

# Measure distance between neutrosophic sets: An evidential approach

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**Abstract.** Due to the efficiency to handle uncertainty information, the single valued neutrosophic set is widely used in multi-criteria decision-making. In MCDM, it is inevitable to measure the distance between two single valued neutrosophic sets. In this paper, an evidence distance for neutrosophic sets is proposed. There are two main contributions of this work. One is a new method to transform the single valued neutrosophic set into basic probability assignment. The other is evidence distance function between two single valued neutrosophic sets. The application in MCDM is illustrated the efficiency of the proposed distance.

**Keywords:** Single valued neutrosophic set, Dempster-Shafer Evidence theory, Evidence distance, Uncertainty, Multi-criteria decision-making

## 1. Introduction

Neutrosophy was introduced by F. Smarandache in 1995, and provides a more flexible way to handle uncertainty information because of its union of the classic set, fuzzy set[52], interval valued fuzzy set[15], intuitionistic fuzzy set[2], etc. A single valued neutrosophic set (SVNS) is composed by three components, truth-membership function, indeterminacy-membership function, and falsity-membership function [43]. Due to its flexibility, SVNS is widely used in decision-making problems [7,17,26,27,28,29,48,49,50,55], in pattern recognition[1], in clustering[17], etc[16,51].

To solve multi-criteria decision-making problems, it is unavoidable to measure similarity or distance between two single valued neutrosophic sets. J. Ye pro-

posed a similarity measure was introduced by utilizing the weighted correlation coefficient or the weighted cosine[48], also, he presented a similarity measure between interval neutrosophic sets[50]. In [4], S. Broumi *et al.* presented distance and similarity measures for interval neutrosophic sets. In [30], similarity measures and entropy of single valued neutrosophic sets were introduced by P. Majumdar and S.K. Samanta. In addition, R. Sahin *et al.* introduced a similarity measure and a entropy of neutrosophic soft sets applied to multi-criteria decision-making problems[33]. In [3], the new similarity measures and entropy of single valued neutrosophic sets were formulated. P. Liu *et al.* introduced a weighted distance measure used in neutrosophic multi-attribute group decision-making[26]. HL. Huang formulated a distance measure of SVNS to propose clustering method and multi-criteria decision-making method[17]. ZP. Tian presented a entropy based on cross-entropy used in multi-criteria decision-making[40]. Besides, many

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other distances, similarity measures or entropy were developed[16,18,24,46].

Though many methods are presented, it is still an open issue to measure the distance between two SVN-S. In this paper, a new evidential distance between SVN-S is proposed. There are two main contributions in this paper. On the one hand, a new method to transform SVN-S into belief probability assignment(BPA) is proposed. On the other hand, the distance of SVN-Ss is measured from the aspect evidential method.

The rest of this paper is organized as follows. In Section 2, some basic concepts on neutrosophic set and evidence theory are introduced. In Section 3, a new distance between two single valued neutrosophic sets is proposed. In Section 4, a numerical example is presented to illustrate the effectiveness of the proposed method. Section 5 concludes the paper.

## 2. Preliminaries

### 2.1. Neutrosophic set

In this subsection, some basic definitions related single valued neutrosophic set in [43] are presented as follows.

**Definition (Single valued neutrosophic set(SVNS))** [43]

Let  $X$  be a space of points (objects), with a generic element in  $X$  denoted by  $x$ . A SVN-S  $A$  is characterized by truth-membership function  $T_A$ , indeterminacy-membership function  $I_A$ , and falsity-membership function  $F_A$ . For each point  $x$  in  $X$ ,  $T_A(x), I_A(x), F_A(x) \in [0, 1]$ .

When  $X$  is continuous, a SVN-S  $A$  can be defined as

$$A = \int \frac{\langle T_A(x), I_A(x), F_A(x) \rangle}{x}, x \in X \quad (1)$$

When  $X$  is discrete, a SVN-S  $A$  can be defined as

$$A = \sum_i \frac{\langle T_A(x_i), I_A(x_i), F_A(x_i) \rangle}{x_i}, x_i \in X \quad (2)$$

**Definition (Complement)** [43]

The complement of a SVN-S  $A$  is denoted by  $C(A)$  and is defined by

$$T_{C(A)}(x) = F_A(x), \quad (3)$$

$$I_{C(A)}(x) = 1 - I_A(x), \quad (4)$$

$$F_{C(A)}(x) = T_A(x) \quad (5)$$

**Definition (Containment)** [43]

A SVN-S  $A$  is contained in the other SVN-S  $B$ , denoted by  $A \subseteq B$ , if and only if

$$T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x) \quad (6)$$

for all  $x$  in  $X$ .

**Definition (Equality)** [43]

Two SVN-Ss  $A$  and  $B$  are equal, written as  $A = B$ , if and only if  $A \subseteq B$  and  $B \subseteq A$ .

**Definition (Union)** [43]

The union of two SVN-Ss  $A$  and  $B$  is a SVN-S  $C$ , written as  $C = A \cup B$ , whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of  $A$  and  $B$  by

$$\begin{aligned} T_C(x) &= \max\{T_A(x), T_B(x)\}, \\ I_C(x) &= \max\{I_A(x), I_B(x)\}, \\ F_C(x) &= \min\{F_A(x), F_B(x)\}, \end{aligned} \quad (7)$$

for all  $x$  in  $X$ .

**Definition (Intersection)** [43]

The intersection of two SVN-Ss  $A$  and  $B$  is a SVN-S  $C$ , written as  $C = A \cap B$ , whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of  $A$  and  $B$  by

$$\begin{aligned} T_C(x) &= \min\{T_A(x), T_B(x)\}, \\ I_C(x) &= \min\{I_A(x), I_B(x)\}, \\ F_C(x) &= \max\{F_A(x), F_B(x)\}, \end{aligned} \quad (8)$$

for all  $x$  in  $X$ .

### 2.2. Evidence theory and evidence distance

Uncertainty information exists everywhere in the real application. There are many math tools to handle uncertainty, such as fuzzy numbers [53,54,44,41,56,47], Z numbers [21], D numbers [31,11,58] and so on. Among these tools, evidence theory is paid more and more attention recently [22,8,12,10]. In this subsection, some definitions in [23,34] are presented as follows.

**Definition (Basic Probability Assignment(BPA)) [34]**

Let  $\Theta$  be a frame of discernment, including  $N$  mutually exclusive and exhaustive elements. A BPA is a function from  $P(\Theta)$  to  $[0, 1]$ , defined as follows

$$m : P(\Theta) \rightarrow [0, 1] \quad (9)$$

where  $P(\Theta)$  is the power set of  $\Theta$ . A BPA should satisfy the following conditions[34]

$$\sum_{A \in P(\Theta)} m(A) = 1, \quad (10)$$

$$m(\phi) = 0. \quad (11)$$

**Definition (Evidence Distance) [23]**

Let  $\Theta$  be a frame of discernment including  $N$  mutually exclusive and exhaustive objects, and  $m_1, m_2$  be two BPAs. The evidence distance between  $m_1$  and  $m_2$  is defined as follows

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}(\mathbf{m}_1 - \mathbf{m}_2)\mathbf{D}(\mathbf{m}_1 - \mathbf{m}_2)^T}, \quad (12)$$

where  $\mathbf{m}$  is a row vector associated with the BPA  $m$ , defined as

$$\mathbf{m} = (m(A_1), m(A_2), \dots, m(A_{2^N})) \quad (13)$$

where  $A_i \in P(\Theta), i = 1, 2, \dots, 2^N$ , when  $i \neq j, A_i \neq A_j$ , and  $\mathbf{D}$  is a  $2^N \times 2^N$  matrix with entries (written as  $ent_{ij}\mathbf{D}$ ) defined below

$$ent_{ij}\mathbf{D} = \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \quad (14)$$

For simplicity, vector  $\mathbf{m}$  in (13) is also called a BPA in the remainder of this paper. It should be noted that evidence distance is widely used to measure the conflicts between BPAs [19,32] and a lots of distance functions are developed from evidential aspect[25,45].

**3. Proposed method**

In this section, a new method to measure distance between SVNNSs is proposed. A key step in the proposed method is to transform SVNNS into BPAs. Actually, how to generate the BPA is still an open issue

[9,22,57]. Best to our knowledge, there is no work to determine BPA with SVNNS.

For simplicity, a mapping from three components of a SVNNS to  $\{1, 2, 3\}$  is constructed, namely

$$f : \begin{cases} T & \rightarrow 1 \\ I & \rightarrow 2 \\ F & \rightarrow 3 \end{cases} \quad (15)$$

For three components of a SVNNS

$$A_1 = \sum \frac{\langle T_1(x), I_1(x), F_1(x) \rangle}{x}$$

and an object  $x$  in  $X$ , a mapping from every component to a row vector is defined as follows

$$f_1 : \begin{cases} T_1(x) & \rightarrow \mathbf{m}_{11}(x) = (T_1(x), 1 - T_1(x)) \\ I_1(x) & \rightarrow \mathbf{m}_{12}(x) = (I_1(x), 1 - I_1(x)) \\ F_1(x) & \rightarrow \mathbf{m}_{13}(x) = (F_1(x), 1 - F_1(x)) \end{cases} \quad (16)$$

Then, a SVNNS  $A_1$  is transformed into three BPAs, presented as follows

$$A_1 \rightarrow \begin{pmatrix} \mathbf{m}_{11}(x) \\ \mathbf{m}_{12}(x) \\ \mathbf{m}_{13}(x) \end{pmatrix} = \begin{pmatrix} T_1(x) & 1 - T_1(x) \\ I_1(x) & 1 - I_1(x) \\ F_1(x) & 1 - F_1(x) \end{pmatrix}$$

Next, for two SVNNSs  $A_1$  and  $A_2$ , three distances can be constructed according to Eq.(12). For simplicity, some symbols are denoted by

$$\begin{aligned} \Delta_1(x) &= \mathbf{m}_{11} - \mathbf{m}_{21} \\ &= (T_1(x) - T_2(x), T_2(x) - T_1(x)) \end{aligned} \quad (17)$$

$$\begin{aligned} \Delta_2(x) &= \mathbf{m}_{12} - \mathbf{m}_{22} \\ &= (I_1(x) - I_2(x), I_2(x) - I_1(x)) \end{aligned} \quad (18)$$

$$\begin{aligned} \Delta_3(x) &= \mathbf{m}_{13} - \mathbf{m}_{23} \\ &= (F_1(x) - F_2(x), F_2(x) - F_1(x)) \end{aligned} \quad (19)$$

Then, three distances are constructed as follows

$$\begin{aligned} d_1 &= \sqrt{\frac{1}{2} \Delta_1 D_1 \Delta_1^T} \\ d_2 &= \sqrt{\frac{1}{2} \Delta_2 D_2 \Delta_2^T} \\ d_3 &= \sqrt{\frac{1}{2} \Delta_3 D_3 \Delta_3^T} \end{aligned} \quad (20)$$

According to Eq.(14),  $D_1, D_2, D_3$  are defined as follows

$$D_1 = D_2 = D_3 = \begin{pmatrix} 1 & 1/3 \\ 1/3 & 1 \end{pmatrix} \quad (21)$$

Respectively,  $d_1, d_2, d_3$  is equal to

$$\begin{aligned} d_1 &= \sqrt{\frac{2}{3}} |T_1(x) - T_2(x)| \\ d_2 &= \sqrt{\frac{2}{3}} |I_1(x) - I_2(x)| \\ d_3 &= \sqrt{\frac{2}{3}} |F_1(x) - F_2(x)| \end{aligned}$$

Consequently, the component of distance between SVNSs can be defined as follows.

**Definition (Component of Distance for SVNS)** Let  $X$  be a space constructed by  $N$  points (objects), with a generic element in  $X$  denoted by  $x$ . Given two SVNSs, named  $A_1, A_2$ . For a element  $x_i$  in  $X$ , the  $i$ th-component with respect to  $x_i$  of the distance between  $A_1$  and  $A_2$  is defined as

$$\begin{aligned} d(x_i) &= d_1 + d_2 + d_3 \\ &= \sqrt{\frac{2}{3}} (|T_1(x_i) - T_2(x_i)| \\ &\quad + |I_1(x_i) - I_2(x_i)| \\ &\quad + |F_1(x_i) - F_2(x_i)|) \end{aligned} \quad (22)$$

Finally, the distance between two SVNSs is defined as follows.

**Definition (Distance for SVNS)** The distance between SVNSs is a weighted distance constructed by its components defined above and a weight vector  $\omega$ . Let

weight vector be  $\omega = (\omega_1, \omega_2, \dots, \omega_N)^T$ , and  $\mathbf{d} = (d(x_1), d(x_2), \dots, d(x_N))$ , hence the distance can be defined as follows

$$d(A_1, A_2) = \mathbf{d}\omega = \sum_{i=1}^N \omega_i d(x_i) \quad (23)$$

**Example** Set  $X$  to  $\{a, b, c\}$ . Given two SVNSs  $A_1$  and  $A_2$  as follows

$$\begin{aligned} A_1 &= \frac{\langle 0.3, 0.3, 0.7 \rangle}{a} + \frac{\langle 0.4, 0.5, 0.6 \rangle}{b} + \frac{\langle 0.7, 0.1, 0.5 \rangle}{c} \\ A_2 &= \frac{\langle 0.2, 0.2, 0.6 \rangle}{a} + \frac{\langle 0.3, 0.1, 0.7 \rangle}{b} + \frac{\langle 0.5, 0.2, 0.0 \rangle}{c} \end{aligned}$$

and weight vector  $\omega = (0.3, 0.3, 0.4)^T$ . Then

$$\begin{aligned} d_{12}(a) &= \sqrt{\frac{2}{3}} \sum_C |C_A(a) - C_B(a)| \\ &= \frac{\sqrt{6}}{10} \approx 0.24495 \\ d_{12}(b) &= \sqrt{\frac{2}{3}} \sum_C |C_A(b) - C_B(b)| \\ &= \frac{\sqrt{6}}{5} \approx 0.48990 \\ d_{12}(c) &= \sqrt{\frac{2}{3}} \sum_C |C_A(c) - C_B(c)| \\ &= \frac{4\sqrt{6}}{15} \approx 0.65320 \end{aligned}$$

Hence, the distance between  $A_1$  and  $A_2$  is

$$\begin{aligned} d(A_1, A_2) &= \mathbf{d}\omega \\ &= \frac{\sqrt{6}}{10} \times 0.3 + \frac{\sqrt{6}}{5} \times 0.3 + \frac{4\sqrt{6}}{15} \times 0.4 \\ &\approx 0.48173 \end{aligned}$$

#### 4. Practical Application

In order to demonstrate the application of the proposed approach, a multi-criteria decision making problem illustrated in [3] is concerned with a manufacturing company which wants to select the best global supplier according to the core competencies of supplier-

s. Suppose that there is a set containing four suppliers  $S = \{S_1, S_2, S_3, S_4\}$  whose core competencies are evaluated by the following four criteria

- $C_1$ . the level of technology innovation,
- $C_2$ . the control ability of flow,
- $C_3$ . the ability of management,
- $C_4$ . the level of service.

Then, the weight vector for the four criteria is  $\omega = (0.25, 0.30, 0.20, 0.25)^T$ . It is useful to define the ideal point to identify the best alternative. For this problem, set  $X = \{C_1, C_2, C_3, C_4\}$  to the space of criteria, the ideal value can be defined as

$$S_0 = \sum_{i=1}^4 \frac{\langle 1, 0, 0 \rangle}{C_i}, C_i \in X$$

When the four possible alternatives with respect to the above four criteria are evaluated by the similar method from the expert, the following single valued neutrosophic decision matrix  $E$  is constructed as follows

$$\begin{pmatrix} \{0.4, 0.2, 0.3\} & \{0.5, 0.1, 0.4\} & \{0.7, 0.1, 0.2\} & \{0.3, 0.2, 0.1\} \\ \{0.4, 0.2, 0.3\} & \{0.3, 0.2, 0.4\} & \{0.9, 0.0, 0.1\} & \{0.5, 0.3, 0.2\} \\ \{0.4, 0.3, 0.1\} & \{0.5, 0.1, 0.3\} & \{0.5, 0.0, 0.4\} & \{0.6, 0.2, 0.2\} \\ \{0.6, 0.1, 0.2\} & \{0.2, 0.2, 0.5\} & \{0.4, 0.3, 0.2\} & \{0.7, 0.2, 0.1\} \end{pmatrix}$$

The entries of  $E(\text{ent}_{i,j} E)$  represent the expert's opinion about an alternative  $S_i$  with respect to the criterion  $C_j$ . Then, the proposed method is used to decide the best supplier in four steps. Step 1. Convert SVNSSs into BPAs. According to Eq.(16), three matrices can be written as follows

$$\begin{aligned} E_1 &= \begin{pmatrix} (0.4, 0.6) & (0.5, 0.5) & (0.7, 0.3) & (0.3, 0.7) \\ (0.4, 0.6) & (0.3, 0.7) & (0.9, 0.1) & (0.5, 0.5) \\ (0.4, 0.6) & (0.5, 0.5) & (0.5, 0.5) & (0.6, 0.4) \\ (0.6, 0.4) & (0.2, 0.8) & (0.4, 0.6) & (0.7, 0.3) \end{pmatrix} \\ E_2 &= \begin{pmatrix} (0.2, 0.8) & (0.1, 0.9) & (0.1, 0.9) & (0.2, 0.8) \\ (0.2, 0.8) & (0.2, 0.8) & (0.0, 1.0) & (0.3, 0.7) \\ (0.3, 0.7) & (0.1, 0.9) & (0.0, 1.0) & (0.2, 0.8) \\ (0.1, 0.9) & (0.2, 0.8) & (0.3, 0.7) & (0.2, 0.8) \end{pmatrix} \\ E_3 &= \begin{pmatrix} (0.3, 0.7) & (0.4, 0.6) & (0.2, 0.8) & (0.1, 0.9) \\ (0.3, 0.7) & (0.4, 0.6) & (0.1, 0.9) & (0.2, 0.8) \\ (0.1, 0.9) & (0.3, 0.7) & (0.4, 0.6) & (0.2, 0.8) \\ (0.2, 0.8) & (0.5, 0.5) & (0.2, 0.8) & (0.1, 0.9) \end{pmatrix} \end{aligned}$$

where  $\text{ent}_{i,j} E_1, \text{ent}_{i,j} E_2, \text{ent}_{i,j} E_3$  are BPAs constructed by  $S_i$  with respect to criterion  $C_j$  according to E-

q.(16). For  $S_0$ , the matrices can be written as

$$\begin{aligned} E_1 &= \begin{pmatrix} (1.0, 0.0) & (1.0, 0.0) & (1.0, 0.0) & (1.0, 0.0) \\ (1.0, 0.0) & (1.0, 0.0) & (1.0, 0.0) & (1.0, 0.0) \\ (1.0, 0.0) & (1.0, 0.0) & (1.0, 0.0) & (1.0, 0.0) \\ (1.0, 0.0) & (1.0, 0.0) & (1.0, 0.0) & (1.0, 0.0) \end{pmatrix} \\ E_2 &= \begin{pmatrix} (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \\ (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \\ (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \\ (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \end{pmatrix} \\ E_3 &= \begin{pmatrix} (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \\ (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \\ (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \\ (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) & (0.0, 1.0) \end{pmatrix} \end{aligned}$$

Step 2. Calculate the components of distance  $d_1, d_2, d_3$  between each  $S_i$  and  $S_0$ . They can be formulated with the form of matrix. These matrices is calculated and presented below

$$\begin{aligned} D'_1 &= \sqrt{\frac{2}{3}} \begin{pmatrix} 0.6 & 0.5 & 0.3 & 0.7 \\ 0.6 & 0.7 & 0.1 & 0.5 \\ 0.6 & 0.5 & 0.5 & 0.4 \\ 0.4 & 0.8 & 0.6 & 0.3 \end{pmatrix} \\ D'_2 &= \sqrt{\frac{2}{3}} \begin{pmatrix} 0.2 & 0.1 & 0.1 & 0.2 \\ 0.2 & 0.2 & 0.0 & 0.3 \\ 0.3 & 0.1 & 0.0 & 0.2 \\ 0.1 & 0.2 & 0.3 & 0.2 \end{pmatrix} \\ D'_3 &= \sqrt{\frac{2}{3}} \begin{pmatrix} 0.3 & 0.4 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.2 & 0.1 \end{pmatrix} \end{aligned}$$

Step 3. Calculate  $d_{0i}(x)$ . Similar to Step 2, they can be formulated as a matrix. The matrix is calculated and presented below

$$D' = D'_1 + D'_2 + D'_3 = \sqrt{\frac{2}{3}} \begin{pmatrix} 1.1 & 1.0 & 0.6 & 1.0 \\ 1.1 & 1.3 & 0.2 & 1.0 \\ 1.0 & 0.9 & 0.9 & 0.8 \\ 0.7 & 1.5 & 1.1 & 0.6 \end{pmatrix}$$

Step 4. Calculate the distance between  $S_i$  and  $S_0$ . According to Eq.(23), the distance between  $S_i$  and  $S_0$  can

be formulated as a column vector as follows

$$\begin{aligned} \mathbf{d} &= \mathbf{D}'\boldsymbol{\omega} \\ &= \sqrt{\frac{2}{3}} \begin{pmatrix} 1.1 & 1.0 & 0.6 & 1.0 \\ 1.1 & 1.3 & 0.2 & 1.0 \\ 1.0 & 0.9 & 0.9 & 0.8 \\ 0.7 & 1.5 & 1.1 & 0.6 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.30 \\ 0.20 \\ 0.25 \end{pmatrix} \\ &= \sqrt{\frac{2}{3}} \begin{pmatrix} 0.945 \\ 0.955 \\ 0.860 \\ 0.995 \end{pmatrix} \approx \begin{pmatrix} 0.7716 \\ 0.7798 \\ 0.7349 \\ 0.8124 \end{pmatrix} \end{aligned}$$

The best supplier of  $S_i$  is defined as the one closest to the  $S_0$ . According to the vector, the rank order of four suppliers is  $S_3 \succ S_1 \succ S_2 \succ S_4$ . Here, the symbol “ $\succ$ ” represents the former supplier is better than the latter one. Hence, the best alternative is  $S_3$ .

## 5. Conclusion

Neutrosophic set has been paid great attention recent years due to its flexibility to handle uncertain information. It's important to measure the distance between single valued neutrosophic set in some uncertainty decision making situations. In this paper, an evidence distance between two SVNS is presented. Based on a new transformation of the SVNS into BPA, the distance is measured from the aspect of evidence theory. The application in MCDM shows the efficiency of the proposed method.

## Acknowledgment

The work is partially supported by National Natural Science Foundation of China (Grant Nos. 61573290, 61503237, 61402546).

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