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Minus Ordering on Fuzzy Neutrosophic Soft Matrices

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Abstract

Aim of this article is to study the idea of minus-ordering on fuzzy neutrosophic soft matrix. It is shown that the minus ordering in the set of all fuzzy neutrosophic soft matrix is a partial ordering. Further some properties of minus ordering on fuzzy neutrosophic soft matrix are discussed.

Keywords: Fuzzy Neutrosophic Soft Set, Fuzzy Neutrosophic Soft Matrix, Fuzzy Neutrosophic Soft Minus Ordering.

1 Introduction

The theory of Fuzzy set was presented by Zadeh [20] in 1965. Intuitionistic fuzzy set introduced by Atanassov [3] can managed the incomplete information considering both the truth membership and falsity membership values. It does not deal the indeterminate and inconsistent information which can be in faith system. The percept of Neutrosophic set was started by Smarandache[16]. Neutrosophic sets and logic are the foundations for many theories which are general than their classical counterparts in fuzzy, intuitionistic fuzzy, paraconsistent sets, dialetheist sets, paradoxist set and

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tautological set. The Neutrosophic set which is handling the problems as T-truth value, I-indeterminate value and F-falsity value. Thus T, I, F be a real standard(or) non-standard subset of $]^{-}0,\ 1^{+}[$ with
$Sup\ \ T = t\_sup,\ \ \ inf\ \ \ T = t\_inf,$
$Sup\ \ \ I = i\_sup,\ \ \ inf\ \ \ I = i\_inf,$
$Sup\ \ \ F = f\_sup,\ \ \ inf\ \ \ F = f\_inf.$ and $n\_sup = t\_sup + i\_sup + f\_sup,$
$n\_inf = t\_inf + i\_inf + f\_inf.$ T, I, F are called neutrosophic components. The superior sums and inferior sums.

$$\begin{gathered} n_{sup} = sup\quad T + sup\quad I + sup\quad F \in\ ]^{-}0,\ 3^{+}[, \\ \text{may be as high as 3 or } 3^{+} \text{ while,} \\ n_{inf} = inf\quad T + inf\quad I + inf\quad F \in\ ]^{-}0, 3^{+}[, \\ \text{may be as low as 0 or } 0^{-}. \end{gathered}$$

The concept of soft set theory was formulated at first by Molodtsov[7] in 1999. In Maji et al., [8] initiated the idea of fuzzy soft sets with the operations of union, intersection, complement of fuzzy soft set. The fuzzy soft set is extended into intuitionistic fuzzy soft set and fuzzy Neutrosophic soft set. In [19]Yong Yang and Chenli J: put forth a matrix representation of fuzzy soft set and used it in decision making problems. Rajarajeswari and Dhanalakshmi [14] innovated the intuitionistic fuzzy soft matrices and started to use in the application of medical diagnosis. Sumathi and Arockiarani [1, 2] displayed new operation on fuzzy Neutrosophic soft matrices. In [11] P. Murugadas introduced maximum g-inverse as well as minimum g-inverse of fuzzy matrix and intuitionistic fuzzy matrices. Meenakshi.AR and Inbam [10] explicate the minus ordering for fuzzy matrices and established that the minus ordering is a partial ordering in the set of all regular fuzzy matrices. Susanta, k.khan and anita pal [17] brought in the theory of generalized inverse for intuitionistic fuzzy matrices. In [18] R.Uma et al., introduced the concept of fuzzy neutrosophic soft matrices of Type-1 and Type-2. The section-2 recalls some basic definition. In section-3 introduced Minus ordering of Fuzzy neutrosophic soft matrices (FNSMs).

Notation:

$\mathcal{N}$ -denotes by the set of all fuzzy Neutrosophic Soft Matrices(FNSMs).
$\mathcal{N}_{mn}$-The set of all FNSMs of order $m \times n$.
$\mathcal{N}_{nn}$-The set of all FNSMs of order $n \times n$.
$\mathcal{N}_{mn}^{+} = \{A \in \mathcal{N}_{mn} | A$ has Moore-Penrose Inverse $\}$.
$\mathcal{N}_{mn}^{-} = \{A \in \mathcal{N}_{mn} | A$ has Generalized-inverse$\}$.

2 Preliminaries

Definition 2.1. *[16]* A neutrosophic set A on the universe of discourse X is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$, where $T, I, F : X \to \ ]^{-}0, 1^{+}[$ and $^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$......(1). From philosophical point of view the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. But in real life application especially in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^{-}0, 1^{+}[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0, 1]$. Therefore we can rewrite equation (1) as $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$. In short an element $\widetilde{a}$ in the neutrosophic set A, can be written as $\widetilde{a} = \langle a^T, a^I, a^F\rangle$, where a^T denotes degree of truth, a^I denotes degree of indeterminacy, a^F denotes degree of falsity such that $0 \leq a^T + a^I + a^F \leq 3$

Example 2.2. Assume that the universe of discourse $X = \{x_1,\ x_2,\ x_3\}$ where $x_1,\ x_2$ and x_3 characterize the quality, reliability, and the price of the objects. It may be further assumed that the values of $\{x_1,\ x_2,\ x_3\}$ are in [0,1] and they are obtained from some investigations of some experts. The experts may impose their opinion in three components viz; the degree of goodness, the degree of indeterminacy and the degree of poorness to explain the characteristics of the objects. Suppose A is a Neutrosophic Set (NS) of X, such that
$A = \{\langle x_1,\ 0.4,\ 0.5,\ 0.3\rangle \langle x_2,\ 0.7,\ 0.2,\ 0.4\rangle, \langle x_3, 0.8,\ 0.3,\ 0.4\rangle\}$ where for x_1 the degree of goodness of quality is 0.4, degree of indeterminacy of quality is 0.5 and degree of falsity of quality is 0.3 etc,. .

Definition 2.3. Let U be the initial universe set and E be a set of parameter. Consider a non-empty set $A, A \subset E$. Let $P(U)$ denote the set of all fuzzy neutrosophic sets of U. The collection (F, A) is termed to be the fuzzy neutrosophic soft set (FNSS) over U, where F is a mapping given by $F : A \to \ P(U)$. Here after we simply consider A as FNSS over U instead of (F, A).

Definition 2.4. *[1]* Let $U = \{c_1,\ c_2,\ ...c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1,\ e_2,\ ...e_m\}$. Let $A \subset E$. A pair (F, A) be a $FNSS$ over U. Then the subset of $U \times E$ is defined by $R_A = \{(u, e);\ e \in A,\ u \in F_A(e)\}$ which is called a relation form of $(F_A\ E)$. The membership function, indeterminacy membership function and non membership function are written by
$T_{R_A} : U \times E \to [0, 1]$, $I_{R_A} : U \times E \to [0, 1]$ and $F_{R_A} : U \times E \to [0, 1]$ where $T_{R_A}(u, e) \in [0, 1]$, $I_{R_A}(u, e) \in [0, 1]$ and $F_{R_A}(u, e) \in [0, 1]$ are the membership value, indeterminacy value and non membership value respectively of $u \in U$ for each $e \in E$.

If $[(T_{ij},\ I_{ij},\ F_{ij})] = [T_{ij}(u_i, e_j),\ I_{ij}(u_i, e_j),\ F_{ij}(u_i, e_j)]$ we define a matrix

$$[\langle T_{ij}\ , I_{ij},\ F_{ij}\rangle]_{m\times n} = \begin{bmatrix} \langle T_{11},\ I_{11},\ F_{11}\rangle & \cdots & \langle T_{1n},\ I_{1n},\ F_{1n}\rangle \\ \langle T_{21},\ I_{21},\ F_{21}\rangle & \cdots & \langle T_{2n},\ I_{2n},\ F_{2n}\rangle \\ \vdots & \vdots & \vdots \\ \langle T_{m1},\ I_{m1},\ F_{m1}\rangle & \cdots & \langle T_{mn},\ I_{mn},\ F_{mn}\rangle \end{bmatrix}$$

Which is called an $m \times n$ $FNSM$ of the FNSS (F_A, E) over U.

FNSMs OF Type-I [18]

Definition 2.5. Let $A = (\langle a^T_{ij}, a^I_{ij}, a^F_{ij}\rangle),\ B = \langle (b^T_{ij}, b^I_{ij}, b^F_{ij}\rangle) \in \mathcal{N}_{m\times n}$. The component wise addition and component wise multiplication is defined as
$A \oplus B = (sup\{a^T_{ij}, b^T_{ij}\},\quad sup\{a^I_{ij}, b^I_{ij}\},\quad inf\{a^F_{ij}, b^F_{ij}\})$
$A \odot B = (inf\{a^T_{ij}, b^T_{ij}\},\quad inf\{a^I_{ij}, b^I_{ij}\},\quad sup\{a^F_{ij}, b^F_{ij}\})$.

Definition 2.6. Let $A \in F_{m\times n},\ B \in \mathcal{N}_{m\times p}$, the composition of A and B is defined as

$$A \circ B = \left(\sum_{k=1}^{n}(a^T_{ik} \wedge\ b^T_{kj}),\quad \sum_{k=1}^{n}(a^I_{ik} \wedge\ b^I_{kj}),\quad \prod_{k=1}^{n}(a^F_{ik} \vee b^F_{kj})\right)$$

equivalently we can write the same as

$$= \left(\bigvee_{k=1}^{n}(a^T_{ik} \wedge b^T_{kj}),\quad \bigvee_{k=1}^{n}(a^I_{ik} \wedge b^I_{kj}),\quad \bigwedge_{k=1}^{n}(a^F_{ik} \vee b^F_{kj})\right).$$

The product $A \circ B$ is defined if and only if the number of columns of A is same as the number of rows of B. A and B are said to be conformable for multiplication. We shall use AB instead of $A \circ B$.
Where $\sum(a^T_{ik} \wedge b^T_{kj})$ means max-min operation and
$\prod(a^F_{ik} \vee b^F_{kj})$ means min-max operation.

3 Minus Ordering of Fuzzy Neutrosophic Soft Matrices

Definition 3.1. If two FNSMs A and X of order $m \times n$ and $n \times m$ respectively satisfies the following $A\{1\} = \{X \in \mathcal{N}_{nm}|\ AXA = A\}$,
$A\{2\} = \{X \in \mathcal{N}_{nm}|\ XAX = X\}$, $A\{3\} = \{X \in \mathcal{N}_{nm}|\ (AX)^t = AX\}$ and
$A\{4\} = \{X \in \mathcal{N}_{nm}|(XA)^t = XA\}$, then $A\{1, 2, 3, 4\}$ is called Moore-Penrose inverse of A which is denoted by A^+, t- denote the transpose.

Definition 3.2. Let $A \in \mathcal{N}_{mn}$ and $X \in \mathcal{N}_{nm}$ satisfying $AXA = A$, then A has a g-inverse. The g-inverse of A is denoted as A^- and $A\{1\}$ is the set of all g-inverse of A.

Definition 3.3. Let $A,\ B \in \mathcal{N}_{mn}$. The T-ordering $A \overset{T}{\leq} B$ is defined as
$A \overset{T}{\leq} B \Leftrightarrow A^tA = A^tB$ and $AA^t = BA^t$.

Definition 3.4. The row space $\mathcal{R}(A)$ is the subspace of the set of all FNSMs of order $m \times n$, generated by the rows of A. Similarly, column space of A is denoted by $\mathcal{C}(A)$ and is generated by the columns of A.

Definition 3.5. For $A \in \mathcal{N}_{mn}^-$ and $B \in \mathcal{N}_{mn}$, the minus ordering denoted as $\bar{<}$ is defined as $A\bar{<}B \Leftrightarrow A^-A = A^-B$ and $AA^- = BA^-$ for some $A^- \in A\{1\}$.
To specify the minus ordering with respect to a particular g-inverse of A let us write $A\bar{<}\ B$ with respect to $X \Leftrightarrow XA = XB$ and $AX = BX$ for $X \in A\{1\}$.

Example 3.6. $A = \begin{pmatrix} \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \\ \langle 0.5 & 0.5 & 0.5\rangle & \langle 0 & 0 & 1\rangle \end{pmatrix}$, $B = \begin{pmatrix} \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \\ \langle 1 & 1 & 0\rangle & \langle 0 & 0 & 1\rangle \end{pmatrix}$
$A^- = \begin{pmatrix} \langle 0 & 0 & 1\rangle & \langle 0.5 & 0.5 & 0.5\rangle \\ \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \end{pmatrix}$, where A^- is the g-inverse of A.
and $A^-A = A^-B$ and $AA^- = BA^-$.
Therefore $A\bar{<}B$.

Remark 3.7. For $A \in \mathcal{N}_{mn}^-$ and $B \in \mathcal{N}_{mn}$, if A^+ exists, then A^+ is unique and $A^+ = A^t$. We have, A^t
$A \overset{T}{\leq} B \Leftrightarrow A \bar{<} B$ with respect to $A^+ \Leftrightarrow A^tA = A^tB$ and $AA^t = BA^t$, which is precisely the Definition 3.3 of T- ordering. Thus "T-ordering" is a special case of minus ordering. However, the converse $A \bar{<} B \Rightarrow A \overset{T}{\leq} B$ need not be true. This is illustrated in the following example.

Example 3.8. $A = \begin{pmatrix} \langle 0 & 0 & 1\rangle & \langle 0 & 0 & 1\rangle \\ \langle 1 & 1 & 0\rangle & \langle 0 & 0 & 1\rangle \end{pmatrix}$, $B = \begin{pmatrix} \langle 1 & 1 & 0\rangle & \langle 0 & 0 & 1\rangle \\ \langle 0 & 0 & 1\rangle & \langle 1 & 1 & 0\rangle \end{pmatrix}$.
Since A^t is a g-inverse of A, A^+ exists and $A^+ = A^t$, also A is
idempotent, A itself is a g-inverse of A, $A = AB = BA \Rightarrow A \bar{<} B$ with respect to A. But $A^tA \neq A^tB$ and $AA^t \neq BA^t$. Hence $A \bar{<} B \nRightarrow A \overset{T}{\leq} B$.

Lemma 3.9. For $A \in \mathcal{N}_{mn}^-$ and $B \in \mathcal{N}_{mn}$, the following are equivalent:
(i). $A \bar{<} B$,
(ii). $A = AA^-B = BA^-A = BA^-B$.

Proof:
(i)⇒ (ii): $A \bar{<} B \Rightarrow AA^- = BA^-$ and $A^-A = A^-B$
for some $A^- \in A\{1\}$.
Now, $A = A(A^-A) = AA^-B$ $A = (AA^-)A = BA^-A$ $A = B(A^-A) = BA^-B$.
(ii)⇒(i):
Let $X = A^-AA^-$ $AXA = A(A^-AA^-)A = (AA^-A)A^-A = A \Rightarrow X \in A\{1\}$.
Now $XA = (A^-AA^-)AA^-B = A^-(AA^-A)A^-B = (A^-AA^-)B = XB$.
Similarly, $AX = BX$.
Hence $A \bar{<} B$ with respect to $X \in A\{1\}$.

Remark 3.10. In general, in the definition of minus ordering $A \bar{<} B$, B need not be regular. This is illustrated in the following example.

Example 3.11. $A = \begin{pmatrix} \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \\ \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \end{pmatrix}$, $B = \begin{pmatrix} \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \\ \langle 0 & 0 & 1\rangle & \langle 1 & 1 & 0\rangle \end{pmatrix}$
Since A is idempotent, A is a regular and A itself is g-inverse of A. Therefore $AXA = A$, then $A = X$. Hence $A \bar{<} B$ with respect to $A = X \in A\{1\}$.
But B is not regular. Therefore $BXB \neq B$.

Theorem 3.12. Let $A,\ B \in \mathcal{N}_{mn}^-$. If $A \bar{<} B$, then $B\{1\} \subseteq A\{1\}$.
Proof: $A \bar{<} B \Rightarrow A = AA^-B = BA^-A$. (by Lemma 3.9).
For $B^- \in B\{1\}$,

$$\begin{aligned} AB^-A &= (AA^-B)B^-(BA^-A) \\ &= AA^-(BB^-B)AA^- \\ &= (AA^-B)A^-A = AA^-A = A. \end{aligned}$$

Hence $AB^-A = A$ for each $B^- \in B\{1\}$. Therefore, $B\{1\} \subseteq A\{1\}$.

Remark 3.13. For complex matrices, the converse of the above Theorem 3.12 hold and this need not be the case for fuzzy neutrosophic soft matrices. This is illustrated in the following example.

Example 3.14. $A = \begin{pmatrix} \langle 0 & 0 & 1\rangle & \langle 0 & 0 & 1\rangle \\ \langle 0.5 & 0.5 & 0.5\rangle & \langle 0 & 0 & 1\rangle \end{pmatrix}$, $B = \begin{pmatrix} \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \\ \langle 0.5 & 0.5 & 0.5\rangle & \langle 0 & 0 & 1\rangle \end{pmatrix}$.
The g-inverse of A *and* B are computed as
$A\{1\} = \{X | X = \begin{pmatrix} \langle a^T & a^I & a^F\rangle & \langle b^T & b^I & b^F\rangle \\ \langle c^T & c^I & c^F\rangle & \langle d^T & d^I & d^F\rangle \end{pmatrix}$ }

$$0 \leq a^T,\ a^I \leq 1,\ a^F \in [0.1]$$

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$$0 \leq b^T,\ b^I \leq 1,\ b^F \in [0,1]$$
$$0.5 \leq c^T,\ c^I \leq 1,\ c^F \in [0,0.5]$$
$$0 \leq d^T,\ d^I \leq 0.5,\ d^F \in [0,1].$$

We have
$B\{1\} = \{X|X = \begin{pmatrix} \langle a^T & a^I & a^F\rangle & \langle b^T & b^I & b^F\rangle \\ \langle c^T & c^I & c^F\rangle & \langle d^T & d^I & d^F\rangle \end{pmatrix}\}$

$$0 \leq b^T,\ b^I,\ b^F \leq 1$$
$$0.5 \leq c^T,\ c^I \leq 1,\ c^F \in (0,0.5)$$
$$0 \leq d^T,\ d^I,\ d^F \leq 1.$$

But there is no $A^- \in A\{1\}$ satisfying $AA^- = BA^-$ and $A^-A = A^-B$.
Hence $A \not\lesssim B$.

Corollary 3.15. If $A \lesssim B$ with B idempotent, then $B \in A\{1\}$.
Proof: Since B is idempotent, B is regular and B itself is a g-inverse of B. Hence $B \in B\{1\}$, by the Theorem 3.12, $B\{1\} \subseteq A\{1\}$. Hence $B \in A\{1\}$.

Remark 3.16. In the above corollary (3.15), the condition on B to be idempotent is essential for $B \in A\{1\}$. This is illustrated in the following example

Example 3.17. *Let* $A = \begin{pmatrix} \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \\ \langle 0.5 & 0.5 & 0.5\rangle & \langle 0 & 0 & 1\rangle \end{pmatrix}$, *and* $B = \begin{pmatrix} \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \\ \langle 1 & 1 & 0\rangle & \langle 0 & 0 & 1\rangle \end{pmatrix}$

B is not idempotent.

$$A\{1\} = \begin{cases} X = \begin{pmatrix} \langle 0 & 0 & 1\rangle & \langle \beta^t & \beta^I & \beta^F\rangle \\ \langle \gamma^T & \gamma^I & \gamma^F\rangle & \langle \alpha^T & \alpha^I & \alpha^F\rangle \end{pmatrix}, 0 \leq \beta^T,\ \beta^I,\ \beta^F \leq 1 \quad and \\ 0.5 \leq \gamma^T,\ \gamma^I,\ \gamma^F \leq 1 \quad and \quad 0 \leq \alpha^T,\ \alpha^I,\ \alpha^F \leq 1. \end{cases}$$

Here $A \lesssim B$ *for* $A^- = \begin{pmatrix} \langle 0 & 0 & 1\rangle & \langle 1 & 1 & 0\rangle \\ \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \end{pmatrix}$
but $B \notin A\{1\}$.

Theorem 3.18. For $A \in \mathcal{N}_{mn}^-$ and $B \in \mathcal{N}_{mn}$ with $A \lesssim B$, we have (i). If $B = B^2$, then $A = A^2$. (ii). If $B^2 = 0$, then $A^2 = 0$.
Proof:

$$\begin{aligned} (i).A^2 = A.\quad A &= (AA^-B)(BA^-A) \qquad (by\ \ Lemma\ 3.9) \\ &= AA^-B^2A^-A \\ &= (AA^-B)A^-A \qquad (by\ \ B = B^2) \\ &= AA^-A \\ &= A. \end{aligned}$$

(ii). $A^2 = AA = (AA^- B)(BA^- A) = AA^-(B^2)A^- A = 0$.

Remark 3.19. In the above Theorem 3.18, if $A \bar{<} B$ with A idempotent then B need not be idempotent. Consider

Example 3.20. $A = \begin{pmatrix} \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \\ \langle 1 & 1 & 0\rangle & \langle 1 & 1 & 0\rangle \end{pmatrix}$, $B = \begin{pmatrix} \langle 0 & 0 & 1\rangle & \langle 1 & 1 & 0\rangle \\ \langle 1 & 1 & 0\rangle & \langle 0 & 0 & 1\rangle \end{pmatrix}$.
Here $B^2 \neq B$, but $A^2 = A$ and $A \bar{<} B$.

Theorem 3.21. For $A,\ B \in \mathcal{N}^+_{m,n}$ the following conditions are equivalent:
i. $A \bar{<} B$ with respect to $A^+ (A \overset{T}{<} B)$.
ii. $B \in A^+\{1,3,4\}$.
iii. $B^+ \in A\{1,3,4\}$.
Proof
(i)⇒(ii) $A \bar{<} B$ with respect to $A^+ \Rightarrow A^+A = A^+B$ and $AA^+ = BA^+$.
Now $A^+ = A^+AA^+ = A^+BA^+ \Rightarrow B \in A^+\{1\}$.
$(A^+B)^T = (A^+A)^T = A^+A = A^+B \Rightarrow B \in A^+\{3\}$.
$(BA^+)^T = (AA^+)^T = AA^+ = BA^+ \Rightarrow B \in A^+\{4\}$.
(ii)⇒(iii): Since $A^+ = A^T$ and $B^+ = B^T$, we have
$B \in A^+\{1,3,4\} \Rightarrow B^+ \in A\{1,3,4\}$.

$$\begin{aligned} (iii) \Rightarrow (i) \quad B^+ \in A\{1,3,4\} \Rightarrow \\ AB^+A = A,\ (AB^+)^T &= AB^+ \\ &= BA^+ \\ \text{and} \quad (B^+A)^T &= B^+A \\ &= A^+B. \end{aligned}$$

$$\begin{aligned} A^+A &= A^+A(B^+A) \\ &= (A^+AA^+)B \\ &= A^+B. \end{aligned}$$

$AA^+ = (AB^+A)A^+ = (AB^+)AA^+ = BA^+AA^+ = BA^+$.
Hence $A \bar{<} B$ with respect to A^+.

Theorem 3.22. Let $A \in (\mathcal{N})^+_{mn}$. Then
$H \bar{<} A^+ \bar{<} G$ for every $G \in A\{1,3,4\}$ and every $H \in A\{2,3,4\}$.
Proof.
$H \in A\{2,3,4\} \Rightarrow H\{1,3,4\} \neq \Phi \Rightarrow H^+$ exists.

Also, $HAH = H$, $(HA)^T = HA$ and $(AH)^T = AH$.
Now, $HAH = H \Rightarrow HH^TA^+ = H$
$\Rightarrow (H^+HH^+)A^+ = H^+H$
$\Rightarrow H^+A^+ = H^+H$.
Similarly, we have $A^+H^+ = HH^+$.
Hence $H \bar{<} A^+$ with respect to $H^- = H^+$.
Now $G \in A\{1, 3, 4\} \Rightarrow A^+ = GAG$
$\Rightarrow AA^+ = (AGA)G = AG$.
Also $A^+A = GAGA = G(AGA) = GA$.
Hence $A^+ \bar{<} G$ with respect to A.

4 Conclusion

As minus ordering has close relationship with g-inverse, it has some special role in relational equation. Recently the hybrid structure neutrosophic soft set is widely used in all areas such as Engineering, Medical Science, Decision Making etc,. Thus the study of minus ordering on fuzzy neutrosophic soft matrices has some future.

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