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Monotone Fuzzy Neutrosophic Soft Eigenspace Structure in Max-Min Algebra

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Abstract. Fuzzy Neutrosophic Soft Matrix (FNSM) A in a max-min Fuzzy Neutrosophic Soft Algebra (FNSA), the set of all increasing Fuzzy Neutrosophic Soft Eigenvectors (FNSEvs), in notation $\mathcal{F}^{\leq}(A)$ is studied.

AMS Subject classification: Primary 03E72; Secondary 15B15.

Keywords: Fuzzy Neutrosophic Soft Set (FNSS), Fuzzy Neutrosophic Soft Matrix(FNSM), Fuzzy Neutrosophic Soft Vector(FNSV), Fuzzy Neutrosophic Soft Eigenvector(FNSEv), Fuzzy Neutrosophic Soft monotone Eigenvector(FNSMEv).

INTRODUCTION

In 1999, Molodtsov [29], initiated the novel concept of soft set theory, which was a completely new approach for modeling uncertainty. The fuzzy set was introduced by Zadeh [41] in 1965 where each element had a degree of membership. The intuitionistic fuzzy set (IFS) was introduced by Atanaasov [2] in 1983 as a generalization of fuzzy set. Systematic investigation in this direction can be found in [4, 10, 14, 40]. Problems analogous to the problems known in linear algebra, like independent of vectors, regularity of matrices, solvability and unique solvability of system of linear equation, finding eigenvectors, eigenvalues, were studied in many subsequent paper.

Kim and Roush [23] introduced the concept of Fuzzy Matrix(FM). FM plays a vital role in various areas in Science and Engendering and solves the problems involving various types of uncertainties [32].

Yang and Ji [38], introduced a matrix representation of fuzzy set and applied it in decision making problems. Bora et.al, [6] introduced the intuitionistic fuzzy soft matrices and applied in the application of a Medical diagnosis. Sumathi and Arokiarani [1] introduced new operation on fuzzy neutrosophic soft matrices. Dhar et.al, [15] have also defined neutrosophic fuzzy matrices and studied square neutrosophic fuzzy matrices. Kavitha et.al, [24] studied the concepts of minimal solution of fuzzy neutrosophic soft matrix. They, also studied on the powers of fuzzy neutrosophic soft matrices in [27]. Uma et.al, [37] introduced two types of fuzzy neutrosophic soft matrices. In this paper we deal with max-min FNSA which have wide applications in the fuzzy neutrosophic soft set theory (the max-min FNSA on the unit real interval is one of most important FNSAs). Questions connected with the solvability and the unique solvability of linear systems in max-min FNSA were studied in [5, 3, 13]. The results were completed for general max-min FNSA in [17, 18]. The FNSEvs of a max-min FNSM can be useful in cluster analysis (see [19]) or in fuzzy reasoning. A procedure for computing the maximal FNSEv of a given max-min FNSM was proposed in [34] and an efficient algorithm was described later in [12]. FNSEvs of max-min FNSMs and their connection with paths in digraphs were investigated in [11, 19, 20, 21]. The eigenproblem in distributive lattices was studied in [34].

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PRELIMINARIES

For the basic definitions and examples of Neutrosophic Set (NS), FNSS, FNSM and fuzzy neutrosophic soft matrices of type-I refer [24-27].

NOTIONS AND NOTATION

In this section we discuss about fuzzy neutrosophic soft eigenspace and notation of increasing FNSEvs, strictly increasing FNSEvs.

By a max-min FNSA $\mathcal{N}$, we mean any linearly ordered set $(\mathcal{N}, \leq)$ with the binary operations of maximum and minimum, denoted by $\oplus$ and $\otimes$. In general, $\mathcal{N}$ need not be bounded. We shall denote by $\mathcal{N}^*$ the bounded algebra derived from $\mathcal{N}$ by adding the least element, or the greatest element (or both), if necessary. If $\mathcal{N}$ itself is bounded, then $\mathcal{N} = \mathcal{N}^*$. The least element in $\mathcal{N}^*$ will be **0** denoted $\langle 0, 0, 1\rangle$, the greatest one **1** denoted by $\langle 1, 1, 0\rangle$ To avoid the trivial case, we assume $0 < I$.

For any natural $n > 0$, $\mathcal{N}_{(n)}$ denotes the set of all n-dimensional columns FNSVs over $\mathcal{N}$, and $\mathcal{N}_{(m,n)}$ denotes the set of all FNSMs of type $m \times n$ over $\mathcal{N}$. We say that a FNSV $\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)}$ is increasing, if $\langle b^T_i, b^I_i, b^F_i\rangle \leq \langle b^T_j, b^I_j, b^F_j\rangle$ holds for any $i, j \in N, i \leq j$. FNSV $\langle b^T, b^I, b^F\rangle$ is strictly increasing, if $\langle b^T_i, b^I_i, b^F_i\rangle < \langle b^T_j, b^I_j, b^F_j\rangle$ whenever $i < j$. The set of all increasing (strictly increasing) FNSVs in $\mathcal{N}_{(n)}$ denoted by $\mathcal{N}^{\leq}_{(n)}$ (by $\mathcal{N}^{<}_{(n)}$). For $\langle x^T, x^I, x^F\rangle, \langle y^T, y^I, y^F\rangle \in \mathcal{N}_{(n)}$, we write $\langle x^T, x^I, x^F\rangle \leq \langle y^T, y^I, y^F\rangle$, if $\langle x^T_i, x^I_i, x^F_i\rangle \leq \langle y^T_i, y^I_i, y^F_i\rangle$ holds for all $i \in N$, and we write $\langle x^T, x^I, x^F\rangle < \langle y^T, y^I, y^F\rangle$, if $\langle x^T, x^I, x^F\rangle \leq \langle y^T, y^I, y^F\rangle$ and $\langle x^T, x^I, x^F\rangle \neq \langle y^T, y^I, y^F\rangle$. The FNSM operations over $\mathcal{N}$ are defined with respect to $\oplus, \otimes$ formally in the same manner as FNSM operations over any field. For a given natural $n > 0$, we use the notation $N = \{1, 2, ..., n\}$.

The set of all permutations on N will be denoted by P_n. Let $A \in \mathcal{N}_{(n,n)}$ and $\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)}$. For $\varphi, \psi \in P_n$, we denote by $A_{\varphi\psi}$ the FNSM created by applying permutation φ to the rows and permutation ψ to the columns of A, and by b_φ we denote the FNSV created by applying permutation φ to $FNSV\langle b^T, b^I, b^F\rangle$

For any square FNSM $A \in \mathcal{N}_{(n,n)}$, the FNSE space of A is defined by

$$\mathcal{F}(A) := \{\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)};\ A \otimes \langle b^T, b^I, b^F\rangle = \langle b^T, b^I, b^F\rangle\}.$$

The FNSVs in $\mathcal{F}(A)$ are called FNSEvs of FNSM A. The set of all increasing FNSEvs is denoted by $\mathcal{F}^{\leq}(A)$, and the set of all strictly increasing FNSEVs of A is denoted by $\mathcal{F}^{<}(A)$.

INTERVALS OF MONOTONE FUZZY NEUTROSOPHIC SOFT EIGGENVECTORS

In this section we discuss for any FNSV $\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)}$ can be permuted to an increasing FNSV, by a suitable permutation. Therefore, in view of Theorem 4.1, the structure of the FNSE space $\mathcal{F}(A)$ of a give $n\times n$ max-min FNSM A can be described by investigating the structure of monotone FNSE space $\mathcal{F}^{\leq}(A)$.

Theorem 4.1 *Let* $A \in \mathcal{N}_{(n,n)}$, $\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)}$ *and* $\varphi \in P_n$.
Then $\langle b^T, b^I, b^F\rangle \in \mathcal{F}(A)$ *if and only if* $\langle b^T, b^I, b^F\rangle_\varphi \in \mathcal{F}(A_{\varphi\varphi})$.

Proof. *Let* ϵ *be the identical permutation on N. It is easy to see that the following formulas are equivalent:*
$A \otimes \langle b^T, b^I, b^F\rangle = \langle b^T, b^I, b^F\rangle$,
$A_{\varphi\epsilon} \otimes \langle b^T, b^I, b^F\rangle = \langle b^T, b^I, b^F\rangle_\varphi$, $A_{\varphi\varphi} \otimes \langle b^T, b^I, b^F\rangle_\varphi = \langle b^T, b^I, b^F\rangle_\varphi$. *By this, the proof is complete.*

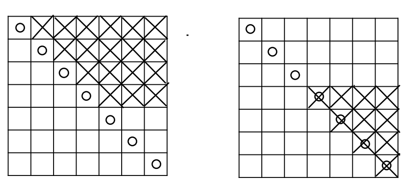

Fig-1

For $A \in \mathcal{N}_{(n,n)}$, *we define FNSVs* $m^*(A)$, $M^*(A) \in \mathcal{N}_{(n)}$ *in the following way as shown in Fig-1. For any* $i \in N$, *we put*

$$m_i^*(A) := \max_{j\le i}\max_{k>j}\langle a_{jk}^T, a_{jk}^I, a_{jk}^F\rangle,\quad M_i^*(A) := \min_{j\ge i}\max_{k\ge j}\langle a_{jk}^T, a_{jk}^I, a_{jk}^F\rangle.$$

In this figure, the FNSM elements used in the above definition are ticked by crosses, the diagonal elements are indicate by circles.

Remark 4.2 If a maximum of an empty set should be computed in the above definition of $m^*(A)$, then we use the fact that, by usual definition, max $\phi = 0$.

Remark 4.3 *The definition of $m^*(A)$ is not new. It was used (in a different notation) for defining trapezoidal FNSMs in [24], and subsequently by other authors.*

Theorem 4.4 Let $A \in \mathcal{N}_{(n,n)}$ and $\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)}$ be a strictly increasing FNSV. Then $\langle b^T, b^I, b^F\rangle \in \mathcal{F}(A)$ if and only if $m^*(A) \le \langle b^T, b^I, b^F\rangle \le M^*(A)$. In formal notation,

$$\mathcal{F}^<(A) = \langle m^*(A),\ M^*(A)\rangle \cap \mathcal{N}^<_{(n)}.$$

Proof. Let use assume first that $\langle b^T, b^I, b^F\rangle \in \mathcal{N}^{\le}{}_{(n)}$ and the inequality $m^*(A) \le \langle b^T, b^I, b^F\rangle \le M^*(A)$ hold true, i.e. $m_i^*(A) \le \langle b_i^T, b_i^I, b_i^F\rangle \le M_i^*(A)$ for every $i \in N$. Let $i \in N$ be a arbitrary, but fixed. For $j < i$,
we have $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle \le \langle b_j^T, b_j^I, b_j^F\rangle \le \langle b_i^T, b_i^I, b_i^F\rangle$, in view of the monotonicity of $\langle b_i^T, b_i^I, b_i^F\rangle$, for $j > i$ we have $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle \le \langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \le m_i^*(A) \le \langle b_i^T, b_i^I, b_i^F\rangle$, and for $j = i$ an obvious inequality $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle = \langle a_{ii}^T, a_{ii}^I, a_{ii}^F\rangle \otimes \langle b_i^T, b_i^I, b_i^F\rangle \le \langle b_i^T, b_i^I, b_i^F\rangle$ holds true. Therefore, $\sum^{\oplus}_{j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle \le \langle b_i^T, b_i^I, b_i^F\rangle$. On other hand we have, in view of the monotonicity, $\sum_{j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle \ge \sum_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle$
$\ge \sum_{j\ge N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_i^T, b_i^I, b_i^F\rangle$
$\ge (\sum_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \otimes \langle b_i^T, b_i^I, b_i^F\rangle$
$= (\max_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \otimes \langle b_i^T, b_i^I, b_i^F\rangle$
$\ge M_i^*(A) \otimes \langle b_i^T, b_i^I, b_i^F\rangle$
$= \langle b_i^T, b_i^I, b_i^F\rangle$. Thus, we have proved that $\sum^{\oplus}_{j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle = \langle b_i^T, b_i^I, b_i^F\rangle$ for arbitrary $i \in N$,, i.e. $\langle b^T, b^I, b^F\rangle \in \mathcal{F}(A)$. We may notice that in the first part of the proof, we have used only the monotonicity of $\langle b^T, b^I, b^F\rangle$, and the strict monotonicity was not assumed.

For the proof of the converse implication, let us suppose that $\langle b^T, b^I, b^F\rangle \in \mathcal{F}^<(A)$, i.e. $\langle b^T, b^I, b^F\rangle$ is strictly increasing and $A \otimes \langle b^T, b^I, b^F\rangle = \langle b^T, b^I, b^F\rangle$. Let $i \in N$ be arbitrary, but fixed. For $j < i$,
we have $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle \le \langle b_j^T, b_j^I, b_j^F\rangle < \langle b_i^T, b_i^I, b_i^F\rangle$ and the equality $\sum_{j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle = \langle b_i^T, b_i^I, b_i^F\rangle$ implies

$$\sum_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle = \langle b_i^T, b_i^I, b_i^F\rangle. \tag{1}$$

Therefore, we have

$$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle \le \langle b_i^T, b_i^I, b_i^F\rangle \text{ for } j \ge i, \tag{2}$$

which implies, in view of the strict monotonicity of $\langle b^T, b^I, b^F\rangle$,

$$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \le \langle b_i^T, b_i^I, b_i^F\rangle \text{ for } j > i, \tag{3}$$

i.e. $\max_{j>i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \le \langle b_i^T, b_i^I, b_i^F\rangle$. As i is arbitrary and $\langle b^T, b^I, b^F\rangle$ increasing, we get similar inequalities
$\max_{k>j}\langle a_{jk}^T, a_{jk}^I, a_{jk}^F\rangle \le \langle b_j^T, b_j^I, b_j^F\rangle \le \langle b_i^T, b_i^I, b_i^F\rangle$ for $j \le i$,

which give $m_i^*(A) \le \langle b_i^T, b_i^I, b_i^F\rangle$. (In fact, a strict inequality $\max_{k>j}\langle a_{jk}^T, a_{jk}^I, a_{jk}^F\rangle \le \langle b_j^T, b_j^I, b_j^F\rangle < \langle b_i^T, b_i^I, b_i^F\rangle$ holds for every $j < i$, since $\langle b^T, b^I, b^F\rangle$ is assumed to be strictly increasing, but we do not use this in our proof.)
Further we have, by (4.1) and by the monotonicity of $\langle b^T, b^I, b^F\rangle$, $\langle b_i^T, b_i^I, b_i^F\rangle = (\sum_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle) \otimes \langle b_i^T, b_i^I, b_i^F\rangle$
$= \sum_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle \otimes \langle b_i^T, b_i^I, b_i^F\rangle$
$= \sum_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_i^T, b_i^I, b_i^F\rangle$
$= (\sum_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \otimes \langle b_i^T, b_i^I, b_i^F\rangle$
$= (\max_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \otimes \langle b_i^T, b_i^I, b_i^F\rangle$ i.e. $\langle b_i^T, b_i^I, b_i^F\rangle \le \max_{j\ge i}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$. Similarly as above, we use the fact that i is arbitrary and $\langle b^T, b^I, b^F\rangle$ is increasing, and we get inequalities

$$\langle b_i^T, b_i^I, b_i^F\rangle \le \langle b_j^T, b_j^I, b_j^F\rangle \le \max_{k\ge j}\langle a_{jk}^T, a_{jk}^I, a_{jk}^F\rangle \text{ for } j \ge i,$$

which imply $\langle b_i^T, b_i^I, b_i^F\rangle \le M_i^*(A)$. (Again we may observe that a strict inequality $\langle b_i^T, b_i^I, b_i^F\rangle < \langle b_j^T, b_j^I, b_j^F\rangle \le \max_{k\ge j}\langle a_{jk}^T, a_{jk}^I, a_{jk}^F\rangle$ holds for every $j > i$.) We have noticed already that the first part of the above proof is valid also for non-strictly increasing FNSV $\langle b^T, b^I, b^F\rangle$. This gives the following theorem.

Theorem 4.5 *Let $A \in \mathcal{N}_{(n,n)}$ and let $\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)}$ be an increasing FNSV. If $m^*(A) \le \langle b^T, b^I, b^F\rangle \le M^*(A)$, then $\langle b^T, b^I, b^F\rangle \in \mathcal{F}(A)$. In formula notation,*

$$\mathcal{F}^{\le}(A) \supseteq \langle m^*(A),\ M^*(A)\rangle \cap \mathcal{N}_{(n)}^{\le}.$$

Remark 4.6 It can be easily seen from the examples in Section-5, that in general, the inclusion sign in Theorem 4.5 cannot be replaced by equality.

In the following theorem, an interval for constant FNSEvs is described. For $A \in \mathcal{N}_{(n,n)}$, we define the value $M(A) \in \mathcal{N}$ as $M(A) := \min_{i\in N}\max_{j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$.

Theorem 4.7 *Let $A \in \mathcal{N}_{(n,n)}$ and let $\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)}$ be a constant FNSV. Then $\langle b^T, b^I, b^F\rangle \in \mathcal{F}(A)$ if and only if $O \le \langle b_1^T, b_1^I, b_1^F\rangle \le M(A)$.*
Proof. *It is easy to verify that, for a constant FNSV $\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)}$ and for any given $i \in N$, the following formulas are equivalent:* $\sum_{j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_j^T, b_j^I, b_j^F\rangle = \langle b_i^T, b_i^I, b_i^F\rangle$,
$\sum_{j\in Ni}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \otimes \langle b_1^T,\ b_1^I, b_1^F\rangle) = \langle b_1^T, b_1^I, b_1^F\rangle$,
$(\sum_{j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \otimes \langle b_1^T, b_1^I, b_1^F\rangle) = \langle b_1^T, b_1^I, b_1^F\rangle$,
$(\max_{j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \otimes \langle b_1^T, b_1^I, b_1^F\rangle) = \langle b_1^T, b_1^I, b_1^F\rangle$,
$\langle b_1^T, b_1^I, b_1^F\rangle \le \max_{j\in N}\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$. *A i is arbitrary, we get $\langle b_1^T, b_1^I, b_1^F\rangle \le M(A)$.*

EXAMPLES

In the case $n = 2$, the theorems from Section 4 can be used describe in detail the structure of the monotone FNSE space $\mathcal{F}^{\le}(A)$ of a given FNSM $A \in \mathcal{N}_{(n,n)}$, and in view of the Theorem 4.1, also the structure of the whole FNSE space. The idea will be demonstrated by three simple examples.
Example: 5.1. Let us consider a closed real interval $\mathcal{N} = (0, 0.8)$, let $n = 2$ and $A \in \mathcal{N}_{(n,n)}$ with

$$A = \begin{bmatrix} \langle 0.7\ \ 0.6\ \ 0.2\rangle & \langle 0.3\ \ 0.4\ \ 0.5\rangle \\ \langle 0.2\ \ 0.3\ \ 0.6\rangle & \langle 0.1\ \ 0.2\ \ 0.7\rangle \end{bmatrix}$$

First, we compute $\mathcal{F}^{<}(A) = \langle m^*(A),\ M^*(A)\rangle \cap \mathcal{N}_{(n)}^{<}$. By definition of $m^*(A),\ M^*(A)$ we get
$m_1^*(A) = \langle 0.3\ \ 0.4\ \ 0.6\rangle$,
$M_1^*(A) = min\{\langle 0.7\ \ 0.6\ \ 0.2\rangle,\ \langle 0.1\ \ 0.2\ \ 0.7\rangle\} = \langle 0.1\ \ 0.2\ \ 0.7\rangle$, $m_2^*(A) = max\{\langle 0.3\ \ 0.4\ \ 0.5\rangle,\ \langle 0\ \ 0\ \ 1\rangle\} = \langle 0.3\ \ 0.4\ \ 0.5\rangle$,

$M_2^*(A) = \langle 0.1\ \ 0.2\ \ 0.7\rangle$
in view of the fact that $\max_{\mathcal{N}} \phi = \langle 0\ \ 0\ \ 1\rangle$.
Thus, every strictly increasing FNSEv
$\langle b^T, b^I, b^F\rangle = (\langle b_1^T, b_1^I, b_1^F\rangle,\ \ \langle b_2^T, b_2^I, b_2^F\rangle) \in \mathcal{F}^<(A)$ should fulfill the inequalities $\langle 0.3\ \ 0.4\ \ 0.5\rangle \leq \langle b_1^T, b_1^I, b_1^F\rangle \leq \langle 0.1\ \ 0.2\ \ 0.7\rangle$,
$\langle 0.3\ \ 0.4\ \ 0.5\rangle \leq \langle b_2^T, b_2^I, b_2^F\rangle \leq \langle 0.1\ \ 0.2\ \ 0.7\rangle$, , which are contradictory, i.e. $\mathcal{F}^<(A) = \phi$.

Second, we take the permutation $\varphi \in P_n$ with $\varphi(1) = 2$ and $\varphi(2) = 1$. We denote $A_{\varphi\varphi} = A' = \begin{bmatrix} \langle 0.1\ \ 0.2\ \ 0.7\rangle & \langle 0.2\ \ 0.3\ \ 0.6\rangle \\ \langle 0.3\ \ 0.4\ \ 0.5\rangle & \langle 0.7\ \ 0.6\ \ 0.2\rangle \end{bmatrix}$
and compute $\mathcal{F}^<(A') = \langle m^*(A'),\ \ M^*(A')\rangle \cap \mathcal{N}_{(n)}^<$. We get
$m_1^*(A') = \langle 0.2\ \ 0.3\ \ 0.6\rangle$,
$M_1^*(A') = min\{\langle 0.2\ \ 0.3\ \ 0.6\rangle, \langle 0.7\ \ 0.6\ \ 0.2\rangle\}e = \langle 0.2\ \ 0.3\ \ 0.6\rangle$, $m_2^*(A') = max\{\langle 0.2\ \ 0.3\ \ 0.6\rangle, \langle 0\ \ 0\ \ 1\rangle = \langle 0.2\ \ 0.3\ \ 0.6\rangle$,
$M_2^*(A') = \langle 0.7\ \ 0.6\ \ 0.2\rangle$. Thus, every strictly increasing FNSEvs
$\langle b_\varphi^T, b_\varphi^I, b_\varphi^F\rangle = \langle b^T, b^I, b^F\rangle' = (\langle b_1^T, b_1^I, b_1^F\rangle',\ \langle b_2^T, b_2^I, b_2^F\rangle') \in \mathcal{F}^<(A')$ must fulfill the inequalities $\langle 0.2\ \ 0.3\ \ 0.6\rangle \leq \langle b_1^T, b_1^I, b_1^F\rangle' \leq \langle 0.2\ \ 0.3\ \ 0.6\rangle$, $\langle 0.2\ \ 0.3\ \ 0.6\rangle \leq \langle b_2^T, b_2^I, b_2^F\rangle' \leq \langle 0.7\ \ 0.6\ \ 0.2\rangle$. In view of Theorem 4.1, the FNSEvs $\langle b^T, b^I, b^F\rangle \in \mathcal{F}(A)$ with $\langle b_1^T, b_1^I, b_1^F\rangle > \langle b_2^T, b_2^I, b_2^F\rangle$ are exactly the FNSVs fulfilling the inequalities $\langle 0.2\ \ 0.3\ \ 0.6\rangle = \langle b_2^T, b_2^I, b_2^F\rangle < \langle b_1^T, b_1^I, b_1^F\rangle \leq \langle 0.7\ \ 0.6\ \ 0.2\rangle$. Finally, we compute the constant FNSEvs.
We have $M(A) = \min\{\langle 0.7\ \ 0.6\ \ 0.2\rangle, \langle 0.2\ \ 0.3\ \ 0.6\rangle\} = \langle 0.2\ \ 0.3\ \ 0.6\rangle$, which implies, according to Theorem 4.7, $\langle 0\ \ 0\ \ 1\rangle \leq \langle b_1^T, b_1^I, b_1^F\rangle = \langle b_2^T, b_2^I, b_2^F\rangle \leq \langle 0.2\ \ 0.3\ \ 0.6\rangle$. The FNSE space $\mathcal{F}(A)$ is shown in Fig-2.

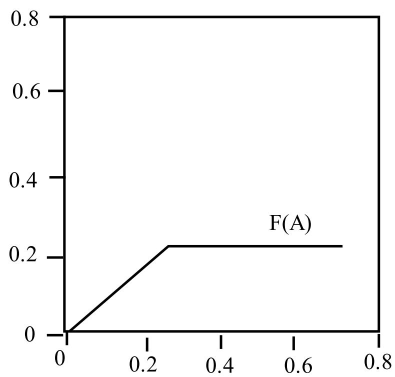

Fig-2

Example 5.2 *Similarly, as in the previous example, we consider* $\mathcal{N} = (0, 0.8)$ *and let* $n = 2$ *FNSM* $A \in \mathcal{N}_{(n,n)}$ *is slightly modified (in one entry):*

$$A = \begin{bmatrix} \langle 0.7\ \ 0.6\ \ 0.2\rangle & \langle 0.3\ \ 0.4\ \ 0.5\rangle \\ \langle 0.2\ \ 0.3\ \ 0.6\rangle & \langle 0.5\ \ 0.6\ \ 0.3\rangle \end{bmatrix}.$$

We begin with computation of $\mathcal{F}^<(A)$. *We get* $m_1^*(A) = \langle 0.3\ \ 0.4\ \ 0.6\rangle$,
$M_1^*(A) = \min\{\langle 0.7\ \ 0.6\ \ 0.2\rangle,\ \langle 0.5\ \ 0.6\ \ 0.3\rangle\} = \langle 0.5\ \ 0.6\ \ 0.3\rangle$, $m_2^*(A) = \max\{\langle 0.3\ \ 0.4\ \ 0.5\rangle,\ \langle 0\ \ 0\ \ 1\rangle\} = \langle 0.3\ \ 0.4\ \ 0.5\rangle$,
$M_2^*(A) = \langle 0.5\ \ 0.6\ \ 0.3\rangle$. *Hence,*
$\mathcal{F}^<(A) = \langle(\langle 0.3\ \ 0.4\ \ 0.5\rangle,\ \langle 0.3\ \ 0.4\ \ 0.5\rangle), (\langle 0.5\ \ 0.6\ \ 0.3\rangle, \langle 0.5\ \ 0.6\ \ 0.3\rangle)\cap \mathcal{N}_{(n)}^< = \{\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)};\ \langle 0.3\ \ 0.4\ \ 0.5\rangle \leq \langle b_1^T, b_1^I, b_1^F\rangle < \langle b_2^T, b_2^I, b_2^F\rangle \leq \langle 0.5\ \ 0.6\ \ 0.3\rangle\}$. *We now use the permutations* $\varphi \in P_n$ *with* $\varphi(1) = 2$ *and* $\varphi(2) = 1$. *We denote*

$$A_{\varphi\varphi} = A' = \begin{bmatrix} \langle 0.5\ \ 0.6\ \ 0.3\rangle & \langle 0.2\ \ 0.3\ \ 0.6\rangle \\ \langle 0.3\ \ 0.4\ \ 0.5\rangle & \langle 0.7\ \ 0.6\ \ 0.2\rangle \end{bmatrix}$$

and compute $m^*(A'),\ \ M^*(A')$. *We get*
$m_1^*(A') = \langle 0.2\ \ 0.3\ \ 0.6\rangle$,
$M_1^*(A') = \min\{\langle 0.5\ \ 0.6\ \ 0.3\rangle, \langle 0.7\ \ 0.6\ \ 0.2\rangle\} = \langle 0.5\ \ 0.6\ \ 0.3\rangle$,
$m_2^*(A') = \max\{\langle 0.2\ \ 0.3\ \ 0.6\rangle, \langle 0\ \ 0\ \ 1\rangle\} = \langle 0.2\ \ 0.3\ \ 0.6\rangle$,

$M_2^*(A^{'}) = \langle 0.7\ \ 0.6\ \ 0.2\rangle$. *Thus,* $\mathcal{F}^{<}(A^{'}) = \langle((\langle 0.2\ \ 0.3\ \ 0.6\rangle,\ \langle 0.2\ \ 0.3\ \ 0.6\rangle,),(\langle 0.5\ \ 0.6\ \ 0.3\rangle,\langle 0.7\ \ 0.6\ \ 0.2\rangle)\cap \mathcal{N}_{(n)}^{<} = \{\langle b^T, b^I, b^F\rangle^{'} \in \mathcal{N}_{(n)};\ \langle 0.2\ \ 0.3\ \ 0.6\rangle \le \langle b_1^T, b_1^I, b_1^F\rangle^{'} \le \langle 0.5\ \ 0.6\ \ 0.3\rangle,\ \ \langle b_1^T, b_1^I, b_1^F\rangle^{'} < \langle b_2^T, b_2^I, b_2^F\rangle \le \langle 0.7\ \ 0.6\ \ 0.2\rangle\}$. *i.e. the strictly decreasing FNSEvs* $\langle b^T, b^I, b^F\rangle \in \mathcal{F}(A)$ *are exactly the FNSvs fulfilling the inequalities* $\langle 0.2\ \ 0.3\ \ 0.6\rangle \le \langle b_2^T, b_2^I, b_2^F\rangle \le \langle 0.5\ \ 0.6\ \ 0.3\rangle,\ \ \langle b_2^T, b_2^I, b_2^F\rangle < \langle b_1^T, b_1^I, b_1^F\rangle \le \langle 0.7\ \ 0.6\ \ 0.2\rangle$. *Third, we compute the bounds for constant FNSEvs. We have*
$M(A) = min\{\langle 0.7\ \ 0.6\ \ 0.2\rangle,\ \ \langle 0.5\ \ 0.6\ \ 0.3\rangle\} = \langle 0.5\ \ 0.6\ \ 0.3\rangle$, *which implies, according to Theorem 4.5,* $\langle 0\ \ 0\ \ 1\rangle \le \langle b_1^T, b_1^I, b_1^F\rangle = \langle b_2^T, b_2^I, b_2^F\rangle \le \langle 0.5\ \ 0.6\ \ 0.3\rangle$. *The FNSE spaces* $\mathcal{F}(A)$ *is shows in Fig-3.*

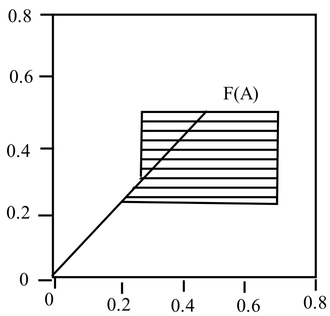

Fig-3

Example 5.3 This example shows the FNSE space of another modified FNSM
$A = \begin{bmatrix} \langle 0.5\ \ 0.6\ \ 0.3\rangle & \langle 0.6\ \ 0.7\ \ 0.1\rangle \\ \langle 0.2\ \ 0.3\ \ 0.6\rangle & \langle 0.5\ \ 0.6\ \ 0.3\rangle \end{bmatrix}$ with $A^{'} = \begin{bmatrix} \langle 0.5\ \ 0.6\ \ 0.3\rangle & \langle 0.2\ \ 0.3\ \ 0.6\rangle \\ \langle 0.6\ \ 0.7\ \ 0.1\rangle & \langle 0.5\ \ 0.6\ \ 0.3\rangle \end{bmatrix}$.

By a similar procedure as in two previous examples, we get expressions for the monotone FNSE spaces

$$\mathcal{F}^{<}(A) = \langle((\langle 0.6\ \ 0.7\ \ 0.1\rangle,\ \langle 0.6\ \ 0.7\ \ 0.1\rangle),(\langle 0.5\ \ 0.6\ \ 0.3\rangle,\langle 0.5\ \ 0.6\ \ 0.3\rangle))\cap \mathcal{N}_{(n)}^{<} = \phi,$$
$$\mathcal{F}^{<}(A)^{'} = \langle((\langle 0.2\ \ 0.3\ \ 0.6\rangle,\ \langle 0.2\ \ 0.3\ \ 0.6\rangle),(\langle 0.5\ \ 0.6\ \ 0.3\rangle,\langle 0.5\ \ 0.6\ \ 0.3\rangle))\cap \mathcal{N}_{(n)}^{<} = \{\langle b^T, b^I, b^F\rangle^{'} \in \mathcal{N}_{(n)};\ \langle 0.2\ \ 0.3\ \ 0.6\rangle \le \langle b_1^T, b_1^I, b_1^F\rangle^{'} < \langle b_2^T, b_2^I, b_2^F\rangle^{'} \le \langle 0.5\ \ 0.6\ \ 0.3\rangle\}.$$

and for the constant FNSE space

$$\{\langle b^T, b^I, b^F\rangle \in \mathcal{N}_{(n)}; \langle 0, 0, 1\rangle \le \langle b_1^T, b_1^I, b_1^F\rangle = \langle b_2^T, b_2^I, b_2^F\rangle \le \langle 0.5\ \ 0.6\ \ 0.3\rangle$$

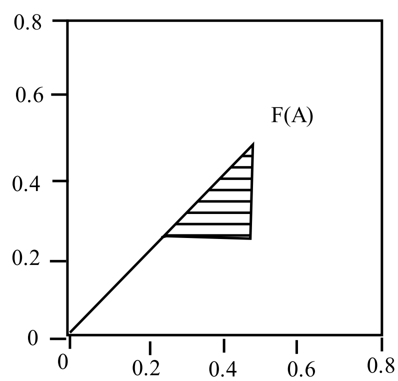

Fig-4

Again, the FNSE space $\mathcal{F}(A)$ is shown in Fig-4.

CONCLUSION

In this paper, the authors presented fuzzy neutrosophic soft eigenvector space and notation of increasing FNSEvs, strictly increasing FNSEvs, intervals of monotone FNSEv with some examples. Then monotone FNSE space structure and Computing the bounds.

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