\inf \mathrm{I}_{A}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf _{\mathrm{I}}^{\mathrm{I}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\sup _{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\sup I_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{C}}^{\mathrm{C}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\inf \mathrm{F}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{F}_{\mathrm{B}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\inf \mathrm{F}_{\mathrm{A}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\inf \mathrm{F}_{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \\
& \left|\operatorname{supF}_{A}^{j}\left(\mathrm{x}_{\mathrm{i}}\right)-\operatorname{supF}_{B}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right| \leq\left|\operatorname{supF}_{A}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)-\sup _{\mathrm{C}}^{\mathrm{j}}\left(\mathrm{x}_{\mathrm{i}}\right)\right|
\end{aligned}
$$

## METHODOLOGY

In this section, it was presented an application of n-valued interval neutrosophic set in medical diagnosis. In a given pathology, suppose $S$ is a set of symptoms, $D$ is a set of diseases and $P$ is a set of patients and let $Q$ be an-valued interval neutrosophic relation from the set of patients to the symptoms. i.e., $\mathrm{Q}(\mathrm{P} \rightarrow \mathrm{S})$ and R be an interval neutrosophic relation from the set of symptoms to the diseases i.e., $R(S \rightarrow D)$ and then the methodology involves three main jobs:

- Determination of symptoms
- Formulation of medical knowledge based on n-valued interval neutrosophic sets and interval neutrosophic sets
- Determination of diagnosis on the basis of various computation techniques of $n$-valued interval neutrosophic sets


## Algorithm:

Step 1: The symptoms of the patients are given to obtain the patient-symptom relation and are noted in Table 1

Step 2: The medical knowledge relating the symptoms with the set of diseases under consideration are given to obtain the symptom-disease relation and are noted in Table 2

Step 3: Table 3 is obtained by calculating average values for Table 1

Step 4: Table 4 is obtained by applying '(1)' between Table 2 and 3
Step 5: The computation $T$ of the relation of patients and diseases is found using '(3)'and '(4)'in Table 4 and are noted in Table 5
Step 6: The computation $T$ of the relation of patients and diseases is found '(5)', '(6)','(7)', '(8)' and '(9)'and are noted in Table 6
Step 7: Finally, the minimum value from Table 5 (grade function) and Table 6 (logarithmic distance) and maximum value from Table 5 (similarity grade function and Table 6 (exponential measure, similarity measure, logarithmic function and exponential function) of each row were selected to find the possibility of the patient affected with the respective disease and then it was concluded that the patient was suffering from the disease

## CASE STUDY ${ }^{21}$

Let there be three patients $P=\left\{P_{1}, P_{2}, P_{3}\right\}$ and the set of symptoms $S=\left\{S_{1}=\right.$ Temperature, $\mathrm{S}_{2}=$ Cough, $\mathrm{S}_{3}=$ Throat pain, $S_{4}=$ Headache, $S_{5}=$ Body pain\}. The n-valued interval neutrosophic relation $\mathrm{Q}(\mathrm{P} \rightarrow \mathrm{S})$ is given as in Table 1. Let the set of diseases $D=\left\{D_{1}=\right.$ Viral fever, $D_{2}=$ Tuberculosis, $D_{3}=$ Typhoid, $D_{4}=$ Throat disease $\}$. The interval neutrosophic relation $R(S \rightarrow D)$ is given as in Table 2.

From Table 5 and 6,it is obvious that, if the doctor agrees, then $P_{1}$ and $P_{3}$ suffers from Viral fever and $P_{2}$ suffers from Throat disease.

| Q | Temperature | Cough | Throat pain | Headache | Body pain |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\bar{P}_{1}$ | [0.3,0.4], [0.4,0.5], | [0.1,0.2], [0.3,0.6], | [0.0,0.5], | [0.2,0.3], | [0.0,0.4], |
|  | [0.3,0.7] | [0.6,0.8] | [0.2,0.6], [0.0,0.4] | [0.3,0.5], [0.0,0.7] | [0.6,0.7], [0.2,0.5] |
|  | [0.0,0.3], [0.1,0.3], | [0.0,0.5], [0.4,0.7], | [0.3,0.4], | [0.4,0.5], | [0.2,0.4], |
|  | [0.0,0.5] | [0.4, 0.5$]$ | [0.2,0.3], [0.3,0.4] | [0.4,0.7], [0.3,0.6] | [0.4,0.5], [0.1,0.2] |
|  | [0.0,0.6], [0.4,0.5], | [0.2,0.3], [0.0,0.5], | [0.0,0.7], | [0.2,0.6], | [0.1,0.3], |
|  | [0.3,0.4] | [0.4,0.6] | [0.3,0.7], [0.3,0.5] | [0.0,0.6], [0.3,0.4] | [0.1,0.3], [0.2,0.3] |
| $P_{2}$ | [0.2,0.03], | [0.5,0.7], [0.0,0.4], | [0.5,0.6], | [0.2,0.5], | [0.2,0.4], |
|  | [0.4,0.5], [0.1,0.2] | [0.7,0.8] | [0.0,0.6], [0.2,0.3] | [0.5,0.6], [0.1,0.5] | [0.4,0.6], [0.1,0.4] |
|  | [0.4,0.5], [0.2,0.5], | [0.6,0.7], [0.0,0.5], | [0.4,0.7], | [0.2,0.3], | [0.0,0.5], |
|  | [0.0,0.3] | [0.4, 0.5$]$ | [0.4,0.6], [0.3,0.4] | [0.2,0.5], [0.5,0.6] | [0.2,0.4], [0.5,0.6] |
|  | [0.6,0.7], [0.4, 0.5], | [0.4,0.6], [0.2,0.7], | [0.1,0.3], | [0.1,0.3], | [0.5,0.7], |
|  | [0.4,0.5] | [0.0,0.3] | [0.2,0.3], [0.5,0.7] | [0.3,0.4], [0.4,0.5] | [0.0,0.7], [0.2,0.4] |
| $P_{3}$ | [0.1,0.3], [0.0,0.5], | [0.2,0.3], [0.0,0.7], | [0.2,0.4], | [0.2,0.3], | [0.0,0.6], |
|  | [0.4,0.6] | [0.1,0.4] | [0.3,0.6], [0.0,0.6] | [0.5,0.6], [0.4, 0.5$]$ | [0.4,0.7], [0.2,0.3] |
|  | [0.1,0.2], [0.3,0.4], | [0.5,0.6], [0.0,0.3], | [0.4,0.5], | [0.2,0.4], | [0.2,0.3], |
|  | [0.2,0.5] | [0.3,0.5] | [0.0,0.3], [0.3,0.4] | [0.0,0.4], [0.2,0.7] | [0.2,0.3], [0.1,0.2] |
|  | [0.2,0.4], [0.4,0.5], | [0.3,0.5], [0.2,0.5], | [0.5,0.7], | [0.4,0.5], | [0.0,0.6], |
|  | [0.3,0.7] | [0.4,0.6] | [0.4,0.6], [0.3,0.7] | [0.2,0.3], [0.3,0.5] | [0.2,0.4], [0.4,0.6] |

Table 2: Symptom-disease relation (using step2)

| $R$ | Viral fever | Tuberculosis | Typhoid | Throat disease |
| :--- | :--- | :--- | :--- | :--- |
| Temperature | $[0.2,0.4],[0.3,0.5],[0.3,0.7]$ | $[0.1,0.4],[0.2,0.6],[0.6,0.7]$ | $[0.0,0.3],[0.4,0.6],[0.0,0.2]$ | $[0.3,0.4],[0.2,0.5],[0.0,0.6]$ |
| Cough | $[0.2,0.4],[0.2,0.3],[0.0,0.5]$ | $[0.3,0.4],[0.2,0.5],[0.7,0.8]$ | $[0.3,0.4],[0.2,0.3],[0.1,0.2]$ | $[0.4,0.5],[0.1,0.3],[0.0,0.5]$ |
| Throat pain | $[0.0,0.4],[0.2,0.4],[0.2,0.4]$ | $[0.0,0.2],[0.3,0.6],[0.6,0.7]$ | $[0.1,0.2],[0.4,0.5],[0.3,0.4]$ | $[0.2,0.4],[0.2,0.5],[0.3,0.7]$ |
| Headache | $[0.4,0.7],[0.0,0.3],[0.3,0.5]$ | $[0.1,0.2],[0.0,0.5],[0.0,0.6]$ | $[0.3,0.4],[0.2,0.3],[0.2,0.5]$ | $[0.0,0.3],[0.3,0.6],[0.2,0.5]$ |
| Body pain | $[0.1,0.4],[0.2,0.5],[0.3,0.4]$ | $[0.5,0.7],[0.4,0.5],[0.2,0.5]$ | $[0.2,0.3],[0.2,0.4],[0.2,0.3]$ | $[0.0,0.4],[0.1,0.2],[0.1,0.3]$ |

Table 3: Average for patient-symptom relation

| Q | Temperature | Cough | Throat pain | Headache | Body pain |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}_{1}$ | $[0.1,0.43],[0.3,0.43]$, | $[0.1,0.33],[0.23,0.6]$, | $[0.1,0.53],[0.23,0.53]$, | $[0.26,0.46],[0.23,0.6]$, | $[0.1,0.36],[0.36,0.5]$, |
|  | $[0.2,0.53]$ | $[0.46,0.63]$ | $[0.2,0.43]$ | $[0.2,0.56]$ | $[0.16,0.33]$ |
| $\mathrm{P}_{2}$ | $[0.4,0.5],[0.33,0.5]$, | $[0.5,0.66],[0.06,0.53]$, | $[0.33,0.53],[0.2,0.5]$, | $[0.16,0.36],[0.33,0.5]$, | $[0.23,0.53],[0.2,0.56]$, |
|  | $[0.16,0.33]$ | $[0.36,0.53]$ | $[0.33,0.46]$ | $[0.33,0.53]$ | $[0.0,0.46]$ |
| $\mathrm{P}_{3}$ | $[0.13,0.3],[0.23,0.46]$, | $[0.33,0.46],[0.06,0.5]$, | $[0.36,0.53],[0.23,0.5]$, | $[0.26,0.4],[0.23,0.43]$, | $[0.3,0.5],[0.26,0.46]$, |
|  | $[0.3,0.6]$ | $[0.26,0.5]$ | $[0.2,0.56]$ | $[0.23,0.36]$ |  |

Table 4: Sup-star composition between symptom-disease relation and average for patient-symptom relation

| $T$ | Viral fever | Tuberculosis | Typhoid | Throat disease |
| :--- | :--- | :--- | :--- | :--- |
| $P_{1}$ | $[0.26,0.46],[0.3,0.5],[0.2,0.4]$ | $[0.1,0.4],[0.36,0.53],[0.2,0.5]$ | $[0.26,0.4],[0.3,0.5],[0.2,0.33]$ | $[0.1,0.4],[0.23,0.6],[0.16,0.33]$ |
| $P_{2}$ | $[0.2,0.4],[0.3,0.5],[0.3,0.46]$ | $[0.3,0.53],[0.2,0.5],[0.26,0.5]$ | $[0.3,0.4],[0.33,0.5],[0.16,0.33]$ | $[0.4,0.5],[0.3,0.5],[0.16,0.46]$ |
| $P_{3}$ | $[0.26,0.4],[0.23,0.46],[0.2,0.4]$ | $[0.3,0.5],[0.26,0.5],[0.23,0.5]$ | $[0.3,0.4],[0.23,0.5],[0.23,0.36]$ | $[0.33,0.46],[0.23,0.5],[0.23,0.36]$ |

Table 5: Grade function and similarity grade function (using step 5 and step 7)

|  | Viral fever | Tuberculosis | Typhoid | Throat disease |
| :--- | :---: | :---: | :---: | :---: |
| Grade function |  |  |  |  |
| $\mathrm{P}_{1}$ | 0.8100 | 0.8930 | 0.8190 | 0.9604 |
| $\mathrm{P}_{2}$ | 0.8464 | 0.7140 | 0.7744 | 0.6400 |
| $\mathrm{P}_{3}$ | 0.6642 | 0.7140 | 0.7225 | 0.7482 |
| Similarity grade function |  |  |  |  |
| $\mathrm{P}_{1}$ | 0.7000 | 0.6850 | 0.6983 | 0.6733 |
| $\mathrm{P}_{2}$ | 0.6933 | 0.7183 | 0.7066 | 0.7333 |
| $\mathrm{P}_{3}$ | 0.7283 | 0.7183 | 0.7166 | 0.7116 |

Table 6: Logarithmic distance, exponential measure, similarity measure, logarithmic function and exponential function(using step 6 and step 7)

| T | Viral fever | Tuberculosis | Typhoid | Throat disease |
| :---: | :---: | :---: | :---: | :---: |
| Logarithmic distance |  |  |  |  |
| $\mathrm{P}_{1}$ | 0.1180 | 0.1208 | 0.1196 | 0.1193 |
| $\mathrm{P}_{2}$ | 0.1208 | 0.1217 | 0.1201 | 0.1191 |
| $\mathrm{P}_{3}$ | 0.1174 | 0.1210 | 0.1187 | 0.1180 |
| Exponential measure |  |  |  |  |
| $\mathrm{P}_{1}$ | 0.8108 | 0.7906 | 0.7991 | 0.8019 |
| $\mathrm{P}_{2}$ | 0.7907 | 0.7827 | 0.7952 | 0.8038 |
| $\mathrm{P}_{3}$ | 0.8145 | 0.7884 | 0.8059 | 0.8126 |
| Similarity measure |  |  |  |  |
| $\mathrm{P}_{1}$ | 0.4226 | 0.4115 | 0.4161 | 0.4175 |
| $\mathrm{P}_{2}$ | 0.4115 | 0.4078 | 0.4142 | 0.4184 |
| $\mathrm{P}_{3}$ | 0.4247 | 0.4106 | 0.4197 | 0.4226 |
| Logarithmic function |  |  |  |  |
| $\mathrm{P}_{1}$ | 0.0310 | 0.0302 | 0.0305 | 0.0306 |
| $\mathrm{P}_{2}$ | 0.0302 | 0.0299 | 0.0304 | 0.0307 |
| $\mathrm{P}_{3}$ | 0.0312 | 0.0301 | 0.0308 | 0.0310 |
| Exponential function |  |  |  |  |
| $\mathrm{P}_{1}$ | 0.1668 | 0.1658 | 0.1662 | 0.1663 |
| $\mathrm{P}_{2}$ | 0.1658 | 0.1655 | 0.1661 | 0.1664 |
| $\mathrm{P}_{3}$ | 0.1670 | 0.1657 | 0.1665 | 0.1668 |

## CONCLUSION

In this study, it was analyzed that the relationship between the set of symptoms found within patients and set of
diseases and employed seven methods (grade function, similarity grade function, logarithmic distance, exponential measure, similarity measure, logarithmic function, exponential function) to find out the disease possibly affected the patient. The techniques considered in this study were more reliable to handle medical diagnosis problems quiet comfortably. The proposed methods had more accuracy than the others and they could handle the limitations and drawbacks of the previous works well.

## SIGNIFICANCE STATEMENTS

This study discovers the relationship between the symptoms found within patients and set of diseases. This study will help the researcher to find out the diseases accurately that impacted the patients. The methods employed are free from the limitations that are commonly found in other studies. Without such limitations, in this study a new theory on image processing, cluster analysis etc., has been developed.

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