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Logarithmic Hybrid Aggregation Operators Based on Single Valued Neutrosophic Sets and Their Applications in Decision Support Systems

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Abstract: Recently, neutrosophic sets are found to be more general and useful to express incomplete, indeterminate and inconsistent information. The purpose of this paper is to introduce new aggregation operators based on logarithmic operations and to develop a multi-criteria decision-making approach to study the interaction between the input argument under the single valued neutrosophic (SVN) environment. The main advantage of the proposed operator is that it can deal with the situations of the positive interaction, negative interaction or non-interaction among the criteria, during decision-making process. In this paper, we also defined some logarithmic operational rules on SVN sets, then we propose the single valued neutrosophic hybrid aggregation operators as a tool for multi-criteria decision-making (MCDM) under the neutrosophic environment and discuss some properties. Finally, the detailed decision-making steps for the single valued neutrosophic MCDM problems were developed, and a practical case was given to check the created approach and to illustrate its validity and superiority. Besides this, a systematic comparison analysis with other existent methods is conducted to reveal the advantages of our proposed method. Results indicate that the proposed method is suitable and effective for decision process to evaluate their best alternative.

Keywords: single valued neutrosophic sets; logarithmic operational laws; logarithmic aggregation operators; MCGDM problems

1. Introduction

The information involves, in most of the real-life decision-making problems are often incomplete, indeterminate and inconsistent. Fuzzy set theory introduced by Zadeh [1] deals with imprecise, inconsistent information. Although fuzzy set information proved to be very handy but it cannot express the information about rejection. Atanassov [2] introduced the intuitionistic fuzzy set (IFS) to bring in non-membership. Non membership function represents degree of rejection. To incorporate indeterminate and inconsistent information, in addition to incomplete information, the concept of neutrosophic set (NS) proposed by Smarandache [3]. A NS generalizes the notion of the classic set, fuzzy set (FS) [1], IFS [2], paraconsistent set [4], dialetheist set, paradoxist set [4], and tautological set [4] to name a few. In NS, indeterminacy is quantified explicitly, and truth, indeterminacy, and falsity memberships are expressed independently. The NS generalizes different types of non-crisp sets but in real scientific and engineering applications the NS and the set-theoretic operators require to be specified. For a detailed study on NS we refer to [5–17].

Related Work

Most of the weighted aggregation operators consider situations in which criteria and preferences of experts are independent, which means that additivity is a main property of these operators. However, in real life decision-making problems, the criteria of the problems are often interdependent or interactive.

Most of the weighted average operators are based on the basic algebraic product and algebraic sum of single valued neutrosophic numbers (SVNNs) which are not the only operations available to model the intersection and union of SVNNs. The logarithmic algebraic product and sum are two good alternatives of algebraic operations which can be used the model intersection and union of SVNNs. Moreover, it is observed that in the literature there is little investigation on aggregation operators utilizing the logarithmic operations on SVNNs. For a detailed review on the applications of logarithmic operations, we refer to [10]. As already mentioned that the single valued neutrosophic set (SVNS) is an effective tool to describe the uncertain, incomplete and indeterminate information. The logarithmic single valued neutrosophic hybrid and logarithmic generalized single valued neutrosophic algebraic operators have the ability to express interactions among the criteria and it can replace the weighted average to aggregate dependent criteria for obtaining more accurate results. Motivated by these, we find it interesting to develop the logarithmic single valued neutrosophic hybrid aggregation operators for decision-making with neutrosophic information.

Also, we proposed the possibility of a degree-ranking technique for SVNNs from the probability point of view, since the ranking of SVNNs is very important for decision-making under the SVN environment. Furthermore, we proposed a multi-criteria decision-making model based on the logarithmic single valued neutrosophic hybrid weighted operators. For study the multi-criteria decision-making models, we refer [18–31].

The aim of writing this paper is to introduce a decision-making method for MCDM problems in which there exist interrelationships among the criteria. The contributions of this research are:

(1) A novel logarithmic operations for neutrosophic information is defined, which can overcome the weaknesses of algebraic operations and obtain the relationship between various SVNNs.

(2) Logarithmic operators for IFSs are extended to logarithmic single-valued neutrosophic hybrid operators and logarithmic generalized single-valued neutrosophic operators, namely, logarithmic single valued neutrosophic hybrid weighted averaging (L-SVNHWA), logarithmic single valued neutrosophic hybrid weighted geometric (L-SVNHWG), logarithmic generalized single-valued neutrosophic weighted averaging (L-GSVNWA) and logarithmic single-valued neutrosophic weighted geometric (L-GSVNWG) to SVNSs, which can overcome the algebraic operators drawbacks.

(3) A decision-making approach to handle the MCDM problems under the neutrosophic informations is introduced.

To attain our research goals which are stated above, the arrangement of the paper is offered as: Section 2 concentrates on basic definitions and operations of existing extensions of fuzzy set theories. In Section 3, some novel logarithmic operational laws of SVNSs are presented. Section 4 defines the logarithmic hybrid aggregation operators for SVNNs. In Section 5, an algorithm for handling the neutrosophic MCDM problem based on the developed logarithmic operators is presented. In Section 5.1, an application to verify the novel method is given and Section 5.2 presents the comparison study about algebraic and logarithmic aggregation operators. Section 6 consists of the conclusion of the study.

2. Preliminaries

This section includes the concepts and basic operations of existing extensions of fuzzy sets to make the study self contained.

Definition 1. [2] For a set \mathfrak{R} , by an intuitionistic fuzzy set in \mathfrak{R} , we have a structure

$$\zeta = \{ \langle P_{\sigma}(r), N_{\sigma}(r) \rangle \mid r \in \mathfrak{R} \}, \quad (1)$$

in which $P_{\sigma} : \mathfrak{R} \rightarrow \Theta$ and $N_{\sigma} : \mathfrak{R} \rightarrow \Theta$ indicate the membership and non-membership grades in \mathfrak{R} , $\Theta = [0, 1]$ be the unit interval. Also the following condition is satisfied by P_{σ} and N_{σ} , $0 \leq P_{\sigma}(r) + N_{\sigma}(r) \leq 1; \forall r \in \mathfrak{R}$. Then ζ is said to be intuitionistic fuzzy set in \mathfrak{R} .

Definition 2. [32] For a set \mathfrak{R} , by a neutrosophic set in \mathfrak{R} , we have a structure

$$\zeta = \{ \langle P_{\sigma}(r), I_{\sigma}(r), N_{\sigma}(r) \rangle \mid r \in \mathfrak{R} \}, \quad (2)$$

in which $P_{\sigma} : \mathfrak{R} \rightarrow \Theta$, $I_{\sigma} : \mathfrak{R} \rightarrow \Theta$ and $N_{\sigma} : \mathfrak{R} \rightarrow \Theta$ indicate the truth, indeterminacy and falsity memberships in \mathfrak{R} , $\Theta =]0^-, 1^+[$. Also the following condition is satisfied by P_{σ} , I_{σ} and N_{σ} , $0^- \leq P_{\sigma}(r) + I_{\sigma}(r) + N_{\sigma}(r) \leq 3^+; \forall r \in \mathfrak{R}$. Then, ζ is said to be neutrosophic set in \mathfrak{R} .

Definition 3. [33] For a set \mathfrak{R} , by a single valued neutrosophic set in \mathfrak{R} , we mean a structure

$$\zeta = \{ \langle P_{\sigma}(r), I_{\sigma}(r), N_{\sigma}(r) \rangle \mid r \in \mathfrak{R} \}, \quad (3)$$

in which $P_{\sigma} : \mathfrak{R} \rightarrow \Theta$, $I_{\sigma} : \mathfrak{R} \rightarrow \Theta$ and $N_{\sigma} : \mathfrak{R} \rightarrow \Theta$ indicate the truth, indeterminacy and falsity memberships in \mathfrak{R} , $\Theta = [0, 1]$. Also the following condition is satisfied by P_{σ} , I_{σ} and N_{σ} , $0 \leq P_{\sigma}(r) + I_{\sigma}(r) + N_{\sigma}(r) \leq 3; \forall r \in \mathfrak{R}$. Then, ζ is said to be a single valued neutrosophic set in \mathfrak{R} . We denote this triplet $\zeta = \langle P_{\sigma}(r), I_{\sigma}(r), N_{\sigma}(r) \rangle$, in whole study called SVN.

Ye [14], Wang et al. [33] and [34] proposed the basic operations of SVN, which are as follows:

Definition 4. [34] For any two SVN $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ and $\zeta_q = \langle P_{\zeta_q}(r), I_{\zeta_q}(r), N_{\zeta_q}(r) \rangle$ in \mathfrak{R} . The union, intersection and compliment are proposed as:

- (1) $\zeta_p \subseteq \zeta_q$ iff $\forall r \in \mathfrak{R}, P_{\zeta_p}(r) \leq P_{\zeta_q}(r), I_{\zeta_p}(r) \geq I_{\zeta_q}(r)$ and $N_{\zeta_p}(r) \geq N_{\zeta_q}(r)$;
- (2) $\zeta_p = \zeta_q$ iff $\zeta_p \subseteq \zeta_q$ and $\zeta_q \subseteq \zeta_p$;
- (3) $\zeta_p \cup \zeta_q = \langle \max(P_{\zeta_p}, P_{\zeta_q}), \min(I_{\zeta_p}, I_{\zeta_q}), \min(N_{\zeta_p}, N_{\zeta_q}) \rangle$;
- (4) $\zeta_p \cap \zeta_q = \langle \min(P_{\zeta_p}, P_{\zeta_q}), \max(I_{\zeta_p}, I_{\zeta_q}), \max(N_{\zeta_p}, N_{\zeta_q}) \rangle$;
- (5) $\zeta_p^c = \langle N_{\zeta_p}, I_{\zeta_p}, P_{\zeta_p} \rangle$.

Definition 5. [13,15,33] For any two SVN $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ and $\zeta_q = \langle P_{\zeta_q}(r), I_{\zeta_q}(r), N_{\zeta_q}(r) \rangle$ in \mathfrak{R} and $\beta \geq 0$. Then the operations of SVN are proposed as:

- (1) $\zeta_p \oplus \zeta_q = \{ P_{\zeta_p} + P_{\zeta_q} - P_{\zeta_p} \cdot P_{\zeta_q}, I_{\zeta_p} \cdot I_{\zeta_q}, N_{\zeta_p} \cdot N_{\zeta_q} \}$;
- (2) $\beta \cdot \zeta_p = \{ 1 - (1 - P_{\zeta_p})^{\beta}, (I_{\zeta_p})^{\beta}, (N_{\zeta_p})^{\beta} \}$;
- (3) $\zeta_p \otimes \zeta_q = \{ P_{\zeta_p} \cdot P_{\zeta_q}, I_{\zeta_p} + I_{\zeta_q} - I_{\zeta_p} \cdot I_{\zeta_q}, N_{\zeta_p} + N_{\zeta_q} - N_{\zeta_p} \cdot N_{\zeta_q} \}$;
- (4) $\zeta_p^{\beta} = \{ (P_{\zeta_p})^{\beta}, 1 - (1 - I_{\zeta_p})^{\beta}, 1 - (1 - N_{\zeta_p})^{\beta} \}$.
- (5) $\beta^{\zeta_p} = \begin{cases} (\beta^{1-P_{\zeta_p}}, 1 - \beta^{I_{\zeta_p}}, 1 - \beta^{N_{\zeta_p}}) & \text{if } \beta \in (0, 1) \\ \left(\left(\frac{1}{\beta} \right)^{1-P_{\zeta_p}}, 1 - \left(\frac{1}{\beta} \right)^{I_{\zeta_p}}, 1 - \left(\frac{1}{\beta} \right)^{N_{\zeta_p}} \right) & \text{if } \beta \geq 1 \end{cases}$

Definition 6. [33] For any three SVN $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$, $\zeta_q = \langle P_{\zeta_q}(r), I_{\zeta_q}(r), N_{\zeta_q}(r) \rangle$ and $\zeta_l = \langle P_{\zeta_l}(r), I_{\zeta_l}(r), N_{\zeta_l}(r) \rangle$ in \mathfrak{R} and $\beta_1, \beta_2 \geq 0$. Then, we have

- (1) $\zeta_p \oplus \zeta_q = \zeta_q \oplus \zeta_p$;
- (2) $\zeta_p \otimes \zeta_q = \zeta_q \otimes \zeta_p$;
- (3) $\beta_1(\zeta_p \oplus \zeta_q) = \beta_1\zeta_p \oplus \beta_1\zeta_q, \beta_1 > 0$;
- (4) $(\zeta_p \otimes \zeta_q)^{\beta_1} = \zeta_p^{\beta_1} \otimes \zeta_q^{\beta_1}, \beta_1 > 0$;
- (5) $\beta_1\zeta_p \oplus \beta_2\zeta_p = (\beta_1 + \beta_2)\zeta_p, \beta_1 > 0, \beta_2 > 0$;
- (6) $\zeta_p^{\beta_1} \otimes \zeta_p^{\beta_2} = \zeta_p^{\beta_1 + \beta_2}, \beta_1 > 0, \beta_2 > 0$;
- (7) $(\zeta_p \oplus \zeta_q) \oplus \zeta_l = \zeta_p \oplus (\zeta_q \oplus \zeta_l)$;
- (8) $(\zeta_p \otimes \zeta_q) \otimes \zeta_l = \zeta_p \otimes (\zeta_q \otimes \zeta_l)$.

Definition 7. [33] For any SVN $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ in \mathfrak{R} . Then score and accuracy values are defined as:

- (1) $\tilde{S}(\zeta_p) = P_{\zeta_p} - I_{\zeta_p} - N_{\zeta_p}$
- (2) $\tilde{A}(\zeta_p) = P_{\zeta_p} + I_{\zeta_p} + N_{\zeta_p}$

The above definitions of score and accuracy functions suggest which SVN is greater than other SVNs. The comparison technique is defined in following definition.

Definition 8. [33] For any SVNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, 2$) in \mathfrak{R} .

Then comparison techniques are proposed as:

- (1) If $\tilde{S}(\zeta_1) < \tilde{S}(\zeta_2)$, then $\zeta_1 < \zeta_2$,
- (2) If $\tilde{S}(\zeta_1) > \tilde{S}(\zeta_2)$, then $\zeta_1 > \zeta_2$,
- (3) If $\tilde{S}(\zeta_1) = \tilde{S}(\zeta_2)$, and
 - (a) $\tilde{A}(\zeta_1) < \tilde{A}(\zeta_2)$, then $\zeta_1 < \zeta_2$,
 - (b) $\tilde{A}(\zeta_1) > \tilde{A}(\zeta_2)$, then $\zeta_1 > \zeta_2$,
 - (c) $\tilde{A}(\zeta_1) = \tilde{A}(\zeta_2)$, then $\zeta_1 \approx \zeta_2$.

Garg and Nancy [10] proposed some logarithmic-based aggregation operators, which are as follows:

Definition 9. [10] For any collection of SVNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, 2, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$. Then, the structure of logarithmic single valued neutrosophic weighted averaging (L-SVNWA) operator is defined as:

$$L - SVNWA (\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{c} 1 - \prod_{p=1}^n (\log_{\sigma_p} P_{\zeta_p})^{\beta_p}, \\ \prod_{p=1}^n (\log_{\sigma_p} (1 - I_{\zeta_p}))^{\beta_p}, \\ \prod_{p=1}^n (\log_{\sigma_p} (1 - N_{\zeta_p}))^{\beta_p} \end{array} \right), \tag{4}$$

where β_p ($p = 1, 2, \dots, n$) are weight vectors with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$.

Definition 10. [10] For any collection of SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, 2, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$. Then, the structure of the logarithmic single-valued neutrosophic-ordered weighted averaging (L-SVNOWA) operator is defined as:

$$L - SVNOWA(\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{c} 1 - \prod_{p=1}^n (\log_{\sigma_p} P_{\zeta_{\eta(p)}})^{\beta_p}, \\ \prod_{p=1}^n (\log_{\sigma_p} (1 - I_{\zeta_{\eta(p)}}))^{\beta_p}, \\ \prod_{p=1}^n (\log_{\sigma_p} (1 - N_{\zeta_{\eta(p)}}))^{\beta_p} \end{array} \right), \tag{5}$$

where β_p ($p = 1, 2, \dots, n$) are weighting vector with $\beta_p \geq 0, \sum_{p=1}^n \beta_p = 1$ and p th largest weighted value is $\zeta_{\eta(p)}$ consequently by total order $\zeta_{\eta(1)} \geq \zeta_{\eta(2)} \geq \dots \geq \zeta_{\eta(n)}$.

Definition 11. [10] For any collection of SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, 2, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$. Then, the structure of logarithmic single-valued neutrosophic-weighted geometric (L-SVNWG) operator is defined as:

$$L - SVNWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{c} \prod_{p=1}^n (1 - \log_{\sigma_p} P_{\zeta_p})^{\beta_p}, \\ 1 - \prod_{p=1}^n (1 - \log_{\sigma_p} (1 - I_{\zeta_p}))^{\beta_p}, \\ 1 - \prod_{p=1}^n (1 - \log_{\sigma_p} (1 - N_{\zeta_p}))^{\beta_p} \end{array} \right), \tag{6}$$

where β_p ($p = 1, 2, \dots, n$) are weight vectors with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$.

Definition 12. [10] For any collection of SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, 2, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$. Then, the structure of logarithmic single valued neutrosophic ordered weighted geometric (L-SVNOWG) operator is defined as:

$$L - SVNOWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{c} \prod_{p=1}^n (1 - \log_{\sigma_p} P_{\zeta_{\eta(p)}})^{\beta_p}, \\ 1 - \prod_{p=1}^n (1 - \log_{\sigma_p} (1 - I_{\zeta_{\eta(p)}}))^{\beta_p}, \\ 1 - \prod_{p=1}^n (1 - \log_{\sigma_p} (1 - N_{\zeta_{\eta(p)}}))^{\beta_p} \end{array} \right), \tag{7}$$

where β_p ($p = 1, 2, \dots, n$) are weighting vector with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$ and p th are the largest weighted value is $\zeta_{\eta(p)}$ consequently by total order $\zeta_{\eta(1)} \geq \zeta_{\eta(2)} \geq \dots \geq \zeta_{\eta(n)}$.

3. Logarithmic Operational Laws

Motivated by the well growing concept of SVNNSs, we introduce some novel logarithmic operational laws for single valued neutrosophic numbers. As in real number systems $\log_{\sigma} 0$ is meaningless and $\log_{\sigma} 1$ is not defined therefore, in our study we take non-empty SVNNSs and $\sigma \neq 1$, where σ is any real number.

Definition 13. For any SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ in \mathfrak{R} . The logarithmic SVNNS is defined as:

$$\log_{\sigma} \zeta_p = \left\{ \left\langle 1 - (\log_{\sigma} P_{\zeta_p}(r)), \log_{\sigma} (1 - I_{\zeta_p}(r)), \log_{\sigma} (1 - N_{\zeta_p}(r)) \right\rangle \mid r \in \mathfrak{R} \right\}, \tag{8}$$

in which $P_\sigma : \mathfrak{R} \rightarrow \Theta$, $I_\sigma : \mathfrak{R} \rightarrow \Theta$ and $N_\sigma : \mathfrak{R} \rightarrow \Theta$ are indicated the truth, indeterminacy and falsity memberships in \mathfrak{R} , $\Theta = [0, 1]$ be the unit interval. Also following condition is satisfied by P_σ, I_σ and N_σ , $0 \leq P_\sigma(r) + I_\sigma(r) + N_\sigma(r) \leq 3; \forall r \in \mathfrak{R}$. Therefore the truth membership grade is

$$1 - \left(\log_\sigma P_{\tilde{\zeta}_p}(r) \right) : \mathfrak{R} \rightarrow \Theta, \text{ such that } 0 \leq 1 - \left(\log_\sigma P_{\tilde{\zeta}_p}(r) \right) \leq 1, \text{ for all } r \in \mathfrak{R}$$

the indeterminacy membership is

$$\log_\sigma \left(1 - I_{\tilde{\zeta}_p}(r) \right) : \mathfrak{R} \rightarrow \Theta, \text{ such that } 0 \leq \log_\sigma \left(1 - I_{\tilde{\zeta}_p}(r) \right) \leq 1, \text{ for all } r \in \mathfrak{R}$$

and falsity membership is

$$\log_\sigma \left(1 - N_{\tilde{\zeta}_p}(r) \right) : \mathfrak{R} \rightarrow \Theta, \text{ such that } 0 \leq \log_\sigma \left(1 - N_{\tilde{\zeta}_p}(r) \right) \leq 1, \text{ for all } r \in \mathfrak{R}.$$

Therefore

$$\begin{aligned} \log_\sigma \tilde{\zeta}_p &= \left\{ \left\langle 1 - \left(\log_\sigma P_{\tilde{\zeta}_p}(r) \right), \log_\sigma \left(1 - I_{\tilde{\zeta}_p}(r) \right), \log_\sigma \left(1 - N_{\tilde{\zeta}_p}(r) \right) \right\rangle | r \in \mathfrak{R} \right\} \\ 0 &< \sigma \leq \min \left\{ P_{\tilde{\zeta}_p}, 1 - I_{\tilde{\zeta}_p}, 1 - N_{\tilde{\zeta}_p} \right\} \leq 1, \sigma \neq 1 \end{aligned}$$

is SVNS.

Definition 14. For any SVNN $\zeta_p = \langle P_{\tilde{\zeta}_p}(r), I_{\tilde{\zeta}_p}(r), N_{\tilde{\zeta}_p}(r) \rangle$ in \mathfrak{R} . If

$$\log_\sigma \tilde{\zeta}_p = \begin{cases} \left(\begin{array}{l} 1 - \left(\log_\sigma P_{\tilde{\zeta}_p}(r) \right), \\ \log_\sigma \left(1 - I_{\tilde{\zeta}_p}(r) \right), \\ \log_\sigma \left(1 - N_{\tilde{\zeta}_p}(r) \right) \end{array} \right) & 0 < \sigma \leq \min \left\{ P_{\tilde{\zeta}_p}, 1 - I_{\tilde{\zeta}_p}, 1 - N_{\tilde{\zeta}_p} \right\} < 1 \\ \left(\begin{array}{l} 1 - \left(\log_{\frac{1}{\sigma}} P_{\tilde{\zeta}_p}(r) \right), \\ \log_{\frac{1}{\sigma}} \left(1 - I_{\tilde{\zeta}_p}(r) \right), \\ \log_{\frac{1}{\sigma}} \left(1 - N_{\tilde{\zeta}_p}(r) \right) \end{array} \right) & 0 < \frac{1}{\sigma} \leq \min \left\{ P_{\tilde{\zeta}_p}, 1 - I_{\tilde{\zeta}_p}, 1 - N_{\tilde{\zeta}_p} \right\} < 1, \\ & \sigma \neq 1 \end{cases} \tag{9}$$

then the function $\log_\sigma \tilde{\zeta}_p$ is known to be a logarithmic operator for SVNS, and its value is said to be logarithmic SVNN (L-SVNN). Here, we take $\log_\sigma 0 = 0, \sigma > 0, \sigma \neq 1$.

Theorem 1. [10] For any SVNN $\zeta_p = \langle P_{\tilde{\zeta}_p}(r), I_{\tilde{\zeta}_p}(r), N_{\tilde{\zeta}_p}(r) \rangle$ in \mathfrak{R} , then $\log_\sigma \tilde{\zeta}_p$ is also be SVNN.

Now, we give some discussion on the basic properties of the L-SVNN.

Definition 15. For any two L-SVNNs $\log_\sigma \tilde{\zeta}_p = \left(\begin{array}{l} 1 - \left(\log_\sigma P_{\tilde{\zeta}_p}(r) \right), \\ \log_\sigma \left(1 - I_{\tilde{\zeta}_p}(r) \right), \\ \log_\sigma \left(1 - N_{\tilde{\zeta}_p}(r) \right) \end{array} \right)$ and $\log_\sigma \tilde{\zeta}_q =$

$\left(\begin{array}{l} 1 - \left(\log_\sigma P_{\tilde{\zeta}_q}(r) \right), \\ \log_\sigma \left(1 - I_{\tilde{\zeta}_q}(r) \right), \\ \log_\sigma \left(1 - N_{\tilde{\zeta}_q}(r) \right) \end{array} \right)$ in \mathfrak{R} and $\beta \geq 0$. Then the logarithmic operations of L-SVNNs are propose as

$$(1) \log_\sigma \tilde{\zeta}_p \oplus \log_\sigma \tilde{\zeta}_q = \left\{ \begin{array}{l} 1 - \left(\log_\sigma P_{\tilde{\zeta}_p}(r) \right) \cdot \left(\log_\sigma P_{\tilde{\zeta}_q}(r) \right), \\ \log_\sigma \left(1 - I_{\tilde{\zeta}_p}(r) \right) \cdot \log_\sigma \left(1 - I_{\tilde{\zeta}_q}(r) \right), \\ \log_\sigma \left(1 - N_{\tilde{\zeta}_p}(r) \right) \cdot \log_\sigma \left(1 - N_{\tilde{\zeta}_q}(r) \right) \end{array} \right\};$$

$$\begin{aligned}
 (2) \beta \cdot \log_{\sigma} \zeta_p &= \left\{ \begin{array}{l} 1 - \left(\log_{\sigma} P_{\zeta_p}(r) \right)^{\beta}, \\ \left(\log_{\sigma} \left(1 - I_{\zeta_p}(r) \right) \right)^{\beta}, \\ \left(\log_{\sigma} \left(1 - N_{\zeta_p}(r) \right) \right)^{\beta} \end{array} \right\}; \\
 (3) \log_{\sigma} \zeta_p \otimes \log_{\sigma} \zeta_q &= \left\{ \begin{array}{l} 1 - \left(\log_{\sigma} P_{\zeta_p}(r) \right) \cdot 1 - \left(\log_{\sigma} P_{\zeta_q}(r) \right), \\ 1 - \left(1 - \log_{\sigma} \left(1 - I_{\zeta_p}(r) \right) \right) \cdot \left(1 - \log_{\sigma} \left(1 - I_{\zeta_q}(r) \right) \right), \\ 1 - \left(1 - \log_{\sigma} \left(1 - N_{\zeta_p}(r) \right) \right) \cdot \left(1 - \log_{\sigma} \left(1 - N_{\zeta_q}(r) \right) \right) \end{array} \right\}; \\
 (4) \left(\log_{\sigma} \zeta_p \right)^{\beta} &= \left\{ \begin{array}{l} \left(1 - \left(\log_{\sigma} P_{\zeta_p}(r) \right) \right)^{\beta}, \\ 1 - \left(1 - \log_{\sigma} \left(1 - I_{\zeta_p}(r) \right) \right)^{\beta}, \\ 1 - \left(1 - \log_{\sigma} \left(1 - N_{\zeta_p}(r) \right) \right)^{\beta} \end{array} \right\}.
 \end{aligned}$$

Theorem 2. [10] For any two L-SVNNs $\log_{\sigma} \zeta_p = \left(\begin{array}{l} 1 - \left(\log_{\sigma} P_{\zeta_p}(r) \right), \\ \log_{\sigma} \left(1 - I_{\zeta_p}(r) \right), \\ \log_{\sigma} \left(1 - N_{\zeta_p}(r) \right) \end{array} \right)$ ($p = 1, 2$) in \mathfrak{R} , with $0 <$

$\sigma \leq \min \{ P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p} \} < 1, \sigma \neq 1, \beta, \beta_1, \beta_2 > 0$ be any real numbers. Then

- (1) $\beta \left(\log_{\sigma} \zeta_1 \oplus \log_{\sigma} \zeta_2 \right) = \beta \log_{\sigma} \zeta_1 \oplus \beta \log_{\sigma} \zeta_2$;
- (2) $\left(\log_{\sigma} \zeta_1 \otimes \log_{\sigma} \zeta_2 \right)^{\beta} = \left(\log_{\sigma} \zeta_1 \right)^{\beta} \otimes \left(\log_{\sigma} \zeta_2 \right)^{\beta}$;
- (3) $\beta_1 \log_{\sigma} \zeta_1 \oplus \beta_2 \log_{\sigma} \zeta_1 = \left(\beta_1 + \beta_2 \right) \log_{\sigma} \zeta_1$;
- (4) $\left(\log_{\sigma} \zeta_1 \right)^{\beta_1} \otimes \left(\log_{\sigma} \zeta_1 \right)^{\beta_2} = \left(\log_{\sigma} \zeta_1 \right)^{\left(\beta_1 + \beta_2 \right)}$;
- (5) $\left(\left(\log_{\sigma} \zeta_1 \right)^{\beta_1} \right)^{\beta_2} = \left(\log_{\sigma} \zeta_1 \right)^{\beta_1 \beta_2}$.

Comparison Technique for L-SVNNs

Definition 16. [10] For any L-SVNN $\log_{\sigma} \zeta_p = \left(\begin{array}{l} 1 - \left(\log_{\sigma} P_{\zeta_p}(r) \right), \\ \log_{\sigma} \left(1 - I_{\zeta_p}(r) \right), \\ \log_{\sigma} \left(1 - N_{\zeta_p}(r) \right) \end{array} \right)$ in \mathfrak{R} . Then score and accuracy

values are define as

- (1) $\tilde{S}(\log_{\sigma} \zeta_p) = 1 - \left(\log_{\sigma} P_{\zeta_p}(r) \right) - \log_{\sigma} \left(1 - I_{\zeta_p}(r) \right) - \left(\log_{\sigma} \left(1 - N_{\zeta_p}(r) \right) \right)$
- (2) $\tilde{A}(\log_{\sigma} \zeta_p) = 1 - \left(\log_{\sigma} P_{\zeta_p}(r) \right) + \log_{\sigma} \left(1 - I_{\zeta_p}(r) \right) + \left(\log_{\sigma} \left(1 - N_{\zeta_p}(r) \right) \right)$

The above defined score and accuracy values suggest which L-SVNN are greater than other L-SVNNs. The comparison technique is defined in the following definition.

Definition 17. For any L-SVNNs $\log_{\sigma} \zeta_p = \left(\begin{array}{l} 1 - \left(\log_{\sigma} P_{\zeta_p}(r) \right), \\ \log_{\sigma} \left(1 - I_{\zeta_p}(r) \right), \\ \log_{\sigma} \left(1 - N_{\zeta_p}(r) \right) \end{array} \right)$ ($p = 1, 2$) in \mathfrak{R} . Then, comparison

technique is proposed as:

- (1) If $\tilde{S}(\log_{\sigma} \zeta_1) < \tilde{S}(\log_{\sigma} \zeta_2)$ then $\log_{\sigma} \zeta_1 < \log_{\sigma} \zeta_2$,
- (2) If $\tilde{S}(\log_{\sigma} \zeta_1) > \tilde{S}(\log_{\sigma} \zeta_2)$ then $\log_{\sigma} \zeta_1 > \log_{\sigma} \zeta_2$,
- (3) If $\tilde{S}(\log_{\sigma} \zeta_1) = \tilde{S}(\log_{\sigma} \zeta_2)$ then
 - (a) $\tilde{A}(\log_{\sigma} \zeta_1) < \tilde{A}(\log_{\sigma} \zeta_2)$ then $\log_{\sigma} \zeta_1 < \log_{\sigma} \zeta_2$,
 - (b) $\tilde{A}(\log_{\sigma} \zeta_1) > \tilde{A}(\log_{\sigma} \zeta_2)$ then $\log_{\sigma} \zeta_1 > \log_{\sigma} \zeta_2$,
 - (c) $\tilde{A}(\log_{\sigma} \zeta_1) = \tilde{A}(\log_{\sigma} \zeta_2)$ then $\log_{\sigma} \zeta_1 \approx \log_{\sigma} \zeta_2$.

4. Logarithmic Aggregation Operators for L-SVNNs

Now, we propose novel logarithmic hybrid aggregation operators for L-SVNNs based on logarithmic operations laws as follows:

4.1. Logarithmic Hybrid Averaging Operator

Definition 18. For any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{ P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p} \} < 1, \sigma \neq 1$. The structure of logarithmic single valued neutrosophic hybrid weighted averaging (L-SVNHWA) operator is

$$L - SVNHWA (\zeta_1, \zeta_2, \dots, \zeta_n) = \sum_{p=1}^n \omega_p \log_{\sigma_p} \zeta_{\eta(p)}^* \tag{10}$$

where β_p ($p = 1, \dots, n$) is the weighting vector with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$ and p th biggest weighted value is $\zeta_{\eta(p)}^*$ ($\zeta_{\eta(p)}^* = n\beta_p \zeta_{\eta(p)}, P \in N$) consequently by total order $\zeta_{\eta(1)}^* \geq \zeta_{\eta(2)}^* \geq \dots \geq \zeta_{\eta(n)}^*$. Also, the associated weights are $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ with $\omega_p \geq 0, \sum_{p=1}^n \omega_p = 1$.

Theorem 3. For any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{ P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p} \} < 1, \sigma \neq 1$. Then by using logarithmic operations and Definition 18, L - SVNHWA is defined as

$$L - SVNHWA (\zeta_1, \zeta_2, \dots, \zeta_n) = \begin{cases} \left(\begin{matrix} 1 - \prod_{p=1}^n \left(\log_{\sigma_p} P_{\zeta_{\eta(p)}^*} \right)^{\omega_p} \\ \prod_{p=1}^n \left(\log_{\sigma_p} \left(1 - I_{\zeta_{\eta(p)}^*} \right) \right)^{\omega_p} \\ \prod_{p=1}^n \left(\log_{\sigma_p} \left(1 - N_{\zeta_{\eta(p)}^*} \right) \right)^{\omega_p} \end{matrix} \right)^{\omega_p} & 0 < \sigma_p \leq \min \left\{ \begin{matrix} P_{\zeta_{p'}}, \\ 1 - I_{\zeta_{p'}}, \\ 1 - N_{\zeta_{p'}} \end{matrix} \right\} < 1 \\ \left(\begin{matrix} 1 - \prod_{p=1}^n \left(\log_{\frac{1}{\sigma_p}} P_{\zeta_{\eta(p)}^*} \right)^{\omega_p} \\ \prod_{p=1}^n \left(\log_{\frac{1}{\sigma_p}} \left(1 - I_{\zeta_{\eta(p)}^*} \right) \right)^{\omega_p} \\ \prod_{p=1}^n \left(\log_{\frac{1}{\sigma_p}} \left(1 - N_{\zeta_{\eta(p)}^*} \right) \right)^{\omega_p} \end{matrix} \right)^{\omega_p} & 0 < \frac{1}{\sigma_p} \leq \min \left\{ \begin{matrix} P_{\zeta_{p'}}, \\ 1 - I_{\zeta_{p'}}, \\ 1 - N_{\zeta_{p'}} \end{matrix} \right\} < 1, \\ & \sigma \neq 1 \end{cases} \tag{11}$$

where β_p ($p = 1, \dots, n$) are weighting vector with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$ and p th biggest weighted value is $\zeta_{\eta(p)}^*$ ($\zeta_{\eta(p)}^* = n\beta_p \zeta_{\eta(p)}, P \in N$) consequently by total order $\zeta_{\eta(1)}^* \geq \zeta_{\eta(2)}^* \geq \dots \geq \zeta_{\eta(n)}^*$. Also the associated weights are $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ with $\omega_p \geq 0, \sum_{p=1}^n \omega_p = 1$.

Proof. Using mathematical induction to prove Equation (3), we proceed as:

(a) For $n = 2$, since

$$\omega_1 \log_{\sigma_1} \zeta_{\eta(1)}^* = \left(\begin{matrix} 1 - \left(\log_{\sigma_1} P_{\zeta_{\eta(1)}^*} \right)^{\omega_1} \\ \left(\log_{\sigma_1} \left(1 - I_{\zeta_{\eta(1)}^*} \right) \right)^{\omega_1} \\ \left(\log_{\sigma_1} \left(1 - N_{\zeta_{\eta(1)}^*} \right) \right)^{\omega_1} \end{matrix} \right)^{\omega_1}$$

and

$$\omega_2 \log_{\sigma_2} \zeta_{\eta(2)}^* = \left(\begin{array}{c} 1 - \left(\log_{\sigma_2} P_{\zeta_{\eta(2)}}^* \right)^{\omega_2}, \\ \left(\log_{\sigma_2} \left(1 - I_{\zeta_{\eta(2)}}^* \right) \right)^{\omega_2}, \\ \left(\log_{\sigma_2} \left(1 - N_{\zeta_{\eta(2)}}^* \right) \right)^{\omega_2} \end{array} \right)$$

Then

$$\begin{aligned} L - SVNHWA (\zeta_1, \zeta_2) &= \omega_1 \log_{\sigma_1} \zeta_{\eta(1)}^* \oplus \omega_2 \log_{\sigma_2} \zeta_{\eta(2)}^* \\ &= \left(\begin{array}{c} 1 - \left(\log_{\sigma_1} P_{\zeta_{\eta(1)}}^* \right)^{\omega_1}, \\ \left(\log_{\sigma_1} \left(1 - I_{\zeta_{\eta(1)}}^* \right) \right)^{\omega_1}, \\ \left(\log_{\sigma_1} \left(1 - N_{\zeta_{\eta(1)}}^* \right) \right)^{\omega_1} \end{array} \right) \oplus \left(\begin{array}{c} 1 - \left(\log_{\sigma_2} P_{\zeta_{\eta(2)}}^* \right)^{\omega_2}, \\ \left(\log_{\sigma_2} \left(1 - I_{\zeta_{\eta(2)}}^* \right) \right)^{\omega_2}, \\ \left(\log_{\sigma_2} \left(1 - N_{\zeta_{\eta(2)}}^* \right) \right)^{\omega_2} \end{array} \right) \\ &= \left(\begin{array}{c} 1 - \left(\log_{\sigma_1} P_{\zeta_{\eta(1)}}^* \right)^{\omega_1} \cdot \left(\log_{\sigma_2} P_{\zeta_{\eta(2)}}^* \right)^{\omega_2}, \\ \left(\log_{\sigma_1} \left(1 - I_{\zeta_{\eta(1)}}^* \right) \right)^{\omega_1} \cdot \left(\log_{\sigma_2} \left(1 - I_{\zeta_{\eta(2)}}^* \right) \right)^{\omega_2}, \\ \left(\log_{\sigma_1} \left(1 - N_{\zeta_{\eta(1)}}^* \right) \right)^{\omega_1} \cdot \left(\log_{\sigma_2} \left(1 - N_{\zeta_{\eta(2)}}^* \right) \right)^{\omega_2} \end{array} \right) \\ &= \left(\begin{array}{c} 1 - \prod_{p=1}^2 \left(\log_{\sigma_p} P_{\zeta_{\eta(p)}}^* \right)^{\omega_p}, \\ \prod_{p=1}^2 \left(\log_{\sigma_p} \left(1 - I_{\zeta_{\eta(p)}}^* \right) \right)^{\omega_p}, \\ \prod_{p=1}^2 \left(\log_{\sigma_p} \left(1 - N_{\zeta_{\eta(p)}}^* \right) \right)^{\omega_p} \end{array} \right). \end{aligned}$$

(b) Now Equation (3) is true for $n = k$,

$$L - SVNHWA (\zeta_1, \zeta_2, \dots, \zeta_k) = \left(\begin{array}{c} 1 - \prod_{p=1}^k \left(\log_{\sigma_p} P_{\zeta_{\eta(p)}}^* \right)^{\omega_p}, \\ \prod_{p=1}^k \left(\log_{\sigma_p} \left(1 - I_{\zeta_{\eta(p)}}^* \right) \right)^{\omega_p}, \\ \prod_{p=1}^k \left(\log_{\sigma_p} \left(1 - N_{\zeta_{\eta(p)}}^* \right) \right)^{\omega_p} \end{array} \right).$$

(c) Now, we prove that Equation (3) for $n = k + 1$, that is

$$L - SVNHWA (\zeta_1, \zeta_2, \dots, \zeta_k) = \sum_{p=1}^k \omega_p \log_{\sigma_p} \zeta_{\eta(p)}^* + \omega_{k+1} \log_{\sigma_{k+1}} \zeta_{\eta(k+1)}^*$$

$$\begin{aligned}
 &L - SVNHWA (\zeta_1, \zeta_2, \dots, \zeta_k) \\
 &= \left(\begin{array}{l} 1 - \prod_{p=1}^k (\log_{\sigma_p} P_{\zeta_{\eta(p)}}^*)^{\omega_p} , \\ \prod_{p=1}^k (\log_{\sigma_p} (1 - I_{\zeta_{\eta(p)}}^*))^{\omega_p} , \\ \prod_{p=1}^k (\log_{\sigma_p} (1 - N_{\zeta_{\eta(p)}}^*))^{\omega_p} \end{array} \right) \oplus \left(\begin{array}{l} 1 - (\log_{\sigma_{k+1}} P_{\zeta_{\eta(k+1)}}^*)^{\omega_{k+1}} , \\ (\log_{\sigma_{k+1}} (1 - I_{\zeta_{\eta(k+1)}}^*))^{\omega_{k+1}} , \\ (\log_{\sigma_{k+1}} (1 - N_{\zeta_{\eta(k+1)}}^*))^{\omega_{k+1}} \end{array} \right) \\
 &= \left(\begin{array}{l} 1 - \prod_{p=1}^{k+1} (\log_{\sigma_p} P_{\zeta_{\eta(p)}}^*)^{\omega_p} , \\ \prod_{p=1}^{k+1} (\log_{\sigma_p} (1 - I_{\zeta_{\eta(p)}}^*))^{\omega_p} , \\ \prod_{p=1}^{k+1} (\log_{\sigma_p} (1 - N_{\zeta_{\eta(p)}}^*))^{\omega_p} \end{array} \right)
 \end{aligned}$$

Thus Equation (3) is true for $n = z + 1$. Hence its satisfies for whole n . Therefore

$$L - SVNHWA (\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{l} 1 - \prod_{p=1}^n (\log_{\sigma_p} P_{\zeta_{\eta(p)}}^*)^{\omega_p} , \\ \prod_{p=1}^n (\log_{\sigma_p} (1 - I_{\zeta_{\eta(p)}}^*))^{\omega_p} , \\ \prod_{p=1}^n (\log_{\sigma_p} (1 - N_{\zeta_{\eta(p)}}^*))^{\omega_p} \end{array} \right) .$$

In a similarly way, if $0 < \frac{1}{\sigma_p} \leq \min \{ P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p} \} < 1, \sigma \neq 1$, we can also obtain

$$L - SVNHWA (\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{l} 1 - \prod_{p=1}^n (\log_{\frac{1}{\sigma_p}} P_{\zeta_{\eta(p)}}^*)^{\omega_p} , \\ \prod_{p=1}^n (\log_{\frac{1}{\sigma_p}} (1 - I_{\zeta_{\eta(p)}}^*))^{\omega_p} , \\ \prod_{p=1}^n (\log_{\frac{1}{\sigma_p}} (1 - N_{\zeta_{\eta(p)}}^*))^{\omega_p} \end{array} \right)$$

which completes the proof. \square

Remark 1. If $\sigma_1 = \sigma_2 = \sigma_3 = \dots = \sigma_n = \sigma$, that is $0 < \sigma \leq \min \{ P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p} \} < 1, \sigma \neq 1$, then $L - SVNHWA$ operator is reduced as follows

$$L - SVNHWA (\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{l} 1 - \prod_{p=1}^n (\log_{\sigma} P_{\zeta_{\eta(p)}}^*)^{\omega_p} , \\ \prod_{p=1}^n (\log_{\sigma} (1 - I_{\zeta_{\eta(p)}}^*))^{\omega_p} , \\ \prod_{p=1}^n (\log_{\sigma} (1 - N_{\zeta_{\eta(p)}}^*))^{\omega_p} \end{array} \right) . \tag{12}$$

Properties

$L - SVNHWA$ operator satisfies some properties are enlist below;

(1) Idempotency: For any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle (p = 1, \dots, n)$ in \mathfrak{R} . Then, if collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle (p = 1, \dots, n)$ are identical, that is

$$L - SVNHWA (\zeta_1, \zeta_2, \dots, \zeta_n) = \zeta. \tag{13}$$

(2) Boundedness: for any collection of SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . $\zeta_p^- = \langle \min_p P_{\zeta_p}^*, \max_p I_{\zeta_p}^*, \max_p N_{\zeta_p}^* \rangle$ and $\zeta_p^+ = \langle \max_p P_{\zeta_p}^*, \min_p I_{\zeta_p}^*, \min_p N_{\zeta_p}^* \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , therefore

$$\zeta_p^- \subseteq L - SVNHWA(\zeta_1, \zeta_2, \dots, \zeta_n) \subseteq \zeta_p^+. \tag{14}$$

(3) Monotonically: for any collection of SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . If $\zeta_{\eta(p)} \subseteq \zeta_{\eta(p)}^*$ for ($p = 1, \dots, n$), then

$$L - SVNHWA(\zeta_1, \zeta_2, \dots, \zeta_n) \subseteq L - SVNHWA(\zeta_1^*, \zeta_2^*, \dots, \zeta_n^*). \tag{15}$$

4.2. Logarithmic Hybrid Geometric Operators

Definition 19. For any collection of SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$. The structure of logarithmic single valued neutrosophic hybrid weighted geometric (L-SVNHWG) operator is

$$L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \prod_{p=1}^n (\log_{\sigma_p} \zeta_{\eta(p)}^*)^{\omega_p} \tag{16}$$

where β_p ($p = 1, \dots, n$) are weight vectors with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$ and p th biggest weighted value is $\zeta_{\eta(p)}^*$ ($\zeta_{\eta(p)}^* = (\zeta_{\eta(p)})^{n\beta_p}, P \in N$) consequently by total order $\zeta_{\eta(1)}^* \geq \zeta_{\eta(2)}^* \geq \dots \geq \zeta_{\eta(n)}^*$. Also associated weights are $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ with $\omega_p \geq 0, \sum_{p=1}^n \omega_p = 1$.

Theorem 4. For any collection of SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$. Then by using logarithmic operations and Definition 19, L-SVNHWG define as

$$L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \left[\begin{array}{l} \left(\prod_{p=1}^n \left(1 - \log_{\sigma_p} P_{\zeta_{\eta(p)}^*} \right)^{\beta_p} \right. \\ \left. 1 - \prod_{p=1}^n \left(1 - \left(\log_{\sigma_p} \left(1 - I_{\zeta_{\eta(p)}^*} \right) \right) \right)^{\beta_p} \right. \\ \left. 1 - \prod_{p=1}^n \left(1 - \left(\log_{\sigma_p} \left(1 - N_{\zeta_{\eta(p)}^*} \right) \right) \right)^{\beta_p} \right) \\ \left(\prod_{p=1}^n \left(1 - \log_{\frac{1}{\sigma_p}} P_{\zeta_{\eta(p)}^*} \right)^{\beta_p} \right. \\ \left. 1 - \prod_{p=1}^n \left(1 - \left(\log_{\frac{1}{\sigma_p}} \left(1 - I_{\zeta_{\eta(p)}^*} \right) \right) \right)^{\beta_p} \right. \\ \left. 1 - \prod_{p=1}^n \left(1 - \left(\log_{\frac{1}{\sigma_p}} \left(1 - N_{\zeta_{\eta(p)}^*} \right) \right) \right)^{\beta_p} \right) \end{array} \right] \tag{17}$$

$$0 < \sigma_p \leq \min \left\{ \begin{array}{l} P_{\zeta_{\eta(p)}^*} \\ 1 - I_{\zeta_{\eta(p)}^*} \\ 1 - N_{\zeta_{\eta(p)}^*} \end{array} \right\} < 1$$

$$0 < \frac{1}{\sigma_p} \leq \min \left\{ \begin{array}{l} P_{\zeta_{\eta(p)}^*} \\ 1 - I_{\zeta_{\eta(p)}^*} \\ 1 - N_{\zeta_{\eta(p)}^*} \end{array} \right\} < 1, \sigma \neq 1$$

where β_p ($p = 1, \dots, n$) are weight vectors with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$ and p th biggest weighted value is $\zeta_{\eta(p)}^*$ ($\zeta_{\eta(p)}^* = (\zeta_{\eta(p)})^{n\beta_p}, P \in N$) consequently by total order $\zeta_{\eta(1)}^* \geq \zeta_{\eta(2)}^* \geq \dots \geq \zeta_{\eta(n)}^*$. Also associated weights are $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ with $\omega_p \geq 0, \sum_{p=1}^n \omega_p = 1$.

Proof. Using mathematical induction to prove Equation (4), we proceed as:

(a) For $n = 2$, since

$$(\log_{\sigma_1} \zeta_1^*)^{\omega_1} = \begin{pmatrix} (1 - \log_{\sigma_1} P_{\zeta_1}^*)^{\omega_1} \\ 1 - (1 - (\log_{\sigma_1} (1 - I_{\zeta_1}^*)))^{\omega_1} \\ 1 - (1 - (\log_{\sigma_1} (1 - N_{\zeta_1}^*)))^{\omega_1} \end{pmatrix}$$

and

$$(\log_{\sigma_2} \zeta_2^*)^{\omega_2} = \begin{pmatrix} (1 - \log_{\sigma_2} P_{\zeta_2}^*)^{\omega_2} \\ 1 - (1 - (\log_{\sigma_2} (1 - I_{\zeta_2}^*)))^{\omega_2} \\ 1 - (1 - (\log_{\sigma_2} (1 - N_{\zeta_2}^*)))^{\omega_2} \end{pmatrix}.$$

Then

$$\begin{aligned} L - SVNHWG(\zeta_1, \zeta_2) &= (\log_{\sigma_1} \zeta_1^*)^{\omega_1} \otimes (\log_{\sigma_2} \zeta_2^*)^{\omega_2} \\ &= \begin{pmatrix} (1 - \log_{\sigma_1} P_{\zeta_1}^*)^{\omega_1} \\ 1 - (1 - (\log_{\sigma_1} (1 - I_{\zeta_1}^*)))^{\omega_1} \\ 1 - (1 - (\log_{\sigma_1} (1 - N_{\zeta_1}^*)))^{\omega_1} \end{pmatrix} \otimes \begin{pmatrix} (1 - \log_{\sigma_2} P_{\zeta_2}^*)^{\omega_2} \\ 1 - (1 - (\log_{\sigma_2} (1 - I_{\zeta_2}^*)))^{\omega_2} \\ 1 - (1 - (\log_{\sigma_2} (1 - N_{\zeta_2}^*)))^{\omega_2} \end{pmatrix} \\ &= \left\{ \begin{matrix} (1 - \log_{\sigma_1} P_{\zeta_1}^*)^{\omega_1} \cdot (1 - \log_{\sigma_2} P_{\zeta_2}^*)^{\omega_2} \\ 1 - (1 - (\log_{\sigma_1} (1 - I_{\zeta_1}^*)))^{\omega_1} \cdot (1 - (\log_{\sigma_2} (1 - I_{\zeta_2}^*)))^{\omega_2} \\ 1 - (1 - (\log_{\sigma_1} (1 - N_{\zeta_1}^*)))^{\omega_1} \cdot (1 - (\log_{\sigma_2} (1 - N_{\zeta_2}^*)))^{\omega_2} \end{matrix} \right\} \\ &= \begin{pmatrix} \prod_{p=1}^2 (1 - \log_{\sigma_p} P_{\zeta_p}^*)^{\omega_p} \\ 1 - \prod_{p=1}^2 (1 - (\log_{\sigma_p} (1 - I_{\zeta_p}^*)))^{\omega_p} \\ 1 - \prod_{p=1}^2 (1 - (\log_{\sigma_p} (1 - N_{\zeta_p}^*)))^{\omega_p} \end{pmatrix}. \end{aligned}$$

(b) Now Equation (4) is true for $n = k$,

$$L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_k) = \begin{pmatrix} \prod_{p=1}^k (1 - \log_{\sigma_p} P_{\zeta_p}^*)^{\omega_p} \\ 1 - \prod_{p=1}^k (1 - (\log_{\sigma_p} (1 - I_{\zeta_p}^*)))^{\omega_p} \\ 1 - \prod_{p=1}^k (1 - (\log_{\sigma_p} (1 - N_{\zeta_p}^*)))^{\omega_p} \end{pmatrix},$$

(c) Now, we prove that Equation (4) for $n = k + 1$, that is

$$L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_k, \zeta_{k+1}) = \prod_{p=1}^k (\log_{\sigma_p} \zeta_p)^{\omega_p} \otimes (\log_{\sigma_{k+1}} \zeta_{k+1})^{\omega_{k+1}}$$

$$\begin{aligned}
 &L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_k, \zeta_{k+1}) \\
 &= \left(\begin{array}{c} \prod_{p=1}^k (1 - \log_{\sigma_p} P_{\zeta_p}^*)^{\omega_p} \\ 1 - \prod_{p=1}^k (1 - (\log_{\sigma_p} (1 - I_{\zeta_p}^*)))^{\omega_p} \\ 1 - \prod_{p=1}^k (1 - (\log_{\sigma_p} (1 - N_{\zeta_p}^*)))^{\omega_p} \end{array} \right) \otimes \left(\begin{array}{c} (1 - \log_{\sigma_p} P_{\zeta_{k+1}}^*)^{\omega_{k+1}} \\ 1 - (1 - (\log_{\sigma_p} (1 - I_{\zeta_{k+1}}^*)))^{\omega_{k+1}} \\ 1 - (1 - (\log_{\sigma_p} (1 - N_{\zeta_{k+1}}^*)))^{\omega_{k+1}} \end{array} \right) \\
 &= \left(\begin{array}{c} \prod_{p=1}^{k+1} (1 - \log_{\sigma_p} P_{\zeta_p}^*)^{\omega_p} \\ 1 - \prod_{p=1}^{k+1} (1 - (\log_{\sigma_p} (1 - I_{\zeta_p}^*)))^{\omega_p} \\ 1 - \prod_{p=1}^{k+1} (1 - (\log_{\sigma_p} (1 - N_{\zeta_p}^*)))^{\omega_p} \end{array} \right)
 \end{aligned}$$

Thus Equation (4) is true for $n = z + 1$. Hence it is satisfied for all n . Therefore

$$L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{c} \prod_{p=1}^n (1 - \log_{\sigma_p} P_{\zeta_p}^*)^{\omega_p} \\ 1 - \prod_{p=1}^n (1 - (\log_{\sigma_p} (1 - I_{\zeta_p}^*)))^{\omega_p} \\ 1 - \prod_{p=1}^n (1 - (\log_{\sigma_p} (1 - N_{\zeta_p}^*)))^{\omega_p} \end{array} \right).$$

In a similar way, if $0 < \frac{1}{\sigma_p} \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$, we can also obtain

$$L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{c} \prod_{p=1}^n (1 - \log_{\frac{1}{\sigma_p}} P_{\zeta_p}^*)^{\omega_p} \\ 1 - \prod_{p=1}^n (1 - (\log_{\frac{1}{\sigma_p}} (1 - I_{\zeta_p}^*)))^{\omega_p} \\ 1 - \prod_{p=1}^n (1 - (\log_{\frac{1}{\sigma_p}} (1 - N_{\zeta_p}^*)))^{\omega_p} \end{array} \right)$$

which completes the proof. \square

Remark 2. If $\sigma_1 = \sigma_2 = \sigma_3 = \dots = \sigma_n = \sigma$, that is $0 < \sigma \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$, then $L - SVNHWG$ operator reduced as follows

$$L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{c} \prod_{p=1}^n (1 - \log_{\sigma} P_{\zeta_p}^*)^{\omega_p} \\ 1 - \prod_{p=1}^n (1 - (\log_{\sigma} (1 - I_{\zeta_p}^*)))^{\omega_p} \\ 1 - \prod_{p=1}^n (1 - (\log_{\sigma} (1 - N_{\zeta_p}^*)))^{\omega_p} \end{array} \right). \tag{18}$$

Properties

$L - SVNHWG$ operator satisfies some properties are enlist below;

(1) Idempotency: for any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle (p = 1, \dots, n)$ in \mathfrak{R} . Then, if collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle (p = 1, \dots, n)$ are identical, that is

$$L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \zeta. \tag{19}$$

(2) Boundedness: for any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . $\zeta_p^- = \langle \min_p P_{\zeta_p}, \max_p I_{\zeta_p}, \max_p N_{\zeta_p} \rangle$ and $\zeta_p^+ = \langle \max_p P_{\zeta_p}, \min_p I_{\zeta_p}, \min_p N_{\zeta_p} \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , therefore

$$\zeta_p^- \subseteq L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_n) \subseteq \zeta_p^+ \tag{20}$$

(3) Monotonically: for any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . If $\zeta_p \subseteq \zeta_p^*$ for ($p = 1, \dots, n$), then

$$L - SVNHWG(\zeta_1, \zeta_2, \dots, \zeta_n) \subseteq L - SVNHWG(\zeta_1^*, \zeta_2^*, \dots, \zeta_n^*) \tag{21}$$

4.3. Generalized Logarithmic Averaging Operator

Definition 20. For any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1$, $\sigma \neq 1$. The structure of logarithmic generalized single-valued neutrosophic weighted averaging (L-GSVNWA) operator is

$$L - GSVNWA(\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\sum_{p=1}^n \beta_p \log_{\sigma_p}(\zeta_p)^\gamma \right)^{\frac{1}{\gamma}} \tag{22}$$

where β_p ($p = 1, \dots, n$) are weighting vector with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$.

Theorem 5. For any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1$, $\sigma \neq 1$, $\gamma \geq 1$. Then by using logarithmic operations and Definition 20, L - GSVNWA define as

$$L - GSVNWA(\zeta_1, \zeta_2, \dots, \zeta_n) = \left\{ \begin{array}{l} \left(\begin{array}{l} \left(1 - \prod_{p=1}^n \left(1 - \left(1 - \left(\log_{\sigma_p} P_{\zeta_p} \right)^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \\ 1 - \left[1 - \prod_{p=1}^n \left(1 - \left(1 - \log_{\sigma_p} \left(1 - I_{\zeta_p} \right)^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \\ 1 - \left[1 - \prod_{p=1}^n \left(1 - \left(1 - \log_{\sigma_p} \left(1 - N_{\zeta_p} \right)^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \end{array} \right)^{\frac{1}{\gamma}} \\ \left(\begin{array}{l} \left(1 - \prod_{p=1}^n \left(1 - \left(1 - \left(\log_{\frac{1}{\sigma_p}} P_{\zeta_p} \right)^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \\ 1 - \left[1 - \prod_{p=1}^n \left(1 - \left(1 - \log_{\frac{1}{\sigma_p}} \left(1 - I_{\zeta_p} \right)^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \\ 1 - \left[1 - \prod_{p=1}^n \left(1 - \left(1 - \log_{\frac{1}{\sigma_p}} \left(1 - N_{\zeta_p} \right)^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \end{array} \right)^{\frac{1}{\gamma}} \end{array} \right. \tag{23}$$

where β_p ($p = 1, \dots, n$) are weighting vector with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$.

Apparently, if we use $\gamma = 1$, then the L - GSVNWA operator is becomes into L - SVNWA operator.

Proof. Theorem 5 take the form by utilized the technique of mathematical induction and procedure is eliminate here. \square

Remark 3. If $\sigma_1 = \sigma_2 = \sigma_3 = \dots = \sigma_n = \sigma$, that is $0 < \sigma \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$, then $L - GSVNWA$ operator reduced as follows

$$L - GSVNWA (\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{c} \left(1 - \prod_{p=1}^n \left(1 - \left(1 - (\log_{\sigma} P_{\zeta_p}) \right)^{\gamma} \right)^{\beta_p} \right)^{\frac{1}{\gamma}}, \\ 1 - \left[1 - \prod_{p=1}^n \left(1 - \left(1 - \log_{\sigma} (1 - I_{\zeta_p}) \right)^{\gamma} \right)^{\beta_p} \right]^{\frac{1}{\gamma}} \\ 1 - \left[1 - \prod_{p=1}^n \left(1 - \left(1 - \log_{\sigma} (1 - N_{\zeta_p}) \right)^{\gamma} \right)^{\beta_p} \right]^{\frac{1}{\gamma}} \end{array} \right). \quad (24)$$

Properties

$L - GSVNWA$ operator satisfies some properties are enlist below;

(1) Idempotency: For any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . Then, if collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) are identical, that is

$$L - GSVNWA (\zeta_1, \zeta_2, \dots, \zeta_n) = \zeta. \quad (25)$$

(2) Boundedness: for any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . $\zeta_p^- = \langle \min_p P_{\zeta_p}, \max_p I_{\zeta_p}, \max_p N_{\zeta_p} \rangle$ and $\zeta_p^+ = \langle \max_p P_{\zeta_p}, \min_p I_{\zeta_p}, \min_p N_{\zeta_p} \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , therefore

$$\zeta_p^- \subseteq L - GSVNWA (\zeta_1, \zeta_2, \dots, \zeta_n) \subseteq \zeta_p^+. \quad (26)$$

(3) Monotonically: for any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . If $\zeta_p \subseteq \zeta_p^*$ for ($p = 1, \dots, n$), then

$$L - GSVNWA (\zeta_1, \zeta_2, \dots, \zeta_n) \subseteq L - GSVNWA (\zeta_1^*, \zeta_2^*, \dots, \zeta_n^*). \quad (27)$$

4.4. Generalized Logarithmic Geometric Operator

Definition 21. For any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p}\} < 1, \sigma \neq 1$. The structure of logarithmic generalized single valued neutrosophic weighted geometric ($L - GSVNWG$) operator is

$$L - GSVNWG (\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\sum_{p=1}^n \left(\log_{\sigma_p} (\zeta_p)^{\gamma} \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \quad (28)$$

where β_p ($p = 1, \dots, n$) are weighting vector with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$.

Theorem 6. For any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , with $0 < \sigma_p \leq \min \{ P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p} \} < 1, \sigma \neq 1, \gamma \geq 1$. Then by using logarithmic operations and definition (21), $L - GSVNWG$ define as

$$L - GSVNWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{l} \left(\begin{array}{l} 1 - \left[1 - \prod_{p=1}^n \left(1 - (\log_{\sigma_p} P_{\zeta_p})^\gamma \right)^{\beta_p} \right]^{\frac{1}{\gamma}}, \\ \left(1 - \prod_{p=1}^n \left(1 - (\log_{\sigma_p} (1 - I_{\zeta_p}))^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \\ \left(1 - \prod_{p=1}^n \left(1 - (\log_{\sigma_p} (1 - N_{\zeta_p}))^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \end{array} \right) \\ \left(\begin{array}{l} 1 - \left[1 - \prod_{p=1}^n \left(1 - (\log_{\frac{1}{\sigma_p}} P_{\zeta_p})^\gamma \right)^{\beta_p} \right]^{\frac{1}{\gamma}}, \\ \left(1 - \prod_{p=1}^n \left(1 - (\log_{\frac{1}{\sigma_p}} (1 - I_{\zeta_p}))^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \\ \left(1 - \prod_{p=1}^n \left(1 - (\log_{\frac{1}{\sigma_p}} (1 - N_{\zeta_p}))^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \end{array} \right) \end{array} \right) \quad \begin{array}{l} 0 < \sigma_p \leq \min \left\{ \begin{array}{l} P_{\zeta_p}, \\ 1 - I_{\zeta_p}, \\ 1 - N_{\zeta_p} \end{array} \right\} < 1 \\ \\ 0 < \frac{1}{\sigma_p} \leq \min \left\{ \begin{array}{l} P_{\zeta_p}, \\ 1 - I_{\zeta_p}, \\ 1 - N_{\zeta_p} \end{array} \right\} < 1, \\ \sigma \neq 1 \end{array} \quad (29)$$

where β_p ($p = 1, \dots, n$) is the weighting vector with $\beta_p \geq 0$ and $\sum_{p=1}^n \beta_p = 1$.

Apparently, if we use $\gamma = 1$, then the $L - GSVNWG$ operator is becomes into $L - SVNWG$ operator.

Proof. Theorem 6 takes the form by utilizing the technique of mathematical induction and the procedure is eliminated here. \square

Remark 4. If $\sigma_1 = \sigma_2 = \sigma_3 = \dots = \sigma_n = \sigma$, that is $0 < \sigma \leq \min \{ P_{\zeta_p}, 1 - I_{\zeta_p}, 1 - N_{\zeta_p} \} < 1, \sigma \neq 1$, then $L - GSVNWG$ operator reduced as follows

$$L - GSVNWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \left(\begin{array}{l} 1 - \left[1 - \prod_{p=1}^n \left(1 - (\log_{\sigma} P_{\zeta_p})^\gamma \right)^{\beta_p} \right]^{\frac{1}{\gamma}}, \\ \left(1 - \prod_{p=1}^n \left(1 - (\log_{\sigma} (1 - I_{\zeta_p}))^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \\ \left(1 - \prod_{p=1}^n \left(1 - (\log_{\sigma} (1 - N_{\zeta_p}))^\gamma \right)^{\beta_p} \right)^{\frac{1}{\gamma}} \end{array} \right). \quad (30)$$

Properties

$L - GSVNWG$ operator satisfies some properties are enlist below;

(1) Idempotency: For any collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . Then, if collection of SVNNs $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) are identical, that is

$$L - GSVNWG(\zeta_1, \zeta_2, \dots, \zeta_n) = \zeta. \quad (31)$$

(2) Boundedness: for any collection of SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . $\zeta_p^- = \langle \min_p P_{\zeta_p}, \max_p I_{\zeta_p}, \max_p N_{\zeta_p} \rangle$ and $\zeta_p^+ = \langle \max_p P_{\zeta_p}, \min_p I_{\zeta_p}, \min_p N_{\zeta_p} \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} , therefore

$$\zeta_p^- \subseteq L - GSVNWG(\zeta_1, \zeta_2, \dots, \zeta_n) \subseteq \zeta_p^+ \tag{32}$$

(3) Monotonically: for any collection of SVNNS $\zeta_p = \langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \rangle$ ($p = 1, \dots, n$) in \mathfrak{R} . If $\zeta_p \subseteq \zeta_p^*$ for ($p = 1, \dots, n$), then

$$L - GSVNWG(\zeta_1, \zeta_2, \dots, \zeta_n) \subseteq L - GSVNWG(\zeta_1^*, \zeta_2^*, \dots, \zeta_n^*) \tag{33}$$

5. Proposed Technique for Solving Decision-Making Problems

This section includes the new approach to decision-making based on the single-valued neutrosophic sets, and we will propose a decision-making matrix as indicated below.

Let $H = (h_1, h_2, \dots, h_m)$ be a distinct collection of m probable alternatives and $Y = (y_1, y_2, \dots, y_n)$ be a finite collection of n criteria, where h_i indicate the i -th alternatives and y_j indicate the j -th criteria. Let $D = (d_1, d_2, \dots, d_t)$ be a finite set of t experts, where d_k indicate the k -th expert. The expert d_k supply her appraisal of an alternative h_i on an attribute y_j as a SVNNS ($i = 1, \dots, m; j = 1, \dots, n$). The expert's information is represented by the SVNS decision-making matrix $D^s = [E_{ip}^{(s)}]_{m \times n}$. Assume that β_p ($p = 1, \dots, m$) is the weight vector of the attribute y_j , where $0 \leq \beta_p \leq 1, \sum_{p=1}^n \beta_p = 1$ and $\psi = (\psi_1, \psi_2, \dots, \psi_m)$ be the weights of the decision makers d_k such that $\psi_k \leq 1, \sum_{k=1}^n \psi_k = 1$.

When we construct the SVNS decision-making matrices, $D^s = [E_{ip}^{(s)}]_{m \times n}$ for decision. Basically, criteria have two types, one is benefit criteria and other one is cost criteria. If the SVNS decision matrices have cost-type criteria metrics $D^s = [E_{ip}^s]_{m \times n}$ can be converted into the normalized SVNS decision matrices, $R^s = [r_{ip}^{(s)}]_{m \times n}$, where $r_{ip}^s = \begin{cases} E_{ip}^s, & \text{for benefit criteria } A_p \\ \bar{E}_{ip}^s, & \text{for cost criteria } A_p \end{cases} \quad j = 1, \dots, n$, and \bar{E}_{ip}^s is the complement of E_{ip}^s . The normalization is not required, if the criteria have the same type.

Step 1: In this step, we get the neutrosophic information, using the all proposed logarithmic aggregation operators to evolute the alternative preference values with associated weights, which are $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ with $\omega_p \geq 0, \sum_{p=1}^n \omega_p = 1$.

Step 2: We find the score value $\tilde{S}(\log_{\sigma} \zeta_p)$ and the accuracy value $\tilde{A}(\log_{\sigma} \zeta_p)$ of the cumulative total preference value h_i ($i = 1, \dots, m$).

Step 3: By definition, we give ranking to the alternatives h_i ($i = 1, \dots, m$) and choose the best alternative which has the maximum score value.

5.1. Numerical Example

Assume that there is a committee which selects five applicable emerging technology enterprises H_g ($g = 1, \dots, 5$), which are given as follows.

- (1) Augmented reality (H_1),
- (2) Personalized medicine (H_2),
- (3) Artificial intelligence (H_3),
- (4) Gene drive (H_4) and
- (5) Quantum computing (H_5).

They assess the possible rising technology enterprises according to the five attributes, which are

- (1) Advancement (D_1),
- (2) Market risk (D_2),
- (3) Financial investments (D_3),

- (4) Progress of science and technology (D_4) and
 (5) Designs (D_5).

To avoid the conflict between them, the decision makers take the attribute weights as $\beta = (0.15, 0.28, 0.20, 0.22, 0.15)^T$. They construct the SVNS decision-making matrix given in Table 1.

Table 1. Emerging Technology Enterprises D^1 .

	D_1	D_2	D_3	D_4	D_5
H_1	(0.5, 0.3, 0.4)	(0.3, 0.2, 0.5)	(0.2, 0.2, 0.6)	(0.4, 0.2, 0.3)	(0.3, 0.3, 0.4)
H_2	(0.7, 0.1, 0.3)	(0.3, 0.2, 0.7)	(0.6, 0.3, 0.2)	(0.2, 0.4, 0.6)	(0.7, 0.1, 0.2)
H_3	(0.5, 0.3, 0.4)	(0.4, 0.2, 0.6)	(0.6, 0.1, 0.2)	(0.3, 0.1, 0.5)	(0.6, 0.4, 0.3)
H_4	(0.7, 0.3, 0.2)	(0.2, 0.2, 0.7)	(0.4, 0.5, 0.2)	(0.2, 0.2, 0.5)	(0.4, 0.5, 0.4)
H_5	(0.4, 0.1, 0.3)	(0.2, 0.1, 0.5)	(0.4, 0.1, 0.5)	(0.6, 0.3, 0.4)	(0.3, 0.2, 0.4)

Since D_1, D_3 are benefit-type criteria and D_2, D_4 is cost type criteria, the normalization is required for these decision matrices. Normalized decision matrices are shown in Table 2.

Table 2. Emerging Technology Enterprises R^1 .

	D_1	D_2	D_3	D_4	D_5
H_1	(0.5, 0.3, 0.4)	(0.5, 0.2, 0.3)	(0.2, 0.2, 0.6)	(0.3, 0.2, 0.4)	(0.3, 0.3, 0.4)
H_2	(0.7, 0.1, 0.3)	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.2)	(0.6, 0.4, 0.2)	(0.7, 0.1, 0.2)
H_3	(0.5, 0.3, 0.4)	(0.6, 0.2, 0.4)	(0.6, 0.1, 0.2)	(0.5, 0.1, 0.3)	(0.6, 0.4, 0.3)
H_4	(0.7, 0.3, 0.2)	(0.7, 0.2, 0.2)	(0.4, 0.5, 0.2)	(0.5, 0.2, 0.2)	(0.4, 0.5, 0.4)
H_5	(0.4, 0.1, 0.3)	(0.5, 0.1, 0.2)	(0.4, 0.1, 0.5)	(0.4, 0.3, 0.6)	(0.3, 0.2, 0.4)

Step 1: Now, we apply all the proposed logarithmic aggregation operators to collective neutrosophic information as follows.

Case 1: Using logarithmic single-valued neutrosophic hybrid weighted averaging aggregation operator, we obtained the results shown in Table 3.

Table 3. Aggregated information using the logarithmic single valued neutrosophic hybrid weighted averaging (L-SVNHWA) operator for $\sigma = 0.3$.

H_1	(0.17624, 0.23432, 0.43885)
H_2	(0.66164, 0.16229, 0.21840)
H_3	(0.52788, 0.18347, 0.32224)
H_4	(0.49410, 0.30962, 0.20985)
H_5	(0.22496, 0.12393, 0.39318)

Case 2: Using Logarithmic single valued neutrosophic hybrid weighted geometric aggregation operator, we obtained the results shown in Table 4.

Table 4. Aggregated information using logarithmic single valued neutrosophic hybrid weighted geometric (L-SVNHWG) operator for $\sigma = 0.1$.

H_1	(0.52472, 0.12638, 0.24189)
H_2	(0.81968, 0.10633, 0.11764)
H_3	(0.74946, 0.11782, 0.17620)
H_4	(0.70685, 0.18942, 0.11685)
H_5	(0.58497, 0.07427, 0.23305)

Step 2: We find the score index $\tilde{S}(\log_{\sigma}\zeta_p)$ and the accuracy index $\tilde{A}(\log_{\sigma}\zeta_p)$ of the cumulative overall preference value h_i ($i = 1, 2, 3, 4, 5$).

Case 1: Using the score of aggregated information for L-SVNHWA operator, we obtained the results shown in Table 5.

Table 5. Score of aggregated information for L-SVNHWA operator.

$\tilde{S}(\log_{0.3}H_1)$	-1.14345	$\tilde{A}(\log_{0.3}H_1)$	0.25985
$\tilde{S}(\log_{0.3}H_2)$	0.30519	$\tilde{A}(\log_{0.3}H_2)$	1.0087
$\tilde{S}(\log_{0.3}H_3)$	-0.02207	$\tilde{A}(\log_{0.3}H_3)$	0.96078
$\tilde{S}(\log_{0.3}H_4)$	-0.08895	$\tilde{A}(\log_{0.3}H_4)$	0.91781
$\tilde{S}(\log_{0.3}H_5)$	-0.76389	$\tilde{A}(\log_{0.3}H_5)$	0.28571

Case 2: Score of Aggregated information for L-SVNHWA Operator, we obtained the results shown in Table 6.

Table 6. Score of aggregated information for L-SVNHWA operator.

$\tilde{S}(\log_{0.1}H_1)$	0.540979	$\tilde{A}(\log_{0.1}H_1)$	0.89888
$\tilde{S}(\log_{0.1}H_2)$	0.810463	$\tilde{A}(\log_{0.1}H_2)$	1.01683
$\tilde{S}(\log_{0.1}H_3)$	0.736126	$\tilde{A}(\log_{0.1}H_3)$	1.01338
$\tilde{S}(\log_{0.1}H_4)$	0.704159	$\tilde{A}(\log_{0.1}H_4)$	0.994506
$\tilde{S}(\log_{0.1}H_5)$	0.618387	$\tilde{A}(\log_{0.1}H_5)$	0.903179

Step 3: We find the best (suitable) alternative which has the maximum score value from the set of alternatives h_i ($i = 1, 2, 3, 4, 5$). Overall preference value and ranking of the alternatives are summarized in Table 7.

Table 7. Overall preference value and ranking of the alternatives.

	$\tilde{S}(H_1)$	$\tilde{S}(H_2)$	$\tilde{S}(H_3)$	$\tilde{S}(H_4)$	$\tilde{S}(H_5)$	Ranking
$L - SVNHWA$	-1.143	0.305	-0.022	-0.088	-0.763	$H_2 > H_3 > H_4 > H_5 > H_1$
$L - SVNHWA$	0.540	0.810	0.736	0.704	0.618	$H_2 > H_3 > H_4 > H_5 > H_1$

5.2. Comparison with Existing Methods

This section consists of the comparative analysis of several existing aggregation operators of neutrosophic information with the proposed logarithmic single valued hybrid weighted aggregation operators. Existing methods for aggregated neutrosophic information are shown in Table 8–11.

Table 8. Average aggregated SVN information.

	SVNWA [35]	SVNOWA [35]	NWA [14]
H_1	(0.3779, 0.2259, 0.4002)	(0.3820, 0.2449, 0.4071)	(0.3779, 0.2314, 0.4223)
H_2	(0.6615, 0.2052, 0.2381)	(0.6663, 0.1801, 0.2430)	(0.6615, 0.2426, 0.2446)
H_3	(0.5656, 0.1763, 0.3131)	(0.5597, 0.1838, 0.3122)	(0.5656, 0.2109, 0.3272)
H_4	(0.5722, 0.2929, 0.2219)	(0.5706, 0.3145, 0.2219)	(0.5722, 0.3348, 0.2338)
H_5	(0.4165, 0.1413, 0.3607)	(0.3960, 0.1373, 0.3696)	(0.4165, 0.1633, 0.4131)

Table 9. Average aggregated SVN information.

	SVNFWA [12]	SVNHWA [11] $\gamma = 2$
H_1	(0.3755, 0.2262, 0.4018)	(0.3725, 0.2264, 0.4033)
H_2	(0.6611, 0.2072, 0.2385)	(0.6608, 0.2086, 0.2388)
H_3	(0.5652, 0.1779, 0.3141)	(0.5648, 0.1790, 0.3149)
H_4	(0.5692, 0.2956, 0.2225)	(0.5663, 0.2978, 0.2230)
H_5	(0.4159, 0.1422, 0.3646)	(0.4151, 0.1427, 0.3680)

Table 10. Average aggregated SVN information.

	SVNHWA [11] $\gamma = 3$	L-SVNWA [10]
H_1	(0.3693, 0.2266, 0.4048)	(0.3130, 0.1753, 0.3544)
H_2	(0.6604, 0.2099, 0.2390)	(0.6486, 0.1989, 0.2313)
H_3	(0.5645, 0.1800, 0.3157)	(0.4989, 0.1733, 0.3321)
H_4	(0.5635, 0.3000, 0.2234)	(0.5585, 0.2736, 0.1942)
H_5	(0.4143, 0.1432, 0.3714)	(0.2849, 0.1249, 0.3758)

Table 11. Average aggregated SVN information.

	L-SVNOWA [10]
H_1	(0.3229, 0.1926, 0.3607)
H_2	(0.6549, 0.1719, 0.2368)
H_3	(0.4896, 0.1823, 0.3303)
H_4	(0.5561, 0.2975, 0.1942)
H_5	(0.2442, 0.1209, 0.3834)

Now, we analyze the ranking of the alternatives according to their aggregated information (in Table 12).

Table 12. Overall ranking of the alternatives.

Existing Operators	Ranking
NWA [14]	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNWA [35]	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNOWA [35]	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNHWG [35]	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNOWG [35]	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNFWA [12]	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNHWA [11] $\gamma = 2$	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNHWA [11] $\gamma = 3$	$H_2 > H_3 > H_4 > H_5 > H_1$
NWG [14]	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNFWG [12]	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNHWG [11] $\gamma = 2$	$H_2 > H_3 > H_4 > H_5 > H_1$
SVNHWG [11] $\gamma = 3$	$H_2 > H_3 > H_4 > H_5 > H_1$
SNWEA [15]	$H_2 > H_3 > H_5 > H_4 > H_1$
L-SVNWA [10]	$H_2 > H_4 > H_3 > H_5 > H_1$
L-SVNOWA [10]	$H_2 > H_4 > H_3 > H_5 > H_1$
L-SVNHWG [10]	$H_2 > H_4 > H_3 > H_1 > H_5$
L-SVNOWG [10]	$H_2 > H_3 > H_4 > H_5 > H_1$
Proposed Operators	Ranking
L-SVNHWA	$H_2 > H_3 > H_4 > H_5 > H_1$
L-SVNHWG	$H_2 > H_3 > H_4 > H_5 > H_1$
L-GSVNWA	$H_2 > H_4 > H_3 > H_5 > H_1$
L-GSVNHWG	$H_2 > H_4 > H_3 > H_1 > H_5$

The best alternative was H_2 . The obtained results utilizing logarithmic single valued neutrosophic hybrid weighted operators and logarithmic generalized single valued neutrosophic weighted operators were same as results shows existing methods. Hence, this study proposed novel logarithmic aggregation operators to aggregate the neutrosophic information more effectively and efficiently. Utilizing the proposed logarithmic aggregation operators, we found the best alternative from a set of alternatives given by the decision maker. Hence the proposed MCDM technique based on logarithmic operators lets us find the best alternative as an applications in decision support systems.

6. Conclusions

In this work, an attempt has been made to present different kinds of logarithmic weighted averaging and geometric aggregation operators based on the single-valued neutrosophic set environment. Earlier, it has been observed that the various aggregation operators are defined under the SVNSS environment where the aggregation operators based on the algebraic or Einstein t-norm and t-conorm. In this paper, we proposed novel logarithmic hybrid aggregation operators and also logarithmic generalized averaging and geometric aggregation operators. Aggregation operators, namely L-SVNHWA, L-SVNHWG, L-GSVNWA and L-GSVNWA are developed under the SVNSS environment and we have studied their properties in detail. Further, depending on the standardization of the decision matrix and the proposed aggregation operators, a decision-making approach is presented to find the best alternative to the SVNSS environment. An illustrative example is taken for illustrating the developed approach, and their results are compared with some of the existing approaches of the SVNSS environment to show the validity of it. From the studies, we conclude that the proposed approach is more generic and suitable for solving the stated problem.

In the future, we shall link the proposed operators with some novel fuzzy sets, like as type 2 fuzzy sets, neutrosophic sets, and so on. Moreover, we may examine if our constructed approach can also be applied in different areas, such as personal evaluation, medical artificial intelligence, energy management and supplier selection evaluation.

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