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# Multi-expert performance evaluation of healthcare institutions using an integrated intuitionistic fuzzy AHP&DEA methodology

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ABSTRACT

Healthcare management and healthcare industry have been one of the popular and complex topics that many researchers and professionals have focused on. This paper proposes a new multi-expert fuzzy approach integrating intuitionistic fuzzy Data Envelopment Analysis (DEA) and intuitionistic fuzzy Analytic Hierarchy Process (IF-AHP) for solving the performance evaluation problem of healthcare institutions. In this paper, intuitionistic fuzzy sets (IFS) have been preferred since they simultaneously provide information on the membership, non-membership, and hesitancy functions. A real life problem is demonstrated to validate the proposed methodology. A total number of 16 hospitals operating in Istanbul have been analyzed based on a broad set of inputs and outputs. Then, a comparison with crisp DEA has been performed.



## 1. Introduction

Performance evaluation has a great importance for decision makers in healthcare resources allocation and service quality improvement [1]. The researchers highlighted that hospital performance evaluation has been gaining importance for a variety of reasons such as increasing health expenditures and competition in the health sector all around the world [2,3]. The World Health Report 2000 by The World Health Organization (WHO) has stated that there has been a widespread concern with regard to measuring health system performances among researchers, policy makers and managers of hospitals [4]. Hospitals are an important component of healthcare organizations and have a significant impact on the success of health care system of a country [5]. For efficient use of resources, measuring performances of the hospitals and improvements needed to be done are sine qua non [6]. Today, the health sector is one of the sectors getting the largest share from public spendings. As stated in the Universal Declaration of Human Rights (Article 25th), many governments invest large amounts on healthcare for sustainable and continuous improvement [7].

The performance of hospitals is measured by various methods in the literature [8]. Data Envelopment Analysis (DEA) is one of the frequently used and most popular approaches that measures the performance of similar units under a multi-output and multi-input environment. Traditional DEA is a deterministic method where all inputs and outputs are quantitative and identified with numerical values. However, there is an inherent complex and uncertain nature of real world applications. In case of imprecise and vague data, people are more likely to prefer making linguistic evaluations rather than using exact numerical values. The fuzzy set theory was initiated for dealing with uncertainty and ambiguity [9]. In the literature, there are different extensions of ordinary fuzzy sets such as type 2 fuzzy sets, fuzzy multi-sets, intuitionistic fuzzy sets, hesitant fuzzy sets, and Pythagorean fuzzy sets. All these extensions intend better representing the vague and imprecise information. Intuitionistic fuzzy sets whose components are membership, non-membership and hesitancy functions are one of the most used extensions for handling vagueness and impreciseness [10]. The superiority of intuitionistic fuzzy sets comes from their ability to deal with hesitancy of decision makers in their decisions. These type of sets successfully model the fuzziness and hesitancy based on well established mathematical theorems. In this study, we utilize intuitionistic fuzzy sets for handling vagueness, imprecision and hesitancy in the performance evaluation of hospitals.

Analytic Hierarchy Analysis (AHP), developed by Saaty [11], is a multi-criteria decision making method, which is based on pairwise comparisons among criteria and alternatives. AHP divides a huge problem into small and easy to handle problems. AHP is one of the most reliable methods in weighting criteria. Classical AHP uses a linguistic scale whose numerical values are between 1 and 9. However, according to Buckley [12], the representation of a linguistic term by an exact number may not reflect the de-

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cision maker's judgments in his/her mind. For instance, a linguistic evaluation such as "Very Strong Importance" is represented by 7 in the scale of classical AHP. However, the decision maker's judgment "Very Strong Importance" may not be clear enough to assign '7'. By "Very Strong Importance", the decision maker might mean *around seven* which corresponds to a fuzzy number such as (6.5, 7, 7.5). It provides a better representation of the decision maker's evaluation. Fuzzy sets are excellent tools for dealing with such type of uncertainties. On the other hand, in some of the fuzzy AHP methods, the evaluations are defuzzified at the initial stages of the method [13]. In those cases, loss of information is caused at the beginning and the advantage of better representation of the uncertainties are not achieved. Fuzzy AHP in our proposed method applies the defuzzification in the latest stage, which reserves the fuzzy information throughout the process. Besides, the intuitionistic fuzzy sets can reflect the hesitancy of decision makers whereas classical sets cannot deal with it.

This study proposes an integrated triangular intuitionistic fuzzy (TIF) AHP and TIF-DEA approach [14] for analyzing and solving hospital performance evaluation problem. Since the inputs and outputs have different impacts on the performances of hospitals, we used AHP method to weigh them. TIF-DEA can deal with both tangible and intangible inputs and outputs. Especially, it can simultaneously evaluate the efficiency of many alternatives. Therefore, we integrated DEA method with AHP method. The employed DEA method is based on Constant Returns to Scale (CRS) model since the scale of economies does not change as the sizes of hospitals increase. In the study, inputs and outputs are evaluated by TIF AHP and the weights obtained from TIF AHP are used in TIF-DEA analysis. To the best of the authors' knowledge, this may be the first study conducting a multi-expert decision making analysis by integrating fuzzy AHP and fuzzy DEA methods employing Triangular Intuitionistic Fuzzy Numbers and applying the proposed approach for a real life hospital performance evaluation problem. In the application section, 16 public and private healthcare institutions operating in a particular city are analyzed based on seven inputs and six outputs. In the study, all inputs and outputs are defined by using intuitionistic fuzzy sets. The findings are then compared with the ones derived from traditional DEA approach based on CRS model.

The rest of the paper is organized as follows: In Section 2, we provide a detailed literature review on performance evaluation of healthcare institutions particularly hospitals. In Section 3, we describe preliminaries on intuitionistic fuzzy sets. In Section 4, we present our intuitionistic fuzzy DEA model. In Section 5, we propose a new integrated multi-expert intuitionistic fuzzy AHP&DEA methodology. In Section 6, we apply the proposed fuzzy integrated methodology to a real life hospital performance evaluation problem and demonstrate the findings of the proposed approach. In this section, we also perform a comparative analysis, illustrate and discuss the results of the analysis with the findings of the proposed method. In the last section we conclude the paper and present future remarks.

## 2. Literature review on performance evaluation of healthcare institutions

There are a lot of papers on the performance evaluation of healthcare institutions in the literature. For instance, O'Neill et al. [15] made the first taxonomy on hospital efficiency by reviewing approximately 79 studies published between 1984 and 2004, and presented studies using Data Envelopment Analysis and other related techniques.

We first classified the papers which analyze healthcare efficiency evaluation problems into 2 groups. The primer is the studies employing classical DEA method and the next is the ones using the fuzzy set theory. The literature review highlights that the classical DEA and fuzzy DEA papers can be also divided into 3 sub-groups:

- The papers employing only classical or fuzzy DEA,
- The papers integrating classical or fuzzy DEA with another approach (stochastic, mathematical models, or simulation) and
- The papers proposing new and modified DEA models for the healthcare performance evaluation problem.

There are many studies applying classical DEA methods. Even though they can cope with large number of inputs and outputs, they are unable to handle the vagueness and impreciseness in the values of inputs and outputs. For instance, Kazley and Ozcan [16] employed Data Envelopment Analysis and windows analysis to analyze the relationship between electronic medical record use and efficiency for small, medium and large hospitals.

The papers integrating DEA models with other approaches can provide the necessary data for the parameters of DEA models. For example, the studies integrating Analytical Hierarchy Process (AHP) with DEA model obtain the weights of inputs & outputs from AHP method and use the weights in the DEA model. Lai et al. [17] suggested integrated knowledge-based system as an benchmarking tool using CCR (Charnes, Cooper and Rhodes) and BCC (Banker, Charnes and Cooper) DEA models for medical centers, followed by regional hospitals and district hospitals in Taiwan. Mitropoulos et al. [21] developed a new approach - a chance constrained DEA (CCDEA) model–integrated it with a stochastic mechanism from Bayesian techniques, and implemented the proposed approach for obtaining efficiency scores of 117 Greek public hospitals. Leleu et al. [19] aimed to assess hospital patient payer mix and technical efficiency and check variations in profits. The researchers applied a weighted DEA and used the data of 138 hospitals locating in Florida for the year 2005. Al-Refaie et al. [5] applied simulation and DEA for improving the performance of emergency department of a hospital in Jordan. For this aim, they focused on reducing the average waiting time of the patients in the emergency department, improving the utilization of nurses, and at the same time increasing the number of served patients. Fang and Li [20] evaluated facility location - allocation problems. The researchers utilized DEA for evaluating the relative efficiency of potential locations and used the solutions obtained from DEA as a goal in a multi objective decision making model. Mitropoulos et al. [18] presented a methodology which combined DEA and integer programming location allocation models as medium and long term decision tools. The authors evaluated efficiencies of health service providers, and used these efficiencies to determine locations of health providers and service allocations. Chowdhury and Zelenyuk [22] analyzed production performance of hospital services in Canada through DEA and used truncated regression estimation with double-bootstrap the significance of determinants. The analysis displayed that some factors such as occupancy rate, rate of unit-producing personnel, outpatient–inpatient ratio, geographic locations, size and teaching status were significant determinants of efficiency. Bahadori et al. [23] integrated DEA and PROMETHEE methods to evaluate performance evaluation of selective wards in a military hospital. Khushalani and Ozcan [24] focused on efficiency of producing quality and hospital sub-divisions from 2009 to 2013 by means of Dynamic Network DEA. In addition to the benefits of employing integrated approaches, this group of papers does not take into account the uncertainties both in inputs and outputs.

Studies modifying classical DEA method generally handle limited number of inputs and outputs related to healthcare problems. We can give Ouellette and Vierstraete [25] as an example of the studies proposing modified classical DEA. The researchers modified classical DEA approach by introducing quasi-fixed inputs and used the method for evaluating efficiency of hospital emergency services in Montreal. Wei et al. [26] applied Super-DEA-R-I

(DEA-R-based input oriented high-efficiency model) for analyzing the medical sectors in Taiwan.

On the other hand, there are also some studies implementing fuzzy methodology in performance evaluation problem in the healthcare literature. These studies aim to capture the vagueness in the evaluations of inputs and outputs and to incorporate the vagueness into the model. For instance, Ebrahimnejad [27] applied a fuzzy DEA approach where the input prices were defined with trapezoidal fuzzy numbers, for evaluating insurance organizations and hospitals. The authors employed the linear ranking functions when comparing the fuzzy numbers and considered four inputs namely "the number of personnel, the total number of computers, the area of the branch and administrative expenses" and four outputs which were "the total number of insured persons, the number of insured person's agreements, the total number of life-pension receivers and the receipt total sum". Costantino et al. [28] suggested using a cross-efficiency fuzzy DEA for solving healthcare systems performance evaluation problems in the southern part of Italy. Chang [29] concentrated on hospital service evaluation in Taiwan. The researcher analyzed two public and three private medical centers based on 33 evaluation criteria and proposed fuzzy VIKOR (Vise Kriterijumska Optimizacija I Kompromisno Resenje) approach with triangular fuzzy numbers.

In the literature, some other studies integrate fuzzy methods with other approaches. Khodaparasti and Maleki [30] presented a new combined fuzzy dynamic model for locating emergency vehicles and ambulance stations in an emergency medical service. In the study, the input and output parameters were defined as fuzzy numbers. Akdag et al. [31] evaluated the hospital service quality by integrating fuzzy AHP, fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Yager's min-max approach. Karadayi and Karsak [3] proposed an imprecise DEA for determining performances of state hospitals operating in 26 districts of Istanbul and obtained Optimistic and Pessimistic Scenario Efficiency Scores. In their study, three inputs which were "number of beds, number of overall staff and operating expenses" and five outputs such as "number of outpatients, number of discharged patients, tangibility and responsiveness" were evaluated. Muriana et al. [32] proposed an expert system for evaluating financial performances of health care structures employing the fuzzy set theory and key performance indicators. Arya and Yadav [33] proposed dual Slack Based Measure (SBM) model with fuzzy DEA and applied the developed model to the health sector. They evaluated the number of doctors, staff nurses and pharmacists as input variables and number of inpatients and outpatients as output variables which were identified with triangular fuzzy numbers. Ameryoun et al. [34] proposed a novel analysis based on DEA and SERVQUAL (Service Quality) for evaluating influence of service quality on hospitals.

Table 1 summarizes the studies

- Implementing only classical or fuzzy DEA approaches,
- Presenting integrated approaches and
- Proposing new and modified classical or fuzzy DEA approaches for measuring the efficiency of the healthcare actors such as hospitals and healthcare insurance companies.

This literature review shows us that healthcare performance evaluation is applied to various healthcare problems with different scopes and it requires both objective and subjective criteria to be used. Handling the vagueness in the subjective criteria is possible through the fuzzy set theory and ordinary fuzzy sets have been often utilized with this aim. The extensions of fuzzy sets have been recently utilized in performance evaluation [36,37]. Among fuzzy extensions, intuitionistic fuzzy sets are the most used extension.

## 3. Preliminaries: intuitionistic fuzzy sets

In the fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. However, the degree of non-membership of an element in a fuzzy set may not be equal to 1 minus the membership degree since there may be some hesitation degree. Therefore, a generalization of fuzzy sets was proposed by Atanassov [38] as intuitionistic fuzzy sets (IFS) which incorporate the degree of hesitation, which is defined as "1′' minus the sum of membership and non-membership degrees.

**Definition 1.** Let $X \neq \emptyset$ be a given set. An intuitionistic fuzzy set in $X$ is an object A given by

$$\tilde{A} = \left\{ \left\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x) \right\rangle ; x \epsilon X \right\}, \tag{1}$$

where $\mu_{\tilde{A}} : X \to [0, 1]$ and $v_{\tilde{A}} : X \to [0, 1]$ satisfy the condition

$$0 \le \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \le 1, \tag{2}$$

for every $x \epsilon X$. Hesitancy is equal to "1-($\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x)$)"

**Definition 2.** Intuitionistic fuzzy numbers (IFNs): An IFN $\tilde{A}$ is defined as follows:

(i) An intuitionistic fuzzy subset of the real number line
(ii) Normal, i.e., there is any $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_0) = 1$ (*so* $v_{\tilde{A}}(x_0) = 0$)
(iii) A convex set for the membership function $\mu_{\tilde{A}}(x)$, i.e.,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda) x_2) \ge min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right) \; \forall x_1, x_2 \in \mathbb{R}, \; \lambda \in [0, 1] \tag{3}$$

(iv) A concave set for the non-membership function $v_{\tilde{A}}(x)$, *i.e.*,

$$v_{\tilde{A}}(\lambda x_1 + (1-\lambda) x_2) \le max\left(v_{\tilde{A}}(x_1), v_{\tilde{A}}(x_2)\right) \; \forall x_1, x_2 \in \mathbb{R}, \; \lambda \in [0, 1]. \tag{4}$$

**Definition 3.** $\alpha$-cut of an intuitionistic fuzzy set is given by the following equations:

For an intuitionistic fuzzy set the $\alpha$-cut is defined by Atanassov [38] as the set

$$\tilde{A}_\alpha = \left\{ x \in X | \mu_{\tilde{A}}(x) \ge \alpha, v_{\tilde{A}}(x) \le 1 - \alpha \right\}. \tag{5}$$

**Definition 4.** Triangular Intuitionistic Fuzzy Numbers (TIFN): A TIFN $\tilde{A}$ is an intuitionistic fuzzy subset in $\mathbb{R}$ with following membership function and non-membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a^L}{a^M - a^L}, \; for\, a^L \le x \le a^M \\ \dfrac{a^U - x}{a^U - a^M}, \; for\, a^M \le x \le a^U \\ 0, \; otherwise \end{cases} \tag{6}$$

and

$$v_{\tilde{A}}(x) = \begin{cases} \dfrac{a^M - x}{a^M - a'^L}, \; for\, a'^L \le x \le a^M \\ \dfrac{x - a^M}{a'^U - a^M}, \; for\, a^M \le x \le a'^U \\ 1, \; otherwise \end{cases} \tag{7}$$

where $a'^L \le a^L \le a^M \le a^U \le a'^U$,

$0 \le \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \le 1$ and TIFN is denoted by $\widetilde{A}_{TIFN} = (a^L, a^M, a^U; a'^L, a^M, a'^U)$ (see Fig. 1).

If $\tilde{A}_{TIFN} = (a^L, a^M, a^U; a'^L, a^M, a'^U)$ and $\tilde{B}_{TIFN} = (b^L, b^M, b^U; b'^L, b^M, b'^U)$ are two TIFNs, then

**Addition** $\tilde{C} = \tilde{A} + \tilde{B}$ is also a TIFN:

$$\tilde{C} = \left( a^L + b^L, \; a^M + b^M, \; a^U + b^U; a'^L + b'^L, \; a^M + b^M, \; a'^U + b'^U \right). \tag{8}$$

**Table 1**
Studies implementing DEA for the healthcare sector.

| Authors | Application area | Classical DEA / Fuzzy DEA | Integrated Methods | Inputs/Outputs |
|---|---|---|---|---|
| Ouellette and Vierstraete [25] | Hospital emergency services in Montreal | Modified Classical DEA with quasi-fixed inputs (CRS and VRS) | NA | Inputs: Hours worked—excluding physicians—and expenditure on furniture and equipment), two quasi-fixed inputs (number of stretchers and full time equivalent number of physicians), Output: Number of cases |
| Kazley and Ozcan [16] | The relationship between electronic medical record use and efficiency for small, medium and large hospitals | Classical DEA (CRS) | DEA with windows analysis | Inputs: Equipment, doctors, and nurses Outputs: Discharges, inpatient days, and staff training |
| Wei et al. [26] | The medical sectors in Taiwan | Modified DEA-R model: Super-DEA-R-I (DEA-R-based input oriented high-efficiency model) | NA | Inputs: Sickbeds and physicians, Outputs: Out-patients, in-patients, and surgeries |
| Mitropoulos et al. [18] | Efficiencies of health service providers and location allocation models | Classical DEA (CCR) | DEA and Integer programming | Inputs: Number of doctors, Number of nurses, Treatment population Outputs: Medical exams, Laboratory tests, Transfers |
| Al-Refaie et al. [5] | Emergency department of a hospital in Jordan | Classical DEA (CCR) | Simulation and DEA | Inputs: Number of nurses, Average time in system Outputs: Nurses' utilization, Number of served patients |
| Leleu et al. [19] | Hospital patient payer mix and technical efficiency using the data of 138 hospitals in Florida for 2005 | Classical weighted DEA | Regression | Inputs: -Medical staff (FTE) -Others personnel (FTE) -Beds Outputs: -Case-mix adjusted total inpatient discharges -Outpatient in estimated days |
| Lindlbauer et al. [35] | German acute care hospitals | Classical DEA (BCC) | DEA with difference-in-difference estimation and genetic matching | Inputs:Physician, Nurse, Supplies, Admin, Nonclinical, clinical, BedsOutputs:The number of treated in patient cases |
| Chowdhury and Zelenyuk [22] | Production performance of hospital services in Canada | Classical DEA (CRS) | -Truncated regression estimation with double-bootstrap-distributional analysis | Inputs: Nursinghours Administrativehours Staffedbeds Medical surgical supplies costs (MSSC) Non-medical surgical supplies costs(NMSSC) Equipment expense(EE) MSSC+NMSSC MSSC+NMSSC+EE Outputs: Ambulatory visits Case-mix weighted inpatient days |
| Mitropoulos et al. [21] | Efficiency of 117 Greek public hospitals | Chance constrained DEA (CCDEA) model | Chance constrained DEA (CCDEA) model with a Bayesian technique | Inputs: Doctors, Other personnel, beds, operating cost Outputs: Inpatient admissions, outpatient visits |

(continued on next page)

**Table 1** (*continued*)

| Authors | Application area | Classical DEA / Fuzzy DEA | Integrated Methods | Inputs/Outputs |
|---|---|---|---|---|
| Lai et al. [17] | Medical centers, regional hospitals and district hospitals in Taiwan | CCR and BCC DEA models | Integrated knowledge-based systems as an benchmarking tool | Inputs:Number of physicians, number of nurses, number of expensive equipment, and diversity of expensive equipment.Outputs:Number of inpatients, number of outpatient, and quality assurance |
| Khushalani and Ozcan [24] | The efficiency of producing quality and hospital sub-divisions | Classical DEA | Dynamic Network DEA | Inputs:<br>Number of beds,<br>Total number of non-physician, Total operating expenses per bed.<br>Outputs:<br>Case-mix adjusted discharges,<br>Total number of surgeries,<br>Emergency visits,<br>Outpatient visits. |
| Ebrahimnejad [27] | Insurance Organizations and Hospitals | DEA (CCR) with trapezoidal fuzzy numbers | NA | Inputs:<br>The number of personals,<br>The total number of computers,<br>The area of the branch,<br>Administrative expenses,<br>Outputs:<br>The total number of insured persons,<br>The number of insured person's agreements,<br>The total number of life-pension receivers,<br>The receipt total sum |
| Costantino et al. [28] | Healthcare systems performance evaluation problem in the southern of Italy | Cross-efficiency fuzzy DEA | NA | Inputs:<br>Units of doctors,<br>Units of nurses,<br>Units of other staff,<br>Number of beds<br>Outputs:<br>Number of discharges,<br>Days of hospitalization,<br>Number of surgeries |
| Khodaparasti and Maleki [30] | Locating emergency vehicles and ambulance stations | Fuzzy DEA | A new combined fuzzy dynamic DEA model | Inputs:<br>Number of existing ambulances,<br>Number of stations,<br>Standard distance coverage,<br>Distances between demand points and sites<br>Outputs:<br>Number of treated patients |
| Karadayi and Karsak [3] | State hospitals operating in 26 districts of Istanbul | Fuzzy DEA | Imprecise DEA with Optimistic and Pessimistic Scenario | Inputs:<br>Number of beds,<br>Number of overall staff,<br>Operating expenses<br>Outputs:<br>Number of outpatients,<br>Number of discharged patients,<br>Number of adjusted surgeries,Tangibility, responsiveness |
| Muriana et al. [32] | Financial performances of health care structures | Fuzzy DEA | DEA with the fuzzy set theory and key performance indicators | |
| Arya and Yadav [33] | Application of the proposed model to the health sector | Fuzzy DEA | Dual slack based measure (SBM) model with fuzzy DEA | Inputs:<br>Sum of number of doctors and staff nurses,<br>Number of pharmacists<br>Output:<br>Number of inpatients<br>Number of outpatients. |

**Multiplication:** $\tilde{C} \cong \tilde{A} \otimes \tilde{B}$ is also a TIFN:

$$\tilde{C} \cong \left(a^L b^L,\ a^M b^M,\ a^U b^U; a'^L b'^L,\ a^M b^M,\ a'^U b'^U\right) \tag{9}$$

**Division:** $\tilde{C} \cong \tilde{A} \oslash \tilde{B}$ is also a TIFN:

$$\tilde{C} \cong \left(a^L / b^U,\ a^M / b^M,\ a^U / b^L; a'^L / b'^U,\ a^M / b^M,\ a'^U / b'^L\right) \tag{10}$$

**Multiplication with a constant:**

$$k \times \tilde{A}_{TIFN} = \left(k \times a^L, k \times a^M, k \times a^U; k \times a'^L, k \times a^M, k \times a'^U\right),\ \ k > 0 \tag{11}$$

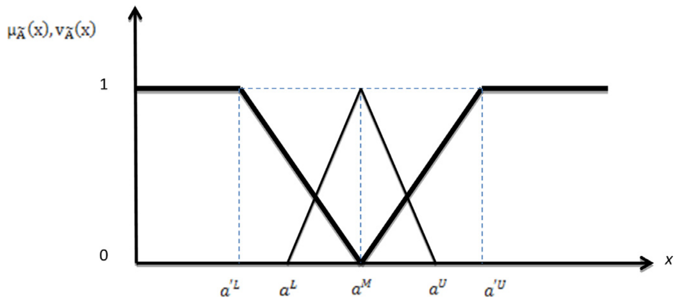


**Fig. 1.** Membership and non-membership functions of TIFN.

### 3.1. Defuzzification of TIFSs

Let $I_i = (a_i^L, a_i^M, a_i^U; a_i'^L, a_i^M, a_i'^U)$ be a triangular intuitionistic fuzzy number. Herein, we propose a new defuzzification function as in Eq. (12).

$$d_f = \frac{a_i^L + a_i^M + a_i^U}{3} + \frac{a_i'^L + a_i^M + a_i'^U}{\tau} \tag{12}$$

where $\tau$ is a very large number. This equation is based on the definition of accuracy function given in Zhang and Liu [39] and Liu et al. [40]. $\tau$ is the non-membership impact factor which determines the effect of non-membership function in the defuzzification. As it gets larger, the effect of non-membership function gets smaller. The magnitude of $\tau$ is determined by decision makers with respect to the type of decision making problem.

### 3.2. Aggregation operators for TIFNs

Suppose $I_i = (a_i^L, a_i^M, a_i^U; a_i'^L, a_i^M, a_i'^U)$ is a set of Triangular Intuitionistic Fuzzy Numbers, then the result is a triangular intuitionistic fuzzy number aggregated by employing Eq. (13) [39].

$$f_m(I_1, I_2, \ldots, I_n) = \left( \left[ 1 - \prod_{i=1}^{n} \left(1 - a_i^L\right)^{w_i},\ 1 - \prod_{i=1}^{n} \left(1 - a_i^M\right)^{w_i},\ 1 - \prod_{i=1}^{n} \left(1 - a_i^U\right)^{w_i} \right], \left[ \prod_{i=1}^{n} \left(a_i'^L\right)^{w_i},\ \prod_{i=1}^{n} \left(a_i^M\right)^{w_i},\ \prod_{i=1}^{n} \left(a_i'^U\right)^{w_i} \right] \right) \tag{13}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $I_i (i = 1,\ 2,\ \ldots, n)$, $w_i \in [0,\ 1]$, $\sum_{i=1}^{n} w_i = 1$.

## 4. Intuitionistic fuzzy Data Envelopment Analysis

Traditional DEA is an efficient performance evaluation method where there exists a multi-output and multi-input environment. Traditional DEA has been extended for various decision-making environments. For instance, Chance Constrained DEA and Stochastic Frontier Analysis DEA are some of these extensions. However, all inputs and outputs are quantitative and identified with numerical values in the traditional DEA method. Usually, the real world problems involve complexity, imprecision and uncertainty. The fuzzy set theory can capture this complexity and vagueness. After the fuzzy set theory has emerged, DEA has been extended to its fuzzy versions. Ordinary fuzzy DEA [41] and Type-2 fuzzy DEA [42] are two of these fuzzy extensions. Ordinary fuzzy sets force the decision makers to define a sharp membership function without considering their hesitancy. Intuitionistic fuzzy DEA can consider the hesitancy of decision makers in the evaluation process. In the following, we first briefly present traditional and ordinary fuzzy DEA and then propose our intuitionistic fuzzy DEA method.

### 4.1. Traditional DEA

The DEA model is used to evaluate the relative efficiency of a given number of homogeneous decision making units (DMUs) such as bank branches and hospitals taking into account multiple input and multiple output, and solve linear programming problems consecutively for each single DMU [43,44]. DEA is defined as a frontier estimator based on a linear programming approach [22]. DEA simultaneously analyzes the efficiency of each unit (here hospital) using inputs to generate outputs by defining the optimal input/output combination. This combination is described as the “best practice frontier” and/or data envelope [45].

In the traditional Data Envelopment Analysis literature, there are various well-known DEA approaches [14,46]. Constant Returns to Scale (CRS) DEA model is preferred when the scale of economies does not change as size of the service facility increases. If inputs and outputs can be selected in such a way that they conform to the proportionality assumption, the discrimination of the CRS model is better when it is compared to other DEA models [47]. In this study, we use input oriented CRS model for the performance comparison of the hospitals. The essence of the CRS model is the maximization of the ratio of weighted multiple outputs to weighted multiple inputs.

CRS model is given in the following [48]:

$$\begin{aligned}
&\min \theta \\
&s.t. \\
&\theta x_{i0} \geq \sum_{j=1}^{N} \lambda_j x_{ij} \\
&y_{r0} \leq \sum_{j=1}^{N} \lambda_j y_{rj} \\
&\lambda_j \geq 0 \quad \forall \mathrm{j} = 1, 2, \ldots, \mathrm{N} \\
&\sum_{\mathrm{j}=1}^{\mathrm{N}} \lambda_j = 1, \quad \theta \geq 0 \\
&i = 1, 2, .., m; \quad j = 1, 2, .., N; \quad r - 1, 2, .., s
\end{aligned} \tag{14}$$

where

| | |
|---|---|
| $x_{i0}$ | Amount of input $i$ for the $o^{\text{th}}$ observed decision making unit |
| $y_{r0}$ | Amount of output $r$ for the $o^{\text{th}}$ observed decision making unit |
| $x_{ij}$ | Amount of input $i$ for the $j^{\text{th}}$ decision making unit |
| $y_{rj}$ | Amount of output $r$ for the $j^{\text{th}}$ decision making unit |
| $\mu_r$ | Weight for the $r^{\text{th}}$ output given by $o^{\text{th}}$ decision making unit |
| $v_i$ | Weight for the $r^{\text{th}}$ input given by $o^{\text{th}}$ decision making unit |
| m | Number of inputs |
| s | Number of outputs |
| N | Number of decision making unit (DMU) |
| $\lambda_j$ | Shadow prices regarding the constraints limiting the efficiency of each DMU |
| $\theta$ | Efficiency. |

### 4.2. Ordinary fuzzy DEA

Kahraman et al. [41] presented one of the first studies proposing fuzzy DEA analysis. In the study, the authors proposed a fuzzy mathematical programming model with multiple attributes using data envelopment analysis. Guo and Tanaka [49] proposed the following fuzzy CCR DEA model to handle the efficiency evaluation problem with fuzzy input and output data.

$$\begin{aligned}&\max_{\mu,v} \mu^t Y_o\\ &s.t.\\ &v^t X_o \approx \tilde{1}\\ &\mu^t Y_j \lesssim v^t X_j \ (j = 1, 2, \ldots, n)\\ &\mu, v \geq 0\end{aligned} \tag{15}$$

where $X_j = (x_j, c_j)$ is an $s$-dimensional fuzzy input vector and $Y_j = (y_j, d_j)$ is an $m$-dimensional fuzzy output vector for the $j^{th}$ decision making unit.

The fuzzy model can be transformed to the following LP problem as in Eq. (16).

$$\begin{aligned}&\max_{\mu,v} \mu^t y_o - (1-h)\mu^t d_o\\ &s.t.\\ &v^t c_o \geq g_o\\ &v^t x_o - (1-h)v^t c_o = 1 - (1-h)e\\ &v^t x_o + (1-h)v^t c_o \leq 1 + (1-h)e\\ &\mu^t y_j - (1-h)\mu^t d_j \leq v^t x_j - (1-h)v^t c_j\\ &\mu^t y_j + (1-h)\mu^t d_j \leq v^t x_j + (1-h)v^t c_j \quad (j = 1, 2, \ldots, N)\\ &\mu, v \geq 0\end{aligned} \tag{16}$$

where $g_o$ indicates the optimal value of the objective function, $0 \leq h \leq 1$ is a predefined possibility level by decision makers, and e is equal to $\max_{j=1,2,\ldots,n}(\max_{k=1,2,\ldots,s} c_{jk}/x_{jk})$.

### 4.3. Proposed triangular intuitionistic fuzzy DEA

In case of vague information on inputs and outputs, intuitionistic fuzzy sets reflecting the hesitancy of decision makers can be utilized. The proposed intuitionistic fuzzy CRS DEA model is presented in Eq. (17).

$$\begin{aligned}&\min \theta\\ &s.t.\\ &\theta \tilde{x}_{i0} \geq \sum_{j=1}^{N} \lambda_j \tilde{x}_{ij}\\ &\tilde{y}_{r0} \leq \sum_{j=1}^{N} \lambda_j \tilde{y}_{rj}\\ &\lambda_j \geq 0 \ \forall j = 1, 2, \ldots, N\\ &\sum_{j=1}^{N} \lambda_j = 1\\ &\theta \geq 0\end{aligned} \tag{17}$$

where

$$\tilde{x}_{ij} = \left(x_{ij}^L, x_{ij}^M, x_{ij}^U; x_{ij}^{\prime L}, x_{ij}^{\prime M}, x_{ij}^{\prime U}\right)$$

$$\tilde{x}_{i0} = \left(x_{i0}^L, x_{i0}^M, x_{i0}^U; x_{i0}^{\prime L}, x_{i0}^{\prime M}, x_{i0}^{\prime U}\right)$$

$$\tilde{y}_{r0} = \left(y_{r0}^L, y_{r0}^M, y_{r0}^U; y_{r0}^{\prime L}, y_{r0}^{\prime M}, y_{r0}^{\prime U}\right)$$

$$\tilde{y}_{rj} = \left(y_{rj}^L, y_{rj}^M, y_{rj}^U; y_{rj}^{\prime L}, y_{rj}^{\prime M}, y_{rj}^{\prime U}\right)$$

Eq. (17) can be expressed as in Eq. (18).

$$\begin{aligned}&\min \theta\\ &s.t.\\ &\left(\theta x_{i0}^L, \theta x_{i0}^M, \theta x_{i0}^U; \theta x_{i0}^{\prime L}, \theta x_{i0}^{\prime M}, \theta x_{i0}^{\prime U}\right)\\ &\geq \left(\sum_{j=1}^{N} \lambda_j x_{ij}^L, \sum_{j=1}^{N} \lambda_j x_{ij}^M, \sum_{j=1}^{N} \lambda_j x_{ij}^U; \ \sum_{j=1}^{N} \lambda_j x_{ij}^{\prime L}, \sum_{j=1}^{N} \lambda_j x_{ij}^{\prime M}, \sum_{j=1}^{N} \lambda_j x_{ij}^{\prime U}\right)\\ &\left(y_{r0}^L, y_{r0}^M, y_{r0}^U; y_{r0}^{\prime L}, y_{r0}^{\prime M}, y_{r0}^{\prime U}\right)\\ &\leq \left(\sum_{j=1}^{N} \lambda_j y_{rj}^L, \sum_{j=1}^{N} \lambda_j y_{rj}^M, \sum_{j=1}^{N} \lambda_j y_{rj}^U; \sum_{j=1}^{N} \lambda_j y_{rj}^{\prime L}, \sum_{j=1}^{N} \lambda_j y_{rj}^{\prime M}, \sum_{j=1}^{N} \lambda_j y_{rj}^{\prime U}\right)\\ &\lambda_j \geq 0 \quad \forall j = 1, 2, \ldots, N\\ &\sum_{j=1}^{N} \lambda_j = 1\\ &\theta \geq 0\end{aligned} \tag{18}$$

## 5. A new integrated intuitionistic fuzzy AHP & DEA methodology

The proposed methodology is composed of an integrated TIF AHP and TIF DEA methods. The criteria, inputs and outputs have been determined based on a comprehensive literature review and viewpoints of the managers of hospitals. The flowchart of the proposed methodology is displayed in Fig. 2.

The steps of the proposed methodology with an illustrative example (IE) are stated as follows:

**Step 1**: Apply the triangular intuitionistic fuzzy AHP method.

**Step 1.1:** Define the problem and establish the hierarchy, goal being at the top, criteria and sub-criteria at the intermediate level and alternatives at the lowest level.

**IE 1.1:** Consider that we have two alternatives (m = 2), two inputs (t = 2) and two outputs (u = 2) in a performance evaluation problem. In the illustrative example, we assume that there is a consensus among the experts.

**Step 1.2:** Construct pairwise comparison matrices for inputs and outputs, and collect expert judgments using TIF scale given in Table 2.

The pairwise evaluation matrix of each expert for obtaining the weights of inputs is as in Eq. (19).

$$\tilde{A}_i^{TIFN} = \begin{bmatrix} (1,1,1;1,1,1) & \tilde{a}_{12,i}^{TIFN} & \cdots & \tilde{a}_{1t,i}^{TIFN}\\ 1/\tilde{a}_{12,i}^{TIFN} & (1,1,1;1,1,1) & \cdots & \tilde{a}_{2t,i}^{TIFN}\\ \vdots & \vdots & \ddots & \vdots\\ 1/\tilde{a}_{1t,i}^{TIFN} & 1/\tilde{a}_{2t,i}^{TIFN} & \cdots & (1,1,1;1,1,1)\end{bmatrix} \tag{19}$$

where $t$ denotes the number of inputs.

The pairwise evaluation matrix of each expert for obtaining the weights of outputs is given in Eq. (20).

$$\tilde{A}_o^{TIFN} = \begin{bmatrix} (1,1,1;1,1,1) & \tilde{a}_{12,o}^{TIFN} & \cdots & \tilde{a}_{1u,o}^{TIFN}\\ 1/\tilde{a}_{12,o}^{TIFN} & (1,1,1;1,1,1) & \cdots & \tilde{a}_{2u,o}^{TIFN}\\ \vdots & \vdots & \ddots & \vdots\\ 1/\tilde{a}_{1u,o}^{TIFN} & 1/\tilde{a}_{2u,o}^{TIFN} & \cdots & (1,1,1;1,1,1)\end{bmatrix} \tag{20}$$

where $u$ denotes the number of outputs.

In Eq. (19), $\tilde{a}_{12,i}^{TIFN}$, for instance, is as follows:

$$\tilde{a}_{12,i}^{TIFN} = \left(a_{12,i}^L, a_{12,i}^M, a_{12,i}^U; a_{12,i}^{\prime L}, a_{12,i}^M, a_{12,i}^{\prime U}\right)$$

and

$$1/\tilde{a}_{12,i}^{TIFN} = \left(\frac{1}{a_{12,i}^U}, \frac{1}{a_{12,i}^M}, \frac{1}{a_{12,i}^L}; \ \frac{1}{a_{12,i}^{\prime U}}, \frac{1}{a_{12,i}^M}, \frac{1}{a_{12,i}^{\prime L}}\right)$$

**IE 1.2:** The pairwise comparison matrices produced by three experts (Exp) including both inputs and outputs are exhibited below.

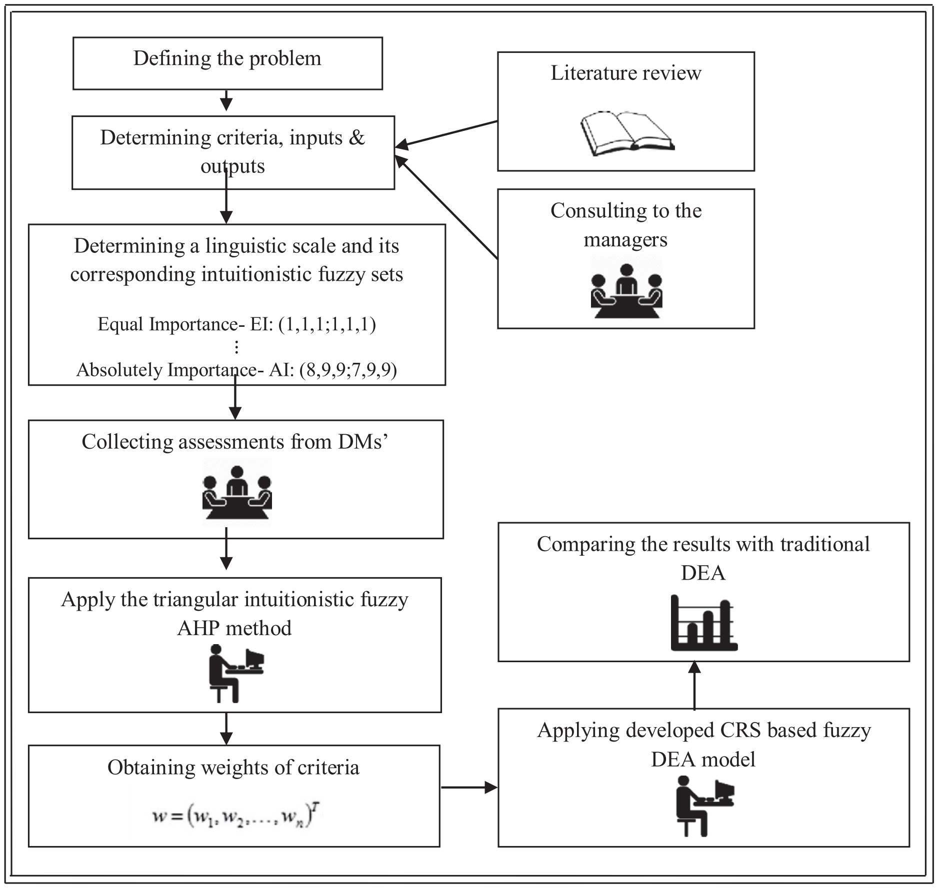


**Fig. 2.** Flowchart of the proposed methodology.

**Table 2**
Scale for pairwise comparisons.

| Importance level | Corresponding intuitionistic fuzzy sets | Reciprocal importance level | Reciprocal intuitionistic fuzzy sets |
|---|---|---|---|
| Equal Importance (EI) | (1, 1, 1; 1, 1, 1) | REI | (1, 1, 1; 1, 1, 1) |
| Weak Importance (WI) | (2, 3, 4; 1, 3, 5) | RWI | $(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}; \frac{1}{5}, \frac{1}{3}, 1)$ |
| Fairly Strong Importance (FSI) | (4, 5, 6; 3, 5, 7) | RFSI | $(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}; \frac{1}{7}, \frac{1}{5}, \frac{1}{3})$ |
| Very Strong Importance (VSI) | (6, 7, 8; 5, 7, 9) | RVSI | $(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}; \frac{1}{9}, \frac{1}{7}, \frac{1}{5})$ |
| Absolute Importance (AI) | (8, 9, 9; 7, 9, 9) | RAI | $(\frac{1}{9}, \frac{1}{9}, \frac{1}{8}; \frac{1}{9}, \frac{1}{9}, \frac{1}{7})$ |

| Inputs | I1 | | | I2 | | | Outputs | O1 | | | O2 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Exp1 | Exp2 | Exp3 | Exp1 | Exp2 | Exp3 | | Exp1 | Exp2 | Exp3 | Exp1 | Exp2 | Exp3 |
| **I1** | EI | EI | EI | WI | FSI | WI | **O1** | EI | EI | EI | FSI | FSI | VSI |
| **I2** | RWI | RFSI | RWI | EI | EI | EI | **O2** | RFSI | RFSI | RVSI | EI | EI | EI |

**Step 1.3:** Examine consistency of the fuzzy pairwise comparison matrices. To this end, the comparison matrix is defuzzified and checked for consistency. If any inconsistency is detected, then the matrix is reformed again.

**IE 1.3:** Using Eq. (12), we defuzzified and checked the consistency of the pairwise comparison matrices in IE 1.2, and found that the comparison matrices are consistent. In fact, a $2 \times 2$ pairwise comparison matrix is always consistent and there is no need to calculate the consistency for this small example.

**Step 1.4:** Calculate geometric mean of each row ($r$) in $\tilde{A}_i^{TIFN}$ and $\tilde{A}_o^{TIFN}$ matrices using Eqs. (21) and (22).

$$\tilde{g}_r = \left[\tilde{a}_{r1}^{TIFN} \otimes \ldots \otimes \tilde{a}_{rn}^{TIFN}\right]^1 / n \tag{21}$$

where

$$\tilde{g}_r^{TIFN} = \left( \begin{array}{c} \left(\prod_{j=1}^{n} a_{rj}^{L}\right)^{\frac{1}{n}}, \left(\prod_{j=1}^{n} a_{rj}^{M}\right)^{\frac{1}{n}}, \left(\prod_{j=1}^{n} a_{rj}^{U}\right)^{\frac{1}{n}}; \\ \left(\prod_{j=1}^{n} a_{rj}^{\prime L}\right)^{1/n}, \left(\prod_{j=1}^{n} a_{rj}^{M}\right)^{1/n}, \left(\prod_{j=1}^{n} a_{rj}^{\prime U}\right)^{1/n} \end{array} \right) \tag{22}$$

**IE 1.4:** Employing Eq. (22), the geometric mean of pairwise comparison matrices for inputs and outputs are obtained as follows:

| Inputs | I1 | I2 |
|---|---|---|
| **I1** | (1,1,1;1,1,1) | (2.52,3.56,4.58;1.44,3.56,5.59) |
| **I2** | (0.22,0.28,0.4;0.18,0.28,0.69) | (1,1,1;1,1,1) |
| Outputs | O1 | O2 |
| **O1** | (1,1,1;1,1,1) | (4.58,5.59,6.6;3.56,5.59,7.61) |
| **O2** | (0.15,0.18,0.22;0.13,0.18,0.28) | (1,1,1;1,1,1) |

**Step 1.5:** Calculate triangular intuitionistic fuzzy weights of the $r^{\text{th}}$ input ($\tilde{w}_{r,i}^{TIFN}$) and the $r^{\text{th}}$ output ($\tilde{w}_{r,o}^{TIFN}$) using Eqs. (23) and (24), respectively.

$$\tilde{w}_{r,i}^{TIFN} = \tilde{g}_{r,i}^{TIFN} \otimes \left[\tilde{g}_{1,i}^{TIFN} \oplus \ldots \oplus \tilde{g}_{2,i}^{TIFN} \oplus \ldots \oplus \tilde{g}_{t,i}^{TIFN}\right]^{-1} \tag{23}$$

$$\tilde{w}_{r,o}^{TIFN} = \tilde{g}_{r,o}^{TIFN} \otimes \left[\tilde{g}_{1,o}^{TIFN} \oplus \ldots \oplus \tilde{g}_{2,o}^{TIFN} \oplus \ldots \oplus \tilde{g}_{u,o}^{TIFN}\right]^{-1} \tag{24}$$

**IE 1.5:** Applying Eqs. (23) and (24) we obtain the following TIF weights of inputs and outputs:

| Inputs | I1 | I2 |
|---|---|---|
| **I1** | (0.72,0.78,0.82;0.59,0.78,0.85) | (0.45,0.78,1.30;0.22,0.78,2.29) |
| **I2** | (0.16,0.22,0.33;0.11,0.22,0.59) | (0.18,0.22,0.28;0.15,0.22,0.41) |
| Outputs | O1 | O2 |
| **O1** | (0.82,0.85,0.87;0.78,0.85,0.88) | (0.60,0.85,1.18;0.41,0.85,1.67) |
| **O2** | (0.12,0.15,0.19;0.1,0.15,0.25) | (0.13,0.15,0.18;0.12,0.15,0.22) |

**Step 1.6:** Defuzzify fuzzy weights to determine importance weights of inputs and outputs using Eq. (12).
**IE 1.6:** Then, we calculated the following normalized and defuzzified TIF weights of inputs and outputs:

| Inputs | $w_{r,i}{}^{TIFN}$ |
|---|---|
| **I1** | 0.78 |
| **I2** | 0.22 |
| Outputs | $w_{r,o}{}^{TIFN}$ |
| **O1** | 0.85 |
| **O2** | 0.15 |

**Step 2:**

**Step 2.1**: Use the scale given in Table 3 to assign the triangular intuitionistic fuzzy preferences into inputs and outputs matrices.

**IE 2.1:** Input and output evaluations of alternatives using Table 3 are presented below:

| Alternatives | I1 | I2 | O1 | O2 |
|---|---|---|---|---|
| **A1** | VH | H | VL | H |
| **A2** | MH | ML | MH | AH |

**Table 3**
Linguistic scale and its corresponding TIFN.

| Linguistic terms | Membership & non-membership |
|---|---|
| Absolutely Low (AL) | (10,18,25;65,70,75) |
| Very Low (VL) | (15,23,30;60,65,70) |
| Low (L) | (20,28,35;55,60,65) |
| Medium Low (ML) | (25,33,40;50,55,60) |
| Equal (E) | (45,50,55;30,38,45) |
| Medium High (MH) | (50,55,60;25,33,40) |
| High (H) | (55,60,65;20,28,35) |
| Very High (VH) | (60,65,70;15,23,30) |
| Absolutely High (AH) | (65,70,75;10,18,25) |
| Exactly Equal (EE) | (50,50,50;50,50,50) |

**Step 2.2:** Obtain TIF decision matrix for each decision maker as illustrated in Eq. (25). This matrix includes the whole inputs and outputs of the DEA model.

$$\tilde{D}_k^{TIFN} = \begin{bmatrix} \tilde{d}_{11,k}^{TIFN} & \tilde{d}_{12,k}^{TIFN} & \cdots & \tilde{d}_{1(n-1),k}^{TIFN} & \tilde{d}_{1n,k}^{TIFN} \\ \tilde{d}_{21,k}^{TIFN} & \tilde{d}_{22,k}^{TIFN} & \cdots & \tilde{d}_{2(n-1),k}^{TIFN} & \tilde{d}_{2n,k}^{TIFN} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{d}_{m1,k}^{TIFN} & \tilde{d}_{m2,k}^{TIFN} & \cdots & \tilde{d}_{m(n-1),k}^{TIFN} & \tilde{d}_{mn,k}^{TIFN} \end{bmatrix}$$

$$\text{for decision maker } k \tag{25}$$

where $m$ is the number of alternatives and $n$ is the total number of inputs and outputs.

**IE 2.2:** Using Table 3, linguistic values are converted to the fuzzy values in the DEA input & output matrix.

| Alternatives | I1 | I2 | O1 | O2 |
|---|---|---|---|---|
| **A1** | (60,65,70;15,23,30) | (55,60,65;20,28,35) | (15,23,30;60,65,70) | (55,60,65;20,28,35) |
| **A2** | (50,55,60;25,33,40) | (25,33,40;50,55,60) | (50,55,60;25,33,40) | (65,70,75;10,18,25) |

**Step 2.3:** Normalize experts' evaluations in the DEA input & output matrices.

$$Norm\ \tilde{d}_{ij}^{TIFN} = \left[\frac{\left(d_{ij}^{L}, d_{ij}^{M}, d_{ij}^{U}; d_{ij}^{\prime L}, d_{ij}^{M}, d_{ij}^{\prime U}\right)}{\max\limits_{\forall i,j} d_{ij}^{\prime U}}\right] \tag{26}$$

**IE 2.3:** The DEA input & output matrix is normalized using Eq. (26) by dividing the values to the maximum value in the linguistic scale.

| Alternatives | I1 | I2 |
|---|---|---|
| **A1** | (0.8,0.87,0.93;0.2,0.31,0.4) | (0.73,0.8,0.87;0.27,0.37,0.47) |
| **A2** | (0.67,0.73,0.8;0.33,0.44,0.53) | (0.33,0.44,0.53;0.67,0.73,0.8) |
| Alternatives | O1 | O2 |
| **A1** | (0.2,0.31,0.4;0.8,0.87,0.93) | (0.73,0.8,0.87;0.27,0.37,0.47) |
| **A2** | (0.67,0.73,0.8;0.33,0.44,0.53) | (0.87,0.93,1;0.13,0.24,0.33) |

**Step 2.4:** Aggregate the normalized input & output matrices considering the weights of experts based on Eq. (13).

$$\tilde{D}_{ag}^{TIFN} = \begin{bmatrix} \tilde{d}_{11}^{TIFN} & \tilde{d}_{12}^{TIFN} & \cdots & \tilde{d}_{1(n-1)}^{TIFN} & \tilde{d}_{1n}^{TIFN} \\ \tilde{d}_{21}^{TIFN} & \tilde{d}_{22}^{TIFN} & \cdots & \tilde{d}_{2(n-1)}^{TIFN} & \tilde{d}_{2n}^{TIFN} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{d}_{m1}^{TIFN} & \tilde{d}_{m2}^{TIFN} & \cdots & \tilde{d}_{m(n-1)}^{TIFN} & \tilde{d}_{mn}^{TIFN} \end{bmatrix} \tag{27}$$

**IE 2.4:** Since there is a consensus among the experts, aggregation operations are not needed. So, this step is not applied in the illustrative example.

**Step 2.5:** Obtain the weighted aggregated input & output matrix by multiplying the matrix in Step 2.4 by the weights of inputs & outputs derived from TIF AHP.

$$\tilde{D}_{ag,w}^{TIFN} = \begin{bmatrix} w_1 \times \tilde{d}_{11}^{TIFN} & w_2 \times \tilde{d}_{12}^{TIFN} & \cdots & w_{(n-1)} \times \tilde{d}_{1(n-1)}^{TIFN} & w_n \times \tilde{d}_{1n}^{TIFN} \\ w_1 \times \tilde{d}_{21}^{TIFN} & w_2 \times \tilde{d}_{22}^{TIFN} & \cdots & w_{(n-1)} \times \tilde{d}_{2(n-1)}^{TIFN} & w_n \times \tilde{d}_{2n}^{TIFN} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ w_1 \times \tilde{d}_{m1}^{TIFN} & w_2 \times \tilde{d}_{m2}^{TIFN} & \cdots & w_{(n-1)} \times \tilde{d}_{m(n-1)}^{TIFN} & w_n \times \tilde{d}_{mn}^{TIFN} \end{bmatrix}$$

$$\tilde{D}_{ag,w}^{TIFN} = \begin{bmatrix} \tilde{d}_{11}^{i} & \cdots & \tilde{d}_{1t}^{i} & \tilde{d}_{11}^{o} & \cdots & \tilde{d}_{1u}^{o} \\ \tilde{d}_{21}^{i} & \cdots & \tilde{d}_{2t}^{i} & \tilde{d}_{21}^{o} & \cdots & \tilde{d}_{2u}^{o} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{d}_{m1}^{i} & \cdots & \tilde{d}_{mt}^{i} & \tilde{d}_{m1}^{o} & \cdots & \tilde{d}_{mu}^{o} \end{bmatrix} \tag{28}$$

where $t + u = n$.

**Table 4**
Explanations of inputs & outputs.

| | Inputs/Outputs | Explanations | References |
|---|---|---|---|
| **INPUTS** | I1: Patient care supplies and other expenses | This input is related to expenses including patient care supplies and other expenses. | Chirikos and Sear [50]; O'Neill et al. [15] |
| | I2: Number of beds | This input pertains to the total number of beds that a hospital possesses. | Karadayi and Karsak [3]; Retzlaff-Roberts et al. [4]; O'Neill et al. [15] |
| | I3: Number of physicians | This input is related to the total number of physicians in each public or private hospital. | Retzlaff-Roberts et al. [4]; O'Neill et al. [15] |
| | I4: Number of nurses | This input is with regard to the total number of nurses working in each public or private hospital. | Lo et al. [51]; O'Neill et al. [15] |
| | I5: Number of other personnel | This input is relevant to the total number of other personnel such as secretary working in each public or private hospital. | Karadayi and Karsak [3]; O'Neill et al. [15] |
| | I6: Technology level | This input indicates capabilities of technology facilities in a hospital. | Jiang et al. [52]; Gok and Sezen [53]; |
| | I7: Service mix | This input represents a range of medical offerings of a hospital. | Harris II et al. [54] |
| **OUTPUTS** | O1: Annual revenues | This output includes annual revenues of hospitals. | Ayanoğlu et al. [6] |
| | O2: Number of outpatient department visits | This output is about the total number of outpatient department visits in each public or private hospital. | Karadayi and Karsak [3]; O'Neill et al. [15]; Chowdhury and Zelenyuk [22] |
| | O3: Overall patient satisfaction | This output describes overall satisfaction of each customer served by each hospital. | Gok and Sezen [53]; Hod et al. [55] Benneyan and Ceyhan [56] |
| | O4: Number of inpatient department admissions | This output informs the total number of inpatient department visits in each public or private hospital. | O'Neill et al. [15]; Jat and Sebastian [57] |
| | O5: Conformance to quality procedures | This output explains conformance level of each hospital with regard to quality procedures. | Morey et al. [58] |
| | O6: Bed usage rate (%) | This output is related to the bed usage rate of each evaluated hospital. | Papadaki et al. [59]; Papadaki et al. [60] |

**Table 5**
The pairwise comparisons of inputs.

| Expert 1 | I1 | I2 | I3 | I4 | I5 | I6 | I7 |
|---|---|---|---|---|---|---|---|
| I1 | EI | WI | RAI | RFSI | WI | RVSI | RWI |
| I2 | RWI | EI | RAI | RFSI | REI | RVSI | RFSI |
| I3 | AI | AI | EI | WI | RFSI | WI | FSI |
| I4 | FSI | FSI | RWI | EI | WI | RWI | WI |
| I5 | RWI | EI | FSI | RWI | EI | RVSI | RFSI |
| I6 | VSI | VSI | RWI | WI | VSI | EI | WI |
| I7 | WI | FSI | RFSI | RWI | FSI | RWI | EI |
| **Expert 2** | **I1** | **I2** | **I3** | **I4** | **I5** | **I6** | **I7** |
| I1 | EI | VSI | RWI | RWI | FSI | RAI | RFSI |
| I2 | RVSI | EI | RAI | RFSI | AI | RVSI | RFSI |
| I3 | WI | AI | EI | FSI | RWI | FSI | VSI |
| I4 | WI | FSI | RFSI | EI | WI | RWI | FSI |
| I5 | RFSI | RAI | WI | RWI | EI | RAI | RFSI |
| I6 | AI | VSI | RFSI | WI | AI | EI | AI |
| I7 | FSI | FSI | RVSI | RFSI | FSI | RAI | EI |
| **Expert 3** | **I1** | **I2** | **I3** | **I4** | **I5** | **I6** | **I7** |
| I1 | EI | FSI | RWI | RVSI | VSI | RFSI | RAI |
| I2 | RFSI | EI | RAI | RFSI | AI | RAI | RVSI |
| I3 | WI | AI | EI | FSI | RWI | FSI | VSI |
| I4 | VSI | FSI | RFSI | EI | WI | RFSI | WI |
| I5 | RVSI | RAI | WI | RWI | EI | RVSI | RFSI |
| I6 | FSI | AI | RFSI | FSI | VSI | EI | FSI |
| I7 | AI | VSI | RVSI | RWI | FSI | RFSI | EI |

**Table 6**
The pairwise comparisons of outputs.

| Expert 1 | O1 | O2 | O3 | O4 | O5 | O6 |
|---|---|---|---|---|---|---|
| O1 | EI | FSI | RWI | WI | REI | VSI |
| O2 | RFSI | EI | RAI | RWI | RFSI | WI |
| O3 | WI | AI | EI | VSI | FSI | AI |
| O4 | RWI | WI | RVSI | EI | RWI | FSI |
| O5 | EI | FSI | RFSI | WI | EI | VSI |
| O6 | RVSI | RWI | RAI | RFSI | RVSI | EI |
| **Expert 2** | **O1** | **O2** | **O3** | **O4** | **O5** | **O6** |
| O1 | EI | VSI | RFSI | FSI | WI | FSI |
| O2 | RVSI | EI | RAI | RFSI | RVSI | WI |
| O3 | FSI | AI | EI | FSI | VSI | RAI |
| O4 | RFSI | FSI | RFSI | EI | RFSI | VSI |
| O5 | RWI | VSI | RVSI | FSI | EI | AI |
| O6 | RFSI | RWI | AI | RVSI | RAI | EI |
| **Expert 3** | **O1** | **O2** | **O3** | **O4** | **O5** | **O6** |
| O1 | EI | FSI | WI | REI | WI | AI |
| O2 | RFSI | EI | AI | RWI | RFSI | WI |
| O3 | RWI | RAI | EI | AI | FSI | VSI |
| O4 | EI | WI | RAI | EI | RWI | FSI |
| O5 | RWI | FSI | RFSI | WI | EI | VSI |
| O6 | RAI | RWI | RVSI | RFSI | RVSI | EI |

**IE 2.5:** The weighted input & output matrix is obtained as follows:

| Alternatives | I1 | I2 |
|---|---|---|
| **A1** | (0.62,0.68,0.73;0.16,0.24,0.31) | (0.16,0.18,0.19;0.06,0.08,0.1) |
| **A2** | (0.52,0.57,0.62;0.26,0.34,0.41) | (0.07,0.1,0.12;0.15,0.16,0.18) |
| **Alternatives** | **O1** | **O2** |
| **A1** | (0.17,0.26,0.34;0.68,0.74,0.79) | (0.11,0.12,0.13;0.04,0.06,0.07) |
| **A2** | (0.57,0.62,0.68;0.28,0.37,0.45) | (0.13,0.14,0.15;0.02,0.04,0.05) |

**Table 7**
The weights of inputs & outputs.

| Weights of inputs | | Weights of outputs | |
|---|---|---|---|
| I1 | 0.068 | O1 | 0.275 |
| I2 | 0.035 | O2 | 0.065 |
| I3 | 0.283 | O3 | 0.309 |
| I4 | 0.165 | O4 | 0.113 |
| I5 | 0.045 | O5 | 0.199 |
| I6 | 0.292 | O6 | 0.039 |
| I7 | 0.112 | | |

**Table 8**
Evaluations of experts for DEA inputs.

| Hospital | I1 | I2 | I3 | I4 | I5 | I6 | I7 |
|---|---|---|---|---|---|---|---|
| H1 | (52,60,68;50,60,70) | (1530,1535,1540;1523,1535,1547) | (1780,1787,1794;1775,1787,1799) | (748,753,758;743,753,763) | (942,950,958;940,950,960) | (60,65,70;15,23,30) | (60,65,70;15,23,30) |
| | (53,5,61,5,69,5;51,5,61,5,71,5) | (1530,1535,1540;1523,1535,1547) | (1777,1784,1791;1772,1784,1796) | (753,758,763;748,758,768) | (944,952,960;942,952,962) | (60,65,70;15,23,30) | (60,65,70;15,23,30) |
| | (54,5,62,5,70,5;52,5,62,5,72,5) | (1525,1530,1535;1518,1530,1542) | (1778,1785,1792;1773,1785,1797) | (750,755,760;745,755,765) | (937,945,953;935,945,955) | (60,65,70;15,23,30) | (55,60,65;20,28,35) |
| H2 | (47,55,63;45,55,65) | (1370,1375,1380;1363,1375,1387) | (1493,1500,1507;1488,1500,1512) | (690,695,700;685,695,705) | (877,885,893;875,885,895) | (55,60,65;20,28,35) | (60,65,70;15,23,30) |
| | (48,56,64;46,56,66) | (1373,1378,1383;1366,1378,1390) | (1488,1495,1502;1483,1495,1507) | (695,700,705;690,700,710) | (879,887,895;877,887,897) | (60,65,70;15,23,30) | (65,70,75;10,18,25) |
| | (49,8,57,8,65,8;47,8,57,8,67,8) | (1375,1380,1385;1368,1380,1392) | (1497,1504,1511;1492,1504,1516) | (695,700,705;690,700,710) | (872,880,888;870,880,890) | (60,65,70;15,23,30) | (55,60,65;20,28,35) |
| H3 | (21,29,37;19,29,39) | (214,219,224;207,219,231) | (343,350,357;338,350,362) | (160,165,170;155,165,175) | (427,435,443;425,435,445) | (60,65,70;15,23,30) | (50,55,60;25,33,40) |
| | (23,5,31,5,39,5;21,5,31,5,41,5) | (217,222,227;210,222,234) | (346,353,360;341,353,365) | (164,169,174;159,169,179) | (425,433,441;423,433,443) | (60,65,70;15,23,30) | (55,60,65;20,28,35) |
| | (23,31,39;21,31,41) | (307,312,317;300,312,324) | (348,355,362;343,355,367) | (167,172,177;162,172,182) | (432,440,448;430,440,450) | (60,65,70;15,23,30) | (60,65,70;15,23,30) |
| H4 | (22,30,38;20,30,40) | (186,191,196;179,191,203) | (328,335,342;323,335,347) | (175,180,185;170,180,190) | (527,535,543;525,535,545) | (65,70,75;10,18,25) | (60,65,70;15,23,30) |
| | (25,33,41;23,33,43) | (210,215,220;203,215,227) | (331,338,345;326,338,350) | (180,185,190;175,185,195) | (522,530,538;520,530,540) | (65,70,75;10,18,25) | (65,70,75;10,18,25) |
| | (24,4,32,4,40,4;22,4,32,4,42,4) | (190,195,200;183,195,207) | (323,330,337;318,330,342) | (180,185,190;175,185,195) | (532,540,548;530,540,550) | (65,70,75;10,18,25) | (65,70,75;10,18,25) |
| H5 | (17,25,33;15,25,35) | (185,190,195;178,190,202) | (293,300,307;288,300,312) | (215,220,225;210,220,230) | (612,620,628;610,620,630) | (65,70,75;10,18,25) | (60,65,70;15,23,30) |
| | (19,5,27,5,35,5;17,5,27,5,37,5) | (182,187,192;175,187,199) | (291,298,305;286,298,310) | (209,214,219;204,214,224) | (614,622,630;612,622,632) | (60,65,70;15,23,30) | (55,60,65;20,28,35) |
| | (19,27,35;17,27,37) | (180,185,190;173,185,197) | (302,309,316;297,309,321) | (219,224,229;214,224,234) | (617,625,633;615,625,635) | (60,65,70;15,23,30) | (55,60,65;20,28,35) |
| H6 | (20,5,28,5,36,5;18,5,28,5,38,5) | (245,250,255;238,250,262) | (118,125,132;113,125,137) | (80,85,90;75,85,95) | (137,145,153;135,145,155) | (20,28,35;55,60,65) | (50,55,60;25,33,40) |
| | (19,8,27,8,35,8;17,8,27,8,37,8) | (242,247,252;235,247,259) | (120,127,134;115,127,139) | (90,95,100;85,95,105) | (141,149,157;139,149,159) | (50,55,60;25,33,40) | (55,60,65;20,28,35) |
| | (16,5,24,5,32,5;14,5,24,5,34,5) | (249,254,259;242,254,266) | (124,131,138;119,131,143) | (90,95,100;85,95,105) | (139,147,155;137,147,157) | (15,23,30;60,65,70) | (20,28,35;55,60,65) |
| H7 | (81,89,97;79,89,99) | (375,380,385;368,380,392) | (338,345,352;333,345,357) | (295,300,305;290,300,310) | (277,285,293;275,285,295) | (60,65,70;15,23,30) | (65,70,75;10,18,25) |
| | (87,95,103;85,95,105) | (415,420,425;408,420,432) | (358,365,372;353,365,377) | (310,315,320;305,315,325) | (281,289,297;279,289,299) | (65,70,75;10,18,25) | (50,55,60;25,33,40) |
| | (59,67,75;57,67,77) | (393,398,403;386,398,410) | (380,387,394;375,387,399) | (282,287,292;277,287,297) | (297,305,313;295,305,315) | (65,70,75;10,18,25) | (55,60,65;20,28,35) |
| H8 | (14,5,22,5,30,5;12,5,22,5,32,5) | (185,190,195;178,190,202) | (273,280,287;268,280,292) | (158,163,168;153,163,173) | (337,345,353;335,345,355) | (55,60,65;20,28,35) | (65,70,75;10,18,25) |
| | (12,5,20,5,28,5;10,5,20,5,30,5) | (180,185,190;173,185,197) | (278,285,292;273,285,297) | (160,165,170;155,165,175) | (334,342,350;332,342,352) | (55,60,65;20,28,35) | (60,65,70;15,23,30) |
| | (16,24,32;14,24,34) | (182,187,192;175,187,199) | (278,285,292;273,285,297) | (153,158,163;148,158,168) | (333,341,349;331,341,351) | (60,65,70;15,23,30) | (55,60,65;20,28,35) |
| H9 | (19,27,35;17,27,37) | (181,186,191;174,186,198) | (303,310,317;298,310,322) | (220,225,230;215,225,235) | (412,420,428;410,420,430) | (15,23,30;60,65,70) | (25,33,40;50,55,60) |
| | (17,8,25,8,33,8;15,8,25,8,35,8) | (184,189,194;177,189,201) | (304,311,318;299,311,323) | (205,210,215;200,210,220) | (409,417,425;407,417,427) | (50,55,60;25,33,40) | (55,60,65;20,28,35) |
| | (21,29,37;19,29,39) | (180,185,190;173,185,197) | (300,307,314;295,307,319) | (215,220,225;210,220,230) | (416,424,432;414,424,434) | (20,28,35;55,60,65) | (15,23,30;60,65,70) |
| H10 | (17,25,33;15,25,35) | (204,209,214;197,209,221) | (213,220,227;208,220,232) | (133,138,143;128,138,148) | (227,235,243;225,235,245) | (55,60,65;20,28,35) | (50,55,60;25,33,40) |
| | (19,4,27,4,35,4;17,4,27,4,37,4) | (207,212,217;200,212,224) | (214,221,228;209,221,233) | (129,134,139;124,134,144) | (222,230,238;220,230,240) | (65,70,75;10,18,25) | (65,70,75;10,18,25) |
| | (16,24,32;14,24,34) | (216,221,226;209,221,233) | (207,214,221;202,214,226) | (130,135,140;125,135,145) | (223,231,239;221,231,241) | (50,55,60;25,33,40) | (60,65,70;15,23,30) |
| H11 | (22,30,38;20,30,40) | (465,470,475;458,470,482 | (293,300,307;288,300,312) | (182,187,192;177,187,197) | (237,245,253;235,245,255) | (20,28,35;55,60,65) | (15,23,30;60,65,70) |
| | (24,7,32,7,40,7;22,7,32,7,42,7) | (462,467,472;455,467,479) | (290,297,304;285,297,309) | (175,180,185;170,180,190) | (233,241,249;231,241,251) | (50,55,60;25,33,40) | (60,65,70;15,23,30) |
| | (24,5,32,5,40,5;22,5,32,5,42,5) | (471,476,481;464,476,488) | (298,305,312;293,305,317) | (184,189,194;179,189,199) | (233,241,249;231,241,251) | (20,28,35;55,60,65) | (20,28,35;55,60,65) |
| H12 | (11,5,19,5,27,5;9,5,19,5,29,5) | (145,150,155;138,150,162) | (219,226,233;214,226,238) | (130,135,140;125,135,145) | (167,175,183;165,175,185) | (15,23,30;60,65,70) | (25,33,40;50,55,60) |
| | (9,5,17,5,25,5;7,5,17,5,27,5) | (150,155,160;143,155,167) | (224,231,238;219,231,243) | (128,133,138;123,133,143) | (167,175,183;165,175,185) | (25,33,40;50,55,60) | (20,28,35;55,60,65) |
| | (14,5,22,5,30,5;12,5,22,5,32,5) | (148,153,158;141,153,165) | (224,231,238;219,231,243) | (127,132,137;122,132,142) | (163,171,179;161,171,181) | (20,28,35;55,60,65) | (15,23,30;60,65,70) |
| H13 | (27,35,43;25,35,45) | (381,386,391;374,386,398) | (386,393,400;381,393,405) | (220,225,230;215,225,235) | (357,365,373;355,365,375) | (50,55,60;25,33,40) | (60,65,70;15,23,30) |
| | (23,2,31,2,39,2;21,2,31,2,41,2) | (387,392,397;380,392,404) | (391,398,405;386,398,410) | (213,218,223;208,218,228) | (354,362,370;352,362,372) | (55,60,65;20,28,35) | (60,65,70;15,23,30) |
| | (25,33,41;23,33,43) | (379,384,389;372,384,396) | (393,400,407;388,400,412) | (223,228,233;218,228,238) | (356,364,372;354,364,374) | (60,65,70;15,23,30) | (60,65,70;15,23,30) |
| H14 | (37,45,53;35,45,55) | (528,533,538;521,533,545) | (543,550,557;538,550,562) | (410,415,420;405,415,425) | (487,495,503;485,495,505) | (15,23,30;60,65,70) | (25,33,40;50,55,60) |
| | (34,42,50;32,42,52) | (530,535,540;523,535,547) | (541,548,555;536,548,560) | (407,412,417;402,412,422) | (484,492,500;482,492,502) | (20,28,35;55,60,65) | (15,23,30;60,65,70) |
| | (34,5,42,5,50,5;32,5,42,5,52,5) | (530,535,540;523,535,547) | (541,548,555;536,548,560) | (405,410,415;400,410,420) | (482,490,498;480,490,500) | (50,55,60;25,33,40) | (55,60,65;20,28,35) |
| H15 | (42,50,58;40,50,60) | (995,1000,1005;988,1000,1012) | (668,675,682;663,675,687) | (384,389,394;379,389,399) | (512,520,528;510,520,530) | (20,28,35;55,60,65) | (25,33,40;50,55,60) |
| | (42,50,58;40,50,60) | (992,997,1002;985,997,1009) | (672,679,686;667,679,691) | (386,391,396;381,391,401) | (515,523,531;513,523,533) | (50,55,60;25,33,40) | (55,60,65;20,28,35) |
| | (43,5,51,5,59,5;41,5,51,5,61,5) | (990,995,1000;983,995,1007) | (673,680,687;668,680,692) | (380,385,390;375,385,395) | (517,525,533;515,525,535) | (20,28,35;55,60,65) | (15,23,30;60,65,70) |
| H16 | (34,42,50;32,42,52) | (195,200,205;188,200,212) | (243,250,257;238,250,262) | (130,135,140;125,135,145) | (152,160,168;150,160,170) | (15,23,30;60,65,70) | (20,28,35;55,60,65) |
| | (34,7,42,7,50,7;32,7,42,7,52,7) | (197,202,207;190,202,214) | (249,256,263;244,256,268) | (134,139,144;129,139,149) | (154,162,170;152,162,172) | (55,60,65;20,28,35) | (65,70,75;10,18,25) |
| | (36,44,52;34,44,54) | (202,207,212;195,207,219) | (246,253,260;241,253,265) | (135,140,145;130,140,150) | (159,167,175;157,167,177) | (15,23,30;60,65,70) | (20,28,35;55,60,65) |

**Step 2.6:** Apply CRS based DEA model given in Section 4.3.

$$\begin{aligned} &\min \theta \\ &s.t. \\ &\theta \tilde{d}^{i}_{e,k} \geq \sum_{j=1}^{m} \lambda_j \tilde{d}^{i}_{j,k} \\ &\tilde{d}^{o}_{e,k} \leq \sum_{j=1}^{m} \lambda_j \tilde{d}^{o}_{j,k} \\ &\lambda_j \geq 0 \ \forall j = 1,2,\ldots,m \\ &\sum_{j=1}^{m} \lambda_j = 1 \\ &\theta \geq 0 \end{aligned} \tag{29}$$

where

$\tilde{d}^{i}_{j,k} = \left(d^{i,L}_{j,k}, d^{i,M}_{j,k}, d^{i,U}_{j,k}; d^{i,'L}_{j,k}, d^{i,'M}_{j,k}, d^{i,'U}_{j,k}\right)$

$\tilde{d}^{i}_{e,k} = \left(d^{i,L}_{e,k}, d^{i,M}_{e,k}, d^{i,U}_{e,k}; d^{i,'L}_{e,k}, d^{i,'M}_{e,k}, d^{i,'U}_{e,k}\right)$

$\tilde{d}^{o}_{e,k} = \left(d^{o,L}_{e,k}, d^{o,M}_{e,k}, d^{o,U}_{e,k}; d^{o,'L}_{e,k}, d^{o,'M}_{e,k}, d^{o,'U}_{e,k}\right)$

$\tilde{d}^{o}_{j,k} = \left(d^{o,L}_{j,k}, d^{o,M}_{j,k}, d^{o,U}_{j,k}; d^{o,'L}_{j,k}, d^{o,'M}_{j,k}, d^{o,'U}_{j,k}\right)$

$\tilde{d}^{i}_{e,k}$ : Amount of input $k^{th}$ for the $e^{th}$ observed decision making unit

$\tilde{d}^{o}_{e,k}$ : Amount of output $k^{th}$ for the $e^{th}$ observed decision making unit

$\tilde{d}^{i}_{j,k}$ : Amount of input $k^{th}$ for the $j^{th}$ decision making unit

$\tilde{d}^{o}_{j,k}$ : Amount of output $k^{th}$ for the $j^{th}$ decision making unit

$\theta$: Efficiency value

$\mu_k$: Weight for the $k^{th}$ output given by $e^{th}$ decision making unit

$v_k$: Weight for the $k^{th}$ input given by $e^{th}$ decision making unit

t : Number of inputs

u : Number of outputs

m : Number of decision making unit (DMU)

**IE 2.6:** When the TIF DEA problem in IE 2.5 is solved by the proposed model in Step 2.6, the following efficiency results are calculated.

| Alternative | Efficiency | Input Slacks | | Output Slacks | |
|---|---|---|---|---|---|
| | | I1 | I2 | O1 | O2 |
| **A1** | 0.72 | 0.00 | 0.04 | 0.28 | 0.00 |
| **A2** | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |

**Step 2.7:** Prioritize the hospitals with respect to their efficiencies ($\theta$) and advise the hospitals considering the values of input & output slacks and input & output targets.

## 6. An application

In this section, we analyze a real life hospital performance evaluation problem considering uncertainties in the decision making environment. A total number of 16 hospitals operating in Istanbul have been evaluated. These hospitals have been chosen among both private and public hospitals. According to their privacy policies, the names of hospitals are not shared. For each hospital, we gather the judgments of three experts from different departments namely procurement, finance, and logistics.

In the study, seven inputs, namely patient care supplies and other expenses, the number of beds, number of nurses, number of physicians, number of other personnel, technology level and service mix and six outputs, namely annual revenue, number of outpatient department visits, overall patient satisfaction, number of

**Table 9**
Aggregated evaluations of experts for DEA inputs.

| Hospital | I1 | I2 | I3 | I4 | I5 | I6 | I7 |
|---|---|---|---|---|---|---|---|
| H1 | (0.59,0.68,0.78;0.55,0.65,0.76) | (0.99,0.99,1;0.98,0.99,1) | (0.99,0.99,0.99;0.99,0.99,1) | (0.98,0.99,0.99;0.97,0.99,1) | (0.98,0.99,1;0.98,0.99,1) | (0.8,0.87,0.93;0.2,0.31,0.4) | (0.78,0.85,0.92;0.22,0.33,0.42) |
| H2 | (0.53,0.62,0.72;0.49,0.6,0.71) | (0.89,0.89,0.89;0.88,0.89,0.9) | (0.83,0.83,0.84;0.83,0.83,0.84) | (0.91,0.91,0.92;0.9,0.91,0.93) | (0.91,0.92,0.93;0.91,0.92,0.93) | (0.78,0.84,0.91;0.22,0.33,0.43) | (0.81,0.88,1;0.19,0.3,0.4) |
| H3 | (0.24,0.33,0.42;0.22,0.32,0.43) | (0.16,0.16,0.16;0.15,0.16,0.17) | (0.19,0.20,0.20;0.19,0.20,0.20) | (0.21,0.22,0.23;0.21,0.22,0.23) | (0.45,0.45,0.46;0.44,0.45,0.46) | (0.8,0.87,0.93;0.2,0.31,0.4) | (0.73,0.8,0.87;0.27,0.38,0.47) |
| H4 | (0.26,0.34,0.43;0.23,0.34,0.45) | (0.13,0.13,0.13;0.12,0.13,0.14) | (0.18,0.19,0.19;0.18,0.19,0.19) | (0.23,0.24,0.25;0.23,0.24,0.25) | (0.55,0.56,0.57;0.55,0.56,0.57) | (0.87,0.93,1;0.13,0.24,0.33) | (0.84,0.91,1;0.16,0.26,0.36) |
| H5 | (0.2,0.29,0.37;0.17,0.28,0.39) | (0.12,0.12,0.12;0.11,0.12,0.13) | (0.16,0.17,0.17;0.16,0.17,0.17) | (0.28,0.29,0.29;0.27,0.29,0.3) | (0.64,0.65,0.66;0.64,0.65,0.66) | (0.83,0.9,1;0.17,0.28,0.37) | (0.76,0.83,0.9;0.24,0.35,0.44) |
| H6 | (0.2,0.29,0.38;0.18,0.29,0.4) | (0.16,0.16,0.17;0.15,0.16,0.17) | (0.07,0.07,0.07;0.06,0.07,0.08) | (0.11,0.12,0.13;0.11,0.12,0.13) | (0.14,0.15,0.16;0.14,0.15,0.16) | (0.41,0.5,0.59;0.59,0.68,0.77) | (0.61,0.68,0.76;0.39,0.5,0.59) |
| H7 | (0.81,0.89,0.98;0.78,0.89,1) | (0.25,0.26,0.26;0.25,0.26,0.26) | (0.20,0.20,0.21;0.20,0.20,0.21) | (0.39,0.39,0.4;0.38,0.39,0.41) | (0.3,0.3,0.31;0.29,0.3,0.31) | (0.84,0.91,1;0.16,0.26,0.36) | (0.78,0.86,1;0.22,0.33,0.42) |
| H8 | (0.16,0.24,0.33;0.13,0.24,0.35) | (0.12,0.12,0.12;0.11,0.12,0.13) | (0.15,0.16,0.16;0.15,0.16,0.16) | (0.21,0.21,0.22;0.2,0.21,0.22) | (0.35,0.36,0.37;0.35,0.36,0.37) | (0.76,0.82,0.89;0.24,0.35,0.45) | (0.81,0.89,1;0.19,0.29,0.39) |
| H9 | (0.21,0.3,0.39;0.18,0.29,0.4) | (0.12,0.12,0.12;0.11,0.12,0.13) | (0.17,0.17,0.18;0.17,0.17,0.18) | (0.28,0.29,0.29;0.27,0.29,0.3) | (0.43,0.44,0.45;0.43,0.44,0.45) | (0.4,0.5,0.58;0.6,0.69,0.77) | (0.47,0.56,0.65;0.53,0.63,0.71) |
| H10 | (0.19,0.27,0.36;0.16,0.27,0.38) | (0.13,0.14,0.14;0.13,0.14,0.15) | (0.12,0.12,0.13;0.11,0.12,0.13) | (0.17,0.18,0.18;0.16,0.18,0.19) | (0.23,0.24,0.25;0.23,0.24,0.25) | (0.77,0.84,1;0.23,0.34,0.44) | (0.78,0.86,1;0.22,0.33,0.42) |
| H11 | (0.26,0.34,0.43;0.23,0.34,0.44) | (0.3,0.3,0.31;0.3,0.3,0.31) | (0.16,0.17,0.17;0.16,0.17,0.17) | (0.24,0.24,0.25;0.23,0.24,0.26) | (0.24,0.25,0.26;0.24,0.25,0.26) | (0.42,0.52,0.6;0.58,0.67,0.75) | (0.49,0.59,0.7;0.51,0.62,0.71) |
| H12 | (0.13,0.22,0.31;0.1,0.21,0.32) | (0.1,0.1,0.1;0.09,0.1,0.11) | (0.12,0.13,0.13;0.12,0.13,0.13) | (0.17,0.17,0.18;0.16,0.17,0.19) | (0.17,0.18,0.19;0.17,0.18,0.19) | (0.26,0.37,0.46;0.74,0.8,0.87) | (0.28,0.38,0.48;0.72,0.79,0.86) |
| H13 | (0.27,0.36,0.45;0.25,0.36,0.46) | (0.25,0.25,0.25;0.24,0.25,0.26) | (0.22,0.22,0.22;0.21,0.22,0.23) | (0.29,0.29,0.3;0.28,0.29,0.31) | (0.37,0.38,0.39;0.37,0.38,0.39) | (0.73,0.8,0.87;0.27,0.38,0.47) | (0.8,0.87,0.93;0.2,0.31,0.4) |
| H14 | (0.38,0.47,0.56;0.36,0.46,0.57) | (0.34,0.35,0.35;0.34,0.35,0.35) | (0.30,0.31,0.31;0.30,0.31,0.31) | (0.53,0.54,0.55;0.53,0.54,0.55) | (0.51,0.51,0.52;0.5,0.51,0.52) | (0.4,0.5,0.58;0.6,0.69,0.77) | (0.47,0.56,0.65;0.53,0.63,0.71) |
| H15 | (0.46,0.55,0.65;0.43,0.54,0.65) | (0.64,0.65,0.65;0.64,0.65,0.65) | (0.37,0.38,0.38;0.37,0.37,0.38) | (0.5,0.51,0.51;0.49,0.51,0.52) | (0.54,0.54,0.55;0.53,0.54,0.56) | (0.42,0.52,0.6;0.58,0.67,0.75) | (0.47,0.56,0.65;0.53,0.63,0.71) |
| H16 | (0.38,0.47,0.56;0.35,0.46,0.57) | (0.13,0.13,0.13;0.12,0.13,0.14) | (0.14,0.14,0.14;0.13,0.14,0.15) | (0.17,0.18,0.19;0.17,0.18,0.19) | (0.16,0.17,0.18;0.16,0.17,0.18) | (0.42,0.52,0.62;0.58,0.67,0.76) | (0.56,0.68,1;0.44,0.56,0.65) |

**Table 10**
Weighted aggregated evaluations for DEA inputs.

| Hospital | I1 | I2 | I3 | I4 | I5 | I6 | I7 |
|---|---|---|---|---|---|---|---|
| H1 | (0.04,0.047,0.054; 0.037,0.045,0.052) | (0.035,0.035,0.035; 0.035,0.035,0.035) | (0.28,0.281,0.282; 0.279,0.281,0.283) | (0.162,0.163,0.164; 0.161,0.163,0.165) | (0.044,0.044,0.045; 0.044,0.044,0.045) | (0.233,0.252,0.271; 0.058,0.089,0.116) | (0.088,0.095,0.103; 0.025,0.037,0.047) |
| H2 | (0.036,0.042,0.049; 0.034,0.041,0.048) | (0.031,0.032,0.032; 0.031,0.032,0.032) | (0.235,0.236,0.237; 0.234,0.236,0.238) | (0.15,0.151,0.152; 0.149,0.151,0.153) | (0.041,0.041,0.042; 0.041,0.041,0.042) | (0.226,0.245,0.265; 0.065,0.096,0.124) | (0.091,0.099,0.112; 0.022,0.034,0.045) |
| H3 | (0.017,0.023,0.029; 0.015,0.022,0.03) | (0.006,0.006,0.006; 0.005,0.006,0.006) | (0.054,0.055,0.057; 0.054,0.055,0.057) | (0.035,0.036,0.037; 0.034,0.036,0.039) | (0.02,0.02,0.021; 0.02,0.02,0.021) | (0.233,0.252,0.271; 0.058,0.089,0.116) | (0.082,0.09,0.098; 0.03,0.042,0.053) |
| H4 | (0.018,0.024,0.03; 0.016,0.023,0.03) | (0.004,0.005,0.005; 0.004,0.005,0.005) | (0.052,0.053,0.054; 0.051,0.053,0.055) | (0.038,0.04,0.041; 0.037,0.04,0.042) | (0.025,0.025,0.025; 0.024,0.025,0.025) | (0.252,0.271,0.291; 0.039,0.07,0.097) | (0.095,0.103,0.112; 0.018,0.03,0.04) |
| H5 | (0.014,0.02,0.026; 0.012,0.019,0.027) | (0.004,0.004,0.004; 0.004,0.004,0.005) | (0.046,0.048,0.049; 0.046,0.048,0.049) | (0.046,0.047,0.048; 0.045,0.047,0.05) | (0.029,0.029,0.029; 0.028,0.029,0.029) | (0.241,0.261,0.291; 0.049,0.081,0.108) | (0.086,0.093,0.101; 0.027,0.039,0.049) |
| H6 | (0.014,0.02,0.026; 0.012,0.02,0.027) | (0.006,0.006,0.006; 0.005,0.006,0.006) | (0.019,0.02,0.021; 0.018,0.02,0.022) | (0.019,0.02,0.021; 0.017,0.02,0.022) | (0.006,0.007,0.007; 0.006,0.007,0.007) | (0.118,0.145,0.171; 0.173,0.199,0.223) | (0.068,0.077,0.086; 0.044,0.056,0.067) |
| H7 | (0.055,0.061,0.067; 0.054,0.061,0.068) | (0.009,0.009,0.009; 0.009,0.009,0.009) | (0.056,0.057,0.058; 0.055,0.057,0.059) | (0.064,0.065,0.066; 0.063,0.065,0.067) | (0.013,0.014,0.014; 0.013,0.014,0.014) | (0.245,0.265,0.291; 0.046,0.077,0.104) | (0.088,0.097,0.112; 0.024,0.037,0.048) |
| H8 | (0.011,0.017,0.023; 0.009,0.016,0.024) | (0.004,0.004,0.004; 0.004,0.004,0.005) | (0.043,0.045,0.046; 0.043,0.045,0.046) | (0.034,0.035,0.036; 0.033,0.035,0.037) | (0.016,0.016,0.016; 0.015,0.016,0.016) | (0.22,0.239,0.259; 0.071,0.102,0.13) | (0.092,0.1,0.112; 0.021,0.033,0.044) |
| H9 | (0.014,0.02,0.026; 0.013,0.02,0.027) | (0.004,0.004,0.004; 0.004,0.004,0.005) | (0.048,0.049,0.05; 0.047,0.049,0.051) | (0.046,0.047,0.048; 0.045,0.047,0.049) | (0.019,0.02,0.02; 0.019,0.02,0.02) | (0.117,0.144,0.17; 0.174,0.201,0.224) | (0.052,0.063,0.074; 0.06,0.071,0.08) |
| H10 | (0.013,0.019,0.025; 0.011,0.019,0.026) | (0.005,0.005,0.005; 0.005,0.005,0.005) | (0.033,0.034,0.036; 0.033,0.034,0.036) | (0.028,0.029,0.03; 0.027,0.029,0.032) | (0.01,0.011,0.011; 0.01,0.011,0.011) | (0.223,0.245,0.291; 0.067,0.1,0.128) | (0.088,0.096,0.112; 0.024,0.037,0.048) |
| H11 | (0.018,0.024,0.03; 0.016,0.023,0.03) | (0.011,0.011,0.011; 0.011,0.011,0.011) | (0.046,0.047,0.048; 0.045,0.047,0.049) | (0.039,0.04,0.041; 0.038,0.04,0.042) | (0.011,0.011,0.012; 0.011,0.011,0.012) | (0.122,0.15,0.175; 0.168,0.194,0.218) | (0.055,0.066,0.079; 0.058,0.07,0.08) |
| H12 | (0.009,0.015,0.021; 0.007,0.014,0.022) | (0.003,0.003,0.004; 0.003,0.003,0.004) | (0.035,0.036,0.037; 0.034,0.036,0.038) | (0.028,0.029,0.03; 0.027,0.029,0.031) | (0.008,0.008,0.008; 0.008,0.008,0.009) | (0.076,0.107,0.135; 0.215,0.234,0.253) | (0.031,0.043,0.054; 0.081,0.089,0.096) |
| H13 | (0.019,0.025,0.031; 0.017,0.024,0.032) | (0.009,0.009,0.009; 0.009,0.009,0.009) | (0.061,0.062,0.064; 0.061,0.062,0.064) | (0.047,0.048,0.049; 0.046,0.048,0.051) | (0.017,0.017,0.017; 0.016,0.017,0.017) | (0.213,0.233,0.254; 0.078,0.109,0.137) | (0.09,0.097,0.105; 0.022,0.034,0.045) |
| H14 | (0.026,0.032,0.038; 0.024,0.032,0.039) | (0.012,0.012,0.012; 0.012,0.012,0.012) | (0.085,0.086,0.088; 0.085,0.086,0.088) | (0.088,0.089,0.09; 0.087,0.089,0.091) | (0.023,0.023,0.023; 0.022,0.023,0.023) | (0.117,0.144,0.17; 0.174,0.201,0.224) | (0.052,0.063,0.074; 0.06,0.071,0.08) |
| H15 | (0.032,0.038,0.044; 0.03,0.037,0.044) | (0.023,0.023,0.023; 0.023,0.023,0.023) | (0.106,0.107,0.108; 0.105,0.107,0.109) | (0.083,0.084,0.085; 0.082,0.084,0.086) | (0.024,0.024,0.025; 0.024,0.024,0.025) | (0.122,0.15,0.175; 0.168,0.194,0.218) | (0.052,0.063,0.074; 0.06,0.071,0.08) |
| H16 | (0.026,0.032,0.038; 0.024,0.031,0.039) | (0.005,0.005,0.005; 0.004,0.005,0.005) | (0.039,0.04,0.041; 0.038,0.04,0.042) | (0.029,0.03,0.031; 0.028,0.03,0.032) | (0.007,0.008,0.008; 0.007,0.008,0.008) | (0.123,0.152,0.18; 0.167,0.196,0.22) | (0.063,0.076,0.112; 0.049,0.063,0.073) |

inpatient department admissions, conformance to quality procedures and bed usage rate, are evaluated. All inputs and outputs are expressed by using Triangular Intuitionistic Fuzzy Sets. Brief explanations of both inputs and outputs are stated in Table 4.

### *6.1. Weighting inputs & outputs using TIF AHP*

In this section, the judgments for inputs and outputs are collected from three experts as linguistic evaluations. Tables 5 and 6 exhibit these pairwise comparisons of inputs and outputs based on the linguistic scale in Table 2.

Table 7 demonstrates the weights of inputs and outputs obtained from the application of TIF AHP method.

### *6.2. Measuring efficiencies of hospitals*

The next operation is to measure efficiencies of the hospitals by using CRS based TIF DEA. Table 8 displays evaluations of three experts for DEA inputs in each cell based on the linguistic scale in Table 3. Table 9 illustrates the normalized aggregated values of these input evaluations by employing Eqs. (13) and (26) while Table 10 presents the weighted aggregated evaluations of inputs as in Eq. (28).

Similarly, evaluations of outputs are collected from the decision makers as in Table 11. Once the evaluations are normalized and the experts' judgments are aggregated, the weighted aggregated evaluations of experts for DEA outputs are obtained as shown in Table 12.

Since the hospital performance evaluation problem is too complex and comprehensive to solve using the model given in Eq. (29) in Step 2.6, we preferred defuzzifying the fuzzy scores in Tables 10 and 12. In Table 13, the defuzzified scores of inputs and outputs are calculated where $\tau$ is set to 1000 in Eq. (12). The value of $\tau$ is set to a large number since the experts want to significantly decrease the effect of non-membership function.

Using the defuzzified values in Table 13, the DEA model has been applied. The solution of the proposed approach is presented in Table 14. According to these results, H3, H5, H6, H8, H9, H10, H12, H13, H14, H15, and H16 are the efficient frontiers. The rest of the hospitals are ranked as H11 > H1 > H4 > H2 > H7. The results reveal that 27% of the efficient frontiers are public hospitals and 60% of the inefficient hospitals are private hospitals.

Slacks exist only for those hospitals identified as inefficient. According to Table 14, for H1, the levels of number of beds (I2), number of physicians (I3), number of nurses (I4), number of other personnel (I5), and technology level (I6) should be reduced by 9.14, 160.2, 67.6, 16.1, and 36.3, respectively. However, despite the reduction in these inputs, it would not achieve efficiency. H1 should also augment annual revenue (O1), Number of outpatient department visits (O2), overall patient satisfaction (O3), conformance to quality procedures (O5), and bed usage rate (O6) by 48.6, 4.89, 67.75, 30.01, and 9.7 respectively. Similar situations in different magnitudes exist for H2, H4, H7, and H11.

### *6.3. Comparison & discussion*

To examine the validity of the obtained results through TIF AHP & DEA, the results of the proposed integrated fuzzy method are compared with the results of traditional DEA. Table 15 gives the finding from both approaches. According to Table 15, the efficiencies of hospitals are found higher in traditional DEA compared to proposed TIF AHP & DEA. It is interesting that H11 is efficient with traditional DEA model whereas it is inefficient with a score of 0.974 using TIF AHP & DEA. It reveals that TIF AHP & DEA presents

**Table 11**
Evaluations of experts for DEA outputs.

| Hospital | O1 | O2 | O3 | O4 | O5 | O6 |
|---|---|---|---|---|---|---|
| H1 | (80,88,96;75,88,101) | (965,975,985;967,975,983) | (50,55,60;25,33,40) | (57,65,73;55,65,75) | (60,65,70;15,23,30) | (90,97,104;85,97,109) |
| | (76,88,100;70,88,106) | (968,978,988;963,978,993) | (55,60,65;20,28,35) | (56,68,80;50,68,86) | (50,55,60;25,33,40) | (90,98,106;83,98,113) |
| | (77,89,101;71,89,107) | (972,980,988;968,980,992) | (55,60,65;20,28,35) | (65,70,75;61,70,79) | (55,60,65;20,28,35) | (90,98,106;84,98,112) |
| H2 | (82,90,98;77,90,103) | (875,885,895;877,885,893) | (50,55,60;25,33,40) | (50,58,66;48,58,68) | (55,60,65;20,28,35) | (90,97,104;85,97,109) |
| | (81,93,105;75,93,111) | (877,887,897;872,887,902) | (55,60,65;20,28,35) | (48,60,72;42,60,78) | (50,55,60;25,33,40) | (90,98,106;83,98,113) |
| | (82,94,106;76,94,112) | (881,889,897;877,889,901) | (55,60,65;20,28,35) | (55,60,65;51,60,69) | (55,60,65;20,28,35) | (87,95,103;81,95,109) |
| H3 | (127,135,143;122,135,148) | (670,680,690;672,680,688) | (65,70,75;10,18,25) | (32,40,48;30,40,50) | (65,70,75;10,18,25) | (81,88,95;76,88,100) |
| | (120,132,144;114,132,150) | (664,674,684;659,674,689) | (65,70,75;10,18,25) | (30,42,54;24,42,60) | (65,70,75;10,18,25) | (79,87,95;72,87,102) |
| | (126,138,150;120,138,156) | (666,674,682;662,674,686) | (65,70,75;10,18,25) | (40,45,50;36,45,54) | (65,70,75;10,18,25) | (82,90,98;76,90,104) |
| H4 | (107,115,123;102,115,128) | (270,280,290;272,280,288) | (20,28,35;55,60,65) | (30,38,46;28,38,48) | (15,23,30;60,65,70) | (77,84,91;72,84,96) |
| | (113,125,137;107,125,143) | (275,285,295;270,285,300) | (15,23,30;60,65,70) | (23,35,47;17,35,53) | (10,18,25;65,70,75) | (74,82,90;67,82,97) |
| | (113,125,137;107,125,143) | (267,275,283;263,275,287) | (15,23,30;60,65,70) | (25,30,35;21,30,39) | (10,18,25;65,70,75) | (77,85,93;71,85,99) |
| H5 | (142,150,158;137,150,163) | (490,500,510;492,500,508) | (65,70,75;10,18,25) | (27,35,43;25,35,45) | (65,70,75;10,18,25) | (78,85,92;73,85,97) |
| | (141,153,165;135,153,171) | (495,505,515;490,505,520) | (60,65,70;15,23,30) | (19,31,43;13,31,49) | (65,70,75;10,18,25) | (75,83,91;68,83,98) |
| | (142,154,166;136,154,172) | (488,496,504;484,496,508) | (65,70,75;10,18,25) | (26,31,36;22,31,40) | (65,70,75;10,18,25) | (76,84,92;70,84,98) |
| H6 | (100,108,116;95,108,121) | (440,450,460;442,450,458) | (20,28,35;55,60,65) | (42,50,58;40,50,60) | (20,28,35;55,60,65) | (76,83,90;71,83,95) |
| | (99,111,123;93,111,129) | (442,452,462;437,452,467) | (55,60,65;20,28,35) | (42,54,66;36,54,72) | (65,70,75;10,18,25) | (73,81,89;66,81,96) |
| | (106,118,130;100,118,136) | (444,452,460;440,452,464) | (55,60,65;20,28,35) | (49,54,59;45,54,63) | (60,65,70;15,23,30) | (72,80,88;66,80,94) |
| H7 | (54,62,70;49,62,75) | (290,300,310;292,300,308) | (25,33,40;50,55,60) | (17,25,33;15,25,35) | (15,23,30;60,65,70) | (70,77,84;65,77,89) |
| | (59,71,83;53,71,89) | (287,297,307;282,297,312) | (25,33,40;50,55,60) | (16,28,40;10,28,46) | (20,28,35;55,60,65) | (72,80,88;65,80,95) |
| | (43,55,67;37,55,73) | (304,312,320;300,312,324) | (20,28,35;55,60,65) | (24,29,34;20,29,38) | (15,23,30;60,65,70) | (67,75,83;61,75,89) |
| H8 | (86,94,102;81,94,107) | (500,510,520;502,510,518) | (55,60,65;20,28,35) | (37,45,53;35,45,55) | (55,60,65;20,28,35) | (83,90,97;78,90,102) |
| | (79,91,103;73,91,109) | (499,509,519;494,509,524) | (60,65,70;15,23,30) | (36,48,60;30,48,66) | (60,65,70;15,23,30) | (83,91,99;76,91,106) |
| | (78,90,102;72,90,108) | (504,512,520;500,512,524) | (65,70,75;10,18,25) | (39,44,49;35,44,53) | (65,70,75;10,18,25) | (86,94,102;80,94,108) |
| H9 | (97,105,113;92,105,118) | (630,640,650;632,640,648) | (25,33,40;50,55,60) | (22,30,38;20,30,40) | (10,18,25;65,70,75) | (82,89,96;77,89,101) |
| | (103,115,127;97,115,133) | (632,642,652;627,642,657) | (55,60,65;20,28,35) | (21,33,45;15,33,51) | (60,65,70;15,23,30) | (84,92,100;77,92,107) |
| | (98,110,122;92,110,128) | (639,647,655;635,647,659) | (60,65,70;15,23,30) | (28,33,38;24,33,42) | (55,60,65;20,28,35) | (82,90,98;76,90,104) |
| H10 | (117,125,133;112,125,138) | (354,364,374;356,364,372) | (60,65,70;15,23,30) | (17,25,33;15,25,35) | (60,65,70;15,23,30) | (73,80,87;68,80,92) |
| | (106,118,130;100,118,136) | (357,367,377;352,367,382) | (55,60,65;20,28,35) | (17,29,41;11,29,47) | (60,65,70;15,23,30) | (76,84,92;69,84,99) |
| | (117,129,141;111,129,147) | (352,360,368;348,360,372) | (15,23,30;60,65,70) | (17,22,27;13,22,31) | (15,23,30;60,65,70) | (75,83,91;69,83,97) |
| H11 | (67,75,83;62,75,88) | (400,410,420;402,410,418) | (25,33,40;50,55,60) | (14,22,30;12,22,32) | (15,23,30;60,65,70) | (78,85,92;73,85,97) |
| | (65,77,89;59,77,95) | (402,412,422;397,412,427) | (60,65,70;15,23,30) | (13,25,37;7,25,43) | (55,60,65;20,28,35) | (78,86,94;71,86,101) |
| | (65,77,89;59,77,95) | (400,408,416;396,408,420) | (60,65,70;15,23,30) | (15,20,25;11,20,29) | (60,65,70;15,23,30) | (79,87,95;73,87,101) |
| H12 | (62,70,78;57,70,83) | (355,365,375;357,365,373) | (15,23,30;60,65,70) | (10,18,26;8,18,28) | (10,18,25;65,70,75) | (78,85,92;73,85,97) |
| | (58,70,82;52,70,88) | (359,369,379;354,369,384) | (10,18,25;65,70,75) | (9,21,33;3,21,39) | (10,18,25;65,70,75) | (78,86,94;71,86,101) |
| | (58,70,82;52,70,88) | (353,361,369;349,361,373) | (60,65,70;15,23,30) | (19,24,29;15,24,33) | (60,65,70;15,23,30) | (77,85,93;71,85,99) |
| H13 | (72,80,88;67,80,93) | (675,685,695;677,685,693) | (50,55,60;25,33,40) | (47,55,63;45,55,65) | (20,28,35;55,60,65) | (86,93,100;81,93,105) |
| | (65,77,89;59,77,95) | (678,688,698;673,688,703) | (55,60,65;20,28,35) | (35,47,59;29,47,65) | (50,55,60;25,33,40) | (83,91,99;76,91,106) |
| | (72,84,96;66,84,102) | (681,689,697;677,689,701) | (55,60,65;20,28,35) | (53,58,63;49,58,67) | (60,65,70;15,23,30) | (86,94,102;80,94,108) |
| H14 | (62,70,78;57,70,83) | (810,820,830;812,820,828) | (20,28,35;55,60,65) | (40,48,56;38,48,58) | (15,23,30;60,65,70) | (88,95,102;83,95,107) |
| | (60,72,84;54,72,90) | (814,824,834;809,824,839) | (50,55,60;25,33,40) | (31,43,55;25,43,61) | (50,55,60;25,33,40) | (88,96,104;81,96,111) |
| | (63,75,87;57,75,93) | (816,824,832;812,824,836) | (50,55,60;25,33,40) | (35,40,45;31,40,49) | (55,60,65;20,28,35) | (89,97,105;83,97,111) |
| H15 | (69,77,85;64,77,90) | (890,900,910;892,900,908) | (50,55,60;25,33,40) | (44,52,60;42,52,62) | (25,33,40;50,55,60) | (88,95,102;83,95,107) |
| | (66,78,90;60,78,96) | (887,897,907;882,897,912) | (50,55,60;25,33,40) | (32,44,56;26,44,62) | (50,55,60;25,33,40) | (89,97,105;82,97,112) |
| | (69,81,93;63,81,99) | (897,905,913;893,905,917) | (55,60,65;20,28,35) | (45,50,55;41,50,59) | (55,60,65;20,28,35) | (89,97,105;83,97,111) |
| H16 | (47,55,63;42,55,68) | (460,470,480;462,470,478) | (15,23,30;60,65,70) | (27,35,43;25,35,45) | (10,18,25;65,70,75) | (88,95,102;83,95,107) |
| | (45,57,69;39,57,75) | (465,475,485;460,475,490) | (50,55,60;25,33,40) | (27,39,51;21,39,57) | (50,55,60;25,33,40) | (89,97,105;82,97,112) |
| | (50,62,74;44,62,80) | (460,468,476;456,468,480) | (50,55,60;25,33,40) | (34,39,44;30,39,48) | (55,60,65;20,28,35) | (89,97,105;83,97,111) |

a slightly more pessimistic approach with respect to traditional DEA model.

Traditional DEA needs less computational effort and it uses deterministic information. The deterministic representation of uncertainty and vagueness decreases the reliability of the obtained solutions. Under certainty, traditional DEA and TIF AHP & DEA may produce same results but traditional DEA is preferred to TIF AHP & DEA since it has computational advantages. Under uncertainty, the accuracy of traditional DEA results is low since it cannot deal with the imprecision in the problem and TIF AHP & DEA is more efficient than traditional DEA since it can incorporate this imprecision and vagueness into the model.

The healthcare performance evaluation problem we consider involves high level of uncertainty. Hence, we preferred TIF AHP & DEA rather than traditional DEA. The results indicate that the efficiency scores of H1, H2, H4, H7, and H11 are different in traditional DEA and TIF AHP & DEA. The experts confirmed that the accuracy of TIF AHP & DEA results is higher than traditional DEA.

## 7. Conclusion

Healthcare is a very complex and one of the high priority application fields. It involves many research areas with a large number of constraints and uncertainties. Uncertainty is an inevitable component of decision making process. Linguistic evaluations are generally preferred when exact numerical values cannot be assigned in multicriteria evaluations. Linguistic variables can be best handled by the fuzzy set theory. One of the extensions of ordinary fuzzy sets is intuitionistic fuzzy sets, which includes the definition of membership, non-membership, and hesitancy altogether.

The proposed triangular intuitionistic fuzzy AHP & DEA model could successfully handle the linguistic evaluations of more than one expert by aggregating and weighting them. It is observed that traditional DEA has an optimistic point of view to the same problem while TIF DEA has relatively a pessimistic point of view better revealing inefficient performances.

**Table 12**
Weighted aggregated evaluations of experts for DEA outputs.

| Hospital | O1 | O2 | O3 |
|---|---|---|---|
| H1 | (0.128,0.145,0.161;0.118,0.144,0.171) | (0.064,0.064,0.065;0.064,0.064,0.065) | (0.219,0.24,0.261;0.09,0.123,0.152) |
| H2 | (0.134,0.151,0.168;0.124,0.151,0.177) | (0.058,0.058,0.059;0.058,0.058,0.059) | (0.219,0.24,0.261;0.09,0.123,0.152) |
| H3 | (0.205,0.221,0.238;0.195,0.221,0.247) | (0.044,0.045,0.045;0.044,0.045,0.045) | (0.268,0.288,0.309;0.041,0.074,0.103) |
| H4 | (0.181,0.198,0.215;0.172,0.198,0.224) | (0.018,0.018,0.019;0.018,0.018,0.019) | (0.07,0.103,0.132;0.239,0.259,0.28) |
| H5 | (0.232,0.249,0.266;0.223,0.249,0.275) | (0.032,0.033,0.034;0.032,0.033,0.034) | (0.262,0.284,0.309;0.047,0.08,0.109) |
| H6 | (0.166,0.183,0.2;0.157,0.183,0.209) | (0.029,0.03,0.03;0.029,0.03,0.03) | (0.186,0.211,0.237;0.124,0.156,0.185) |
| H7 | (0.086,0.103,0.12;0.075,0.102,0.128) | (0.019,0.02,0.021;0.019,0.02,0.021) | (0.097,0.13,0.159;0.212,0.233,0.253) |
| H8 | (0.134,0.151,0.167;0.124,0.15,0.176) | (0.033,0.034,0.034;0.033,0.034,0.034) | (0.248,0.27,0.309;0.061,0.095,0.124) |
| H9 | (0.162,0.179,0.196;0.153,0.179,0.205) | (0.042,0.042,0.043;0.042,0.042,0.043) | (0.2,0.226,0.254;0.109,0.142,0.171) |
| H10 | (0.187,0.204,0.22;0.177,0.203,0.229) | (0.023,0.024,0.025;0.023,0.024,0.025) | (0.207,0.233,0.26;0.102,0.137,0.167) |
| H11 | (0.108,0.125,0.142;0.098,0.125,0.151) | (0.026,0.027,0.028;0.026,0.027,0.028) | (0.209,0.236,0.264;0.1,0.134,0.163) |
| H12 | (0.098,0.115,0.131;0.088,0.115,0.141) | (0.023,0.024,0.025;0.023,0.024,0.025) | (0.142,0.175,0.21;0.167,0.201,0.228) |
| H13 | (0.115,0.131,0.148;0.105,0.131,0.157) | (0.045,0.045,0.046;0.044,0.045,0.046) | (0.219,0.24,0.261;0.09,0.123,0.152) |
| H14 | (0.101,0.118,0.135;0.092,0.118,0.144) | (0.054,0.054,0.055;0.053,0.054,0.055) | (0.168,0.193,0.218;0.141,0.173,0.2) |
| H15 | (0.112,0.128,0.145;0.102,0.128,0.154) | (0.059,0.059,0.06;0.059,0.059,0.06) | (0.213,0.233,0.254;0.096,0.129,0.158) |
| H16 | (0.077,0.094,0.111;0.068,0.094,0.12) | (0.03,0.031,0.032;0.03,0.031,0.032) | (0.163,0.188,0.213;0.146,0.178,0.206) |
| Hospital | O4 | O5 | O6 |
| H1 | (0.085,0.096,0.108;0.078,0.096,0.113) | (0.148,0.162,0.176;0.05,0.072,0.091) | (0.032,0.035,0.038;0.03,0.035,0.04) |
| H2 | (0.073,0.084,0.096;0.067,0.084,0.101) | (0.142,0.155,0.169;0.057,0.078,0.096) | (0.032,0.035,0.037;0.03,0.035,0.039) |
| H3 | (0.049,0.06,0.072;0.042,0.06,0.077) | (0.172,0.185,0.199;0.026,0.048,0.066) | (0.029,0.032,0.034;0.027,0.032,0.036) |
| H4 | (0.038,0.05,0.062;0.031,0.049,0.066) | (0.032,0.053,0.072;0.167,0.18,0.193) | (0.027,0.03,0.033;0.025,0.03,0.035) |
| H5 | (0.035,0.047,0.058;0.028,0.046,0.063) | (0.172,0.185,0.199;0.026,0.048,0.066) | (0.027,0.03,0.033;0.025,0.03,0.035) |
| H6 | (0.063,0.075,0.086;0.057,0.074,0.091) | (0.139,0.159,0.199;0.059,0.083,0.102) | (0.026,0.029,0.032;0.024,0.029,0.034) |
| H7 | (0.027,0.038,0.05;0.021,0.038,0.055) | (0.044,0.065,0.084;0.155,0.168,0.181) | (0.025,0.028,0.03;0.023,0.028,0.032) |
| H8 | (0.053,0.065,0.077;0.047,0.065,0.082) | (0.159,0.173,0.199;0.039,0.061,0.08) | (0.03,0.033,0.035;0.028,0.033,0.037) |
| H9 | (0.034,0.045,0.057;0.028,0.045,0.062) | (0.121,0.139,0.158;0.078,0.101,0.12) | (0.029,0.032,0.035;0.027,0.032,0.037) |
| H10 | (0.024,0.036,0.048;0.019,0.036,0.052) | (0.138,0.155,0.173;0.06,0.083,0.102) | (0.027,0.029,0.032;0.024,0.029,0.034) |
| H11 | (0.02,0.032,0.043;0.014,0.032,0.048) | (0.123,0.141,0.159;0.075,0.098,0.117) | (0.028,0.031,0.033;0.026,0.031,0.035) |
| H12 | (0.018,0.029,0.041;0.01,0.029,0.046) | (0.088,0.109,0.132;0.111,0.133,0.151) | (0.028,0.03,0.033;0.026,0.03,0.035) |
| H13 | (0.066,0.078,0.089;0.057,0.076,0.093) | (0.121,0.138,0.156;0.078,0.1,0.118) | (0.03,0.033,0.036;0.028,0.033,0.038) |
| H14 | (0.052,0.064,0.076;0.045,0.062,0.08) | (0.111,0.127,0.144;0.088,0.109,0.127) | (0.032,0.034,0.037;0.029,0.034,0.039) |
| H15 | (0.059,0.071,0.083;0.051,0.069,0.087) | (0.117,0.133,0.149;0.082,0.102,0.12) | (0.032,0.034,0.037;0.03,0.034,0.039) |
| H16 | (0.042,0.053,0.065;0.036,0.053,0.07) | (0.108,0.125,0.142;0.091,0.112,0.131) | (0.032,0.034,0.037;0.03,0.034,0.039) |

**Table 13**
DEA inputs & outputs.

| Hospital | INPUTS | | | | | | | OUTPUTS | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | I1 | I2 | I3 | I4 | I5 | I6 | I7 | O1 | O2 | O3 | O4 | O5 | O6 |
| H1 | 48 | 36 | 290 | 168 | 45 | 255 | 97 | 149 | 66 | 243 | 99 | 164 | 36 |
| H2 | 44 | 32 | 243 | 155 | 42 | 248 | 102 | 155 | 60 | 243 | 87 | 158 | 36 |
| H3 | 23 | 6 | 57 | 37 | 21 | 255 | 91 | 228 | 46 | 291 | 62 | 187 | 33 |
| H4 | 24 | 5 | 54 | 41 | 26 | 273 | 104 | 204 | 19 | 110 | 51 | 58 | 31 |
| H5 | 20 | 4 | 49 | 49 | 30 | 267 | 94 | 257 | 34 | 287 | 48 | 187 | 31 |
| H6 | 20 | 6 | 21 | 20 | 7 | 151 | 79 | 189 | 31 | 216 | 77 | 168 | 30 |
| H7 | 63 | 9 | 59 | 67 | 14 | 269 | 100 | 106 | 21 | 136 | 40 | 69 | 28 |
| H8 | 17 | 4 | 46 | 36 | 16 | 242 | 102 | 155 | 35 | 278 | 67 | 179 | 34 |
| H9 | 21 | 4 | 50 | 49 | 20 | 149 | 65 | 185 | 44 | 231 | 47 | 142 | 33 |
| H10 | 19 | 5 | 35 | 30 | 11 | 256 | 100 | 210 | 25 | 237 | 37 | 158 | 30 |
| H11 | 24 | 11 | 49 | 41 | 12 | 155 | 69 | 128 | 28 | 240 | 33 | 144 | 32 |
| H12 | 15 | 4 | 37 | 30 | 8 | 113 | 45 | 118 | 25 | 182 | 30 | 114 | 31 |
| H13 | 25 | 9 | 64 | 50 | 17 | 237 | 98 | 135 | 47 | 243 | 80 | 141 | 34 |
| H14 | 33 | 13 | 89 | 92 | 24 | 149 | 65 | 121 | 56 | 198 | 65 | 131 | 35 |
| H15 | 39 | 24 | 110 | 86 | 25 | 155 | 65 | 132 | 61 | 237 | 73 | 136 | 35 |
| H16 | 33 | 5 | 41 | 31 | 8 | 158 | 86 | 97 | 32 | 193 | 55 | 128 | 35 |

Even DEA is an effective optimization technique to evaluate the performance of hospitals, it has some limitations which need to be underlined. When the number of inputs and outputs are relatively large and the number of alternative hospitals is low, discriminatory power of the DEA is limited.

For further research, we suggest other extensions of ordinary fuzzy sets such as hesitant fuzzy sets, fuzzy multisets, or non-stationary fuzzy sets to be used for the solution of the same problem. Additionally, the recently proposed sets such as neutrosophic sets and Pythagorean fuzzy sets can be used to deal with uncertainty with a different perspective.

Besides, Electronic Health Record (EHR) systems, which keep patients' historical information electronically, can provide more reliable data for the proposed method to be used in the performance evaluation of the hospitals [61,62].

**Table 14**
DEA results, efficiencies, input slacks, and output slacks.

| Hospital | Efficiency | INPUTS | | | | | | | OUTPUTS | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | I1 | I2 | I3 | I4 | I5 | I6 | I7 | O1 | O2 | O3 | O4 | O5 | O6 |
| H1 | 0.954 | 0.00 | 9.14 | 160.2 | 67.6 | 16.1 | 36.3 | 0.00 | 48.6 | 4.89 | 67.75 | 0.00 | 30.01 | 9.7 |
| H2 | 0.847 | 0.00 | 12.6 | 119.2 | 46.9 | 12.9 | 23.4 | 0.00 | 23.9 | 0.00 | 16.12 | 0.00 | 20.21 | 6.1 |
| H3 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H4 | 0.946 | 4.24 | 0.00 | 9.98 | 0.00 | 4.08 | 44.48 | 15.56 | 0.00 | 12.39 | 142.30 | 0.00 | 109.40 | 0.00 |
| H5 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H6 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H7 | 0.515 | 15.93 | 0.97 | 0.05 | 9.56 | 0.00 | 24.38 | 0.00 | 14.84 | 3.76 | 37.30 | 0.00 | 46.84 | 0.00 |
| H8 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H9 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H10 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H11 | 0.974 | 3.07 | 5.93 | 0.00 | 1.96 | 0.50 | 0.00 | 5.22 | 31.3 | 5.05 | 0.00 | 10.44 | 8.78 | 9.41 |
| H12 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H13 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H14 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H15 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| H16 | 1.000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

**Table 15**
Comparison of TIF AHP & DEA and traditional DEA.

| Hospital | TIF AHP & DEA Results | Traditional DEA results | Difference |
|---|---|---|---|
| H1 | 0.954 | 0.921 | −0.034 |
| H2 | 0.848 | 0.902 | 0.054 |
| H3 | 1 | 1 | 0 |
| H4 | 0.946 | 0.971 | 0.026 |
| H5 | 1 | 1 | 0 |
| H6 | 1 | 1 | 0 |
| H7 | 0.515 | 0.555 | 0.040 |
| H8 | 1 | 1 | 0 |
| H9 | 1 | 1 | 0 |
| H10 | 1 | 1 | 0 |
| H11 | 0.974 | 1 | 0.026 |
| H12 | 1 | 1 | 0 |
| H13 | 1 | 1 | 0 |
| H14 | 1 | 1 | 0 |
| H15 | 1 | 1 | 0 |
| H16 | 1 | 1 | 0 |

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