symmetry

MDPI

Article

Introduction to Non-Standard Neutrosophic Topology

Mohammed A. Al Shumrani [1] and Florentin Smarandache [2,*]

[1] Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia; maalshmrani1@kau.edu.sa
[2] Mathematics Department, University of New Mexico, Gallup, NM 87301, USA
* Correspondence: smarand@unm.edu

Received: 10 May 2019; Accepted: 20 May 2019; Published: 23 May 2019

Abstract: For the first time we introduce non-standard neutrosophic topology on the extended non-standard analysis space, called non-standard real monad space, which is closed under neutrosophic non-standard infimum and supremum. Many classical topological concepts are extended to the non-standard neutrosophic topology, several theorems and properties about them are proven, and many examples are presented.

Keywords: non-standard analysis; extended non-standard analysis; monad; binad; left monad closed to the right; right monad closed to the left; pierced binad; unpierced binad; non-standard neutrosophic mobinad set; neutrosophic topology; non-standard neutrosophic topology

1. Introduction to Non-Standard Analysis

The purpose of this study is to initiate for the first time a new field of research, called non-standard neutrosophic algebraic structures, and we start with non-standard neutrosophic topology (NNT) in this paper. Being constructed on the set of hyperreals, that includes the infinitesimals, NNT can further be utilized in neutrosophic calculus applications.

As a branch of mathematical logic, non-standard analysis [1] deals with *hyperreal numbers*, which include *infinitesimals* and *infinities*.

The introduction of infinitesimals in calculus has been debated philosophically in the history of mathematics since the time of G. W. Leibniz, with pros and cons. Many mathematicians prefer the *epsilon-delta* use in calculus concepts' definitions and theorems' proofs.

By the 1960s Abraham Robinson had developed non-standard analysis [2] in a more rigorous way.

Besides calculus, non-standard analysis found applications in mathematical physics, mathematical economics, and in probability theory.

In 1998, Smarandache [3] used non-standard analysis in philosophy and in neutrosophic logic, in order to differentiate between *absolute truth* (which is truth in all possible worlds, according to Leibniz), and *relative truth* (which is, according to the same Leibniz, truth in at least one world). Let T represent the neutrosophic truth value, I the neutrosophic indeterminacy value, and F the neutrosophic falsehood value, with $T, I, F \in [^-0, 1^+]$. Then T (*absolute truth*) = $1^+ = \mu\ (1^+)$, while T (*relative truth*) = 1. This is analogously for *absolute falsehood* vs. *relative falsehood*, and *absolute indeterminacy* vs. *relative indeterminacy.*

Then he extended [3] the use of non-standard analysis to neutrosophic set (*absolute membership/indeterminacy/nonmembership* vs. *relative membership/indeterminacy/nonmembership* respectively) and to neutrosophic probability (*absolute occurrence/indeterminate occurrence/nonoccurence of an event* vs. *relative occurrence/indeterminate occurrence/nonoccurence of an event*, respectively).

We next recall several notions and results from classical non-standard analysis [2] that are needed to defining and developing the non-standard neutrosophic topology.

The set R^* of *nonstandard reals* (or *hyperreals*) is the generalization of the real numbers (R).

The *transfer principle* states that first-order statements that are valid in R are also valid in R^*.

$$R^* \text{ includes the infinites and the infinitesimals, which on the } \textit{hyperreal number line} \text{ may be represented as } 1/\varepsilon = \omega/1. \tag{1}$$

An *infinite* (or infinite number) (ω) is a number that is greater than anything:

$$1 + 1 + 1 + \ldots\ + 1 \text{ (for any number of finite terms)} \tag{2}$$

The infinitesimals are reciprocals of infinites.

An *infinitesimal* (or infinitesimal number) (ε) is a number ε such that $|\varepsilon| < 1/n$, for any non-null positive integer n.

An infinitesimal is so small that it cannot be measured, and it is very close to zero.

The infinitesimal in absolute value, is a number smaller than anything nonzero positive number.

In calculus one uses the infinitesimals.

$$\text{By } R_+{}^* \text{ we denote the set of positive non-zero hyperreal numbers.} \tag{3}$$

Left Monad {for simplicity, denoted [2] by (^-a) or only ^-a} was defined as:

$$\mu(^-a) = (^-a) = {}^-a = \overline{a} = \{a - x, x \in R_+{}^* | x \text{ is } infinitesimal\} \tag{4}$$

Right Monad {for simplicity, denoted [2] by (a^+) or only by a^+} was defined as:

$$\mu(a^+) = (a^+) = a^+ = \overset{+}{a} = \{a + x, x \in R_+{}^* | x \text{ is } infinitesimal\} \tag{5}$$

μ (a) is a *monad* (*halo*) of an element $a \in R^*$, which is formed by a subset of numbers infinitesimally close (to the left-hand side, or right-hand side) to a.

1.1. Non-Standard Analysis's First Extension

In 1998, Smarandache [3] introduced the pierced binad.

Pierced binad {for simplicity, denoted by ($^-a^+$) or only $^-a^+$} was defined as:

$$\begin{aligned} &\mu(^-a^+) = (^-a^+) = {}^-a^+ = \overset{-+}{a} = \\ &= \{a - x, x \in R_+{}^* | x \text{ is } infinitesimal\} \cup \{a + x, x \in R_+{}^* | x \text{ is } infinitesimal\} \\ &= \{a \pm x, x \in R_+{}^* | x \text{ is } infinitesimal\} \end{aligned} \tag{6}$$

This extension was needed in order to be able to do union aggregations of non-standard neutrosophic sets, where a left monad μ ($^-$a) had to be united with a right monad μ (a$^+$), as such producing a pierced binad: μ ($^-$a) $\cup$ μ (a$^+$) $=_N$ μ ($^-$a$^+$). Without this pierced binad we would not have been able to define the non-standard neutrosophic operators.

1.2. Non-Standard Analysis's Second Extension

Smarandache [4,5] introduced at the beginning of 2019 for the first time, the left monad closed to the right, the right monad closed to the left, and unpierced binad, defined as below:

Left Monad Closed to the Right

$$\begin{aligned} &\mu(\overset{-0}{a}) = (\overset{-0}{a}) = \overset{-0}{a} = \{a - x | x = 0, or\ x \in R_+{}^* \text{ and } x \text{ is } infinitesimal\} = \mu(^-a) \cup \{a\} = (^-a) \cup \{a\} \\ &= {}^-a \cup \{a\} \end{aligned} \tag{7}$$

Right Monad Closed to the Left

$$\mu(\overset{0+}{a}) = (\overset{0+}{a}) = \overset{0+}{a} = \{a + x | x = 0, or\ x \in R_+{}^*\ and\ x\ is\ infinitesimal\} = \mu(a^+) \cup \{a\} = (a^+) \cup \{a\} = a^+ \cup \{a\} \tag{8}$$

Unpierced Binad

$$\begin{aligned}\mu(\overset{-0+}{a}) = (\overset{-0+}{a}) = \overset{-0+}{a} &= \{a - x |\ x \in R_+{}^*\ and\ x\ is\ infinitesimal\} \cup \{a + x |\ x \in R_+{}^*\ and\ x\ is\ infinitesimal\} \cup \{a\} \\ &= \{a \pm x | x = 0, or\ x \in R_+{}^*\ and\ x\ is\ infinitesimal\} \\ &= \mu({}^-a^+) \cup \{a\} = ({}^-a^+) \cup \{a\} = {}^-a^+ \cup \{a\}\end{aligned} \tag{9}$$

Therefore, as seen, the element $\{a\}$ has been included in both the left and right monads, and also in the pierced binad respectively.

All monads and binads are subsets of R^*.

This second extension was done in order to be able to compute the non-standard aggregation operators (negation, conjunction, disjunction, implication, equivalence) in non-standard neutrosophic logic, set, and probability, and now we need them in non-standard neutrosophic topology.

1.3. The Best Notations for Monads and Binads

For any standard real number $a \in R$, we employ the following notations for monads and binads:

$$\overset{m}{a} \in \{a, \overset{-}{a}, \overset{-0}{a}, \overset{+}{a}, \overset{0+}{a}, \overset{-+}{a}, \overset{-0+}{a}\} \text{ and by convention } \overset{0}{a} = a; \tag{10}$$

where

$$m \in \{^-, {}^{-0}, {}^+, {}^{+0}, {}^{-+}, {}^{-0+}\} = \{^0, {}^-, {}^{-0}, {}^+, {}^{+0}, {}^{-+}, {}^{-0+}\}; \tag{11}$$

thus "m" written above the standard real number "a" means: a standard real number (^{0}, or nothing above), or a left monad ($^-$), or a left monad closed to the right ($^{-0}$), or a right monad ($^+$), or a right monad closed to the left ($^{0+}$), or a pierced binad ($^{-+}$), or a unpierced binad ($^{-0+}$) respectively.

Neutrosophic notations will have an index $_N$ associated to each symbol, for example: the classical symbol < (less than), becomes $<_N$ (neutrosophically less than, i.e., some indeterminacy is involved, especially with respect to infinitesimals, monads and binads).

Similarly for: $\cap$ and $\cap_N$, $\wedge$ and $\wedge_N$ etc.

1.4. Non-Standard Neutrosophic Inequalities

We have the following *neutrosophic non-standard inequalities* (taking into account the definitions of infinitesimals, monads and binads):

$$({}^-a) <_N a <_N (a^+) \tag{12}$$

because

$$\forall x \in R_+^*, a - x < a < a + x \tag{13}$$

where x is a (nonzero) positive infinitesimal.

The converse also is true:

$$(a^+) >_N \text{a} >_N (-a) \tag{14}$$

Similarly:

$$({}^-a) \leq_N ({}^-a^+) \leq_N (a^+) \tag{15}$$

To prove it, we rely on the fact that $({}^-a^+) = ({}^-a) \cup (a^+)$ and the number a is in between the subsets (on the real number line) ${}^-a = (a - \varepsilon, a)$ and $a^+ = (a, a + \varepsilon)$, so:

$$({}^-a) \leq_N ({}^-a) \cup (a^+) \geq_N (a^+) \tag{16}$$

Conversely, it is neutrosophically true too:

$$(a^{+}) \geq_N ({}^{-}a) \cup (a^{+}) \geq_N ({}^{-}a) \tag{17}$$

$$\text{Also, } \overset{-}{a} \leq_N \overset{-0}{a} \leq_N a \leq_N \overset{0+}{a} \leq_N \overset{+}{a} \text{ and } \overset{-}{a} \leq_N \overset{-+}{a} \leq_N \overset{-0+}{a} \leq_N \overset{+}{a} \tag{18}$$

Conversely, they are also neutrosophically true:

$$\overset{+}{a} \geq_N \overset{0+}{a} \geq_N a \geq_N \overset{-0}{a} \geq_N \overset{-}{a} \text{ and } \overset{+}{a} \geq_N \overset{-0+}{a} \geq_N \overset{-+}{a} \geq_N \overset{-}{a} \text{ respectively.} \tag{19}$$

Let a, b be two standard real numbers. If $a > b$, which is (standard) classical real inequality, then we have:

$$a >_N ({}^{-}b), a >_N (b^{+}), a >_N ({}^{-}b^{+}), a >_N \overset{-0}{b}, a >_N \overset{0+}{b}, a >_N \overset{-0+}{b}; \tag{20}$$

$$({}^{-}a) >_N b, ({}^{-}a) >_N ({}^{-}b), ({}^{-}a) >_N (b^{+}), ({}^{-}a) >_N ({}^{-}b^{+}), \overset{-}{a} >_N \overset{-0}{b}, \overset{-}{a} >_N \overset{0+}{b}, \overset{-}{a} >_N \overset{-0+}{b}; \tag{21}$$

$$(a^{+}) >_N b, (a^{+}) >_N b({}^{-}b), (a^{+}) >_N b(b^{+}), (a^{+}) >_N b({}^{-}b^{+}), \overset{+}{a} >_N \overset{-0}{b}, \overset{+}{a} >_N \overset{0+}{b}, \overset{+}{a} >_N \overset{-0+}{b}; \tag{22}$$

$$({}^{-}a^{+}) >_N b, ({}^{-}a^{+}) >_N ({}^{-}b), ({}^{-}a^{+}) >_N (b^{+}), ({}^{-}a^{+}) >_N ({}^{-}b^{+}), \text{etc.} \tag{23}$$

No non-standard order relationship between a and $({}^{-}a^{+})$,

$$\text{nor between } a \text{ and } ({}^{-0}a^{+}). \tag{24}$$

1.5. Neutrosophic Infimum and Neutrosophic Supremum

1.5.1. Neutrosophic Infimum

Let $(S, <_N)$ be a set, which is neutrosophically partially ordered, and let M be a subset of S.

The neutrosophic infimum of M, denoted by $inf_N\ (M)$, is the neutrosophically greatest element in S, which is neutrosophically less than or equal to all elements of M.

1.5.2. Neutrosophic Supremum

Let $(S, <_N)$ be a set, which is neutrosophically partially ordered, and let M be a subset of S.

The neutrosophic supremum of M, denoted by $sup_N\ (M)$, is the neutrosophically smallest element in S, which is neutrosophically greater than or equal to all elements of M.

The neutrosophic infimum and supremum are both extensions of the classical infimum and supremum respectively, using the *transfer principle* from the real set R to the neutrosophic real *MoBiNad* set NR_{MB} defined below.

1.5.3. Property

$$\begin{array}{l}\text{If } \overset{m_1}{a}, \overset{m_2}{b} \text{ are left monads, right monads, pierced binads, or unpierced monads,} \\ \text{then both } \inf_N\{\overset{m_1}{a}, \overset{m_2}{b}\} \text{ and } \sup_N\{\overset{m_1}{a}, \overset{m_2}{b}\} \text{ are left monads or right monads.}\end{array} \tag{25}$$

1.6. Non-Standard Real MoBiNad Set

MoBiNad [3] etymologically comes from **mo***nad* + **bi***nad*.

Let R and R^* be the set of standard real numbers, and respectively the set of hyper-reals (or non-standard reals) that contains the infinitesimals and infinites.

The Non-standard Real MoBiNad Set [2] is built as follows:

$$NR_{MB} = {}_N\left\{ \begin{array}{c} \varepsilon, \omega, a, (^{-}a), (^{-}a^{0}), (a^{+}), (^{0}a^{+}), (^{-}a^{+}), (^{-}a^{0+}) \mid \text{where } \varepsilon \text{ are infinitesimals,} \\ \text{with } \varepsilon \in \mathbb{R}^{*}; \omega = 1/\varepsilon \text{ are infinites, with } \omega \in \mathbb{R}^{*}; \text{and } a \text{ are real numbers, with } a \in \mathbb{R} \end{array} \right\} \quad (26)$$

or,

$$NR_{MB} = {}_N\left\{ \varepsilon, \omega, \overset{m}{a} \;\middle|\; \begin{array}{c} \text{where } \varepsilon, \omega \in \mathbb{R}^{*}, \varepsilon \text{ are infinitesimals, } \omega = \frac{1}{\varepsilon} \text{ are infinitesimals;} \\ a \in \mathbb{R}; \text{ and } m \in \{, ^{-}, ^{-0}, ^{+}, ^{+0}, ^{-+}, ^{-0+}\} \end{array} \right\} \quad (27)$$

As a set, NR_{MB} is closed under addition, subtraction, multiplication, division [except division by $\overset{m}{a}$, with $a = 0$ and $m \in \{, ^{-}, ^{-0}, ^{+}, ^{0+}, ^{-+}, ^{-0+}\}$], and power

$$\{\left(\overset{m_1}{a}\right)^{\left(\overset{m_2}{b}\right)} \text{ with: either } a > 0, \text{ or a } \leq 0 \text{ but } b = \frac{p}{r} \text{ (irreducible fraction) and } p, r \text{ are positive integers with } r \text{ an odd number}\}. \quad (28)$$

1.7. Remark

The neutrosophic infimum and neutrosophic maximum are well-defined on the Non-standard Real MoBiNad Set NR_{MB}, in the sense that we can compute inf_N and sup_N of any subset of NR_{MB}.

1.8. Non-Standard Real Open Monad Unit Interval

Since there is no relationship of order between a and $^{-}a^{+}$, not between a and $(^{-0}a^{+})$, and we need a total order relationship on the set of non-standard real numbers, we remove all binads and keep only the open left monads and open right monads [we also remove the monads closed to one side].

$$]^{-}0,\ 1^{+}[_M = \{a,\ \varepsilon, ^{-}a, a^{+} | a \in [0,1], \varepsilon \in R^{*}, \varepsilon > 0\}. \quad (29)$$

where a is subunitary real number, and ε is an infinitesimal number.

The non-standard neutrosophic unit interval $]^{-}0, 1^{+}[_M$ includes the previously defined $]^{-}0, 1^{+}[$ as follows:

$$]^{-}0, 1^{+}[=_N (^{-}0) \cup [0,1] \cup (1^{+}) \subset_N]^{-}0, 1^{+}[_M \quad (30)$$

where the index $_M$ means that the interval includes all open monads and infinitesimals between $^{-}0$ and 1^{+}.

2. General Monad Neutrosophic Set

Let U be a universe of discourse, and S ⊂ U be a subset. Then, a *Neutrosophic Set* is a set for which each element x from S has a degree of membership (T), a degree of indeterminacy (I), and a degree of non-membership (F), with T, I, F standard or non-standard real monad subsets or infinitesimals, neutrosophically included in or equal to the nonstandard real monad unit interval $]^{-}, ^{+}[_M$, or

$$T, I, F \subseteq_N]^{-}0, 1^{+}[_M \quad (31)$$

where

$$^{-}0 \leq_N inf_N T + inf_N I + inf_N F \leq_N sup_N T + sup_N I + sup_N F \leq 3^{+}. \quad (32)$$

2.1. Non-Standard Neutrosophic Set

Let us consider the above general definition of general neutrosophic set, and assume that at least one of T, I, or F (the neutrosophic components) is a non-standard real monad subset or infinitesimal, neutrosophically included in or equal to $]^{-}0, 1^{+}[_M$, where

$$^{-}0 \leq_N inf_N T + inf_N I + inf_N F \leq_N sup_N T + sup_N I + sup_N F \leq 3^{+}, \quad (33)$$

we have a non-standard neutrosophic set.

2.2. Non-Standard Fuzzy t-Norm and Fuzzy t-Conorm

Let T_1, and T_2, $\in]^{-}0, 1^{+}[_M$, be nonstandard real numbers (infinitesimals, or open monads), or standard (classical) real numbers, such that at least one of them is a non-standard real number. T_1 and T_2 are non-standard fuzzy degrees of membership. Then one has:

The non-standard fuzzy t-norms:

$$T_1 /\backslash_F\, T_2 = inf_N \,\{T_1, T_2\} \tag{34}$$

The non-standard fuzzy t-conorms:

$$T_1 \backslash/_F\, T_2 = sup_N \,\{T_1, T_2\} \tag{35}$$

2.3. Aggregation Operators on Non-Standard Neutrosophic Set

Let T_1, I_1, F_1 and $T_2, I_2, F_2 \in]^{-}0, 1^{+}[_{MB}$, be nonstandard real numbers (infinitesimals, or monads), or standard (classical) real numbers, such that at least one of them is a non-standard real number.

Non-Standard Neutrosophic Conjunction

$$\begin{gathered}(T_1, I_1, F_1) \wedge_N (T_2, I_2, \mathrm{F}_2) = (T_1 \wedge_F T_2, I_1 \vee_F I_2, F_1 \vee_F F_2) = \\ (inf_N\,(T_1, T_2), sup_N\,(I_1, I_2), sup_N\,(F_1, F_2))\end{gathered} \tag{36}$$

Non-Standard Neutrosophic Disjunctions

$$\begin{gathered}(T_1, I_1, F_1) \vee_N (T_2, I_2, F_2) = (T_1 \vee_F T_2, I_1 \wedge_F I_2, F_1 \wedge_F F_2) = \\ (sup_N\,(T_1, T_2), inf_N\,(I_1, I_2), inf_N\,(F_1, F_2))\end{gathered} \tag{37}$$

Non-Standard Neutrosophic Complement/Negation

We may use the notations $\mathrm{C_N}$ or $\neg_N$ for the neutrosophic complement.

$$\mathrm{C_N}(T_1, I_1, F_1) = {}_{N}\neg_N(T_1, I_1, F_1) =_N (F_1, I_1, T_1). \tag{38}$$

Non-Standard Neutrosophic Inclusion/Inequality

$$(T_1, I_1, F_1) \leq_N (T_2, I_2, F_2)\ iff\ T_1 \leq_N T_2, I_1 \geq_N I_2, F_1 \geq_N F_2. \tag{39}$$

Let $A, B \in P\,(X)$, if $A \subseteq_N B$ then B is called a *neutrosophic superset* of A.

Non-standard Neutrosophic Equality

$$(T_1, I_1, F_1) =_N (T_2, I_2, F_2)\ \mathit{iff}\ (T_1, I_1, F_1) \leq_N (T_2, I_2, F_2)\ \mathit{and}\ (T_2, I_2, F_2) \leq_N (T_1, I_1, F_1). \tag{40}$$

Non-Standard Monad Neutrosophic Universe of Discourse

We now introduce for the first time the non-standard neutrosophic universe.

Definition 1. *A general set U, defined such that each element $x \in U$ has neutrosophic coordinates of the form $x(T_x, I_x, F_x)$, such that T_x represents the degree of truth-membership of the element x with respect to set U, I_x represents the degree of indeterminate-membership of the element x with respect to the set U, and F_x represents the degree of false-membership of the element x with respect to the set U; where T_x, I_x, and F_x are non-standard or standard subsets of the neutrosophic real monad set NR_M, but at least one of all of them is non-standard (i.e., contains infinitesimals, or open monads).*

Single-Valued Non-Standard Neutrosophic Topology

Let U be a single-valued non-standard neutrosophic universe of discourse, i.e., for all $x \in U$, their neutrosophic components T_x, I_x, F_x are single-values (either real numbers, or infinitesimals, or open monads) belonging to $]^{-}0, 1^{+}[$

Definition 2. *Let X be a non-standard neutrosophic subset of U. The neutrosophic empty-set, denoted by* $\boldsymbol{0}_N$ = $(^{-}0, 1^{+}, 1^{+})$, *is a set* $\Phi_N \subset X$ *whose all elements have the non-standard neutrosophic components equal to* $(^{-}0, 1^{+}, 1^{+})$. *The whole set, denoted by* $1_N = (1^{+}, ^{-}0, ^{-}0)$, *is a set* $W_N \subset X$ *whose all elements have the non-standard neutrosophic components equal to* $(1^{+}, ^{-}0, ^{-}0)$.

Definition 3. *Let X be a non-standard neutrosophic set. Let* $A = (T_1, I_1, F_1)$ *and* $B = (T_2, I_2, F_2)$ *be non-standard neutrosophic numbers. Then:*

$$\mathrm{A} \cap \mathrm{B} = (\inf_N (\mathrm{T}_1, \mathrm{T}_2), \sup_N (\mathrm{I}_1, \mathrm{I}_2), \sup_N (\mathrm{F}_1, \mathrm{F}_2)) \quad (41)$$

$$\mathrm{A} \cup \mathrm{B} = (\sup_N (\mathrm{T}_1, \mathrm{T}_2), \inf_N (\mathrm{I}_1, \mathrm{I}_2), \inf_N (\mathrm{F}_1, \mathrm{F}_2)) \quad (42)$$

$$\mathrm{C}_N\mathrm{A} = (\mathrm{F}_1, \mathrm{I}_1, \mathrm{T}_1) \quad (43)$$

Definition 4. *Let X be a non-standard neutrosophic set. Let A(X) be the family of all non-standard neutrosophic sets in X. Let* $\tau \subseteq A(X)$ *be a family of non-standard neutrosophic sets in X. Then* τ *is called a Non-standard Neutrosophic Topology on X, if it satisfies the following axioms:*

(i) $\boldsymbol{0}_N$ *and* $\boldsymbol{1}_N$ *are in* τ.
(ii) *The intersection of the elements of any finite subcollection of* τ *is in* τ.
(iii) *The union of the elements of any subcollection of* τ *is in* τ.

The pair (X, τ*) is called a non-standard neutrosophic topological space. All members of* τ *are called non-standard neutrosophic open sets in X.*

Example 1. *Let X be a non-standard neutrosophic set. Let* τ *be the set consisting of* $\boldsymbol{0}_N$ *and* $\boldsymbol{1}_N$. *Then* τ *is a topology on X. It is called the non-standard neutrosophic trivial topology.*

Example 2. *Let X be a non-standard neutrosophic set. Let A be a non-standard neutrosophic set in X. Let* $\tau = \{\boldsymbol{0}_N, \boldsymbol{1}_N, A\}$. *Then it can be easily shown that* τ *is a topology on X.*

Example 3. *Let X be a non-standard neutrosophic set. Let A and B be non-standard neutrosophic sets in X such that A is a neutrosophic superset of B. Let* $\tau = \{\boldsymbol{0}_N, \boldsymbol{1}_N, A, B\}$. *Then since* $A \cap B = B$ *and* $A \cup B = A$ *we deduce that* τ *is a topology on X.*

Example 4. *Let X be a non-standard neutrosophic set. Suppose we have a nested sequence*

$$A_1 \subseteq A_2 \subseteq A_3 \subseteq \ldots \subseteq A_{n-1} \subseteq A_n \subseteq \quad (44)$$

of non-standard neutrosophic sets in X such that each A_n *is a neutrosophic superset of* A_{n-1} *for each*

$$n \in \{1, 2, 3, \ldots\}.$$

Let $\tau = \{\boldsymbol{0}_N, \boldsymbol{1}_N, A_n\colon n \in N\}$. *Then since* $A_i \cap_N A_j = A_i$ *and* $A_i \cup_N A_j = A_j$ *for each i less than j, we deduce that* τ *is a topology on X.*

Example 5. *Let X be a non-standard neutrosophic infinite set:*

$$X = \left(x_{m,n,p} \left(\left(\overset{+}{0.7} \right)^m, (0.2)^n, \left(\overset{-}{0.6} \right)^p \right), x_{m,n,p} \in X; m, n, p \in \{1, 2, \ldots\} \right) \quad (45)$$

Let M_{100} be a family of subsets of X, such that each member $A_{m,n,p}$ of the family has:

$$m, n, p \in \{1, 2, \ldots, 100\}. \tag{46}$$

Then $\tau = \{\mathbf{0}_N, \mathbf{1}_N, M_{100}\}$ is a non-standard neutrosophic topology.

Proof. Any monad $\left(\overset{m}{a}\right)$ raised to the integer power $k > 0$, is equal to the monad of a^k:

$$\left(\overset{m}{a}\right)^k = \left(\overset{m}{a^k}\right) \tag{47}$$

Let's consider two non-standard neutrosophic elements from X:

$$x_{m_1,n_1,p_1}\left(\left(\overset{+}{0.7}\right)^{m_1}, (0.2)^{n_1}, \left(\overset{-}{0.6}\right)^{p_1}\right) \text{ and } x_{m_2,n_2,p_2}\left(\left(\overset{+}{0.7}\right)^{m_2}, (0.2)^{n_2}, \left(\overset{-}{0.6}\right)^{p_2}\right) \tag{48}$$

where

$$m_1, n_1, p_1, m_2, n_2, p_2 \in \{1, 2, \ldots, 100\}. \tag{49}$$

It is sufficient to prove that their non-standard neutrosophic finite intersection and the random union of elements from M_{100} are in M_{100}.

$$\begin{aligned} & x_{m_1,n_1,p_1} \cap_N \; x_{m_2,n_2,p_2} =_N \left(\inf_N\{\left(\overset{+}{0.7}\right)^{m_1}, \left(\overset{+}{0.7}\right)^{m_2}\},\right. \\ & \left.\mathrm{SUP}_N\{(0.2)^{n_1}, (0.2)^{n_2}\}, \mathrm{SUP}_N\{\left(\overset{-}{0.6}\right)^{p_1}, \left(\overset{-}{0.6}\right)^{p_2}\}\right) \\ & = \left(\left(\overset{+}{0.7}\right)^{\max\{m_1,m_2\}}, (0.2)^{\min\{n_1,n_2\}}, \left(\overset{-}{0.6}\right)^{\min\{p_1,p_2\}}\right) \in M_{100} \end{aligned} \tag{50}$$

because also $max\{m_1, m_2\}, min\{n_1, n_2\}, min\{p_1, p_2\} \in M_{100}$. (51)

$$\begin{aligned} & \bigcup_{m,n,p\in(\psi_1,\psi_2,\psi_3)\subseteq\{1,2,\ldots,100\}^3} \{x_{m,n,p}\left(\left(\overset{+}{0.7}\right)^{m}, (0.2)^{n}, \left(\overset{-}{0.6}\right)^{p}\right)\} \\ & = \left(\left(\overset{+}{0.7}\right)^{\min\{m, m\in\psi_1\}}, (0.2)^{\max\{n, n\in\psi_2\}}, \left(\overset{-}{0.6}\right)^{\max\{p, p\in\psi_3\}}\right) \in M_{100} \end{aligned} \tag{52}$$

□

Definition 5. *Let X be a non-standard neutrosophic set. Suppose that τ and τ′ are two topologies on X such that τ ⊂ τ ′. Then we say that τ′ is finer than τ.*

Example 6. *Let X be a non-standard neutrosophic set. Let A and B be non-standard neutrosophic sets in X such that A is a neutrosophic superset of B. Let $\tau = \{\mathbf{0}_N, \mathbf{1}_N, A\}$ and $\tau' = \{\mathbf{0}_N, \mathbf{1}_N, B\}$.*

Then τ′ is finer than τ.

Example 7. *Let's consider the above Example 5. In addition to M_{100}, let's define L_{100} as follows:*

$$L_{100} = \{x_{m,n,p}\left(\left(\overset{+}{0.7}\right)^{m}, (0.2)^{n}, \left(\overset{-}{0.6}\right)^{p}\right), x_{m,n,p} \in X; m, n, p \in \{2, 4, 6, \ldots, 100\}\} \tag{53}$$

The non-standard neutrosophic topology $\tau = \{\mathbf{0}_N, \mathbf{1}_N, M_{100}\}$ is a finer non-standard neutrosophic topology than the non-standard neutrosophic topology $\tau' = \{\mathbf{0}_N, \mathbf{1}_N, L_{100}\}$.

Definition 6. *The subset Z of a non-standard neutrosophic topological space X is called a non-standard neutrosophic closed set if its complement* C_N *(Z) is open in X.*

Example 8. *Let Y be a non-standard neutrosophic infinite set*

$$Y = \{y_{m,n}\left(\left(\overset{+}{0.5}\right)^m, \left(\overset{-}{0.1}\right)^n, \left(\overset{+}{0.5}\right)^m\right), y_{m,n} \in Y; m, n \in \{1, 2, \ldots\}\} \tag{54}$$

and P (Y) the power set of Y.

Let $\tau \subseteq P(Y)$ *be a non-standard neutrosophic topology.*

Each non-standard neutrosophic set $A \in \tau$ *is a non-standard neutrosophic open set and closed set in the same time, because its non-standard neutrosophic complement* $C_N(A) = A$.

Proof. For any $y_{m,n} \in Y$ one has:

$$C_N\left(y_{m,n}\right) = C_n\left(\left(\overset{+}{0.5}\right)^m, \left(\overset{-}{0.1}\right)^n, \left(\overset{+}{0.5}\right)^m\right) = \left(\left(\overset{+}{0.5}\right)^m, \left(\overset{-}{0.1}\right)^n, \left(\overset{+}{0.5}\right)^m\right) = y_{m,n} \tag{55}$$

□

Theorem 1. *Unlike in classical topology, the non-standard neutrosophic empty-set* $\mathbf{0}_N$ *and the non-standard neutrosophic whole set* $\mathbf{1_N}$ *are not necessarily closed, since they are not the non-standard neutrosophic complement of each other.*

Proof.

$$C_N\ ({}^{-}0, 1^+, 1^+) =_N (1^+, 1^+, {}^{-}0) \neq (1^+, {}^{-}0, {}^{-}0), \text{ and reciprocally:} \tag{56}$$

$$C_N\ (1^+, {}^{-}0, {}^{-}0) =_N ({}^{-}0, {}^{-}0, 1^+) \neq ({}^{-}0, 1^+, 1^+). \tag{57}$$

Theorem 2. *In a non-stardard neutrosophic topology there may be non-standard neutrosophic sets which are both open and closed set.*

Proof. See the above Example 8. □

Theorem 3. *Unlike in classical topology, the intersection of two non-standard neutrosophic closed sets is not necessarily a non-standard neutrosophic closed set. Moreover, the union of two non-standard neutrosophic closed sets is not necessarily a non-standard neutrosophic closed set.*

Proof. Consider Example 3 above.

$$\text{Let } A = (T_2, I_2, F_2) \text{ and } B = (T_1, I_1, F_1). \text{ Note that } C_N A = (F_2, I_2, T_2) \text{ and } C_N B = (F_1, I_1, T_1). \tag{58}$$

$$\text{Then } C_N A \cap_N C_N B = (F_2, I_1, T_2). \tag{59}$$

$$\text{Since } C_N\ (C_N A \cap_N C_N B) = (T_2, I_1, F_2) \tag{60}$$

is not non-standard neutrosophic open set in X, we have that $C_N A \cap_N C_N B$ is not a non-standard neutrosophic closed set in X. Also,

$$C_N A \cap_N C_N B = (F_1, I_2, T_1). \tag{61}$$

$$\text{Since } C_N\ (C_N A \cap_N C_N B) = (T_1, I_2, F_1) \tag{62}$$

is not non-standard neutrosophic open set in X, we have that $C_N A \cap_N C_N B$ is not a non-standard neutrosophic closed set in X. □

General Remark 1. *Since the non-standard neutrosophic aggregation operators (conjunction, disjunction, complement) needed in non-standard neutrosophic topology, are defined by* ***classes of operators*** *(not by exact unique operators) respectively, the classical topological space theorems and properties extended (by the transfer principle) to the non-standard neutrosophic topological space may be valid for some non-standard neutrosophic operators, but invalid for other classes of neutrosophic aggregation operators.*

Even worth, due to the fact that non-standard neutrosophic conjunction/disjunction/complement are, in addition, based on fuzzy t-norms and fuzzy t-conorms, which are not fixed either, but characterized by classes!

{Similarly for fuzzy and intuitionistic fuzzy aggregation operators.}

For example, the neutrosophic intersection$/\backslash_N$ can be defined in 2 ways:

$$(T_1, I_1, F_1) /\backslash_N (T_2, I_2, F_2) = (T_1 /\backslash_F T_2, I_1 /\backslash_F I_2, F_1 /\backslash_F F_2) \tag{63}$$

And

$$(T_1, I_1, F_1) /\backslash_N (T_2, I_2, F_2) = (T_1 /\backslash_F T_2, I_1 \backslash/_F I_2, F_1 /\backslash_F F_2). \tag{64}$$

In turn, the fuzzy t-norms ($/\backslash_F$) and fuzzy t-conorm ($\backslash/_F$) are also defined in many ways; for example I know at least 3 types of fuzzy t-norms:

$$a /\backslash_F b = min\ \{a, b\} \tag{65}$$

$$a /\backslash_F b = ab \tag{66}$$

$$a /\backslash_F b = max\ \{a + b - 1, 0\} \tag{67}$$

and 3 types of fuzzy t-conorms:

$$a \backslash/_F b = max\ \{a, b\} \tag{68}$$

$$a /\backslash_F b = a + b - ab \tag{69}$$

$$a /\backslash_F b = min\ \{a + b, 1\} \tag{70}$$

therefore there exist at least 2·3·3 = 18 possibilities to define the neutrosophic *t-norm* ($/\backslash_N$).

There exist at least the same number 18 of possibilities of defining the neutrosophic *t-conorm* ($\backslash/_N$).

From these 18 possibilities of defining$/\backslash_N$ and $\backslash/_N$ for some of them the classical topological theorems extended to non-standard neutrosophic topology may be valid, for others invalid.

Definition 7. *Let (X, τ) be a nonstandard neutrosophic topological space. Let A be a non-standard neutrosophic set in X. Then the Non-standard Neutrosophic Closure of A is the intersection of all non-standard neutrosophic closed supersets of A, and we denote it by cl_N (A). The Non-standard Neutrosophic Closure of A is the smallest nonstandard neutrosophic closed set in X that neutrosophically includes A.*

Example 9. *Let X be a non-standard neutrosophic set:*

$$X = \{x_1(\overset{-}{0}.4, \overset{+}{0}.1, \overset{-}{0}.5), x_2(\overset{-}{0}.5, \overset{+}{0}.1, \overset{-}{0}.4), x_3(\overset{-}{0}.5, \overset{+}{0}.1, \overset{-}{0}.5)\} \tag{71}$$

and the following non-standard neutrosophic topology:

$$\tau = \{\Phi_N, 1_N, A_1\{x_1(\overset{-}{0}.4, \overset{+}{0}.1, \overset{-}{0}.5), A_2\{x_2(\overset{-}{0}.5, \overset{+}{0}.1, \overset{-}{0}.4), A_3\{x_3(\overset{-}{0}.5, \overset{+}{0}.1, \overset{-}{0}.5)\}\} \tag{72}$$

where

$$\Phi_N = \{x_1(\overset{-}{0}, \overset{+}{1}, \overset{+}{1}), x_2(\overset{-}{0}, \overset{+}{1}, \overset{+}{1}), x_3(\overset{-}{0}, \overset{+}{1}, \overset{+}{1}), 1_N = x_1(\overset{+}{1}, \overset{-}{0}, \overset{-}{0}), x_1(\overset{+}{1}, \overset{-}{0}, \overset{-}{0}), x_1(\overset{+}{1}, \overset{-}{0}, \overset{-}{0})\} \tag{73}$$

Proof. τ is a non-standard neutrosophic topology because:

$$A_1 \cap_N A_2 = A_1, A_1 \cap_N A_3 = A_1, A_2 \cap_N A_3 = A_3 \tag{74}$$

$$A_1 \cup_N A_2 = A_2, A_1 \cup_N A_3 = A_3, A_2 \cup_N A_3 = A_2, A_1 \cup_N A_2 \cup_N A_3 = A_2. \tag{75}$$

(X, τ) is a non-standard neutrosophic topological space.

The non-standard neutrosophic sets A_1, A_2, A_3 are open sets since they belong to τ.

A_2 is the non-standard neutrosophic complement of A_1, or C_N $(A_2) = A_1$, therefore A_2 is a non-standard neutrosophic closed set in X.

A_3 is the non-standard neutrosophic complement of A_3 (itself), or C_N $(A_3) = A_3$, therefore A_3 is also a non-standard neutrosophic closed set in X.

A_2 and A_3 are nonstandard neutrosophic supersets of A_1, since $A_1 \subset A_2$ and $A_1 \subset A_3$.

Whence, the *Non-standard Neutrosophic Closure* of A_1 is the intersection of its non-standard neutrosophic closed supersets A_2 and A_3, or

$$cl_N\ (A_1) =_N A_2 \cap_N \mathrm{A}_3 =_{\mathrm{N}} A_3 \tag{76}$$

□

Definition 8. *The Non-standard Neutrosophic Interior of A is the union of all non-standard neutrosophic open subsets of A that are contained in A, and we denote it by* int_N *(A).*

The Non-standard Neutrosophic Interior of A is the largest non-standard neutrosophic open set in X that is neutrosophically included into A.

Example 10. *Into the previous Example 9, let's compute* int_N (A_2).

$$A_1 \text{ and } A_3 \text{ are non-standard neutrosophic open sets in } X\text{, with } A_1 \subset_N A_2 \text{ and } A_3 \subset_N A_2 \tag{77}$$

Whence

$$\mathrm{int_N}\ (\mathrm{A}_2) = \mathrm{A}_1 \cup_{\mathrm{N}} \mathrm{A}_3 = \mathrm{A}_3. \tag{78}$$

Definition 9. *Let (X, τ) be a non-standard neutrosophic topological space, and let $Y \subseteq_N X$ be a non-standard neutrosophic subset of X. Then the collection $\tau_Y = \{O \cap_N Y, O \in \tau\}$ is a topology on Y. It is called the non-standard neutrosophic subspace topology and Y is called a non-standard neutrosophic subspace of X.*

Example 11. *In the same previous Example 9, let's take $Y = A_3 \subset X$, and the non-standard neutrosophic subspace topology*

$$\tau_Y = \{\Phi_N, 1_N, A_3, \{(\overset{-}{0.5}, \overset{+}{0.1}, \overset{-}{0.5})\}\} \tag{79}$$

Then Y is a non-standard neutrosophic topological subspace of X.

Definition 10. *Let X and Y be two non-standard neutrosophic topological spaces. A map f:*

$$X \to Y \tag{80}$$

is said to be non-standard neutrosophic continuous map if for each non-standard neutrosophic open set A in Y, the set f^{-1} (A) is a non-standard neutrosophic open set in X.

Example 12. *Let X be a non-standard neutrosophic space. Let Y be a non-standard neutrosophic subspace of X. Then the inclusion map i:* $Y \to X$ *is non-standard neutrosophic continuous.*

Example 13. *Let X be a non-standard neutrosophic set. Suppose that* τ *and* τ' *are two non-standard neutrosophic topologies on X such that* τ' *is finer than* τ*. Then the identity map id:* $(X, \tau') \to (X, \tau)$ *is obviously non-standard neutrosophic continuous.*

Definition 11. *Let* (X_1, τ_1) *and* (X_2, τ_2) *be two non-standard neutrosophic topological spaces. Then* $\tau_1 \times \tau_2 =_N \{U \times V : U \in \tau_1, V \in \tau_2\}$ *defines a topology on the product*

$$X_1 \times X_2 \tag{81}$$

The topology $\tau_1 \times \tau_2$ *is called non-standard neutrosophic product topology.*

3. Development of Neutrosophic Topologies

Since the first definition of neutrosophic topology and neutrosophic topological space [3] in 1998, the neutrosophic topology has been developed tremendously in multiple directions and has added new topological concepts such as: neutrosophic crisp topological [6–9], neutrosophic crisp α-topological spaces [10], neutrosophic soft topological k-algebras [11–13], neutrosophic nano ideal topological structure [14], neutrosophic soft cubic set in topological spaces [15], neutrosophic alpha m-closed sets [16], neutrosophic crisp bi-topological spaces [17], ordered neutrosophic bi-topological space [18], neutrosophic frontier and neutrosophic semi-frontier [19], neutrosophic topological functions [20], neutrosophic topological manifold [21], restricted interval valued neutrosophic topological spaces [22], smooth neutrosophic topological spaces [23], nω–closed sets in neutrosophic topological spaces [24], and other topological properties [25,26], arriving now to the neutrosophic topology extended to the non-standard analysis space.

4. Conclusions

We have introduced for the first time the non-standard neutrosophic topology, non-standard neutrosophic toplogical space and subspace constructed on the non-standard unit interval$]{-0}, 1{+}[_M$ that is formed by real numbers and positive infinitesimals and open monads, together with several concepts related to them, such as: non-standard neutrosophic open/closed sets, non-standard neutrosophic closure and interior of a given set, and non-standard neutrosophic product topology. Several theorems were proven and non-standard neutrosophic examples were presented.

Non-standard neutrosophic topology (NNT) is initiated now for the first time. It is a neutrosophic topology defined on the *set of hyperreals*, while the previous neutrosophic topologies were initiated and developed on the *set of reals*.

The novelty of NNT is its possibility to be used in calculus due to the involvement of infinitesimals, while the previous neutrosophic topologies could not be used due to lack of infinitesimals.

Thus, the paper has contributed to the foundation of a new field of study, called non-standard neutrosophic topology.

As future work, we intend to study more non-standard neutrosophic algebraic structures, such as: non-standard neutrosophic group, non-standard neutroosphic ring and field, non-standard neutrosophic vector space and so on.

Author Contributions: The authors contributed in the following way: methodology by M.A.A.S.; and conceptualization by F.S.

Funding: This project was funded by the Deanship of Scientific Research (DSR) at King Abdulaziz University, Jeddah, under grant No. KEP-34-130-38. The authors, therefore, acknowledge with thanks DSR for technical and financial support.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Insall, M.; Weisstein, E.W. Nonstandard Analysis. From MathWorld—A Wolfram Web Resource. Available online: http://mathworld.wolfram.com/NonstandardAnalysis.html (accessed on 10 May 2019).
2. Robinson, A. *Non-Standard Analysis*; Princeton University Press: Princeton, NJ, USA, 1996.
3. Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics. 1998. Available online: http://fs.unm.edu/eBook-Neutrosophics6.pdf (accessed on 21 April 2019).
4. Smarandache, F. *Extended Nonstandard Neutrosophic Logic., Set, and Probability Based on Extended Nonstandard Analysis*; 31 pages, arXiv; Cornell University: Ithaca, NY, USA, 11 March 2019.
5. Smarandache, F. Extended Nonstandard Neutrosophic Logic, Set, and Probability Based on Extended Nonstandard Analysis. *arXiv* **2019**, arXiv:1903.04558.
6. AL-Nafee, A.B.; Al-Hamido, R.K.; Smarandache, F. Separation Axioms in Neutrosophic Crisp Topological Spaces. *Neutrosophic Sets Syst.* **2019**, *25*, 25–32. [CrossRef]
7. Al-Omeri, W. Neutrosophic crisp Sets via Neutrosophic crisp Topological Spaces. *Neutrosophic Sets Syst.* **2016**, *13*, 96–104. [CrossRef]
8. Salama, A.A.; Smarandache, F.; ALblowi, S.A. New Neutrosophic Crisp Topological Concepts. *Neutrosophic Sets Syst.* **2014**, *4*, 50–54. [CrossRef]
9. Salama, A.A.; Smarandache, F.; Kroumov, V. Neutrosophic Crisp Sets & Neutrosophic Crisp Topological Spaces. *Neutrosophic Sets Syst.* **2014**, *2*, 25–30. [CrossRef]
10. Salama, A.A.; Hanafy, I.M.; Elghawalby, H.; Dabash, M.S. Neutrosophic Crisp α-Topological Spaces. *Neutrosophic Sets Syst.* **2016**, *12*, 92–96. [CrossRef]
11. Akram, M.; Gulzar, H.; Smarandache, F. Neutrosophic Soft Topological K-Algebras. *Neutrosophic Sets Syst.* **2019**, *25*, 104–124. [CrossRef]
12. Mukherjee, A.; Datta, M.; Smarandache, F. Interval Valued Neutrosophic Soft Topological Spaces. *Neutrosophic Sets Syst.* **2014**, *6*, 18–27. [CrossRef]
13. Bera, T.; Mahapatra, N.K. On Neutrosophic Soft Topological Space. *Neutrosophic Sets Syst.* **2018**, *19*, 3–15. [CrossRef]
14. Parimala, M.; Karthika, M.; Jafari, S.; Smarandache, F.; Udhayakumar, R. Neutrosophic Nano ideal topological structure. *Neutrosophic Sets Syst.* **2019**, *24*, 70–76. [CrossRef]
15. Cruz, R.A.; Irudayam, F.N. Neutrosophic Soft Cubic Set in Topological Spaces. *Neutrosophic Sets Syst.* **2018**, *23*, 23–44. [CrossRef]
16. Mohammed, F.M.; Matar, S.F. Fuzzy Neutrosophic Alpha ^{m}-Closed Sets in Fuzzy Neutrosophic Topological Spaces. *Neutrosophic Sets Syst.* **2018**, *21*, 56–65. [CrossRef]
17. Hamido, R.K.; Gharibah, T. Neutrosophic Crisp Bi-Topological Spaces. *Neutrosophic Sets Syst.* **2018**, *21*, 66–73. [CrossRef]
18. Devi, R.N.; Dhavaseelan, R.; Jafari, S. On Separation Axioms in an Ordered Neutrosophic Bitopological Space. *Neutrosophic Sets Syst.* **2017**, *18*, 27–36. [CrossRef]
19. Iswarya, P.; Bageerathi, K. A Study on Neutrosophic Frontier and Neutrosophic Semi-frontier in Neutrosophic Topological Spaces. *Neutrosophic Sets Syst.* **2017**, *16*, 6–15. [CrossRef]
20. Arokiarani, I.; Dhavaseelan, R.; Jafari, S.; Parimala, M. On Some New Notions and Functions in Neutrosophic Topological Spaces. *Neutrosophic Sets Syst.* **2017**, *16*, 16–19. [CrossRef]
21. Salama, A.A.; El Ghawalby, H.; Ali, S.F. Topological Manifold Space via Neutrosophic Crisp Set Theory. *Neutrosophic Sets Syst.* **2017**, *15*, 18–21. [CrossRef]
22. Mukherjee, A.; Datta, M.; Sarkar, S. Restricted Interval Valued Neutrosophic Sets and Restricted Interval Valued Neutrosophic Topological Spaces. *Neutrosophic Sets Syst.* **2016**, *12*, 45–53. [CrossRef]
23. El-Gayyar, M.K. Smooth Neutrosophic Topological Spaces. *Neutrosophic Sets Syst.* **2016**, *12*, 65–72. [CrossRef]
24. Santhi, R.; Udhayarani, N. Nω –Closed Sets in Neutrosophic Topological Spaces. *Neutrosophic Sets Syst.* **2016**, *12*, 114–117. [CrossRef]

25. Thao, N.X.; Smarandache, F. Standard neutrosophic rough set and its topologies properties. *Neutrosophic Sets Syst.* **2016**, *14*, 65–70. [CrossRef]
26. Sweety, C.A.C.; Arockiarani, I. Topological structures of fuzzy neutrosophic rough sets. *Neutrosophic Sets Syst.* **2015**, *9*, 50–57. [CrossRef]