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Multi-Criteria Decision Making Approach Using the Fuzzy Measures for Environmental Improvement under Neutrosophic Environment

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Abstract

The uncertainty, incomplete and inconsistent information can lead to some difficulties of decision making under the single valued neutrosophic set (SVNS) environment. Information measure plays an important role in SVNS theory, which has received more and more attention in recent years. In this study, we develop a multi-attribute decision making (MADM) method based on the single valued neutrosophic information measures. Under the single valued neutrosophic environment, three axiomatic definitions of information measures are first introduced, including entropy, similarity measure and cross-entropy. Then, we construct some information measure formulas on the basis of the cosine function. The relationship among the entropy, similarity measure and cross-entropy is discussed, from which we find that three information measures can be transformed by each other. Moreover, an approach to single valued neutrosophic MADM is proposed, which is based on the constructed information measure formulas. Finally, a numerical example for city pollution evaluation is provided to explore the applicability and effectiveness of the proposed method. Results show that the proposed MADM approach can derive the more accurate decision making results and obtain the reasonable and credible ranking result in some cases.

Keywords: single valued neutrosophic set, entropy, similarity measure, cross-entropy, decision making

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INTRODUCTION

Since Zadeh introduced his remarkable theory of fuzzy sets (FSs) (Zadeh 1965), it has been applied successfully in various fields. However, FS is a set with each element only has a membership degree which is represented by a real number between zero and one. In order to overcome the lack of knowledge of non-membership degrees, the concept of intuitionistic fuzzy sets (IFSs) put forward by Atanassov (1986, 2000), which is a generalization of the FSs. The introduction of IFSs proved to be very meaningful and practical, and has been found to be highly useful to deal with incomplete information. In IFSs, the data information is expressed by means of 2-tuples, and each 2-tuples simultaneously take into account the membership degree and non-membership degree. The sum of membership degree and non-membership degree of each 2-tuple is less than or equal to 1 (Atanassov 1989). To accommodate more complex environment, Atanassov and Gargov further introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov et al. 1989), whose components are intervals rather than exact numbers. The introduction of IFSs and IVIFSs proved to be very meaningful and practical, and have been found to be highly useful to cope with uncertainty and vagueness (Hu et al. 2015, Meng et al. 2015, Onar et al. 2015, Wu and Chiclana 2014, Zhou et al. 2014, 2016).

IFSs and IVIFSs can handle incomplete information, however, in real decision making, the decision information is often incomplete, indeterminate and inconsistent, which cannot be deal with by IFSs and IVIFSs. Therefore, Smarandache (Smarandache 1999, 2003) originally introduced the concept of neutrosophic sets (NSs) from philosophical point of view. The NSs simultaneously take into account the truth membership, the indeterminacy membership and the falsity membership, and they are independent. Owing to the NS is difficult to apply in real science and engineering fields, then Wang et al. (2010) proposed the single valued neutrosophic set (SVNS), which is a subclass of NSs.

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Motivated by the concepts of hesitant fuzzy entropy, similarity measures and cross-entropy, we introduce three axiomatic definitions of information measures for single valued neutrosophic values (SVNVs), and then we construct some information measure formulas based on cosine function. The relationship among these information measures for SVNVs is discussed. Moreover, an approach to MADM is investigated.

The rest of the paper is organized as follows. In Section 2, we review some related work of the SVNSs. Section 3 introduces the axiomatic definitions of entropy, similarity measures and cross-entropy for SVNVs, and constructs several single valued neutrosophic information measure formulas. In Section 3, we also study the relationship among these information measures of SVNVs. Section 4 develops an approach to MADM with the constructed information measure formulas, a numerical example is presented to illustrate the application of the developed method. Finally, some conclusions and future research possibilities are provided in Section 5.

STATE OF THE ART

Entropy, similarity measures, and cross-entropy are three important research topics in the fuzzy theory, which have been widely used in practical applications (Wei et al. 2011), such as decision-making, information fusion system, medical diagnosis and image processing. Entropy is very important for measuring uncertain information. Since its appearance, entropy has received great attentions. Zadeh first introduced the fuzzy entropy (Zadeh 1968) to measure the fuzziness of decision making information. Moreover, Luca and Termini (1972) presented the axioms with which the fuzzy entropy should comply, and defined the entropy of a FS. Based on the ratio of intuitionistic fuzzy cardinalities, Szmidt and Kacprzyk (2001) given the axiomatic requirements of intuitionistic fuzzy entropy measure and introduced a non-probabilistic-type entropy measure for IFSs. Ye (2010) proposed two entropy measures for IVIFSs and established an entropy weighted model to determine the entropy weights. Based on the continuous ordered weighted averaging (COWA) operator, Jin et al. and Ye (2010) investigated an interval-valued intuitionistic fuzzy continuous weighted entropy, and then an approach is developed to cope with interval-valued intuitionistic fuzzy MADM problems. Majumdar and Samant (2014) introduced an entropy to measure the uncertainty involved in a SVNV.

Similarity measures and cross-entropy are mainly used to measure the discrimination information. Up to now, a lot of research has been done about this issue (Grzegorzewski 2004, Hung et al. 2007, Ye 2017, Zhou et al. 2013, 2014). Liu (1992) gave the axiomatic definitions of entropy, distance measure, and similarity measure of FSs and systematically discussed their basic relations. Vlachos and Sergiadis (2007) introduced the concept intuitionistic fuzzy cross-entropy, and discussed relations between cross-entropy and entropy. Beliakov et al. (2014) investigated a new approach for defining similarity measures for IFSs, in which a similarity measure has two components indicating the similarity and hesitancy aspects. Based on the Jaccard, Dice, and cosine similarity measures in vector space, Ye (2014) proposed three vector similarity measures between SVNSs to obtain the ranking order of all alternatives in MADM problems. Ye (2015) constructed the modified cosine similarity measures for SVNSs on the basis of cosine function. With the help of the distance between two SVNSs, Majumdar and Samant (2009) presented several similarity measures for SVNSs and discussed their characteristics. Under the single valued neutrosophic environment, Ye (2014) proposed a cross entropy to establish a MADM method. The relationship among the entropy, similarity measures and cross-entropy has attracted many attentions. Zhang et al. and Zeng and Li (2006) showed that entropies and similarity measures of IVFSs can be transformed by each other.

From above analysis, we can see that information measures are very useful tools to cope with uncertainty and vagueness. On the one hand, it is known that uncertainty, incomplete and inconsistent information exists in human decision making process. Therefore, just as FSs, IFSs and IVIFSs, researches on the entropy, similarity measures and cross-entropy for SVNSs are the important issues. On the other hand, more and more MADM methods and theories have been developed on the basis of SVNSs. To the best of our knowledge, there are few studies focused on the relationship among the entropy, similarity measures and cross-entropy for SVNSs. Therefore, it is necessary and meaningful to study some issues. For example, what is it like the expression of the single valued neutrosophic information measures? What is the relationship among the single valued neutrosophic information measures?

MATERIAL AND METHODS

SVNSs

In this section, we review some basic concepts related to SVNSs, which will be used in the rest of the paper.

Definition 1: Smarandache (1999) Let X be a universal set, with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $T_A(x): X \to]^-0, 1^+[$, $I_A(x): X \to]^-0, 1^+[$ and $F_A(x): X \to]^-0, 1^+[$, and the sum of $T_A(x), I_A(x)$ and $F_A(x)$ satisfies the condition $^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

In order to apply NS easily in science and engineering applications, Wang et al. and Wang et al. (2010) presented the concept of SVNSs, which is an instance of the NS.

Definition 2: Wang et al. (2010) Let X be a universal set, with a generic element in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, then a SVNS A can be denoted by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$, where $T_A(x)$, $I_A(x), F_A(x) \in [0,1]$, and $T_A(x) + I_A(x) + F_A(x) \in [0,3]$.

For convenience, we refer to $\alpha = \langle T_\alpha, I_\alpha, F_\alpha\rangle$ as a single valued neutrosophic value (SVNV), which is a basic unit of SVNS. Let $\widetilde{\Omega}$ be the set of all the SVNVs in X.

Definition 3: Peng et al. (2014) Let $\alpha = \langle T_\alpha, I_\alpha, F_\alpha\rangle$ be a SVNV, then the complement of $\alpha$is denoted by $\alpha^c$and $\alpha^c = \langle 1 - T_\alpha, 1 - I_\alpha, 1 - F_\alpha\rangle$.

Let $\alpha = \langle T_\alpha, I_\alpha, F_\alpha\rangle \triangleq \langle \alpha_1, \alpha_2, \alpha_3\rangle$, then $\alpha^c = \langle 1 - \alpha_1, 1 - \alpha_2, 1 - \alpha_3\rangle$, i.e., $\alpha_t^c = 1 - \alpha_t, t = 1,2,3$.

Single Valued Neutrosophic Information Measures

Single valued neutrosophic entropy

The entropy of a SVNS is defined by Majumdar and Samanta as follows (Majumdar, 2014):

Definition 4: The entropy of a SVNS $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$ is a function $\varepsilon: A \to [0,1]$ which satisfies the following axioms:

(i) $\varepsilon(A) = 0$ if A is a crisp set;

(ii) $\varepsilon(A) = 1$ if $\langle x, T_A(x), I_A(x), F_A(x)\rangle = \langle 0.5,0.5,0.5\rangle$ for $\forall x \in X$;

(iii) $\varepsilon(A) = \varepsilon(A^c)$;

(iv) $\varepsilon(A) \geq \varepsilon(B)$, if A more uncertain than B, i.e.,

$T_A(x) + F_A(x) \leq T_B(x) + F_B(x)$ and $|I_A(x) - I_{A^c}(x)| \leq |I_B(x) - I_{B^c}(x)|$.

However, in some situations, the axiomatic requirement (iv) in Definition 4 might be impractical. This is demonstrated in Example 1.

Example 1: Let $A = \{\langle x, 1,0,0\rangle | x \in X\}$ and $B = \{\langle x, 0.5,0,0.6\rangle | x \in X\}$ be two SVNSs. According to the axiomatic requirement (iv) in Definition 4, since $T_A(x) + F_A(x) = 1 + 0 = 1 < 1.1 = 0.5 + 0.6 = T_B(x) + F_B(x)$ and $|I_A(x) - I_{A^c}(x)| = 1 = |I_B(x) - I_{B^c}(x)|$, which indicates that A is more uncertain than B, then we have $\varepsilon(A) \geq \varepsilon(B)$. However, it is clear that $A = \{\langle x, 1,0,0\rangle | x \in X\}$ is a crisp set, then the entropy of A is $\varepsilon(A) = 0$ and A less uncertain than B. Therefore, the contradiction exists in the Definition 4, and Definition 4 is unreasonable.

In this case, the definition of entropy for SVNSs needs to be improved. In the following, we first introduce the axiomatic definition of entropy for SVNVs, and then investigate an entropy formula of a SVNV.

Definition 5: An entropy on SVNV $\alpha = \langle \alpha_1, \alpha_2, \alpha_3\rangle$ is a function $E: \widetilde{\Omega} \to [0,1]$, which satisfying the following axiomatic requirements:

(E1) $E(\alpha) = 0$, if and only if $\alpha_t = 0$ or $\alpha_t = 1, t = 1,2,3$;

(E2) $E(\alpha) = 1$, if and only if $\langle \alpha_1, \alpha_2, \alpha_3\rangle = \langle 0.5,0.5,0.5\rangle$;

(E3) $E(\alpha) = E(\alpha^c)$;

(E4) $E(\alpha) \leq E(\beta)$, if β more uncertain than α, i.e.,

$\alpha_t \leq \beta_t$ when $\beta_t - \beta_t^c \leq 0, t = 1,2,3,$

or

$\alpha_t \geq \beta_t$ when $\beta_t - \beta_t^c \geq 0, t = 1,2,3.$

Based on the cosine function, an information measure formula for SVNVs is constructed as follows:

$$E_1(\alpha) = \frac{1}{3(\sqrt{2}-1)}\sum_{t=1}^{3}\left(\sqrt{2}cos\frac{\alpha_t - \alpha_t^c}{4}\pi - 1\right) \quad (1)$$

In what follows, we show that $E_1(\alpha)$ is an entropy measure of SVNV α.

Theorem 1. The mapping $E_1(\alpha)$, defined by Eq. (1), is an entropy measure for SVNV α.

Proof. In order for Eq. (1) to be qualified as a sensible measure of single valued neutrosophic entropy, it must satisfy the conditions (E1)-(E4) in Definition 5.

Let $f(x) = \frac{1}{\sqrt{2}-1}\left(\sqrt{2}\cos\frac{\pi}{4}x - 1\right), x \in [-1,1]$, then we have

$$\frac{df(x)}{dx} = -\frac{\sqrt{2}\pi}{4(\sqrt{2}-1)}sin\frac{\pi}{4}x \quad (2)$$

If $x \in [-1,0]$, then $\frac{df(x)}{dx} \geq 0$, which means that $f(x)$ is an increasing function of x, for $x \in [-1,0]$; If $x \in [0,1]$, then $\frac{df(x)}{dx} \leq 0$, which means that $f(x)$ is a decreasing function of x, for $x \in [0,1]$. Since $f(x) \in [0,1]$, then $f_{\min}(x) = 0$, if and only if $x = -1$ or $x = 1$; $f_{\max}(x) = 1$, if and only if $x = 0$.

(E1) If $\alpha_t = 0$ or $\alpha_t = 1, t = 1,2,3$, then $\alpha_t - \alpha_t^c = -1$ or $\alpha_t - \alpha_t^c = 1, t = 1,2,3$. From the above analysis, we have $E_1(\alpha) = 0$.

On the other hand, assume that $E_1(\alpha) = 0$.

As $\alpha_t - \alpha_t^c = \alpha_t - (1-\alpha_t) = 2\alpha_t - 1$ and $0 \leq \alpha_t \leq 1, t = 1,2,3$, then we have $\alpha_t - \alpha_t^c \in [-1,1], t = 1,2,3$. Therefore, every term in the summation of $E_1(\alpha)$ is non-negative. While $E_1(\alpha) = 0$, then every term should be zero in $E_1(\alpha)$, i.e.,

$$\frac{1}{\sqrt{2}-1}\left(\sqrt{2}cos\frac{\alpha_t - \alpha_t^c}{4}\pi - 1\right) = 0, t = 1,2,3 \quad (3)$$

From the above analysis, we know that Eq. (3) holds, if and only if $\alpha_t - \alpha_t^c = -1$ or $\alpha_t - \alpha_t^c = 1, t = 1,2,3$. Hence, $\alpha_t = 0$ or $\alpha_t = 1, t = 1,2,3$.

(E2) If $\langle\alpha_1, \alpha_2, \alpha_3\rangle = \langle 0.5,0.5,0.5\rangle$, we have $\alpha_t - \alpha_t^c = 0, t = 1,2,3$, then based on Eq. (1), $E_1(\alpha) = 0$ is obtained.

On the other hand, from the above analysis, we have $\alpha_t - \alpha_t^c \in [-1,1], t = 1,2,3$, it is obvious that $0 \leq E_1(\alpha) \leq 1$. If $E_1(\alpha) = 1$, then $\alpha_t - \alpha_t^c = 0, t = 1,2,3$. It follows that $\alpha_t = 0.5, t = 1,2,3$, i.e., $\langle\alpha_1, \alpha_2, \alpha_3\rangle = \langle 0.5,0.5,0.5\rangle$.

(E3) Since $\alpha^c = \langle T_{\alpha^c}, I_{\alpha^c}, F_{\alpha^c}\rangle = \langle 1-\alpha_1, 1-\alpha_2, 1-\alpha_3\rangle$, then $(\alpha^c)^c = \alpha$. Thus

$$\begin{aligned} E_1(\alpha^c) &= \frac{1}{3(\sqrt{2}-1)}\sum_{t=1}^{3}\left(\sqrt{2}cos\frac{\alpha_t^c - (\alpha_t^c)^c}{4}\pi - 1\right) \\ &= \frac{1}{3(\sqrt{2}-1)}\sum_{t=1}^{3}\left(\sqrt{2}\cos\frac{\alpha_t^c - \alpha_t}{4}\pi - 1\right) \\ &= \frac{1}{3(\sqrt{2}-1)}\sum_{t=1}^{3}\left(\sqrt{2}\cos\frac{\alpha_t - \alpha_t^c}{4}\pi - 1\right) \\ &= E_1(\alpha) \end{aligned} \quad (4)$$

(E4) Assume that $\alpha_t \leq \beta_t$ when $\beta_t - \beta_t^c \leq 0, t = 1,2,3$, then $1 \geq 1-\alpha_t \geq 1-\beta_t \geq 0$, i.e., $1 \geq \alpha_t^c \geq \beta_t^c \geq 0, t = 1,2,3$. It follows that

$$-1 \leq \alpha_t - \alpha_t^c \leq \beta_t - \beta_t^c \leq 0, t = 1,2,3 \quad (5)$$

Notice that

$$f(x) = \frac{1}{\sqrt{2}-1}\left(\sqrt{2}cos\frac{\pi}{4}x - 1\right) \quad (6)$$

is an increasing function of x, for $x \in [-1,0]$, therefore $E_1(\alpha) \leq E_1(\beta)$.

Similarly, if $\alpha_t \geq \beta_t$ when $\beta_t - \beta_t^c \geq 0, t = 1,2,3$, we have $E(\alpha) \leq E(\beta)$. This completes the proof of Theorem 1.

Definition 6: Suppose that α is a SVNV, then $E_1(\alpha)$, defined by Eq. (1), is called the entropy of SVNV α.

Single valued neutrosophic similarity measure

In this subsection, we give the axiomatic definition of similarity measure of the SVNVs, and then develop similarity measure formula for SVNVs.

Definition 7: Suppose that α and β are two SVNVs, the similarity measure between α and β, denoted as $S(\alpha,\beta)$, should satisfy the following axiomatic requirements:

(S1) $S(\alpha,\beta) = 0$, if and only if $\alpha_t - \beta_t = 1$ or $\alpha_t - \beta_t = -1, t = 1,2,3$;

(S2) $S(\alpha,\beta) = 1$, if and only if $\langle\alpha_1, \alpha_2, \alpha_3\rangle = \langle\beta_1, \beta_2, \beta_3\rangle$;

(S3) $S(\alpha,\beta) = S(\beta,\alpha)$;

(S4) $S(\alpha,\gamma) \leq S(\alpha,\beta), S(\alpha,\gamma) \leq S(\beta,\gamma)$, if $\alpha_t \leq \beta_t \leq \gamma_t$ or $\alpha_t \geq \beta_t \geq \gamma_t, t = 1,2,3$.

Let $\alpha, \beta \in \widetilde{\Omega}$, based on the cosine function, an information measure formula for SVNVs α and β is established as follows:

$$S_1(\alpha,\beta) = \frac{1}{3(\sqrt{2}-1)} \sum_{t=1}^{3} \left(\sqrt{2} cos \frac{\alpha_t - \beta_t}{4} \pi - 1\right) \quad (7)$$

Then we have the following theorem.

Theorem 2: Suppose that α and β are two SVNVs, then the mapping $S_1(\alpha,\beta)$, defined by Eq. (7), is the similarity measure between α and β.

Proof. Now we testify that $S_1(\alpha,\beta)$ satisfies the four axiomatic requirements listed in Definition 3.4.

According to Theorem 1, we know that $f(x) = \frac{1}{\sqrt{2}-1}\left(\sqrt{2}\cos\frac{\pi}{4}x - 1\right)$ is an increasing function of x, for $x \in [-1,0]$; $f(x)$ is a decreasing function of x, for $x \in [0,1]$. Moreover, $f_{\min}(x) = 0$, if and only if $x = -1$ or $x = 1$; $f_{\max}(x) = 1$, if and only if $x = 0$.

(S1) If $\alpha_t - \beta_t = 1$ or $\alpha_t - \beta_t = -1, t = 1,2,3$, then $S_1(\alpha,\beta) = 0$ is definitely validated according to Eq. (7).

Since $\alpha_t, \beta_t \in [0,1], t = 1,2,3$, then $\alpha_t - \beta_t \in [-1,1]$, which implies that every term in the summation of $S_1(\alpha,\beta)$ is non-negative. Suppose that $S_1(\alpha,\beta) = 0$, then every term should equal zero, i.e.,

$$\frac{1}{\sqrt{2}-1}\left(\sqrt{2} cos \frac{\alpha_t - \beta_t}{4}\pi - 1\right) = 0, t = 1,2,3 \quad (8)$$

From the above analysis, Eq. (8) holds, if and only if $\alpha_t - \beta_t = 1$ or $\alpha_t - \beta_t = -1, t = 1,2,3$.

(S2) If $\langle\alpha_1,\alpha_2,\alpha_3\rangle = \langle\beta_1,\beta_2,\beta_3\rangle$, by Eq. (7), it is obvious that $S_1(\alpha,\beta) = 1$.

Assume that $S_1(\alpha,\beta) = 1$, then it is deduced that every term should equal one, i.e.,

$$\frac{1}{\sqrt{2}-1}\left(\sqrt{2} cos \frac{\alpha_t - \beta_t}{4}\pi - 1\right) = 1, t = 1,2,3 \quad (9)$$

and Eq. (9) holds, if and only if $\alpha_t - \beta_t = 0, t = 1,2,3$. Hence, $\langle\alpha_1,\alpha_2,\alpha_3\rangle = \langle\beta_1,\beta_2,\beta_3\rangle$.

(S3) Since $\cos x = \cos(-x)$ for $\forall x \in R$, then we have

$$\begin{aligned} S_1(\alpha,\beta) &= \frac{1}{3(\sqrt{2}-1)} \sum_{t=1}^{3} \left(\sqrt{2} cos \frac{\alpha_t - \beta_t}{4}\pi - 1\right) \\ &= \frac{1}{3(\sqrt{2}-1)} \sum_{t=1}^{3} \left(\sqrt{2}\cos\left(-\frac{\alpha_t - \beta_t}{4}\pi\right) - 1\right) \\ &= \frac{1}{3(\sqrt{2}-1)} \sum_{t=1}^{3} \left(\sqrt{2}\cos\frac{\beta_t - \alpha_t}{4}\pi - 1\right) \\ &= S_1(\beta,\alpha) \end{aligned} \quad (10)$$

(S4) Suppose that

$$\begin{gathered} 0 \leq \alpha_t \leq \beta_t \leq \gamma_t \leq 1, t = 1,2,3, \\ \text{then } -1 \leq \alpha_t - \gamma_t \leq \alpha_t - \beta_t \leq 0 \\ \text{and } -1 \leq \alpha_t - \gamma_t \leq \beta_t - \gamma_t \leq 0, t = 1,2,3 \end{gathered} \quad (11)$$

As $f(x) = \frac{1}{\sqrt{2}-1}\left(\sqrt{2}\cos\frac{\pi}{4}x - 1\right)$ is an increasing function of x, for $x \in [-1,0]$, therefore

$$S_1(\alpha,\gamma) \leq S_1(\alpha,\beta), S_1(\alpha,\gamma) \leq S_1(\beta,\gamma) \quad (12)$$

Similarly, if $\alpha_t \geq \beta_t \geq \gamma_t, t = 1,2,3$, we have $S_1(\alpha,\gamma) \leq S_1(\alpha,\beta), S_1(\alpha,\gamma) \leq S_1(\beta,\gamma)$.

This completes the proof of Theorem 2.

Definition 8. Suppose that α and β are two SVNVs, then $S_1(\alpha,\beta)$, defined by Eq. (7), is called the similarity measure between α and β.

Single valued neutrosophic cross-entropy

In the following, we shall propose the axiomatic definition of single valued neutrosophic cross-entropy, and then construct a cross-entropy formula between SVNVs.

Definition 9. Suppose that α and β are two SVNVs, the single valued neutrosophic cross-entropy between α and β, denoted as $C(\alpha,\beta)$, should satisfy the following two axiomatic requirements:

(C1) $C(\alpha,\beta) \geq 0$;

(C2) $C(\alpha,\beta) = 0$ if $\langle\alpha_1,\alpha_2,\alpha_3\rangle = \langle\beta_1,\beta_2,\beta_3\rangle$.

Let $\alpha, \beta \in \widetilde{\Omega}$, based on the cosine function, an information measure formula between SVNVs α and β is developed as follows:

$$C_1(\alpha,\beta) = 1 - \frac{1}{3(\sqrt{2}-1)}\sum_{t=1}^{3}\left(\sqrt{2}cos\frac{\alpha_t-\beta_t}{4}\pi - 1\right) \quad (13)$$

Theorem 3. Suppose that α and β are two SVNVs, then the mapping $C_1(\alpha,\beta)$, defined by Eq. (13), is the cross-entropy between α and β.

Proof. The proof of Theorem 3 is similar to that of Theorem 2, it is easy to know that the mapping $C_1(\alpha,\beta)$ satisfy the axiomatic requirements (C1)-(C2) listed in Definition 9. So it is omitted here.

Definition 10. Suppose that α and β are two SVNVs, then $C_1(\alpha,\beta)$, defined by Eq. (13), is called the cross-entropy between α and β.

Relationship among the Single Valued Neutrosophic Information Measures

In this section, we study the interrelations among the single valued neutrosophic entropy, similarity measure and cross-entropy.

Theorem 4. Let α be a SVNV, then $S(\alpha,\alpha^c)$ is a single valued neutrosophic entropy, i.e.

$$E(\alpha) = S(\alpha,\alpha^c) \quad (14)$$

Proof. It is sufficient to show that $S(\alpha,\alpha^c)$ satisfies the requirements (E1)-(E4) listed in Definition 3.2.

(E1) $E(\alpha) = 0 \Leftrightarrow S(\alpha,\alpha^c) = 0 \Leftrightarrow \alpha_t - \alpha_t^c = 1$

or $\alpha_t - \alpha_t^c = -1, t = 1,2,3$, i.e.,

$\alpha_t - (1-\alpha_t) = 1$ or

$$\alpha_t - (1-\alpha_t) = -1, t = 1,2,3 \quad (15)$$

Therefore, Eq. (15) holds, if and only if

$\alpha_t = 0$ or $\alpha_t = 1, t = 1,2,3$.

(E2) $E(\alpha) = 1 \Leftrightarrow S(\alpha,\alpha^c) = 1 \Leftrightarrow \langle\alpha_1,\alpha_2,\alpha_3\rangle = \langle\alpha_1^c,\alpha_2^c,\alpha_3^c\rangle$

$\Leftrightarrow \langle\alpha_1,\alpha_2,\alpha_3\rangle = \langle 1-\alpha_1, 1-\alpha_2, 1-\alpha_3\rangle$

$\Leftrightarrow \alpha_t = 1-\alpha_t, t = 1,2,3 \Leftrightarrow \alpha_t = 0.5, t = 1,2,3$

$$\Leftrightarrow \langle\alpha_1,\alpha_2,\alpha_3\rangle = \langle 0.5,0.5,0.5\rangle \quad (16)$$

(E3) $E(\alpha^c) = S(\alpha^c,(\alpha^c)) = S(\alpha^c,\alpha) = S(\alpha,\alpha^c) = E(\alpha)$.

(E4) If $\alpha_t \le \beta_t$ when $\beta_t - \beta_t^c \le 0, t = 1,2,3$, then $\beta_t - (1-\beta_t) \le 0, t = 1,2,3$, i.e., $\beta_t \le 1-\beta_t, t = 1,2,3$, and we have

$$\alpha_t \le \beta_t \le 1-\beta_t \le 1-\alpha_t, t = 1,2,3 \quad (17)$$

i.e.

$$\alpha_t \le \beta_t \le \beta_t^c \le \alpha_t^c, t = 1,2,3 \quad (18)$$

According to the axiomatic requirement (S4) in Definition 3.4, it is deduced that

$$S(\alpha,\alpha^c) \le S(\beta,\alpha^c) \le S(\beta,\beta^c) \quad (19)$$

then

$E(\alpha) \le E(\beta)$.

Similarly, if $\alpha_t \ge \beta_t$ when $\beta_t - \beta_t^c \ge 0, t = 1,2,3$, one can obtain that $E(\alpha) \le E(\beta)$, which completes the proof of Theorem 4.

According to the Theorem 4, then we have the following corollary.

Corollary 1. Let α be a SVNV, then

$$E_1(\alpha) = S_1(\alpha,\alpha^c) \quad (20)$$

Now we discuss the relationship between single valued neutrosophic similarity measure and single valued neutrosophic cross-entropy.

Theorem 5. Let α and β be two SVNVs, then $1 - S(\alpha,\beta)$ is a single valued neutrosophic cross-entropy, i.e.,

$$C(\alpha,\beta) = 1 - S(\alpha,\beta) \quad (21)$$

Proof. It is sufficient to prove that $1 - S(\alpha,\beta)$ holds the two conditions of Definition 3.6.

(C1) Since single valued neutrosophic similarity measure $S(\alpha,\beta) \in [0,1]$, then

$$C(\alpha,\beta) = 1 - S(\alpha,\beta) \in [0,1] \quad (22)$$

thus we can obtain $C(\alpha,\beta) \ge 0$.

(C2) $C(\alpha,\beta) = 0 \Leftrightarrow 1 - S(\alpha,\beta) = 0 \Leftrightarrow S(\alpha,\beta) = 1$. By the axiomatic requirement (S2) of Definition 9, $S(\alpha,\beta) = 1$, if and only if $\langle\alpha_1,\alpha_2,\alpha_3\rangle = \langle\beta_1,\beta_2,\beta_3\rangle$.

Corollary 2. Let α and β be two SVNVs, then

$$C_1(\alpha,\beta) = 1 - S_1(\alpha,\beta) \quad (23)$$

By Theorem 4.1 and Theorem 5, we have the following theorem:

Theorem 6. Let α be a SVNV, then $1 - C(\alpha,\alpha^c)$ is a single valued neutrosophic entropy, i.e.,

$$E(\alpha) = 1 - C(\alpha,\alpha^c) \quad (24)$$

Corollary 3. Let α be a SVNV, then

$$E_1(\alpha) = 1 - C_1(\alpha, \alpha^c) \tag{25}$$

An Approach to MADM

With the rapid development of society and economy, the MADM problems facing DM are becoming more complicated, uncertain and fuzzy than ever, and the uncertainty, imprecise, incomplete and inconsistent information is included. Then SVNS can represent these information. In what follows, we present a handling method for MADM problems under single valued neutrosophic environment.

Assume that $X = \{X_1, X_2, \cdots, X_m\}$ is the set of alternatives, $C = \{C_1, C_2, \cdots, C_n\}$ is a collection of attributes. The DM is required to provide the information that the alternative X_i satisfies the attribute C_j, then the decision making information can be represented as a SVNV $\alpha_{ij} = \langle \alpha_1^{ij}, \alpha_2^{ij}, \alpha_3^{ij} \rangle$. When all the performances of the alternatives are provided, the single valued neutrosophic decision matrix $D = (\alpha_{ij})_{m\times n}$ can be constructed. Assume that the weight vector of attribute is $W = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$, where $0 \le w_j \le 1, j = 1,2,\cdots,n$, and $\sum_{j=1}^{n} w_j = 1$. In the process MADM, sometimes, the information about attribute weights is completely unknown or incompletely known because of lack of knowledge or data, the influence of the decision environment and the expert's limited expertise about the decision making problem domain (Jin et al., 2016).

The method to determine the attribute weights

In order to get the optimal alternatives, first of all, we propose a method to determine the weight vector of attributes based on the entropy and cross-entropy.

One the one hand, considering of the entropy of the attribute C_j, the averaging entropy $E(C_j)$ of the attribute C_j is given as:

$$E(C_j) = \frac{1}{m}\sum_{i=1}^{m} E_1(\alpha_{ij}) \tag{26}$$

and each $E_1(\alpha_{ij})$ can be calculated by Eq. (1). According to the entropy theory, we know that the entropy of an attribute is smaller across alternatives, which implies that it can provide DM with the effective information, and the attribute should be assigned a bigger weight (Jin et al. 2016).

On the other hand, for the attribute C_j, the averaging cross-entropy of the alternative X_i to all the other alternatives can be given as:

$$\frac{1}{m-1}\sum_{k=1,k\neq i}^{m} C_1(\alpha_{ij}, \alpha_{kj}) \tag{27}$$

and the averaging cross-entropy for the attribute C_j can be given as:

$$C(C_j) = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{m-1}\sum_{k=1,k\neq i}^{m} C_1(\alpha_{ij}, \alpha_{kj})\right) \tag{28}$$

which can be described as the divergence measures among all alternatives under the attribute C_j, and each $C_1(\alpha_{ij}, \alpha_{kj})$ can be calculated by Eq. (13). As we all know that cross-entropy of an attribute is bigger across alternatives, it can provide DM with the useful information. Therefore, the attribute should be assigned a bigger weight.

If the information about weight $w_j = \frac{1-E(C_j)+C(C_j)}{\sum_{j=1}^{n}(1-E(C_j)+C(C_j))}, j = 1,2,\cdots,n$ of the attribute $w_j = \frac{1-E(C_j)+C(C_j)}{\sum_{j=1}^{n}(1-E(C_j)+C(C_j))}, j = 1,2,\cdots,n$ is completely unknown, we utilize the following entropy weight approach to determine attribute weights:

$$w_j = \frac{1-E(C_j)+C(C_j)}{\sum_{j=1}^{n}\left(1-E(C_j)+C(C_j)\right)}, j = 1,2,\cdots,n \tag{29}$$

If the information about weight $MaxE_W = \sum_{j=1}^{n} w_j\left(1-E(C_j)+C(C_j)\right)$ of the attribute $MaxE_W = \sum_{j=1}^{n} w_j\left(1-E(C_j)+C(C_j)\right)$ is partly known by DM, we construct the following optimization model to get the optimal weight vector:

$$MaxE_W = \sum_{j=1}^{n} w_j\left(1-E(C_j)+C(C_j)\right)$$

$$= \sum_{j=1}^{n} w_j\left(1 - \frac{1}{m}\sum_{i=1}^{m} E_1(\alpha_{ij}) + \frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{m-1}\sum_{k=1,k\neq i}^{m} C_1(\alpha_{ij}, \alpha_{kj})\right)\right)$$

$$s.t.\begin{cases} W \in \Omega, \\ \sum_{j=1}^{n} w_j = 1, \\ w_j \ge 0, j = 1,2,\cdots,n. \end{cases} \tag{30}$$

where Ω represents the set of incomplete information about attribute weights.

An approach to MADM with information measures

In MADM problems, the concepts of ideal and anti-ideal alternatives may not exist, but it does provide a useful tool to select the best alternative(s). Let $X^+ = \{\alpha_1^+, \alpha_2^+, \cdots, \alpha_n^+\}$ and $X^- = \{\alpha_1^-, \alpha_2^-, \cdots, \alpha_n^-\}$ be the ideal alternative and anti-ideal alternative, respectively, where $\alpha_j^+ = \langle 1,0,0 \rangle, \alpha_j^- = \langle 0,1,1 \rangle, j = 1,2, \cdots, n$.

Based on the above analysis, we develop an approach to MADM under single valued neutrosophic environment, the main steps are as follows:

Step 1: If all the attributes $C_j (j = 1,2, \cdots, n)$ are of the benefit types, then the attribute values need not be normalized. Otherwise, we transform the single valued neutrosophic decision matrix $D = (\alpha_{ij})_{m \times n}$ into the normalized single valued neutrosophic decision matrix $\widetilde{D} = (\tilde{\alpha}_{ij})_{m \times n}$, where

$$\tilde{\alpha}_{ij} = \begin{cases} \alpha_{ij}, & for\ benefit\ attribute\ C_j \\ \alpha_{ij}^c, & for\ cost\ attribute\ C_j \end{cases}, i = 1,2, \cdots, m, j = 1,2, \cdots, n \quad (31)$$

Step 2: Utilize Eq. (29) or model (30) to determine the weight vector of attribute: $\widetilde{D} = (\tilde{\alpha}_{ij})_{m \times n}$.

Step 3: Based on the decision matrix $\widetilde{D} = (\tilde{\alpha}_{ij})_{m \times n}$, we calculate the similarity measures between the alternative X_i and the ideal alternative X^+ and the anti-ideal alternative X^- as follows:

$$S^+(X_i) = \sum_{j=1}^{n} w_j S_1(\tilde{\alpha}_{ij}, \alpha_j^+), i = 1,2, \cdots, m \quad (32)$$

$$S^-(X_i) = \sum_{j=1}^{n} w_j S_1(\tilde{\alpha}_{ij}, \alpha_j^-), i = 1,2, \cdots, m \quad (33)$$

where $S_1(\tilde{\alpha}_{ij}, \alpha_j^+)$ and $S_1(\tilde{\alpha}_{ij}, \alpha_j^-)$ can be calculated by Eq. (7);

Step 4: Compute the closeness degree of the alternative X_i to the ideal alternative by using

$$T(X_i) = \frac{S^+(X_i)}{S^+(X_i) + S^-(X_i)}, i = 1,2, \cdots, m \quad (34)$$

Step 5: Rank all the closeness degree $T(X_i)(i = 1,2, \cdots, m)$ in descending order;

Step 6: Select the best alternative(s) in accordance with the closeness degree $T(X_i)$ $(i = 1,2, \cdots, m)$. The best alternative(s) is the one with $\max_i T(X_i)$;

Step 7: End.

RESULTS AND DISCUSSION

With the development of economy and acceleration of urbanization in China, fog-haze is becoming more and more frequent weather phenomenon, which is directly related to the ecological environment (Jin et al., 2016). Fog-haze pollution is particularly serious in some cities, especially Beijing (X_1), Shanghai (X_2), Wuhan (X_3), Nanjing (X_4), Guangzhou (X_5), and the scientists found that the fog-haze is evaluated by means of four main attributes, i.e., C_1: mass concentration of PM2.5, C_2: the concentration of gaseous pollutants, C_3: meteorological conditions and C_4: geographical conditions. Consider the case that the ministry of environmental protection of China wants to know the most serious city polluted by fog-haze according to the main attributes of fog-haze, then a domain committee evaluates the five cities $X_i (i = 1,2,3,4,5)$ with respect to the above four attribute $C_j (j = 1,2,3,4)$ using the SVNVs $\alpha_{ij} = \langle \alpha_1^{ij}, \alpha_2^{ij}, \alpha_3^{ij} \rangle$. The single valued neutrosophic decision matrix $D = (\alpha_{ij})_{5 \times 4}$ can be constructed as follows:

$$D = \begin{pmatrix} \langle 0.4,0.6,0.0 \rangle & \langle 0.3,0.2,0.5 \rangle & \langle 0.1,0.3,0.7 \rangle & \langle 0.4,0.3,0.3 \rangle \\ \langle 0.7,0.3,0.0 \rangle & \langle 0.2,0.2,0.6 \rangle & \langle 0.0,0.1,0.9 \rangle & \langle 0.1,0.1,0.8 \rangle \\ \langle 0.1,0.2,0.7 \rangle & \langle 0.2,0.4,0.4 \rangle & \langle 0.8,0.2,0.3 \rangle & \langle 0.2,0.3,0.6 \rangle \\ \langle 0.2,0.1,0.8 \rangle & \langle 0.2,0.4,0.5 \rangle & \langle 0.8,0.1,0.4 \rangle & \langle 0.2,0.2,0.7 \rangle \\ \langle 0.3,0.4,0.3 \rangle & \langle 0.6,0.3,0.1 \rangle & \langle 0.2,0.1,0.7 \rangle & \langle 0.2,0.2,0.6 \rangle \end{pmatrix}$$

In the following, to select the most serious city polluted by fog-haze, we utilize the proposed method to deal with this single valued neutrosophic MADM problems. The main steps are as follows:

Step 1: Since all the attributes $C_j (j = 1,2,3,4)$ are of the cost types, then we transform $D = (\alpha_{ij})_{5 \times 4}$ into the normalized single valued neutrosophic decision matrix $\widetilde{D} = (\tilde{\alpha}_{ij})_{5 \times 4}$ by using Eq. (31):

$$\widetilde{D} = \begin{pmatrix} \langle 0.6,0.4,1.0 \rangle & \langle 0.7,0.9,0.5 \rangle & \langle 0.9,0.7,0.3 \rangle & \langle 0.6,0.7,0.7 \rangle \\ \langle 0.3,0.7,1.0 \rangle & \langle 0.8,0.8,0.4 \rangle & \langle 1.0,0.9,0.1 \rangle & \langle 0.9,0.9,0.2 \rangle \\ \langle 0.9,0.8,0.3 \rangle & \langle 0.8,0.6,0.6 \rangle & \langle 0.2,0.8,0.7 \rangle & \langle 0.8,0.7,0.4 \rangle \\ \langle 0.8,0.9,0.2 \rangle & \langle 0.8,0.6,0.5 \rangle & \langle 0.2,0.9,0.6 \rangle & \langle 0.8,0.8,0.3 \rangle \\ \langle 0.7,0.6,0.7 \rangle & \langle 0.4,0.7,0.9 \rangle & \langle 0.8,0.9,0.3 \rangle & \langle 0.8,0.8,0.4 \rangle \end{pmatrix}$$

Step 2: Because the information about weight w_j of the attribute $C_j, j = 1,2,3,4$ is completely unknown, then we utilize Eq. (29) to calculate the weight vector of attribute as follows:

$w_1 = 0.2107, w_2 = 0.3011, w_3 = 0.1067, w_4 = 0.3815$.

Step 3: Utilize Eqs. (32) and (33) to determine the similarity measures between the city X_i and the ideal city X^+ and the anti-ideal city X^-:

$$S^+(X_1) = 0.3317, S^+(X_2) = 0.3845, S^+(X_3) = 0.5319, S^+(X_4) = 0.4486, S^+(X_5) = 0.3627,$$

$$S^-(X_1) = 0.4481, S^-(X_2) = 0.3977, S^-(X_3) = 0.2885, S^-(X_4) = 0.3419, S^-(X_5) = 0.4240.$$

Step 4: By using Eq. (34), we obtain the closeness degree $T(X_i)(i = 1,2,3,4,5)$ of the city X_i to the ideal city:

$$T(X_1) = 0.4254, T(X_2) = 0.4916, T(X_3) = 0.6483, T(X_4) = 0.5675, T(X_5) = 0.4610.$$

Step 5: Since $T(X_3) > T(X_4) > T(X_2) > T(X_5) > T(X_1)$, then we obtain the ranking of $X_i(i = 1,2,3,4,5)$ is $X_3 \succ X_4 \succ X_2 \succ X_5 \succ X_1$, and the most serious city polluted by fog-haze is X_3.

Step 6: End.

In the following, in order to validate the proposed MADM method, we conduct a comparative study with the method proposed by Ye and Fu (2016). Based on the tangent function and Hausdorff distance, Ye and Fu (2016) developed a new similarity measure to calculate the deviations between each alternative and the ideal alternative, and then obtain the most desirable alternative. Now, we utilize the method to deal with the aforementioned problem and select the most serious city polluted by fog-haze.

Step 1': See Step1.

Step 2': Utilizing the following similarity measure formula between SVNS A and B (i.e., Eq. (6)):

$$T(A,B) = 1 - \frac{1}{n}\sum_{j=1}^{n} tan\left(\frac{\pi}{4} \cdot max\{|T_A(x_j) - T_B(x_j)|, |I_A(x_j) - I_B(x_j)|, |F_A(x_j) - F_B(x_j)|\}\right) \quad (35)$$

we can obtain the similarity measures $T(X_i, X^+)(i = 1,2,3,4,5)$ between the city X_i and the ideal city X^+:

$$T(X_1, X^+) = 0.2301, T(X_2, X^+) = 0.1413, T(X_3, X^+) = 0.3561,$$
$$T(X_4, X^+) = 0.2639, T(X_5, X^+) = 0.2381.$$

Step 3': According to the results obtained by the similarity measures, we have

$$T(X_3, X^+) > T(X_4, X^+) > T(X_1, X^+) > T(X_5, X^+) > T(X_2, X^+),$$

then the ranking of all the cities $X_i(i = 1,2,3,4,5)$ is $X_3 \succ X_4 \succ X_1 \succ X_5 \succ X_2$. Therefore, the most serious city polluted by fog-haze is X_3.

Compared with the method proposed by Ye and Fu (2016), although the developed approach in this paper and that of Ye and Fu (2016) produce the same result that the most serious city polluted by fog-haze is X_3, the method generates a little different ranking of the five cities with our approach. In fact, according to the normalized single valued neutrosophic decision matrix $\widetilde{D} = (\tilde{\alpha}_{ij})_{5\times4}$, we have $\tilde{\alpha}_{21} < \tilde{\alpha}_{11}, \tilde{\alpha}_{22} > \tilde{\alpha}_{12}, \tilde{\alpha}_{23} > \tilde{\alpha}_{13}$, $\tilde{\alpha}_{24} > \tilde{\alpha}_{14}$ and $\tilde{\alpha}_{21} < \tilde{\alpha}_{51}, \tilde{\alpha}_{22} > \tilde{\alpha}_{52}, \tilde{\alpha}_{23} > \tilde{\alpha}_{53}, \tilde{\alpha}_{24} > \tilde{\alpha}_{54}$, which means that X_2 is preferred to X_1 and X_5. Therefore, our MADM approach is more rational than that of Ye and Fu (2016) in this case.

Through the above example, we find that compared with the method developed by Ye and Fu (2016) to derive the most serious city polluted by fog-haze, our proposed approach has some advantages.

(1) The application range of the MADM approach proposed in this paper is wider than that of existing methods, such as that of Ye and Fu (2016). Moreover, the proposed MADM method can manage problems in which the decision making information is SVNVs.

(2) Our approach focuses on the information measures, the method developed by Ye and Fu (2016) focuses on the distance measures which are based on the deviations among the decision information, and both of them are suitable to deal with the situations in which the weight vector of the alternatives is unknown. However, the ranking result obtained by the proposed MADM approach is reasonable and credible in some cases.

(3) In the process of decision making, the ranking results are obtained by the proposed MADM approach, which takes all the decision making information into account. However, that proposed by Ye and Fu (2016) leads to information loss, because the Hausdorff distance is used and some middle values are ignored. Therefore, the proposed MADM approach in this paper can derive the more accurate results.

(4) We obtain some interesting theorem results which indicate the close relationship among entropy, similarity measures and cross-entropy.

CONCLUSIONS

At present, many information measures are applied to MADM problems, but they could not be used to deal with the MADM problems with neutrosophic information. A single valued neutrosophic set is an instance of neutrosophic set which can be used in real scientific and engineering applications. Under single valued neutrosophic environment, this paper introduces three axiomatic definitions of information measures, including the entropy, similarity measures and cross-entropy. We construct some information measure formulas on the basis of the cosine function. Furthermore, the relationship among the single valued neutrosophic information measures is discussed. Moreover, we utilize the proposed information measures to develop an approach to cope with MADM problems. Finally, a numerical example is provided to demonstrate the effectiveness of the presented approach.

However, there are still lots of work to be done in our future research. Based on the results in this paper, we will focus on investigate single valued neutrosophic linguistic information measures and apply the single valued neutrosophic information measures to solve practical applications in other areas such as pattern recognition, information fusion system, and image processing.

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