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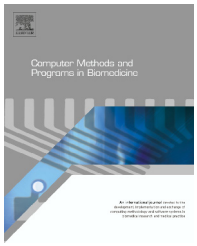

Multi-period medical diagnosis method using a single valued neutrosophic similarity measure based on tangent function

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ABSTRACT

Because of the increased volume of information available to physicians from advanced medical technology, the obtained information of each symptom with respect to a disease may contain truth, falsity and indeterminacy information. Since a single-valued neutrosophic set (SVNS) consists of the three terms like the truth-membership, indeterminacy-membership and falsity-membership functions, it is very suitable for representing indeterminate and inconsistent information. Then, similarity measure plays an important role in pattern recognition and medical diagnosis. However, existing medical diagnosis methods can only handle the single period medical diagnosis problem, but cannot deal with the multi-period medical diagnosis problems with neutrosophic information. Hence, the purpose of this paper was to propose similarity measures between SVNSs based on tangent function and a multi-period medical diagnosis method based on the similarity measure and the weighted aggregation of multi-period information to solve multi-period medical diagnosis problems with single-valued neutrosophic information. Then, we compared the tangent similarity measures of SVNSs with existing similarity measures of SVNSs by a numerical example about pattern recognitions to indicate the effectiveness and rationality of the proposed similarity measures. In the multi-period medical diagnosis method, we can find a proper diagnosis for a patient by the proposed similarity measure between the symptoms and the considered diseases represented by SVNSs and the weighted aggregation of multi-period information. Then, a multi-period medical diagnosis example was presented to demonstrate the application of the proposed diagnosis method and to indicate the effectiveness of the proposed diagnosis method by the comparative analysis. The diagnosis results showed that the developed multi-period medical diagnosis method can help doctors make a proper diagnosis by the comprehensive information of multi-periods.

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1. Introduction

In medical diagnosis problems, due to the complexity of various diseases and the lack of knowledge or data about the problem domain, crisp data are sometimes unavailable as medical diagnosis contains lots of uncertainties. Uncertainty is an important phenomenon of medical diagnosis problems. A symptom is an uncertain indication of a disease. Hence, the uncertainty characterizes a relation between symptoms and diseases. Thus, working with the uncertainties leads us to accurate decision making in medical diagnosis problems. In most of the medical diagnosis problems, there exist some patterns, and then experts make decision according to the similarity between an unknown sample and the basic patterns. However, the arguments introduced from an unknown sample may be vague or fuzzy in nature. To represent incomplete and uncertainty information, Zadeh [1] firstly proposed fuzzy set theory. Its characteristic is that a membership degree is assigned to each element in the set. Since then, various extensions of this concept have been introduced by many researchers. For example, Atanassov [2] extended fuzzy sets to intuitionistic fuzzy sets (IFSs). The prominent characteristic of IFS is that a membership degree and a non-membership degree are assigned to each element in the set. Since there is a number of uncertainties in medical diagnoses, they are the most interesting and fruitful areas of applications in intuitionistic fuzzy set theory. Hence, various medical diagnosis methods have been presented under the general framework of IFSs [3,4]. Recently, Ye [5] put forward cosine similarity measures for IFSs and applied them to pattern recognition and medical diagnosis. Hung [6] introduced an intuitionistic fuzzy likelihood-based measurement and applied it to medical diagnosis and bacteria classification problems. Tian [7] developed the cotangent similarity measure of IFSs and applied it to medical diagnosis.

However, IFSs can handle only incomplete and uncertainty information but not indeterminate and inconsistent information which exists usually in real situations. To deal with this kind of indeterminate and inconsistent information, Smarandache [8] originally proposed the concept of a neutrosophic set from a philosophical point of view. A neutrosophic set A in a universal set X is characterized independently by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. However, since the defined range of the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ in a neutrosophic set A is the non-standard unit interval $]^-0, 1^+[$, the neutrosophic set is only used for philosophical applications, but it is difficult to apply it to science and engineering areas. To use it easily, the defined range of its functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ can be restrained to the normal standard real unit interval [0, 1]. For this purpose, Wang et al. [9] introduced the concept of a single valued neutrosophic set (SVNS), which is a subclass of the neutrosophic set and a generalization of IFS. Because of the increased volume of information available to physicians from advanced medical technology, the obtained information of each symptom with respect to a disease may contain truth, falsity and indeterminacy information. Since a SVNS consists of the three terms like the truth-membership, indeterminacy-membership and falsity-membership functions, it is very suitable for representing indeterminate and inconsistent information. However, similarity measure is a key role in the analysis and research of medical diagnosis, pattern recognition, machine learning, decision making, and clustering analysis in uncertainty environment. Therefore, some researchers have proposed various similarity measures of SVNSs and mainly applied them to decision making. For instance, Majumdar and Samanta [10] introduced several similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [11] further proposed the distance-based similarity measure of SVNSs and applied it to group decision making problems with single valued neutrosophic information. Furthermore, Ye [12] proposed three vector similarity measures for simplified neutrosophic sets (SNSs), including the Jaccard, Dice and cosine similarity measures for SVNSs and interval neutrosophic sets (INSs), and applied them to multicriteria decision-making problems with simplified neutrosophic information, which contains single-valued and interval neutrosophic information. Although simplified neutrosophic sets (SVNSs and INSs) have been successfully applied to decision making [11–18], they are scarcely applied to medical diagnosis problems.

In medical diagnosis, recently, Ye [19] proposed the improved cosine similarity measures of SVNSs and INSs based on cosine function and applied them to medical diagnosis problems. Because the symptoms and inspecting data of some disease may be changed in different time intervals, one of the medical diagnosis questions is whether only by taking a single period inspection we can reach a proper conclusion for a particular patient with a particular decease or not. Sometimes he or she may show symptoms of different diseases also. Then, how can we reach a proper diagnosis for the particular patient by taking one inspection? One solution may be to examine the patient through multi-periods (dynamic inspection in different time intervals) and to realize comprehensive diagnosis for the patient corresponding to the dynamic inspecting information. In this case, multi-period medical diagnosis (i.e. dynamic medical diagnosis) needs a comprehensive diagnosis method. However, the existing medical diagnosis methods [3–7,19] can only handle single period diagnosis problems, but cannot deal with comprehensive medical diagnosis problems in the multi-periods (dynamic diagnosis problems). To handle the multi-period medical diagnosis problem, this paper proposes new similarity measures of SVNSs based on tangent function and a multi-period medical diagnosis method based on the proposed similarity measure and the weighted aggregation of multi-period information to help doctors make a proper diagnosis for a patient.

The rest of the article is organized as follows. Section 2 introduces some basic concepts of SVNSs and similarity measures for SVNSs. Section 3 puts forward similarity measures of SVNSs based on tangent function and weighted similarity measures of SVNSs and investigates their properties. In Section 4, we propose a multi-period medical diagnosis method based on the proposed similarity measure and the weighted aggregation of

multi-period information. In Section 5, a multi-period medical diagnosis example is provided to illustrate the application of the multi-period medical diagnosis method and to indicate the effectiveness by the comparative analysis. Conclusions and further research are given in Section 6.

2. Some basic concepts of SVNSs and similarity measures

Smarandache [8] originally presented a neutrosophic set theory. A neutrosophic set A in a universal set X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. Its three functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$, such that $T_A(x): X \to]^{-}0, 1^{+}[$, $I_A(x): X \to]^{-}0, 1^{+}[$, and $F_A(x): X \to]^{-}0, 1^{+}[$. Hence, the sum of the three functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $^{-}0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^{+}$.

However, Smarandache [8] introduced the neutrosophic set from a philosophical point of view as a generalization of fuzzy set, IFS, and interval-valued IFS. But it is difficult to apply the neutrosophic set to practical problems. To apply it easily in science and engineering fields, Wang et al. [9] introduced the concept of SVNS, which is a subclass of the neutrosophic set, and gave the following definition.

Definition 1 ([9]). Let X be a universal set. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. Then, a SVNS A can be denoted by

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle | x \in X \right\}$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each x in X. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

For a SVNS A in X, the triplet $\langle T_A(x), I_A(x), F_A(x) \rangle$ is called single valued neutrosophic value (SVNV), which is a fundamental element in a SVNS A. For convenience, we can simply denote $a = \langle T_A, I_A, F_A \rangle$ as a SVNV in A.

Let $A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle | x \in X \right\}$ and $B = \left\{ \left\langle x, T_B(x), I_B(x), F_B(x) \right\rangle | x \in X \right\}$ be two SVNSs in X. Then, Wang et al. [9] gave some relations:

(1) Complement: $A^c = \left\{ \left\langle x, F_A(x), 1 - I_A(x), T_A(x) \right\rangle | x \in X \right\}$;
(2) Inclusion: $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for any x in X;
(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Let $A = \left\{ \left\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j) \right\rangle | x_j \in X \right\}$ and $B = \left\{ \left\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j) \right\rangle | x_j \in X \right\}$ be any two SVNSs in $X = \{x_1, x_2, \ldots, x_n\}$. Then, Ye [12] presented the Jaccard, Dice, and cosine similarity measures between SVNSs A and B in vector space, respectively, as follows:

$$S_J(A, B) = \frac{1}{n}\sum_{j=1}^{n} \frac{T_A(x_j)T_B(x_j) + I_A(x_j)I_B(x_j) + F_A(x_j)F_B(x_j)}{[(T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)) + (T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)) - (T_A(x_j)T_B(x_j) + I_A(x_j)I_B(x_j) + F_A(x_j)F_B(x_j))]}, \quad (1)$$

$$S_D(A, B) = \frac{1}{n}\sum_{j=1}^{n} \frac{2(T_A(x_j)T_B(x_j) + I_A(x_j)I_B(x_j) + F_A(x_j)F_B(x_j))}{(T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)) + (T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j))}, \quad (2)$$

$$S_C(A, B) = \frac{1}{n}\sum_{j=1}^{n} \frac{T_A(x_j)T_B(x_j) + I_A(x_j)I_B(x_j) + F_A(x_j)F_B(x_j)}{\sqrt{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)}\sqrt{T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)}}. \quad (3)$$

The similarity measures of $S_k(A, B)$ (k = J, D, C) satisfy the following properties [12]:

(P1) $0 \le S_k(A, B) \le 1$;
(P2) $S_k(A, B) = S_k(B, A)$;
(P3) $S_k(A, B) = 1$ if $A = B$, i.e., $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, and $F_A(x_j) = F_B(x_j)$ for $x_j \in X$.

To overcome some disadvantages of cosine similarity measures in vector space, Ye [19] introduced two improved similarity measures between SVNSs A and B based on cosine function, respectively, as follows:

$$C_1(A, B) = \frac{1}{n}\sum_{j=1}^{n} \cos\left[\pi \frac{\max(|T_A(x_j) - T_B(x_j)|, |I_A(x_j) - I_B(x_j)|, |F_A(x_j) - F_B(x_j)|)}{2}\right], \quad (4)$$

$$C_2(A, B) = \frac{1}{n}\sum_{j=1}^{n} \cos\left[\frac{\pi(|T_A(x_j) - T_B(x_j)| + |I_A(x_j) - I_B(x_j)| + |F_A(x_j) - F_B(x_j)|)}{6}\right], \quad (5)$$

Then, the cosine similarity measures of $C_k(A, B)$ (k = 1, 2) satisfy the following properties (P1)–(P4) [19]:

(P1) $0 \le C_k(A, B) \le 1$;
(P2) $C_k(A, B) = 1$ if and only if $A = B$;
(P3) $C_k(A, B) = C_k(B, A)$;
(P4) If C is a SVNS in X and $A \subseteq B \subseteq C$, then $C_k(A, C) \le C_k(A, B)$ and $C_k(A, C) \le C_k(B, C)$.

3. Tangent function-based similarity measures of SVNSs

3.1. *Tangent similarity measures between SVNSs*

Similarity measure is usually an important mathematical tool in pattern recognition, medical diagnosis, clustering analysis, and decision making. Therefore, the subsection proposes similarity measures between SVNSs based on tangent function and investigates their properties.

Motivated by the improved cosine similarity measures of SVNSs [19] and based on the tangent function, we propose the tangent similarity measures of SVNSs.

Definition 2. Let $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle \mid x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle \mid x_j \in X\}$ be any two SVNSs in $X = \{x_1, x_2, \ldots, x_n\}$. Based on the tangent function, we define the following similarity measures between A and B:

$$T_1(A,B) = 1 - \frac{1}{n}\sum_{j=1}^{n} \tan\left[\frac{\pi}{4}\max(|T_A(x_j) - T_B(x_j)|, |I_A(x_j) - I_B(x_j)|, |F_A(x_j) - F_B(x_j)|)\right], \quad (6)$$

$$T_2(A,B) = 1 - \frac{1}{n}\sum_{j=1}^{n} \tan\left[\frac{\pi}{12}(|T_A(x_j) - T_B(x_j)| + |I_A(x_j) - I_B(x_j)| + |F_A(x_j) - F_B(x_j)|)\right]. \quad (7)$$

Then, the similarity measures of $T_k(A, B)$ $(k = 1, 2)$ have the following proposition.

Proposition 1. For two SVNSs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the similarity measures of $T_k(A, B)$ $(k = 1, 2)$ should satisfy the following properties (P1)–(P4):

(P1) $0 \le T_k(A, B) \le 1$;
(P2) $T_k(A, B) = 1$ if and only if $A = B$;
(P3) $T_k(A, B) = T_k(B, A)$;
(P4) If C is a SVNS in X and $A \subseteq B \subseteq C$, then $T_k(A, C) \le T_k(A, B)$ and $T_k(A, C) \le T_k(B, C)$.

Proofs. (P1) Since the value of the tangent function $\tan(x)$ is within [0, 1] in $x \in [0, \pi/4]$, the similarity measure value based on the tangent function also is within [0, 1] according to Eqs. (6) and (7). Hence, there is $0 \le T_k(A, B) \le 1$ for $k = 1, 2$.
(P2) When $A = B$, this implies $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Then $|T_A(x_j) - T_B(x_j)| = 0$, $|I_A(x_j) - I_B(x_j)| = 0$, and $|F_A(x_j) - F_B(x_j)| = 0$. Thus $\tan(0) = 0$, Hence $T_k(A, B) = 1$ for $k = 1, 2$.
If $T(A, B) = 1$, then it must satisfy $\tan(0) = 0$. This implies $|T_A(x_j) - T_B(x_j)| = 0$, $|I_A(x_j) - I_B(x_j)| = 0$ and $|F_A(x_j) - F_B(x_j)| = 0$. Then, there are $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, and $F_A(x_j) = F_B(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Hence $A = B$.
(P3) Proof is straightforward.
(P4) If $A \subseteq B \subseteq C$, then this implies $T_A(x_j) \le T_B(x_j) \le T_C(x_j)$, $I_A(x_j) \ge I_B(x_j) \ge I_C(x_j)$ and $F_A(x_j) \ge F_B(x_j) \ge F_C(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Thus, we have the following inequalities:

$$|T_A(x_j) - T_B(x_j)| \le |T_A(x_j) - T_C(x_j)|, |T_B(x_j) - T_C(x_j)| \le |T_A(x_j) - T_C(x_j)|,$$

$$|I_A(x_j) - I_B(x_j)| \le |I_A(x_j) - I_C(x_j)|, |I_B(x_j) - I_C(x_j)| \le |I_A(x_j) - I_C(x_j)|,$$

$$|F_A(x_j) - F_B(x_j)| \le |F_A(x_j) - F_C(x_j)|, |F_B(x_j) - F_C(x_j)| \le |F_A(x_j) - F_C(x_j)|.$$

Hence, $T_k(A, C) \le T_k(A, B)$ and $T_k(A, C) \le T_k(B, C)$ for $k = 1, 2$ since the tangent function is an increasing function within $[0, \pi/4]$. Therefore, we complete the proofs of these properties. □

If we consider the weight of each element x_j for $x_j \in X = \{x_1, x_2, \ldots, x_n\}$ and assume that the weight of an element x_j is w_j $(j = 1, 2, \ldots, n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then we can introduce the following weighted tangent similarity measures between SVNSs A and B:

$$T_{W1}(A,B) = 1 - \sum_{j=1}^{n}\left\{w_j \tan\left[\frac{\pi}{4}\max(|T_A(x_j) - T_B(x_j)|, |I_A(x_j) - I_B(x_j)|, |F_A(x_j) - F_B(x_j)|)\right]\right\}, \quad (8)$$

$$T_{W2}(A,B) = 1 - \sum_{j=1}^{n}\left\{w_j \tan\left[\frac{\pi}{12}(|T_A(x_j) - T_B(x_j)| + |I_A(x_j) - I_B(x_j)| + |F_A(x_j) - F_B(x_j)|\frac{\pi}{4}\max)\right]\right\}. \quad (9)$$

Especially, when $w_j = 1/n$ for $j = 1, 2, \ldots, n$, Eqs. (8) and (9) reduce to Eqs. (6) and (7) respectively.

Obviously, the weighted tangent similarity measures also have the following proposition.

Proposition 2. For two SVNSs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, assume that the weight of an element x_j is w_j $(j = 1, 2, \ldots, n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Then, the weighted tangent similarity measures of $T_{Wk}(A, B)$ for $k = 1, 2$ should satisfy the following properties (P1)–(P4):

(P1) $0 \le T_{Wk}(A, B) \le 1$;
(P2) $T_{Wk}(A, B) = 1$ if and only if $A = B$;
(P3) $T_{Wk}(A, B) = T_{Wk}(B, A)$;
(P4) If C is a SVNS in X and $A \subseteq B \subseteq C$, then $T_{Wk}(A, C) \le T_{Wk}(A, B)$ and $T_{Wk}(A, C) \le T_{Wk}(B, C)$.

By the similar proof method of Proposition 1, we can give the proofs of these properties. They are not repeated here.

3.2. *Comparison of some similarity measures*

For the comparison of the similarity measures based on the tangent function with existing similarity measure formulae (1)–(5) [12,19] in single-valued neutrosophic setting, a numerical example is presented to demonstrate the effectiveness and rationality of the proposed tangent similarity measures of SVNSs.

Let us consider two SVNSs A and B in $X = \{x\}$ and compare the proposed tangent similarity measures with the existing similarity measures [12,19] for pattern recognitions. By applying Eqs. (1)–(7), the similarity measure results for the pattern recognitions are indicated by the numerical example, as shown in Table 1.

For the results of Table 1, the cosine similarity measure S_c in [12] not only cannot carry out the recognition between Case 1 and Case 5 but also produces an unreasonable phenomenon

Table 1 – Similarity measure values of Eqs. (1)–(7).

	Case 1	Case 2	Case 3	Case 4	Case 5
A	⟨x, 0.2, 0.3, 0.4⟩	⟨x, 0.3, 0.2, 0.4⟩	⟨x, 1, 0, 0⟩	⟨x, 1, 0, 0⟩	⟨x, 0.4, 0.2, 0.6⟩
B	⟨x, 0.2, 0.3, 0.4⟩	⟨x, 0.4, 0.2, 0.3⟩	⟨x, 0, 1, 1⟩	⟨x, 0, 0, 0⟩	⟨x, 0.2, 0.1, 0.3⟩
$S_J(A, B)$	1	0.9333	**0**	**0**	0.6667
$S_D(A, B)$	1	0.9655	**0**	**0**	0.8000
$S_C(A, B)$	**1**	0.9655	0	**null**	**1**
$C_1(A, B)$	1	0.9877	**0**	**0**	0.8910
$C_2(A, B)$	1	0.9945	0	0.8660	0.9511
$T_1(A, B)$	1	0.9213	**0**	**0**	0.7599
$T_2(A, B)$	1	0.9476	0	0.7321	0.8416

The bold values indicate difficult recognition with respect to the same measure values.

for Case 5 and an undefined (unmeaningful) phenomenon for Case 4. Then, we can see that the similarity measures S_J, S_D, C_1 and T_1 cannot carry out the recognition between Case 3 and Case 4, while the similarity measures C_2 and T_2 can carry out all recognitions. Hence, the similarity measures C_2 and T_2 demonstrate stronger discrimination among them and are superior to other similarity measures, while the cosine similarity measure S_C in vector space demonstrates bad discrimination among them and is inferior to other similarity measures. Obviously, the similarity measures C_2 and T_2 are very suitable for handling pattern recognition and medical diagnosis problems. Therefore, the similarity measure T_2 will be applied to multi-period medical diagnosis problems in this paper.

4. Multi-period medical diagnosis method

Due to more and more complexity of real medical diagnoses, a lot of information available to physicians from modern medical technology is often incomplete, indeterminate and inconsistent information. However, by only taking single period inspection, one is difficult to reach a proper conclusion from a particular patient with a particular decease. Sometimes he or she may also show the symptoms of different diseases. Then, how can we give a proper conclusion? One solution may be to examine the patient across different periods and to realize a comprehensive diagnosis for the patient. To do so, we present a multi-period medical diagnosis method to help doctors make a proper diagnosis based on the comprehensive information of multi-periods.

Let $S = \{S_1, S_2, \ldots, S_m\}$ be a set of symptoms, $T = \{t_1, t_2, \ldots, t_q\}$ be a set of periods, and $D = \{D_1, D_2, \ldots, D_n\}$ be a set of considered diseases. For a patient P_s with various symptoms, we can indicate characteristic values between the patient and symptoms in multi-period medical diagnosis problems, which are shown in Table 2. Then, Table 3 shows characteristic values between symptoms and the considered diseases.

In Table 2, $C_j(t_k)$ denotes the characteristic value between a patient P_s and the jth symptom S_j ($j = 1, 2, \ldots, m$) in the kth period t_k for $k = 1, 2, \ldots, q$. Obviously, if $q = 1$, the medical diagnosis problem is usually a single period medical diagnosis problem [19].

In Table 3, C_{ij} denotes the characteristic value between the jth symptom S_j ($j = 1, 2, \ldots, m$) and the ith considered disease D_i ($i = 1, 2, \ldots, n$).

For a multi-period medical diagnosis problem with single-valued neutrosophic information, the characteristic values between a patient and symptoms are denoted by the form of a SVNV $C_j(t_k) = \langle T_j(t_k), I_j(t_k), F_j(t_k)\rangle$ and the characteristic values between symptoms and the considered diseases are denoted by the form of a SVNV $C_{ij} = \langle T_{ij}, I_{ij}, F_{ij}\rangle$ for convenience. If we consider that the weight vector of the symptoms is $\boldsymbol{w} = (w_1, w_2, \ldots, w_m)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{m} w_j = 1$ and the weight vector of the periods is $\boldsymbol{\omega} = (\omega(t_1), \omega(t_2), \ldots, \omega(t_q))$ with $\omega(t_k) \in [0, 1]$ and $\sum_{k=1}^{q} \omega(t_k) = 1$.

Since the tangent similarity measure of Eq. (9) demonstrates stronger discrimination mentioned above, we apply it to multi-period medical diagnosis problems. Thus, the diagnosis steps are given as follows:

Step 1: Calculate the similarity measure between a patient P_s and the considered disease D_i ($i = 1, 2, \ldots, n$) in each period t_k ($k = 1, 2, \ldots, q$) by the following formula:

$$T_{Wi}(P_s, t_k) = 1 - \sum_{j=1}^{m} \left\{ w_j \tan\left[\frac{\pi}{12}(|T_j(t_k) - T_{ij}| + |I_j(t_k) - I_{ij}| + |F_j(t_k) - F_{ij}|)\right]\right\}. \quad (10)$$

Table 2 – Characteristic values between a patient P_s and symptoms.

	t_k	S_1	S_2	...	S_m
P_s	t_1	$C_1(t_1)$	$C_2(t_1)$	...	$C_m(t_1)$
	t_2	$C_1(t_2)$	$C_2(t_2)$		$C_m(t_2)$
	...	...	...		...
	t_q	$C_1(t_q)$	$C_2(t_q)$		$C_m(t_q)$

Table 3 – Characteristic values between symptoms and the considered diseases.

	S_1	S_2	...	S_m
D_1	C_{11}	C_{12}	...	C_{1m}
D_2	C_{21}	C_{22}	...	C_{2m}
...	...	...	...	...
D_n	C_{n1}	C_{n2}	...	C_{nm}

Table 4 – Characteristic values between symptoms and the considered diseases represented by SVNVs.

	S_1 (temperature)	S_2 (headache)	S_3 (stomach pain)	S_4 (cough)	S_5 (chest pain)
D_1 (viral fever)	⟨0.4, 0.6, 0.0⟩	⟨0.3, 0.2, 0.5⟩	⟨0.1, 0.3, 0.7⟩	⟨0.4, 0.3, 0.3⟩	⟨0.1, 0.2, 0.7⟩
D_2 (malaria)	⟨0.7, 0.3, 0.0⟩	⟨0.2, 0.2, 0.6⟩	⟨0.0, 0.1, 0.9⟩	⟨0.7, 0.3, 0.0⟩	⟨0.1, 0.1, 0.8⟩
D_3 (typhoid)	⟨0.3, 0.4, 0.3⟩	⟨0.6, 0.3, 0.1⟩	⟨0.2, 0.1, 0.7⟩	⟨0.2, 0.2, 0.6⟩	⟨0.1, 0.0, 0.9⟩
D_4 (gastritis)	⟨0.1, 0.2, 0.7⟩	⟨0.2, 0.4, 0.4⟩	⟨0.8, 0.2, 0.0⟩	⟨0.2, 0.1, 0.7⟩	⟨0.2, 0.1, 0.7⟩
D_5 (stenocardia)	⟨0.1, 0.1, 0.8⟩	⟨0.0, 0.2, 0.8⟩	⟨0.2, 0.0, 0.8⟩	⟨0.2, 0.0, 0.8⟩	⟨0.8, 0.1, 0.1⟩

Table 5 – Characteristic values between four patients and symptoms represented by SVNVs.

	t_k	S_1 (temperature)	S_2 (headache)	S_3 (stomach pain)	S_4 (cough)	S_5 (chest pain)
	t_1	⟨0.8, 0.6, 0.5⟩	⟨ 0.5, 0.4, 0.3⟩	⟨0.2, 0.1, 0.3⟩	⟨0.7, 0.6, 0.3⟩	⟨0.4, 0.3, 0.2⟩
P1	t_2	⟨0.7, 0.3, 0.2⟩	⟨0.6, 0.3, 0.2⟩	⟨0.3, 0.2, 0.4⟩	⟨0.6, 0.5, 0.2⟩	⟨0.6, 0.5, 0.3⟩
	t_3	⟨0.5, 0.2, 0.4⟩	⟨0.6, 0.3, 0.4⟩	⟨0.3, 0.3, 0.5⟩	⟨0.4, 0.3, 0.2⟩	⟨0.6, 0.4, 0.4⟩
	t_1	⟨0.6, 0.6, 0.1⟩	⟨0.1, 0.2, 0.6⟩	⟨0.3, 0.2, 0.8⟩	⟨0.6, 0.2, 0.3⟩	⟨0.2, 0.3, 0.7⟩
P_2	t_2	⟨ 0.5, 0.4, 0.2⟩	⟨0.2, 0.2, 0.6⟩	⟨0.2, 0.1, 0.7⟩	⟨0.8, 0.3, 0.1⟩	⟨0.1, 0.1, 0.8⟩
	t_3	⟨0.8, 0.3, 0.1⟩	⟨0.2, 0.1, 0.5⟩	⟨0.1, 0.1, 0.9⟩	⟨0.7, 0.2, 0.0⟩	⟨0.1, 0.1, 0.8⟩
	t_1	⟨0.3, 0.1, 0.2⟩	⟨0.3, 0.2, 0.2⟩	⟨0.7, 0.6, 0.7⟩	⟨0.3, 0.2, 0.2⟩	⟨0.4, 0.4, 0.3⟩
P_3	t_2	⟨0.4, 0.2, 0.2⟩	⟨0.5, 0.1, 0.3⟩	⟨0.4, 0.2, 0.2⟩	⟨(0.5, 0.3, 0.3⟩	⟨0.6, 0.3, 0.2⟩
	t_3	⟨0.8, 0.7, 0.6⟩	⟨0.7, 0.5, 0.5⟩	⟨0.4, 0.1, 0.1⟩	⟨(0.7, 0.3, 0.4⟩	⟨0.7, 0.4, 0.5⟩
	t_1	⟨0.2, 0.1, 0.7⟩	⟨0.2, 0.3, 0.7⟩	⟨0.2, 0.2, 0.7⟩	⟨0.2, 0.1, 0.8⟩	⟨0.8, 0.2, 0.1⟩
P_4	t_2	⟨0.1, 0.1, 0.6⟩	⟨0.1, 0.2, 0.8⟩	⟨0.2, 0.1, 0.8⟩	⟨0.3, 0.0, 0.9⟩	⟨0.7, 0.1, 0.2⟩
	t_3	⟨0.1, 0.1, 0.8⟩	⟨0.1, 0.2, 0.7⟩	⟨0.3, 0.1, 0.8⟩	⟨0.2, 0.1, 0.9⟩	⟨0.9, 0.1, 0.1⟩

Step 2: Obtain the weighted aggregation values of M_{Ti} (P_s) for $i = 1, 2, \ldots, n$ by the following formula:

$$M_{Ti}(P_s) = \sum_{k=1}^{q} T_{Wi}(P_s, t_k)\omega(t_k). \tag{11}$$

Step 3: Give a proper diagnosis for the patient P_s according to the maximum weighted aggregation value.
Step 4: End.

5. Multi-period medical diagnosis example

Since physicians can obtain a lot of information from modem medical technology, medical diagnosis contains a lot of incomplete, uncertainty and inconsistent information. Then, SVNS is a very suitable tool for expressing and handling it. In this section, we provide a medical diagnosis example to demonstrate the application of the proposed multi-period medical diagnosis method and to indicate the effectiveness by the comparative analysis.

In the following example, we shall discuss the medical diagnosis problem adapted from [19].

Let $D = \{D_1, D_2, D_3, D_4, D_5\} = \{$viral fever, malaria, typhoid, gastritis, stenocardia$\}$ be a set of diseases and $S = \{S_1, S_2, S_3, S_4, S_5\} = \{$temperature, headache, stomach pain, cough, chest pain$\}$ be a set of symptoms. Then characteristic values between symptoms and the considered diseases are represented by the form of SVNVs, which are shown in Table 4 [19].

In the medical diagnosis, assume that there are four patients. We have to take three different samples from each patient P_s $(s = 1, 2, 3, 4)$ in three periods. For example, the SVNV of a symptom S_1 for a patient P_s is given as $C_1(t_1) = \langle 0.8, 0.6, 0.5\rangle$ in the first period t_1, which indicates that the characteristic value $C_1(t_1)$ between the patient P_s and the symptom S_1 is the truth degree 0.8, falsity degree 0.5 and indeterminacy degree 0.6. Thus, the characteristic values between the four patients and the five symptoms are represented by SVNVs, which can be constructed as Table 5.

If we consider that the weight vector of the five symptoms is $\boldsymbol{w} = (1/5, 1/5, 1/5, 1/5, 1/5)$ and the weight vector of the three periods is $\boldsymbol{\omega} = (0.25, 0.35, 0.4)$. Thus, the proposed multi-period medical diagnosis method can be applied to the diagnosis problem.

5.1. *Multi-period medical diagnosis based on the tangent similarity measure*

To illustrate the proposed multi-period medical diagnostic process of the example, we give the following diagnosis steps for the patient P_1 in detail:

Step 1: Calculate the similarity measures between the patient P_1 and the considered disease D_i in each period t_k for $i = 1, 2, 3, 4, 5$ and $k = 1, 2, 3$ by Eq. (10), as shown in Table 6.
Step 2: Obtain the weighted aggregation values of $M_{Ti}(P_1)$ for $i = 1, 2, 3, 4, 5$ by applying Eq. (11): $M_{T1}(P_1) = \mathbf{0.8183}$,

Table 6 – Similarity measure values of $T_{Wi}(P_1, t_k)$.

	t_1	t_2	t_3
$T_{W1}(P_1, t_k)$	0.8035	0.7974	0.8458
$T_{W2}(P_1, t_k)$	0.7760	0.7904	0.7865
$T_{W3}(P_1, t_k)$	0.7741	0.7885	0.8178
$T_{W4}(P_1, t_k)$	0.7298	0.7141	0.7758
$T_{W5}(P_1, t_k)$	0.6944	0.6900	0.7539

Table 7 – Diagnosis results based on the tangent similarity measure and the cosine similarity measure.

	D_1 (viral fever)	D_2 (malaria)	D_3 (typhoid)	D_4 (gastritis)	D_5 (stenocardia)	Diagnosis result
$M_{Ti}(P_1)$	**0.8183**	0.7853	0.7966	0.7427	0.7167	Viral fever
$M_{Ti}(P_2)$	0.8985	**0.9409**	0.8315	0.7451	0.7220	Malaria
$M_{Ti}(P_3)$	**0.8058**	0.7554	0.7738	0.7701	0.7230	Viral fever
$M_{Ti}(P_4)$	0.7491	0.7214	0.7692	0.8036	**0.9562**	Stenocardia
$M_{Ci}(P_1)$	**0.9301**	0.9020	0.8990	0.8678	0.8419	Viral fever
$M_{Ci}(P_2)$	0.9774	**0.9883**	0.9263	0.8474	0.8253	Malaria
$M_{Ci}(P_3)$	**0.9169**	0.8764	0.8899	0. 8941	0.8519	Viral fever
$M_{Ci}(P_4)$	0.8610	0.8211	0.8688	0.8967	**0.9951**	Stenocardia

The bold values indicate the largest measure values, which show the proper diagnosis results.

$M_{T2}(P_1) = 0.7852$, $M_{T3}(P_1) = 0.7966$, $M_{T4}(P_1) = 0.7427$, and $M_{T5}(P_1) = 0.7167$.

Step 3: The patient P_1 suffers from viral fever according to the maximum weighted aggregation value ($M_{T1}(P_1) = 0.8183$).

For the patients P_2, P_3 and P_4, by the similar diagnosis steps we can obtain the weighted aggregation values of M_{Ti} (P_s) for $s = 2, 3, 4$ and $i = 1, 2, 3, 4, 5$. Then, all results are shown in Table 7.

5.2. *Comparative analysis*

To illustrate the effectiveness of the proposed multi-period medical diagnosis method, we provide a comparison with the multi-period medical diagnosis method based on the cosine similarity measure of Eq. (5) since the similarity measure of Eq. (5) has stronger discrimination mentioned above.

For convenient comparison, assume that Eq. (10) is replaced by the existing cosine measure formula (5) [19] in Step 1 as the following form:

$$S_{Ci}(P_s, t_k) = \frac{1}{m}\sum_{j=1}^{m}\left\{\cos\left[\frac{\pi}{6}(|T_j(t_k) - T_{ij}| + |I_j(t_k) - I_{ij}| + |F_j(t_k) - F_{ij}|)\right]\right\}, \quad (12)$$

In Step 2, the weighted aggregation values of M_{Ci} (P_s) for $i = 1, 2, \ldots, n$ is calculated by the following formula:

$$M_{Ci}(P_s) = \sum_{k=1}^{q} S_{Ci}(P_s, t_k)\omega(t_k). \quad (13)$$

Then, according to Eqs. (12) and (13), the weighted aggregation values of $M_{Ci}(P_s)$ for $s = 1, 2, 3, 4$ and $i = 1, 2, 3, 4, 5$ can be obtained, which are shown in Table 7.

From the results of Table 7, since the largest measure values indicate the proper diagnosis, the two patients P_1 and P_3 suffer from viral fever, the patient P_2 suffers from malaria, and the patient P_4 suffers from stenocardia. Obviously, the diagnosis results based on the tangent similarity measure and the cosine similarity measure are identical and show the effectiveness of the multi-period medical diagnosis method proposed in this paper. However, the two similarity measures imply rationality to be used for the multi-period medical diagnosis problem because the tangent similarity measure and the improved cosine similarity measure indicate stronger discrimination in Section 3.

Compared with the medical diagnosis methods in [3–7,19], the multi-period medical diagnosis in this paper is a comprehensive medical diagnosis method with examining a patient through multi-periods, which obtains the diagnosis conclusion based on the tangent similarity measure of SVNSs and the information aggregation of multi-periods. However, the medical diagnosis methods in [3–7,19] cannot handle the multi-period medical diagnosis problem with neutrosophic information. Since the single period medical diagnosis problem is a special case of the multi-period medical diagnosis problem, the proposed multi-period medical diagnosis method can deal with the medical diagnosis problems with intuitionistic fuzzy information and single valued neutrosophic information in [3–7,19]. Furthermore, the multi-period medical diagnosis method presented in this paper is superior to the single period medical diagnosis method proposed in [19] because the later is difficult to give a proper diagnosis for a particular patient with a particular decease in some situations and the former has to examine the patient through multi-periods and to consider the weighted information aggregation of multi-periods in order to obtain a proper conclusion for the patient. From the multi-period medical diagnosis point of view, the proposed multi-period medical diagnosis method is more suitable and more reasonable to find a proper disease diagnosis than the existing medical diagnosis methods.

6. Conclusion

This paper proposed the similarity measures for SVNSs based on the tangent function and the weighted tangent similarity measure of SVNSs introduced by considering the importance of each element, and then investigated their properties. Further, we developed a multi-period medical diagnosis method based on the proposed similarity measure and the weighted aggregation of multi-period information. Finally, a medical diagnosis example with single valued neutrosophic information was provided to demonstrate the application of the developed multi-period medical diagnosis method and to indicate the effectiveness by the comparative analysis. The diagnosis results showed that the developed multi-period medical diagnosis method can help doctors make a proper diagnosis by the comprehensive information of multi-periods.

In the further work, it is necessary to apply the similarity measure of SVNSs to other areas such as pattern recognition,

image processing, and clustering analysis. We also shall extend the proposed multi-period medical diagnosis method to multi-period decision making problems with neutrosophic information.

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