Interval, Double-valued Neutrosophic Intuitionistic Fuzzy Graph

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Abstract - The notion of interval, double-valued neutrosophic intuitionistic fuzzy sets is a generalization of fuzzy sets, intuitionistic fuzzy sets, interval valued intuitionistic fuzzy sets. We apply the concept of interval, double-valued neutrosophic intuitionistic fuzzy sets an instance of neutrosophic intuitionistic fuzzy sets, to graphs. We introduce certain types of strong interval, double-valued neutrosophic intuitionistic fuzzy graphs and investigate some of their properties with proofs and examples.

Keywords - Interval, double-valued neutrosophic intuitionistic fuzzy graphs, Strong interval, double-valued neutrosophic intuitionistic fuzzy graphs.

I. INTRODUCTION

Neutrosophic sets (NSs) proposed by Smarandache [4, 5] are powerful mathematical tools for dealing with incomplete, indeterminate and inconsistent information in real world. They are a generalization of fuzzy sets [12], intuitionistic fuzzy sets [9, 11], interval valued fuzzy set [6] and interval-valued intuitionistic fuzzy sets theories[10].

The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i), and falsity-membership function (f) independently, which are within the real standard or nonstandarded unit interval [0,1].

The same authors [3, 14] introduced as well the concept of interval valued neutrosophic sets. The extension of fuzzy graph [1,2] theory have been developed by several researchers, including intuitionistic fuzzy graphs, considering the vertex sets and edge sets as intuitionistic fuzzy sets. In interval valued fuzzy graphs, the vertex sets and edge sets are considered as interval valued fuzzy sets. In interval valued intuitionistic fuzzy graphs, the vertex sets and edge sets are regarded as interval valued intuitionistic fuzzy sets.

But, when the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs and their extensions fail. In order to overcome the failure, Smarandache [3] defined four main categories of neutrosophic graphs: I-edge neutrosophic graph, I-vertex neutrosophic graph, (t,i,f)-edge neutrosophic graph and (t,i,f)-vertex neutrosophic graph. Later on, Broumi et al. introduced another neutrosophic graph model. This model allows the attachment of truth-membership (t), indeterminacy-membership (i) and falsity-membership (f) degrees both to vertices and edges. A neutrosophic graph model that generalizes the fuzzy graph and intuitionistic fuzzy graph is called Strong interval, double-valued neutrosophic intuitionistic fuzzy graphs.

Strong interval, double-valued neutrosophic intuitionistic fuzzy graphs are developed and some interesting properties are explored.
2. PRELIMINARIES

DEFINITION 2.1

A neutrosophic fuzzy graph with underlying set $V$ is defined to be a pair $N_{G}=(A,B)$ where

1. The functions $T_{A}: V \to [0,1]$, $I_{A}: V \to [0,1]$ and $F_{A}: V \to [0,1]$ denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_{i} \in V$, respectively and $0 \leq T_{A}(v_{i}) + I_{A}(v_{i}) + F_{A}(v_{i}) \leq 1$ .......(1)

2. $E \subseteq V \times V$ where, functions $T_{B}: V \times V \to [0,1]$, $I_{B}: V \times V \to [0,1]$ and $F_{B}: V \times V \to [0,1]$ are defined by

$$T_{B}(v_{i},v_{j}) \leq T_{A}(v_{i}).T_{A}(v_{j}) \quad \ldots \quad (2)$$

$$I_{B}(v_{i},v_{j}) \leq I_{A}(v_{i}).I_{A}(v_{j}) \quad \ldots \quad (3)$$

$$F_{B}(v_{i},v_{j}) \leq F_{A}(v_{i}).F_{A}(v_{j}) \quad \ldots \quad (4)$$

for all $v_{i},v_{j} \in V$ where \( \bullet \) means the ordinary multiplication denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_{i},v_{j}) \in E$ respectively, where $0 \leq T_{B}(v_{i},v_{j}) + I_{B}(v_{i},v_{j}) + F_{B}(v_{i},v_{j}) \leq 1$ for all $(v_{i},v_{j}) \in E$ (i,j=1,2,3,…n)

![Figure 1- Neutrosophic fuzzy graph](image)

we call “A” the neutrosophic fuzzy vertex set of V, “B” the neutrosophic fuzzy edge set of E, respectively.

DEFINITION 2.2

An interval valued intuitionistic fuzzy graph with underlying V is defined to be a pair $G=(A,B)$ where

1) The function $M_{A}: V \to D[0,1]$ and $N_{A}: V \to D[0,1]$ denote the degree of membership and non-membership of the element $x \in V$ respectively, such that $0 \leq M_{A}(x) + N_{A}(x) \leq 1$ for all $x \in V$.

2) The functions $M_{B}: E \subseteq V \times V \to D[0,1]$ and $N_{B}: E \subseteq V \times V \to D[0,1]$ are defined by
\[ M_{BL}(x, y) \leq \min(M_{AL}(x), M_{AL}(y)), \]
\[ N_{BL}(x, y) \geq \max(N_{AL}(x), N_{AL}(y)), \]
\[ M_{BU}(x, y) \leq \min(M_{AU}(x), M_{AU}(y)), \]
\[ N_{BU}(x, y) \geq \max(N_{AU}(x), N_{AU}(y)), \]

Such that \( 0 \leq M_{BU}(x, y) + N_{BU}(x, y) \leq 1 \), for all \((x, y) \in E\).

**DEFINITION 2.3**

A double valued neutrosophic fuzzy graph with underlying set \( V \) is defined to be a pair \( G = (A, B) \) where

1. The functions \( T_A : V \rightarrow [0,1], \ I_A : V \rightarrow [0,1] \) and \( F_A : V \rightarrow [0,1] \) denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element \( v_i \in V \) respectively and \( 0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 1 \) for all \( v_i \in V(i = 1,2,3,\ldots,n) \)

2. The functions \( T_B : E \subseteq V \times V \rightarrow [0,1], \ I_B : E \subseteq V \times V \rightarrow [0,1] \) and \( F_B : E \subseteq V \times V \rightarrow [0,1] \) are defined by
   \[ T_B(v_i, v_j) \leq \min(T_A(v_i), T_A(v_j)) \]
   \[ I_B(v_i, v_j) \leq \min(I_A(v_i), I_A(v_j)) \]
   \[ F_B(v_i, v_j) \geq \max(F_A(v_i), F_A(v_j)) \]

Denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge \((v_i, v_j) \in E\) respectively, where \( 0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 1 \) for all \((v_i, v_j) \in E\) \((i, j=1,2,3,\ldots,n)\)

we call “A” the double valued neutrosophic fuzzy vertex set of \( V \), “B” the double valued neutrosophic fuzzy edge set of \( E \), respectively.

Note that \( B \) is a symmetric double valued neutrosophic relation on \( A \). We use the notation \((v_i, v_j)\) for an element of \( E \). Thus, \( G=(A,B) \) is a double valued neutrosophic graph \( G^*=(V,E) \) if :

\[ T_B(v_i, v_j) \leq \min(T_A(v_i), T_A(v_j)) \]
\[ I_B(v_i, v_j) \leq \min(I_A(v_i), I_A(v_j)) \]
\[ F_B(v_i, v_j) \geq \max(F_A(v_i), F_A(v_j)) \]

for all \((v_i, v_j) \in E\)
Figure 2 - Double valued neutrosophic fuzzy graph

3. Interval, Double-Valued Neutrosophic Intuitionistic Fuzzy Graphs

DEFINITION 3.1

A interval, double-valued neutrosophic intuitionistic fuzzy graph (IDVNIF-graph) with underlying set \( V \) is defined to be a pair \( G=(A,B) \) where

1. \( V=\{v_1,v_2,\ldots,v_n\} \) such that \( T_{AL}:V\rightarrow[0,1] \), \( T_{AU}:V\rightarrow[0,1] \), \( I_{AL}:V\rightarrow[0,1] \), \( I_{AU}:V\rightarrow[0,1] \), and \( F_{AL}:V\rightarrow[0,1] \), \( F_{AU}:V\rightarrow[0,1] \) denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element \( v_i\in V \), respectively, and

\[ 0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 1 \text{ for every } v_i \in V. \]

2. The functions \( T_{BL}:V \times V\rightarrow[0,1] \), \( T_{BU}:V \times V\rightarrow[0,1] \), \( I_{BL}:V \times V\rightarrow[0,1] \), \( I_{BU}:V \times V\rightarrow[0,1] \) and \( F_{BL}:V \times V\rightarrow[0,1] \), \( F_{BU}:V \times V\rightarrow[0,1] \), such that

\[
T_{BL}(v_i,v_j) \leq \min[T_{AL}(v_i),T_{AL}(v_j)] \\
T_{BU}(v_i,v_j) \leq \min[T_{AU}(v_i),T_{AU}(v_j)] \\
I_{BL}(v_i,v_j) \leq \min[I_{AL}(v_i),I_{AL}(v_j)] \\
I_{BU}(v_i,v_j) \leq \min[I_{AU}(v_i),I_{AU}(v_j)] \\
F_{BL}(v_i,v_j) \geq \max[F_{BL}(v_i),F_{BL}(v_j)] \\
F_{BU}(v_i,v_j) \geq \max[F_{BU}(v_i),F_{BU}(v_j)]
\]

And

Denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the degree \( (v_i,v_j) \in E \) respectively, where

\[ 0 \leq T_B(v_i,v_j) + I_B(v_i,v_j) + F_B(v_i,v_j) \leq 1 \text{ for all } (v_i,v_j) \in E \]

EXAMPLE

Consider a graph \( G^* \) such that \( V=\{v_1,v_2,v_3\} \), \( E=\{v_1v_2,v_2v_3,v_3v_1\} \). Let \( A \) be a interval, double-valued neutrosophic intuitionistic fuzzy subset of \( V \) and \( B \) a double-valued neutrosophic intuitionistic fuzzy subset of \( E \) denoted by
In figure 3

(i) \((x,[0.2,0.4],[0.5,0.7],[0.4,0.6])\) is a interval, double-valued neutrosophic intuitionistic fuzzy vertex.

(ii) \((xy,[0.1,0.2],[0.4,0.3],[0.4,0.7])\) is interval, double-valued neutrosophic intuitionistic fuzzy edge.

(iii) \((x,[0.2,0.4],[0.5,0.7],[0.4,0.6])\) and \((y,[0.6,0.7],[0.2,0.3],[0.2,0.3])\) are interval, double-valued neutrosophic intuitionistic fuzzy adjacent vertices.

(iv) \((xy,[0.1,0.2],[0.4,0.3],[0.4,0.7])\) and \((yz,[0.1,0.1],[0.2,0.3],[0.8,0.9])\) are interval, double-valued neutrosophic intuitionistic fuzzy adjacent edge.

**DEFINITION 3.2**

A interval, double-valued neutrosophic intuitionistic fuzzy graph \(G=(A,B)\) of \(G^*=(V,E)\) is called strong interval, double-valued neutrosophic intuitionistic fuzzy graph if

\[
\begin{align*}
T_{BL}(v_i,v_j) &= \min(T_{AL}(v_i),T_{AL}(v_j)) \\
T_{BU}(v_i,v_j) &= \min(T_{AU}(v_i),T_{AU}(v_j)) \\
I_{BL}(v_i,v_j) &= \min(I_{AL}(v_i),I_{AL}(v_j)) \\
I_{BU}(v_i,v_j) &= \min(I_{AU}(v_i),I_{AU}(v_j)) \quad \text{and} \\
F_{BL}(v_i,v_j) &= \max(F_{AL}(v_i),F_{AL}(v_j)) \\
F_{BU}(v_i,v_j) &= \max(F_{AU}(v_i),F_{AU}(v_j)) 
\end{align*}
\]

for all \((v_i,v_j) \in E\).

**DEFINITION 3.3**

The complement of a interval, double-valued neutrosophic intuitionistic fuzzy graph \(G(A,B)\) on \(G^*=(V,E)\) is a interval, double-valued neutrosophic intuitionistic fuzzy graph \(\overline{G}\) on \(G^*\) where

1. \(\overline{A} = A\)
2. \(\overline{T_{AL}}(v_i) = T_{AL}(v_i)\),
   \(\overline{T_{AU}}(v_i) = T_{AU}(v_i)\),
\[ T_{AL}(v_i) = I_{AL}(v_i), \]
\[ T_{AU}(v_i) = I_{AU}(v_i), \]
\[ F_{AL}(v_i) = F_{AL}(v_i), \]
\[ F_{AU}(v_i) = F_{AU}(v_i) \text{ for all } v_i \in V. \]

3. \[ T_{BL}(v_i, v_j) = \min[T_{AL}(v_i), T_{AL}(v_j)] - T_{BL}(v_i, v_j), \]
\[ T_{BU}(v_i, v_j) = \min[T_{AU}(v_i), T_{AU}(v_j)] - T_{BU}(v_i, v_j), \]
\[ I_{BL}(v_i, v_j) = \min[I_{AL}(v_i), I_{AL}(v_j)] - I_{BL}(v_i, v_j), \]
\[ I_{BU}(v_i, v_j) = \min[I_{AU}(v_i), I_{AU}(v_j)] - I_{BU}(v_i, v_j) \text{ and} \]
\[ F_{BL}(v_i, v_j) = \max[F_{AL}(v_i), F_{AL}(v_j)] - F_{BL}(v_i, v_j), \]
\[ F_{BU}(v_i, v_j) = \max[F_{AU}(v_i), F_{AU}(v_j)] - F_{BU}(v_i, v_j) \text{ for all } (v_i, v_j) \in E. \]

**REMARK**

If \( G=(V, E) \) is a interval, double-valued neutrosophic intuitionistic fuzzy graph on \( G^* \). Then from above definition, it follow that \( \overline{G} \) is given by the interval, double-valued neutrosophic intuitionistic fuzzy graph \( \overline{G}=(V, \overline{E}) \) on \( G^* \) where

\[ \overline{V} = V \text{ and} \]
\[ \overline{T}_{BL}(v_i, v_j) = \min[T_{AL}(v_i), T_{AL}(v_j)] - T_{BL}(v_i, v_j), \]
\[ \overline{T}_{BU}(v_i, v_j) = \min[T_{AU}(v_i), T_{AU}(v_j)] - T_{BU}(v_i, v_j), \]
\[ \overline{I}_{BL}(v_i, v_j) = \min[I_{AL}(v_i), I_{AL}(v_j)] - I_{BL}(v_i, v_j), \]
\[ \overline{I}_{BU}(v_i, v_j) = \min[I_{AU}(v_i), I_{AU}(v_j)] - I_{BU}(v_i, v_j) \text{ and} \]
\[ \overline{F}_{BL}(v_i, v_j) = \max[F_{AL}(v_i), F_{AL}(v_j)] - F_{BL}(v_i, v_j), \]
\[ \overline{F}_{BU}(v_i, v_j) = \max[F_{AU}(v_i), F_{AU}(v_j)] - F_{BU}(v_i, v_j) \text{ for all } (v_i, v_j) \in E. \]

Thus \( \overline{T}_{BL} = T_{BL}, \overline{T}_{BU} = T_{BU}, \overline{I}_{BL} = I_{BL}, \overline{I}_{BU} = I_{BU} \text{ and } \overline{F}_{BL} = F_{BL}, \overline{F}_{BU} = F_{BU} \) on \( V \), where \( E=(T_{BL}, T_{BU}, I_{BL}, I_{BU}, F_{BL}, F_{BU}) \) is the interval, double-valued neutrosophic intuitionistic relation on \( V \). For any interval, double-valued neutrosophic intuitionistic fuzzy graph \( G \). \( \overline{G} \) is strong interval, double-valued neutrosophic intuitionistic fuzzy graph and \( G \subseteq \overline{G} \).
PROPOSITION 3.1

\[ G = \overline{G} \] if and only if \( G \) is a strong interval, double-valued neutrosophic intuitionistic fuzzy graph

Proof:

Strong interval, double-valued neutrosophic intuitionistic fuzzy graph, \( G = (A, B) \) defined on a graph \( G^* = (V, E) \) such that \( V = \{a, b, c, d\} \), \( E = \{ab, bc, cd, da\} \). \( A \) is an strong interval, double-valued neutrosophic intuitionistic fuzzy set of \( V \).

\[ A = \{(a, [0.4, 0.6], [0.2, 0.1], [0.8, 0.9]), (b, [0.2, 0.3], [0.6, 0.4], [0.9, 0.9]), (c, [0.6, 0.8], [0.4, 0.3], [0.7, 0.5]), (d, [0.1, 0.2], [0.1, 0.1], [0.5, 0.6])\} \]

\[ B = \{(ab, [0.2, 0.3], [0.2, 0.1], [0.9, 0.9]), (bc, [0.2, 0.3], [0.4, 0.3], [0.9, 0.9]), (cd, [0.1, 0.2], [0.1, 0.1], [0.8, 0.9])\} \}

\[ B^* = \{(ac, [0.4, 0.6], [0.2, 0.1], [0.8, 0.9]), (bd, [0.1, 0.2], [0.1, 0.1], [0.9, 0.9])\} \]

Figure 4- \( G \) : Strong Interval, Double-valued Neutrosophic Intuitionistic Fuzzy Graph

Figure 5- \( \overline{G} \) : Strong Interval, Double-valued Neutrosophic Intuitionistic Fuzzy Graph
DEFINITION 3.4

A strong interval, double-valued neutrosophic intuitionistic fuzzy graph $G$ is called self complementaray if $G \cong \overline{G}$. Where $\overline{G}$ is the complement of interval, double-valued neutrosophic intuitionistic fuzzy graph $G$.

PROPOSITION 3.2

Let $G_1$ and $G_2$ be strong interval, double-valued neutrosophic intuitionistic fuzzy graph, $\overline{G_1} \cong \overline{G_2}$ (isomorphism).

Proof

Assume that $G_1$ and $G_2$ are isomorphic, there exist a bijective map $f : v_1 \rightarrow v_2$ satisfying

$$T_{AL}(v_i) = T_{AL}(f(v_i)),$$
$$I_{AL}(v_i) = I_{AL}(f(v_i)),$$
$$F_{AL}(v_i) = F_{AL}(f(v_i)) \text{ for all } v_i \in V$$

And

$$T_{BL}(v_i, v_j) = T_{BL}(f(v_i), f(v_j)),$$
$$I_{BL}(v_i, v_j) = I_{BL}(f(v_i), f(v_j)),$$
$$F_{BL}(v_i, v_j) = F_{BL}(f(v_i), f(v_j)) \text{ for all } (v_i, v_j) \in E.$$
\[
\begin{align*}
F_{BL}(v_i, v_j) &= \min[T_{AL}(v_i), T_{AL}(v_j)] - T_{BL}(v_i, v_j) \\
&= \min[T_{AU}(f(v_i)), T_{AU}(f(v_j))] - T_{BU}(f(v_i), f(v_j)), \\
&= \overline{T_{BU}}(f(v_i), f(v_j)). \\
I_{BL}(v_i, v_j) &= \min[I_{AL}(v_i), I_{AL}(v_j)] - I_{BL}(v_i, v_j) \\
&= \min[I_{AU}(f(v_i)), I_{AU}(f(v_j))] - I_{BU}(f(v_i), f(v_j)), \\
&= \overline{I_{BU}}(f(v_i), f(v_j)). \\
F_{BL}(v_i, v_j) &= \max[F_{AL}(v_i), F_{AL}(v_j)] - F_{BL}(v_i, v_j) \\
&= \max[F_{AU}(f(v_i)), F_{AU}(f(v_j))] - F_{BU}(f(v_i), f(v_j)), \\
&= \overline{F_{BU}}(f(v_i), f(v_j)).
\end{align*}
\]

for all \((v_i, v_j) \in E\). Hence \(G_1 \cong G_2\) (isomorphism).

**PROPOSITION 3.3**

The complement of strong interval, double-valued neutrosophic intuitionistic fuzzy graph is a strong interval, double-valued neutrosophic intuitionistic fuzzy graph with no edge.

**Proof**

Let \(G=(A,B)\) be a strong interval, double-valued neutrosophic intuitionistic fuzzy graph so

\[
\begin{align*}
F_{BL}(v_i, v_j) &= \min[T_{AL}(v_i), T_{AL}(v_j)] - T_{BL}(v_i, v_j), \forall i, j, \ldots, n. \\
&= \min[T_{AL}(v_i), T_{AL}(v_j)] - \min[T_{AL}(v_i), T_{AL}(v_j)] \text{ for all } i, j, \ldots, n \\
&= 0 \text{ for all } i, j, \ldots, n. \\
I_{BL}(v_i, v_j) &= \min[I_{AL}(v_i), I_{AL}(v_j)] - I_{BL}(v_i, v_j), \forall i, j, \ldots, n. \\
&= \min[I_{AL}(v_i), I_{AL}(v_j)] - \min[I_{AL}(v_i), I_{AL}(v_j)] \text{ for all } i, j, \ldots, n \\
&= 0 \text{ for all } i, j, \ldots, n.
\end{align*}
\]
\[ I_{BU}(v_i, v_j) = \min[J_{AU}(v_i), I_{AU}(v_j)] - I_{BU}(v_i, v_j), \forall i, j \ldots \ldots n. \]

\[ = \min[J_{AU}(v_i), I_{AU}(v_j)] - \min[J_{AU}(v_i), I_{AU}(v_j)] \text{ for all } i,j, \ldots n \]

\[ = 0 \text{ for all } i,j, \ldots n. \]

\[ F_{BL}(v_i, v_j) = \max[F_{AU}(v_i), F_{AL}(v_j)] - F_{BL}(v_i, v_j), \forall i, j \ldots \ldots n \]

\[ = \max[F_{AU}(v_i), F_{AL}(v_j)] - \max[F_{AU}(v_i), F_{AL}(v_j)] \text{ for all } i,j, \ldots n \]

\[ = 0 \text{ for all } i,j, \ldots n. \]

\[ F_{BU}(v_i, v_j) = \max[F_{AU}(v_i), F_{BU}(v_j)] - F_{BU}(v_i, v_j), \forall i, j \ldots \ldots n \]

\[ = \max[F_{AU}(v_i), F_{BU}(v_j)] - \max[F_{AU}(v_i), F_{BU}(v_j)] \text{ for all } i,j, \ldots n \]

\[ = 0 \text{ for all } i,j, \ldots n. \]

Thus \[ [I_{BL}(v_i, v_j), T_{BL}(v_i, v_j), I_{BU}(v_i, v_j), I_{BU}(v_i, v_j), F_{BL}(v_i, v_j), F_{BU}(v_i, v_j)] = 0. \]

Hence the edge set of \( \tilde{G} \) is empty. If \( G \) is a strong interval, double-valued neutrosophic intuitionistic fuzzy graph.

**IV CONCLUSION**

Strong interval, double-valued neutrosophic intuitionistic fuzzy sets is a generalization of the notion of fuzzy sets, intuitionistic fuzzy set, Interval valued fuzzy sets, Interval valued intuitionistic fuzzy sets and double valued neutrosophic sets.

In this paper we have defined for the first time interval, double-valued neutrosophic intuitionistic fuzzy graph. In future study, we plan to extend our research to interval, double-valued neutrosophic intuitionistic soft fuzzy graph.

**REFERENCES**


