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Multiple Attribute Decision Making Algorithm via Picture Fuzzy Nano Topological Spaces

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Abstract: Picture fuzzy nano topological spaces is an extension of intuitionistic fuzzy nano topological spaces. Every decision in life ends with an answer such as yes or no, or true or false, but we have an another component called abstain, which we have not yet considered. This work is a gateway to study such a problem. This paper motivates an enquiry of the third component—abstain—in practical problems. The aim of this paper is to investigate the contemporary notion of picture fuzzy nano topological spaces and explore some of its properties. The stated properties are quantified with numerical data. Furthermore, an algorithm for Multiple Attribute Decision-Making (MADM) with an application regarding the file selection of building material under uncertainty by using picture fuzzy nano topological spaces is developed. As a practical problem, a comparison table is presented to show the difference between the novel concept and the existing methods.

Keywords: picture fuzzy topology; picture fuzzy nano topological spaces; picture fuzzy nano-closed sets; picture fuzzy interior and closure

MSC: 54A05; 54A40; 03D45

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1. Introduction

Multiple Attribute Decision-Making (MADM) is a method that specifically considers the best possible alternatives. In medieval times, decisions were made without coping with data uncertainties, which could lead to a potential outcome. Inadequate outcomes had real-life organizational conditions. The results would be ambivalent, undefined, or wrong if we deduced the result of obtained data without hesitation. MADM played an important role in real life problem such as management, diagnosing diseases, economics and industries. Each time, hundreds of decisions are taken by each decision maker to execute the major part of his/her work but it should be a logical judgment. MADM is used to solve complex and complex problems with various parameters for this. In MADM, the issue must be identified by defining potential alternatives, evaluating each alternative on the basis of the criteria set by the decision-maker or community of decision-makers and, ultimately, choosing the best alternative. A variety of valuable mathematical methods, such as fuzzy sets, neutrosophic sets, and soft sets, were developed to tackle the complexities and complexity of MADM problems.

Fuzzy set theory was introduced by Zadeh [1]. MADM algorithm via rough fuzzy information was introduced and developed by Zafer et al. [2]. Among different generalized FSs, in sight of IFSs introduced by Atanassov [3], lacking a logical scheme to effectively process inconsistent and indeterminate knowledge embedded in practical situations, Smarandache [4] introduced the structure of neutrosophic logic, and NSs, which could be viewed as a generalized form of fuzzy logic and intuitionistic fuzzy logic, FSs, and IFSs. Compared with IFSs, through incorporating an indeterminacy membership function which is focused on separately, NSs are able to effectively express the informations of inconsistency, incompleteness, and indeterminacy. NS has been applied in scientific and engineering fields. Salama [5] introduced neutrosophic topological space and its operation. Later, Salama [6] introduced and investigated neutrosophic closed sets and its continuous functions. Properties of closed and continuous functions of the same are presented in his work. Parimala et al. [7] studied neutrosophic $\alpha\psi$ homeomorphism in neutrosophic topological spaces. Parimala et al. [8] established a pre-open set and continuous functions in neutrosophic topology. Parimala et al. [9] implemented the concept of ideal in neutrosophic nano topological space. Parimala et al. [6] and Alharbi [10] proposed an algorithm for decision making. Parimala et al. [11] introduced new type of closed set called neutrosophic $\alpha\psi$ closed set in neutrosophic topological space.

One of extensions of IFS is picture fuzzy set (PFS) and it was developed by Cuong [3]. In PFS theory, there are three components, namely positive, neutral, and negative terms and sum of three components is less than or equal to one. simultaneously, PFS was applied in scientific and engineering fields. The development of PFS can be found in many fields [6,12–14]. In real life, there is more than one option for every decision. For instance, a person wants to purchase a motor bike or a car or both or neither. Membership function related to purchase a motor bike, negative membership function related to purchase a car because the person did not choose a motor bike. Neutrality is related to purchase both car and motor bike. Refusal related to purchase neither a car nor a motorbike. PFS deals with these kind of cases. Lellis Thivagar et al. [6] developed the concept of neutrosophic topology (NT) in the literature. It gained the attention of researchers to develop the theory of nano topology and via picture fuzzy sets [15,16].

FS theory talks about the membership of an object, IFS consider both membership and non-membership of an object. PFS theory added a component to the IFS theory, called abstinence. There is a choice called abstinence, different from yes or no choices, in decision making. Problems related to these cases are interesting and develop a theory for the same motived many researched to pay attention on it. Wei [17] solved multiple attribute decision-making (MADM) problems using picture fuzzy cross-entropy method. Thong and Son [13] proposed a hybrid method called Automatic Picture Fuzzy Clustering (AFC-PFS) between Particle Swarm Optimization and fuzzy clustering algorithms on Picture Fuzzy set. Wei [14] presented new similarity measures between PFSs based on the cosine function between PFSs. Wei [18] presented a similarity measure between PFS and applied the same on building material and minerals field recognition. Wang et al. [19] proposed a tool for risk ranking problems under picture fuzzy environment. Garg [20] presented some aggregation operators for PFSs and investigated application of decision making approach based on proposed picture fuzzy aggregation operator. Peng and Dai [21] proposed an algorithm for PFS and applied in decision making based on new distance measure. In this era, several mathematicians have been focusing on correlation coefficients, similarity measures, aggregation operators, topological spaces, and applications for decision-making. These structures provide different formula for different sets and have better solution to decision-making problems. It has various applications in different fields, such as medical diagnosis, pattern recognition, social sciences, artificial intelligence, business, and decision making problems with multi-attributes.

Motivation and Objective

The notion of PFS [12], nano topological spaces [6], and neutrosophic complex topological space [6] motivates us to propose this novel notion of picture fuzzy nano topological space and apply this notion in the MADM problem. The expanded and hybrid motivation and goal work is given in the entire manuscript, step by step. We make sure other hybrid systems of FS are special PFS cases, under some necessary circumstances. The robustness, durability, superiority and simplicity of our proposed model and algorithms are discussed.

This model is the most common type and is used to collect large-scale data in Artificial Intelligence, Engineering and medical applications. Similar research can be easily replicated in the future with other methods and different forms of hybrid structure.

The scheme of this manuscript is organized as follows. Section 2, gives the preliminary definitions of the literature in NS are discussed. In Section 3, we introduced a novel idea of PFNTSs and established some of its operations such as interior and closure with the help of illustrations, and defined a score function. In Section 4, We proposed an algorithm and flowchart for MADM problem. In Section 5, As a numerical example, we established a method for the solution of MADM problem related to civil engineering (material selection) using PFNTS. We also presented the efficiency, advantage, consistency and validity of the algorithms proposed. With some current methodologies we provided a brief overview and comparative review of our proposed approach. The conclusion of this work is essentially summed up and future scope of research are presented in Section 6.

2. Preliminaries

The definitions from [12,20] are used in sequel.

Definition 1 ([12]). *A PFS on $\mathfrak{S}$ is defined as:*

$$\mathfrak{P} = \{\zeta, \mathfrak{m}(\zeta), \mathfrak{a}(\zeta), \mathfrak{n}(\zeta) | \zeta \in \mathfrak{S}\}$$

where $\mathfrak{m}, \mathfrak{a}, \mathfrak{n} : \mathfrak{S} \to [0,1]$ are called membership, abstinence and non-membership functions. The condition for a PFS is that the sum of all three functions must lie within the unit interval $[0,1]$ and the degree of refusal is defined as $\mathfrak{r}() = 1 - (\mathfrak{m}(\zeta) + \mathfrak{a}(\zeta) + \mathfrak{n}(\zeta))$. A triplet $(\mathfrak{m}, \mathfrak{a}, \mathfrak{n})$ is called a picture fuzzy number (PFN)

Definition 2 ([20]). *Two objects $\mathfrak{S}_1 = \{(\zeta, \langle \mathfrak{m}_{\mathfrak{S}_1}(\zeta), \mathfrak{a}_{\mathfrak{S}_1}(\zeta), \mathfrak{n}_{\mathfrak{S}_1}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$ and $\mathfrak{S}_2 = \{(\zeta, \langle \mathfrak{m}_{\mathfrak{S}_2}(\zeta), \mathfrak{a}_{\mathfrak{S}_2}(\zeta), \mathfrak{n}_{\mathfrak{S}_2}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}$ are two picture fuzzy sets defined on $\mathfrak{U}$, the universe of discourse, and their union and intersection are defined and denoted as follows*

1. *The union of $\mathfrak{S}_1$ and $\mathfrak{S}_2$ is*

$$\mathfrak{S}_1 \cup \mathfrak{S}_2 = \{(\zeta, \langle \mathfrak{m}_{\mathfrak{S}_1}(\zeta) \vee \mathfrak{m}_{\mathfrak{S}_2}(\zeta), \mathfrak{a}_{\mathfrak{S}_1}(\zeta) \wedge \mathfrak{a}_{\mathfrak{S}_2}(\zeta), \mathfrak{n}_{\mathfrak{S}_1}(\zeta) \wedge \mathfrak{n}_{\mathfrak{S}_2}(\zeta)\rangle) : \zeta \in \mathfrak{U}\},$$

2. *The intersection of $\mathfrak{S}_1$ and $\mathfrak{S}_2$ is*

$$\mathfrak{S}_1 \cap \mathfrak{S}_2 = \{(\zeta, \langle \mathfrak{m}_{\mathfrak{S}_1}(\zeta) \wedge \mathfrak{m}_{\mathfrak{S}_2}(\zeta), \mathfrak{a}_{\mathfrak{S}_1}(\zeta) \vee \mathfrak{a}_{\mathfrak{S}_2}(\zeta), \mathfrak{n}_{\mathfrak{S}_1}(\zeta) \vee \mathfrak{n}_{\mathfrak{S}_2}(\zeta)\rangle) : \zeta \in \mathfrak{U}\},$$

3. *The symmetric difference of $\mathfrak{S}_1$ and $\mathfrak{S}_2$ is*

$$\mathfrak{S}_1 - \mathfrak{S}_2 = \{(\zeta, \langle \mathfrak{m}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta), \mathfrak{a}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta), \mathfrak{n}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta)\rangle) : \zeta \in \mathfrak{U}\}, \textit{ where}$$

$$\mathfrak{m}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta) = 0 \vee \mathfrak{m}_{\mathfrak{S}_1} - \mathfrak{m}_{\mathfrak{S}_2}, \mathfrak{n}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta) = 0 \vee \mathfrak{n}_{\mathfrak{S}_1} - \mathfrak{n}_{\mathfrak{S}_2},$$

$$\mathfrak{a}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta) = \begin{cases} 1 - \mathfrak{m}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta) - \mathfrak{n}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta), if\ \mathfrak{a}_{\mathfrak{S}_1}(\zeta) > \mathfrak{a}_{\mathfrak{S}_2}(\zeta) \\ \{1 + \mathfrak{a}_{\mathfrak{S}_1}(\zeta) - \mathfrak{a}_{\mathfrak{S}_2}(\zeta)\} \wedge \{1 - \mathfrak{m}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta) - \mathfrak{n}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\zeta)\}, if\ \mathfrak{a}_{\mathfrak{S}_1}(\zeta) \leq \mathfrak{a}_{\mathfrak{S}_2}(\zeta) \end{cases}$$

4. *$\mathfrak{S}_1 \subseteq \mathfrak{S}_2$ if and only if*

$$\mathfrak{m}_{\mathfrak{S}_1}(\zeta) \leq \mathfrak{m}_{\mathfrak{S}_2}(\zeta), \mathfrak{a}_{\mathfrak{S}_1}(\zeta) \geq \mathfrak{a}_{\mathfrak{S}_2}(\zeta) \textit{ and } \mathfrak{n}_{\mathfrak{S}_1}(\zeta) \geq \mathfrak{n}_{\mathfrak{S}_2}(\zeta), \forall \zeta \in \mathfrak{U}.$$

Definition 3 ([12]). *Let $\mathfrak{S}$ be a family of PFS on $U \neq \emptyset$. Then $(X, \mathfrak{S})$ is called a picture fuzzy topological space if it satisfies the following:*

- *0_S and 1_S are member of $\mathfrak{S}$.*
- *Arbitrary union of picture fuzzy set S in $\mathfrak{S}$ if each S in $\mathfrak{S}$*
- *Finite intersection of picture fuzzy set S in $\mathfrak{S}$ if each S in $\mathfrak{S}$*

Definition 4 ([12]). *Let $\mathfrak{R}$ and $\mathfrak{S}$ be the equivalence relation and picture fuzzy set, respectively defined on universe of discourse $\mathfrak{U}$. The membership $\mathfrak{m}_S$, the abstinence $\mathfrak{a}_S$ and nonmembership $\mathfrak{n}_S$ are the components of $\mathfrak{S}$. The approximation space $(\mathfrak{U}, \mathfrak{R})$ has three components, namely lower $\mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{S})$, upper $\mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{S})$, and boundary approximation $\mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{S})$ where*

(i) *The upper approximation of S with respect to R is denoted by $\mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{S})$, i.e.,* $\mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{S}) = \{\langle \zeta, \mathfrak{m}_{\overline{\mathfrak{R}}\mathfrak{S}}(\zeta), \mathfrak{a}_{\overline{\mathfrak{R}}\mathfrak{S}}(\zeta), \mathfrak{n}_{\overline{\mathfrak{R}}\mathfrak{S}}(\zeta)\rangle | \xi \in [\zeta]_R, \zeta \in \mathfrak{U}\}$

(ii) *The lower approximation of S with respect to R is the set is denoted by $\mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{S})$, i.e.,* $\mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{S}) = \{\langle \zeta, \mathfrak{m}_{\underline{\mathfrak{R}}\mathfrak{S}}(\zeta), \mathfrak{a}_{\underline{\mathfrak{R}}\mathfrak{S}}(\zeta), \mathfrak{n}_{\underline{\mathfrak{R}}\mathfrak{S}}(\zeta)\rangle | \xi \in [\zeta]_R, \zeta \in \mathfrak{U}\}$

(iii) *The boundary region of S with respect to R is the set of all objects which can be classified neither as S nor as not S with respect to R and is denoted by $\mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{S})$. $\mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{S}) = \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{S}) - \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{S})$, where*

$$\mathfrak{m}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \wedge_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{m}_{(\mathfrak{A})}(\xi), \mathfrak{m}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \vee_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{m}_{(\mathfrak{A})}(\xi)$$

$$\mathfrak{a}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \vee_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{a}_{(\mathfrak{A})}(\xi), \mathfrak{a}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \wedge_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{a}_{(\mathfrak{A})}(\xi)$$

$$\mathfrak{n}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \vee_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{n}_{(\mathfrak{A})}(\xi), \mathfrak{n}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\zeta) = \wedge_{\xi \in [\zeta]_{\mathfrak{R}}} \mathfrak{n}_{(\mathfrak{A})}(\xi).$$

3. Picture Fuzzy Nano Topological Spaces

Definition 5. *Let the Universe be $\mathfrak{U}$, equivalence relation on the non-void set $\mathfrak{S} \subseteq \mathfrak{U}$ be $\mathfrak{R}$ and if $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\mathfrak{p}}, 1_{\mathfrak{p}}, \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{A})\}$, where $\mathfrak{A} \subseteq \mathfrak{S}$ and $\tau_{\mathfrak{R}}$ satisfies the following axioms:*

1. $0_{\mathfrak{p}}, 1_{\mathfrak{p}} \in \tau_{\mathfrak{R}}$
2. *If $\mathfrak{A}_{\mathfrak{i}} \in \tau_{\mathfrak{R}}(\mathfrak{A})$, for $\mathfrak{a} = 1, 2, 3, ..$, then*

$$\bigcup_{\mathfrak{a}=1}^{\infty} \mathfrak{A}_{\mathfrak{i}} \in \tau_{\mathfrak{R}}(\mathfrak{A})$$

3. *If $\mathfrak{A}_{\mathfrak{i}} \in \tau_{\mathfrak{R}}(\mathfrak{A})$, for $\mathfrak{a} = 1, 2, 3, ..n$, then*

$$\bigcap_{\mathfrak{a}=1}^{n} \mathfrak{A}_{\mathfrak{i}} \in \tau_{\mathfrak{R}}(\mathfrak{A})$$

then $\tau_{\mathfrak{R}}(\mathfrak{A})$ is termed as PFNTS on $\mathfrak{S}$ with respect to $\mathfrak{A}$. where the picture fuzzy sets $1_{\mathfrak{p}}$ and $0_{\mathfrak{p}}$ are defined by $1_{\mathfrak{p}} = \{(\zeta, \langle 1, 0, 0\rangle) : \zeta \in \mathfrak{U}\}$ and $0_{\mathfrak{p}} = \{(\zeta, \langle 0, 0, 1\rangle) : \zeta \in \mathfrak{U}\}$ respectively. Whereas, we call $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{A}))$ as PFNTS. The components of $\tau_{\mathfrak{R}}(\mathfrak{A})$ are called PFNOS.
The complement $\mathfrak{A}^c$ of a PFNOS $\mathfrak{A}$ in a PFNTS. $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{A}))$ is called a PFNCS in $\mathfrak{S}$.

Example 1. *Suppose we have a IC-chip company. The company has three workers in the fabrication section. Each employee in this plant gets 10 (in hundreds) IC-chip components, that to be fabricated every day. The number of chips fabricated, incomplete, and damaged by the employee is denoted by the membership value, the abstinence value, and the negative membership value of the PFS respectively and their major, minor and border approximations are given below:*
Let $\mathfrak{S} = \{\mathfrak{e}_1, \mathfrak{e}_2, \mathfrak{e}_3\}$ be the universe of discourse. Let $\mathfrak{S}/\mathfrak{R} = \{\{\mathfrak{e}_1, \mathfrak{e}_2\}, \{\mathfrak{e}_3\}\}$ be an equivalence relation on $\mathfrak{S}$ and $\mathfrak{A} = \{\langle \mathfrak{e}_1, (0.4, 0.1, 0.3)\rangle, \langle \mathfrak{e}_2, (0.5, 0.3, 0.1)\rangle, \langle \mathfrak{e}_3, (0.1, 0.4, 0.2)\rangle\}$ be a picture fuzzy set on $\mathfrak{S}$, then
$\mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{e}_1, (0.4, 0.3, 0.3)\rangle, \langle \mathfrak{e}_2, (0.4, 0.3, 0.3)\rangle, \langle \mathfrak{e}_3, (0.1, 0.4, 0.2)\rangle\}$,
$\mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{e}_1, (0.5, 0.1, 0.1)\rangle, \langle \mathfrak{e}_2, (0.5, 0.1, 0.1)\rangle, \langle \mathfrak{e}_3, (0.1, 0.4, 0.2)\rangle\}$ *and*
$\mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{e}_1, (0.2, 0.8, 0.0)\rangle, \langle \mathfrak{e}_2, (0.2, 0.8, .0.0)\rangle, \langle \mathfrak{e}_3, (0, 0.9, 0.1)\rangle\}$.
$\mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}) \cup \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{e}_1, (0.5, 0.1, 0.1)\rangle, \langle \mathfrak{e}_2, (0.5, 0.1, 0.1)\rangle, \langle \mathfrak{e}_3, (0.1, 0.4, 0.2)\rangle\} = \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A})$
$\mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}) \cap \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle \mathfrak{e}_1, (0.4, 0.3, 0.3)\rangle, \langle \mathfrak{e}_2, (0.4, 0.3, 0.3)\rangle, \langle \mathfrak{e}_3, (0.1, 0.4, 0.2)\rangle\} = \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A})$
$0_{\mathfrak{p}} \cap \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A}) = 0_{\mathfrak{p}}$, $0_{\mathfrak{p}} \cap \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}) = 0_{\mathfrak{p}}$, $0_{\mathfrak{p}} \cap \mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{A}) = 0_{\mathfrak{p}}$,
$0_{\mathfrak{p}} \cup \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{PFU}_{\mathfrak{R}}$, $0_{\mathfrak{p}} \cup \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{PFL}_{\mathfrak{R}}$, $0_{\mathfrak{p}} \cup \mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{PFB}_{\mathfrak{R}}$,
$1_{\mathfrak{p}} \cap \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{PFU}_{\mathfrak{R}}$, $1_{\mathfrak{p}} \cap \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{PFL}_{\mathfrak{R}}$, $1_{\mathfrak{p}} \cap \mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{PFB}_{\mathfrak{R}}$,

$1_{\mathfrak{p}} \cup \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A}) = 1_{\mathfrak{p}}$, $1_{\mathfrak{p}} \cup \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}) = 1_{\mathfrak{p}}$, $1_{\mathfrak{p}} \cup \mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{A}) = 1_{\mathfrak{p}}$,
Therefore, $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\mathfrak{p}}, 1_{\mathfrak{p}}, \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{A})\}$ *forms a topology.*

Remark 1. *In PFNTS, the picture fuzzy nano border will be a non-void set. Since the symmetric difference between picture fuzzy nano major and picture fuzzy nano minor approximations is defined here as the maximum and minimum of the values in the picture fuzzy sets.*

Proposition 1. *Let* $\mathfrak{U}$ *be a non-void universe and* $\mathfrak{A}$ *be a picture fuzzy set on* $\mathfrak{U}$*. Then the following statements hold:*

1. *The collection* $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\mathfrak{p}}, 1_{\mathfrak{p}}\}$, *is the in-discrete picture fuzzy nano topology on* $\mathfrak{U}$.
2. *If* $\mathfrak{PFL}_{\mathfrak{R}} = \mathfrak{PFU}_{\mathfrak{R}} = \mathfrak{PF}_{\mathfrak{R}}$, *then the picture fuzzy nano topology is* $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\mathfrak{p}}, 1_{\mathfrak{p}}, \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{A})\}$.
3. *If* $\mathfrak{PFL}_{\mathfrak{R}} = \mathfrak{PFB}_{\mathfrak{R}}$, *then* $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\mathfrak{p}}, 1_{\mathfrak{p}}, \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{PFU}_{\mathfrak{R}}(\mathfrak{A})\}$ *is a picture fuzzy nano topology.*
4. *If* $\mathfrak{PFU}_{\mathfrak{R}} = \mathfrak{PFB}_{\mathfrak{R}}$, *then the picture fuzzy nano topology is* $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\mathfrak{p}}, 1_{\mathfrak{p}}, \mathfrak{PFL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{PFB}_{\mathfrak{R}}(\mathfrak{A})\}$

Definition 6. *Let* $(\mathfrak{U}; \tau_{\mathfrak{R}})$ *be any PFNTS with respect to picture fuzzy subset of* $\mathfrak{U}$ *and let* $\mathfrak{A}$ *be a picture fuzzy nano set in* $\mathfrak{S}$*. Then the picture fuzzy nano interior and picture fuzzy nano closure of* $\mathfrak{A}$ *are defined as follows:*

1. $\mathfrak{A}^{o} = \cup\{\mathfrak{G} : \mathfrak{G}$ *is a PFNOS in* $\mathfrak{S}$ *and* $\mathfrak{G} \subseteq \mathfrak{A}\}$,
2. $\mathfrak{A}^{-} = \cap\{\mathfrak{G} : \mathfrak{G}$ *is a PFNCS in* $\mathfrak{S}$ *and* $\mathfrak{G} \supseteq \mathfrak{A}\}$.

Remark 2. *For any picture fuzzy nano set* $\mathfrak{A}$ *in* $(\mathfrak{U}; \tau_{\mathfrak{R}})$, *we have*

1. $[\mathfrak{A}^{c}]^{-} = [\mathfrak{A}^{o}]^{c}$.
2. $[\mathfrak{A}^{c}]^{o} = [\mathfrak{A}^{-}]^{c}$.
3. $\mathfrak{A}$ *is a PFNCS if and only if* $\mathfrak{A}^{-} = \mathfrak{A}$.
4. $\mathfrak{A}$ *is a PFNOS if and only if* $\mathfrak{A}^{o} = \mathfrak{A}$.
5. $\mathfrak{A}^{-}$ *is a PFNCS in* $\mathfrak{U}$.
6. $\mathfrak{A}^{o}$ *is a PFNOS in* $\mathfrak{U}$.

Theorem 1. *Let* $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$ *be a picture fuzzy nano topological space with respect to* $\mathfrak{S}$ *where* $\mathfrak{S}$ *is a picture fuzzy subset of* $\mathfrak{U}$ *. Let* $\mathfrak{A}_1$ *and* $\mathfrak{A}_2$ *be picture fuzzy subsets of* $\mathfrak{U}$*. Then the following statements hold:*

1. $\mathfrak{A} \subseteq \mathfrak{A}^{-}$.
2. $\mathfrak{A}$ *is picture fuzzy nano closed if and only if* $\mathfrak{A}^{-} = \mathfrak{A}$.
3. $0_{\mathfrak{p}}^{-} = 0_{\mathfrak{p}}$ *and* $1_{\mathfrak{p}}^{-} = 1_{\mathfrak{p}}$.
4. $\mathfrak{A}_1 \subseteq \mathfrak{A}_2 \Rightarrow \mathfrak{A}_1{}^{-} \subseteq \mathfrak{A}_2{}^{-}$.
5. $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^{-} = \mathfrak{A}_1{}^{-} \cup \mathfrak{A}_2{}^{-}$.
6. $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^{-} = \mathfrak{A}_1{}^{-} \cap \mathfrak{A}_2{}^{-}$.
7. $(\mathfrak{A}^{-})^{-} = \mathfrak{A}^{-}$.

Proof.

1. By definition of picture fuzzy nano closure, $\mathfrak{A} \subseteq \mathfrak{A}^{-}$
2. If $\mathfrak{A}$ is a picture fuzzy nano closed set, then $\mathfrak{A}$ is the smallest picture fuzzy nano closed set containing itself and hence $\mathfrak{A}^{-} = \mathfrak{A}$. Conversely, if $\mathfrak{A}^{-} = \mathfrak{A}$, then $\mathfrak{A}$ is the smallest picture fuzzy nano closed set containing itself and hence $\mathfrak{A}$ is a picture fuzzy nano closed set.
3. Since $0_{\mathfrak{p}}$ and $1_{\mathfrak{p}}$ are picture fuzzy nano closed sets in $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$, $0_{\mathfrak{p}}^{-} = 0_{\mathfrak{p}}$ and $1_{\mathfrak{p}}^{-} = 1_{\mathfrak{p}}$.
4. If PFN set $\mathfrak{A}_1$ is a subset of PFN set $\mathfrak{A}_2$, since PFN set $\mathfrak{A}_2$ is a subset of $\mathfrak{A}_2{}^{-}$, then PFN set $\mathfrak{A}_1$ is a subset of $\mathfrak{A}_2{}^{-}$, i.e., $\mathfrak{A}_2{}^{-}$ is a PFNCS containing $\mathfrak{A}_1$. However, $\mathfrak{A}_1{}^{-}$ is the smallest PFNCS containing $\mathfrak{A}_1$. Therefore, $\mathfrak{A}_1{}^{-} \subseteq \mathfrak{A}_2{}^{-}$
5. Since PFN set $\mathfrak{A}_1$ is a subset of union of two PFN sets$\mathfrak{A}_1$ and $\mathfrak{A}_2$ and PFN set $\mathfrak{A}_2$ is a subset of union of two PFN sets $\mathfrak{A}_1$ and $\mathfrak{A}_2$, $\mathfrak{A}_1{}^{-} \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^{-}$. Then closure of

PFN set $\mathfrak{A}_1$ is a subset of closure of union of two PFN sets $\mathfrak{A}_1$ and $\mathfrak{A}_2$ and closure of PFN set $\mathfrak{A}_2$ is a subset of closure of union of two PFN sets $\mathfrak{A}_1$ and $\mathfrak{A}_2$. Therefore, union of closure of PFN sets $\mathfrak{A}_1{}^-, \mathfrak{A}_2{}^-$ is a subset of closure of union of $(\mathfrak{A}_1{}^-, \mathfrak{A}_2)^-$. By the fact that $\mathfrak{A}_1 \cup \mathfrak{A}_2 \subseteq \mathfrak{A}_1{}^- \cup \mathfrak{A}_2{}^-$, and since $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^-$ is the smallest picture fuzzy nano closed set containing $\mathfrak{A}_1 \cup \mathfrak{A}_2$, so $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^- \subseteq \mathfrak{A}_1{}^- \cup \mathfrak{A}_2{}^-$. Thus, $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^- = \mathfrak{A}_1{}^- \cup \mathfrak{A}_2{}^-$.

6. Since $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_1$ and $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_2$, $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^- \subseteq \mathfrak{A}_1{}^- \cap \mathfrak{A}_2{}^-$.
7. Since $\mathfrak{A}^-$ is a picture fuzzy nano closed set, then $(\mathfrak{A}^-)^- = \mathfrak{A}^-$.

□

Theorem 2. $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$ *be a picture fuzzy nano topological space with respect to* $\mathfrak{S}$ *where* $\mathfrak{S}$ *is a picture fuzzy subset of* $\mathfrak{U}$. *Let* $\mathfrak{A}$ *be a picture fuzzy subset of* $\mathfrak{U}$. *Then*

1. $1_{\mathfrak{p}} - \mathfrak{A}^o = (1_{\mathfrak{p}} - \mathfrak{A})^-$.
2. $1_{\mathfrak{p}} - \mathfrak{A}^- = (1_{\mathfrak{p}} - \mathfrak{A})^o$.

Theorem 3. *Let* $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$ *be a picture fuzzy nano topological space with respect to* $\mathfrak{S}$ *where* $\mathfrak{S}$ *is a picture fuzzy subset of* $\mathfrak{U}$. *Let* $\mathfrak{A}_1$ *and* $\mathfrak{A}_2$ *be picture fuzzy subsets of* $\mathfrak{U}$. *Then the following statements hold:*

1. $\mathfrak{A}$ *is picture fuzzy nano open if and only if* $\mathfrak{A}^o = \mathfrak{A}$.
2. $0_{\mathfrak{p}}^o = 0_{\mathfrak{p}}$ *and* $1_{\mathfrak{p}}^o = 1_{\mathfrak{p}}$.
3. $\mathfrak{A}_1 \subseteq \mathfrak{A}_2 \Rightarrow \mathfrak{A}_1^o \subseteq \mathfrak{A}_2^o$.
4. $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^o = \mathfrak{A}_1^o \cup \mathfrak{A}_2^o$.
5. $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^o = \mathfrak{A}_1^o \cap \mathfrak{A}_2^o$.
6. $(\mathfrak{A}^o)^o = \mathfrak{A}^o$.

Proof.

1. $\mathfrak{A}$ is a picture fuzzy nano open set if and only if $1_{\mathfrak{p}} - \mathfrak{A}$ is a picture fuzzy nano closed set, if and only if $(1_{\mathfrak{p}} - \mathfrak{A})^- = 1_{\mathfrak{p}} - \mathfrak{A}$, if and only if $1_{\mathfrak{p}} - (1_{\mathfrak{p}} - \mathfrak{A})^- = \mathfrak{A}$ if and only if $\mathfrak{A}^o = \mathfrak{A}$.
2. Since $0_{\mathfrak{p}}$ and $1_{\mathfrak{p}}$ are picture fuzzy nano open sets in $(\mathfrak{U}; \tau_{\mathfrak{R}})(\mathfrak{S})$, $0_{\mathfrak{p}}^o = 0_{\mathfrak{p}}$ and $1_{\mathfrak{p}}^o = 1_{\mathfrak{p}}$.
3. If $\mathfrak{A}_1 \subseteq \mathfrak{A}_2$, since $\mathfrak{A}_2 \supseteq \mathfrak{A}_2^o$, then $\mathfrak{A}_1 \supseteq \mathfrak{A}_2^o$, i.e., $\mathfrak{A}_2^o$ is a picture fuzzy nano open set containing $\mathfrak{A}_1$. However, $\mathfrak{A}_1^o$ is the largest picture fuzzy nano open set contained in $\mathfrak{A}_1$. Therefore, $\mathfrak{A}_1{}^o \subseteq \mathfrak{A}_2{}^o$
4. Since $\mathfrak{A}_1 \subseteq \mathfrak{A}_1 \cup \mathfrak{A}_2$ and $\mathfrak{A}_2 \subseteq \mathfrak{A}_1 \cup \mathfrak{A}_2$, $\mathfrak{A}_1^o \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^o$ and $\mathfrak{A}_2^o \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^o$. Therefore, $\mathfrak{A}_1^o \cup \mathfrak{A}_2^o \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^o$. By the fact that $\mathfrak{A}_1 \cup \mathfrak{A}_2 \subseteq \mathfrak{A}_1^o \cup \mathfrak{A}_2^o$, and since $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^o$ is the largest picture fuzzy nano open set containing $\mathfrak{A}_1 \cup \mathfrak{A}_2$, so $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^o \subseteq \mathfrak{A}_1^o \cup \mathfrak{A}_2^o$. Thus, $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^o = \mathfrak{A}_1^o \cup \mathfrak{A}_2^o$.
5. Since $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_1$ and $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_2$, $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^o \subseteq \mathfrak{A}_1^o \cap \mathfrak{A}_2^o$.
6. Since $\mathfrak{A}^o$ is a picture fuzzy nano open set, then $(\mathfrak{A}^o)^o = \mathfrak{A}^o$.

□

Definition 7. *Let* $\mathfrak{A}_i = \{\mathfrak{m}_i, \mathfrak{a}_i, \mathfrak{n}_i\}, i = 1, 2, 3, ...k$ *be PFSs, then the score function and the accuracy function of* $\mathfrak{A} = \{\mathfrak{A}_i s\}$ *are defined as* $\mathfrak{Sr}_{\mathfrak{p}}(\mathfrak{A}) = \frac{1}{k}\sum_{i=1}^{k}[\frac{1}{2}\{1 + 2\mathfrak{m}_i - \mathfrak{a}_i/2 - \mathfrak{n}_i\}]$, *and* $\mathfrak{H}_{\mathfrak{p}}(\mathfrak{A}) = \frac{1}{k}\sum_{i=1}^{k}[\mathfrak{m}_i + \mathfrak{a}_i + \mathfrak{n}_i]$.

Let $\mathfrak{A}_1 = \{\mathfrak{m}_1, \mathfrak{a}_1, \mathfrak{n}_1\}$ *and* $\mathfrak{A}_2 = \{\mathfrak{m}_2, \mathfrak{a}_2, \mathfrak{n}_2\}$ *be two PFSs, the following comparison rules are used*

1. *if* $\mathfrak{Sr}_{\mathfrak{p}}(\mathfrak{A}_1) > \mathfrak{Sr}_{\mathfrak{p}}(\mathfrak{A}_2)$, *then* $\mathfrak{A}_1 \succ \mathfrak{A}_2$
2. *if* $\mathfrak{Sr}_{\mathfrak{p}}(\mathfrak{A}_1) < \mathfrak{Sr}_{\mathfrak{p}}(\mathfrak{A}_2)$, *then if* $\mathfrak{A}_1 \prec \mathfrak{A}_2$
3. $\mathfrak{Sr}_{\mathfrak{p}}(\mathfrak{A}_1) = \mathfrak{Sr}_{\mathfrak{p}}(\mathfrak{A}_2)$, *then*
   - (i) *if* $\mathfrak{H}_{\mathfrak{p}}(\mathfrak{A}_1) > \mathfrak{H}_{\mathfrak{p}}(\mathfrak{A}_2)$, *then* $\mathfrak{A}_1 \succ \mathfrak{A}_2$
   - (ii) *if* $\mathfrak{H}_{\mathfrak{p}}(\mathfrak{A}_1) = \mathfrak{H}_{\mathfrak{p}}(\mathfrak{A}_2)$, *then* $\mathfrak{A}_1 \sim \mathfrak{A}_2$

4. Picture Fuzzy Nano Topology in Multiple Attribute Decision-Making

MADM is a procedure for seeking a best solution that has the highest degree of satisfaction from a set of possible alternative solutions. These types of MADM problems arise in a many real-time situations, and they are characterized by multiple attributes.

The proposed algorithm deals with abstinence of an object other than a yes or no choice, while the other algorithm in fuzzy set theory and intuitionistic fuzzy set theory failed to handle these cases. This algorithm shows how picture fuzzy nano topology is influenced in decision making. The procedure we carried out in the algorithm is simple and is an elementary one to handle.

A novel picture fuzzy nano topological approach is presented in this section for decision-making problems with picture fuzzy information. A methodological procedure for selecting the right alternatives and attributes in the decision-making environment is proposed as the following necessary steps.

Proposed Algorithm and Flowchart

The flow chart of proposed Algorithm 1 for MADM is given in Figure 1.

Algorithm 1: Ideal decision making with PFTSs

Input part:
Step-1: Consider the universe of discourse (set of objects) $\mathfrak{O}$, the set of alternatives $\mathfrak{E}$, the set of decision attributes $\mathfrak{D}$.
Step-2: Construct a picture fuzzy matrix of alternative verses objects and object verses decision attributes.
Calculation part:
Step-3: Construct the picture fuzzy topologies $\mathfrak{C}^*_{\tau_1\eta}$ and $\mathfrak{C}^*_{\tau_2\xi}$.
Step-4: Find the score and accuracy values by Definition 2.3 of each of the entries of the PFNTS.
Conclusion Part:
Step-5: Organize the complex neutrosopic score values of the alternatives $\mathfrak{G}_1 \leq \mathfrak{G}_2 \leq .. \leq \mathfrak{G}_\beta$ and the attributes $\mathfrak{H}_1 \leq \mathfrak{H}_2 \leq .. \leq \mathfrak{H}_\gamma$. Choose the attribute $\mathfrak{H}_\gamma$ for the alternative $\mathfrak{G}_1$ and $\mathfrak{H}_\gamma - 1$ for the alternative $\mathfrak{G}_2$ etc.
If $\beta \leq \gamma$, then ignore $\mathfrak{H}_\xi$, where $\xi = 1, 2, \beta - \gamma$.

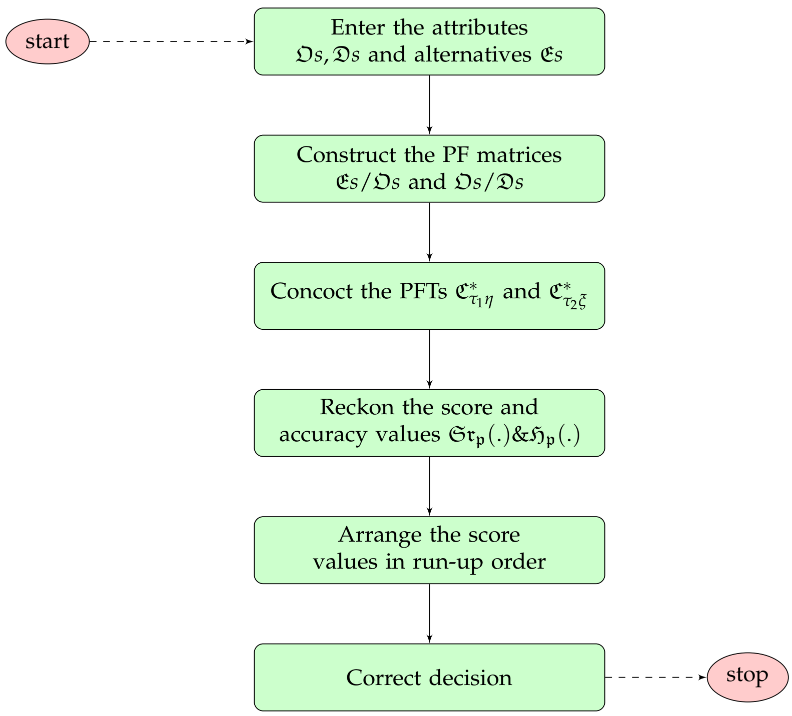

Figure 1. Flow chart representation of Algorithm 1.

The flow chart of proposed Algorithm for MADM is given in Figure 2.

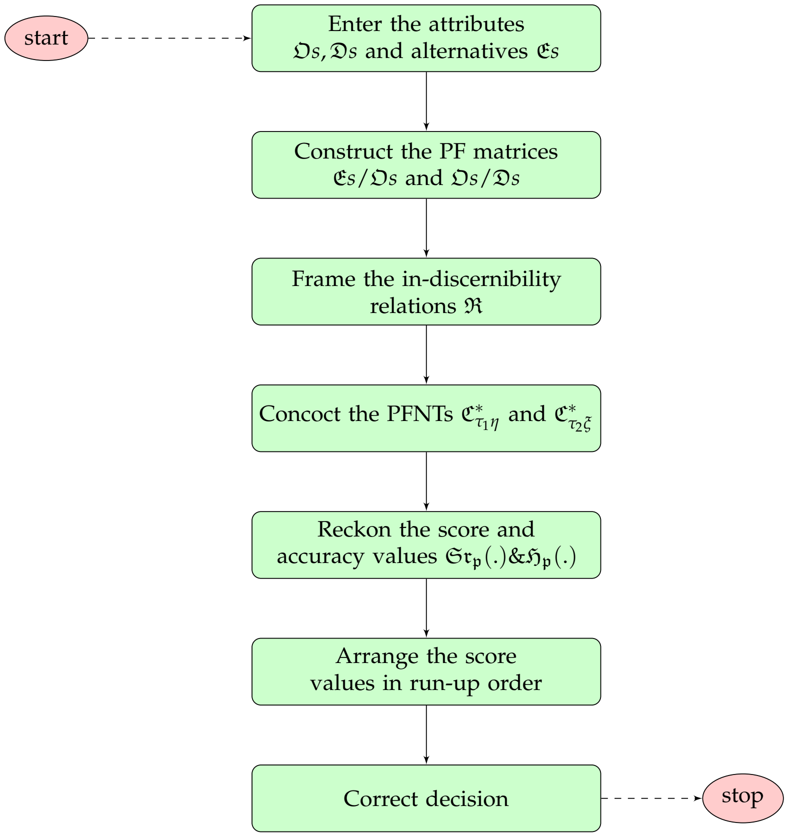

Figure 2. Flow chart representation of Algorithm 2.

Algorithm 2: Ideal decision making with PFNTSs

Input part:
Step-1: Consider the universe of discourse (set of objects) $\mathfrak{O}$, the set of alternatives $\mathfrak{E}$, the set of decision attributes $\mathfrak{D}$.
Step-2: Construct a picture fuzzy matrix of alternative verses objects and object verses decision attributes.
Calculation part:
Step-3: Frame the in-discernibility relation $\mathfrak{R}$ on $\mathfrak{O}$.
Step-4: Construct the picture fuzzy nano topologies $\mathfrak{C}^*_{\tau_1\eta}$ and $\mathfrak{C}^*_{\tau_2\xi}$.
Step-5: Find the score and accuracy values by Definition 2.3 of each of the entries of the PFNTS.
Conclusion Part:
Step-6: Organize the complex neutrosopic score values of the alternatives $\mathfrak{G}_1 \leq \mathfrak{G}_2 \leq .. \leq \mathfrak{G}_\beta$ and the attributes $\mathfrak{H}_1 \leq \mathfrak{H}_2 \leq .. \leq \mathfrak{H}_\gamma$. Choose the attribute $\mathfrak{H}_\gamma$ for the alternative $\mathfrak{G}_1$ and $\mathfrak{H}_\gamma - 1$ for the alternative $\mathfrak{G}_2$ etc. If $\beta \leq \gamma$, then ignore $\mathfrak{H}_\xi$, where $\xi = 1, 2, \beta - \gamma$.

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5. Numerical Example

The proposed algorithms helps the builder to find the suitable building material. The method of classifying various sets of features of the material for flooring under a single form is very critical and complicated. In certain realistic circumstances, each dimension has the possibility within a form of the picture fuzzy sets. Therefore, further abstinence is involved in the medical diagnosis. Complicated situations are addressed by picture fuzzy nano topologies. This strategy is generally more versatile when it comes to less places of abstinence, and easier to use. With a score function between builder versus feature requirement and features versus material type, the proposed algorithms of picture fuzzy topological spaces and picture fuzzy nano topological spaces has the right choice of selection of material in picture fuzzy milieu.

First, we solve the material problem by using first PFTS-MADM method as given in Algorithm 1.

The key feature of this suggested method is that it appraises the factual participation, specific indeterminate and misrepresentation of each dimension in the form of a picture fuzzy set.

Step-1: Let $\mathfrak{B} = \{\mathfrak{b}_1, \mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_4\}$ be the set of builders, $\mathfrak{FM} = \{\mathfrak{f}_1, \mathfrak{f}_2, \mathfrak{f}_3, \mathfrak{f}_4\}$ be the set of flooring material and $\mathfrak{X} = \{\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5\}$ be the set of feature space. The following are the different types of flooring materials: line concrete, flag stones, Marble, Ceramic.

Our work is to analyze the builder's choice and decide on the flooring type of material suitable for them in a picture fuzzy environment.

Step-2: Frame the matrix of picture fuzzy system of relationship between builders and features and the matrix of picture fuzzy system of relationship between features and flooring material are given in Tables 1 and 2 respectively.

Table 1. The picture fuzzy system of relationship between builders and features.

	$\mathfrak{b}_1$	$\mathfrak{b}_2$	$\mathfrak{b}_3$	$\mathfrak{b}_4$
ζ_1	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.4, 0.2, 0.2\rangle$	$\langle 0.2, 0.4, 0.3\rangle$	$\langle 0.3, 0.3, 0.2\rangle$
ζ_2	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.3, 0.2, 0.3\rangle$	$\langle 0.6, 0.2, 0.1\rangle$	$\langle 0.5, 0.1, 0.2\rangle$
ζ_3	$\langle 0.3, 0.4, 0.2\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.6, 0.2, 0.1\rangle$	$\langle 0.2, 0.6, 0.1\rangle$
ζ_4	$\langle 0.9, 0.0, 0.1\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.3, 0.2, 0.5\rangle$	$\langle 0.4, 0.2, 0.3\rangle$
ζ_5	$\langle 0.3, 0.1, 0.5\rangle$	$\langle 0.6, 0.2, 0.1\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.7, 0.0, 0.2\rangle$

Table 2. The picture fuzzy system of relationship between features and flooring material.

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5
$\mathfrak{f}_1$	$\langle 0.3, 0.1, 0.3\rangle$	$\langle 0.4, 0.3, 0.2\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.3, 0.4, 0.2\rangle$	$\langle 0.5, 0.2, 0.2\rangle$
$\mathfrak{f}_2$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.4, 0.2, 0.2\rangle$	$\langle 0.2, 0.4, 0.3\rangle$	$\langle 0.3, 0.3, 0.2\rangle$	$\langle 0.6, 0.1, 0.2\rangle$
$\mathfrak{f}_3$	$\langle 0.9, 0.1, 0.0\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.3, 0.2, 0.3\rangle$	$\langle 0.6, 0.0, 0.0\rangle$	$\langle 0.4, 0.2, 0.3\rangle$
$\mathfrak{f}_4$	$\langle 0.5, 0.1, 0.1\rangle$	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.7, 0.0, 0.2\rangle$	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.5, 0.2, 0.2\rangle$

Step-3: Construct the picture fuzzy topological spaces for each builder and each flooring material with respect to the features as follows:

Picture fuzzy topologies for builders are $\mathfrak{C}^*_{\tau_1\eta}$

1. $\tau_1(\mathfrak{b}_1) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.7, 0.1, 0.1\rangle, \langle 0.5, 0.2, 0.1\rangle, \langle 0.3, 0.4, 0.2\rangle, \langle 0.9, 0.0, 0.1\rangle, \langle 0.3, 0.1, 0.5\rangle, \langle 0.3, 0.2, 0.5\rangle, \langle 0.3, 0.4, 0.5\rangle, \langle 0.5, 0.1, 0.1\rangle, \langle 0.3, 0.1, 0.2\rangle\}$
2. $\tau_1(\mathfrak{b}_2) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.4, 0.2, 0.2\rangle, \langle 0.3, 0.2, 0.3\rangle, \langle 0.6, 0.1, 0.3\rangle, \langle 0.6, 0.1, 0.2\rangle, \langle 0.6, 0.2, 0.1\rangle, \langle 0.4, 0.2, 0.3\rangle, \langle 0.6, 0.2, 0.3\rangle, \langle 0.6, 0.2, 0.2\rangle, \langle 0.6, 0.1, 0.1\rangle\}$
3. $\tau_1(\mathfrak{b}_3) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.2, 0.4, 0.3\rangle, \langle 0.6, 0.2, 0.1\rangle, \langle 0.3, 0.2, 0.5\rangle, \langle 0.5, 0.2, 0.1\rangle, \langle 0.2, 0.4, 0.3\rangle, \langle 0.3, 0.2, 0.3\rangle\}$
4. $\tau_1(\mathfrak{b}_4) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.3, 0.3, 0.2\rangle, \langle 0.5, 0.1, 0.2\rangle, \langle 0.2, 0.6, 0.1\rangle, \langle 0.4, 0.2, 0.3\rangle, \langle 0.7, 0.0, 0.2\rangle, \langle 0.2, 0.6, 0.2\rangle, \langle 0.3, 0.3, 0.3\rangle, \langle 0.2, 0.6, 0.3\rangle, \langle 0.3, 0.3, 0.1\rangle, \langle 0.4, 0.2, 0.2\rangle, \langle 0.5, 0.1, 0.1\rangle, \langle 0.4, 0.2, 0.1\rangle, \langle 0.7, 0.0, 0.1\rangle\}$

Picture fuzzy topologies for flooring material are $\mathfrak{C}^*_{\tau_2\zeta}$

1. $\tau_2(\mathfrak{f}_1) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.3, 0.1, 0.3\rangle, \langle 0.4, 0.3, 0.2\rangle, \langle 0.6, 0.1, 0.3\rangle, \langle 0.3, 0.4, 0.2\rangle, \langle 0.5, 0.2, 0.2\rangle \langle 0.3, 0.3, 0.3\rangle, \langle 0.3, 0.4, 0.3\rangle, \langle 0.3, 0.2, 0.3\rangle, \langle 0.4, 0.3, 0.3\rangle, \langle 0.5, 0.2, 0.3\rangle, \langle 0.4, 0.1, 0.2\rangle, \langle 0.3, 0.1, 0.2\rangle, \langle 0.5, 0.1, 0.2\rangle, \langle 0.6, 0.1, 0.2\rangle\}$
2. $\tau_2(\mathfrak{f}_2) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.7, 0.1, 0.1\rangle, \langle 0.4, 0.2, 0.2\rangle, \langle 0.2, 0.4, 0.3\rangle, \langle 0.3, 0.3, 0.2\rangle, \langle 0.6, 0.1, 0.2\rangle\}$
3. $\tau_2(\mathfrak{f}_3) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.9, 0.1, 0.0\rangle, \langle 0.6, 0.2, 0.2\rangle, \langle 0.3, 0.2, 0.3\rangle, \langle 0.6, 0.0, 0.0\rangle, \langle 0.4, 0.2, 0.3\rangle, \langle 0.6, 0.1, 0.0\rangle, \langle 0.9, 0.0, 0.0\rangle\}$
4. $\tau_2(\mathfrak{f}_4) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.5, 0.1, 0.1\rangle, \langle 0.7, 0.2, 0.1\rangle, \langle 0.7, 0.0, 0.2\rangle, \langle 0.5, 0.3, 0.1\rangle, \langle 0.5, 0.2, 0.2\rangle, \langle 0.5, 0.2, 0.1\rangle, \langle 0.5, 0.1, 0.2\rangle, \langle 0.7, 0.2, 0.2\rangle, \langle 0.5, 0.3, 0.2\rangle, \langle 0.7, 0.1, 0.1\rangle, \langle 0.7, 0.0, 0.1\rangle, \langle 0.7, 0.2, 0.1\rangle\}$

Step-5: Computation of picture fuzzy score functions for the builders and flooring materials are done as in step-5 of the algorithm are as follows:

Score values for the builders are

$$\mathfrak{S}_{cr}(\mathfrak{b}_1) = 0.7773, \mathfrak{S}_{cr}(\mathfrak{b}_2) = 0.735, \mathfrak{S}_{cr}(\mathfrak{b}_3) = 0.675, \mathfrak{S}_{cr}(\mathfrak{b}_4) = 0.8477$$

Score values for the flooring materials are

$$\mathfrak{S}_{cr}(\mathfrak{f}_1) = 0.7328, \mathfrak{S}_{cr}(\mathfrak{f}_2) = 0.775, \mathfrak{S}_{cr}(\mathfrak{f}_3) = 0.9554, \mathfrak{S}_{cr}(\mathfrak{f}_4) = 0.9667$$

As the score functions are not equal, the accuracy function does not need to be calculated.

Step-6: Arrange the picture fuzzy score values for the alternatives $\mathfrak{b}_1, \mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_4$ and the attributes $\mathfrak{f}_1, \mathfrak{f}_2, \mathfrak{f}_3, \mathfrak{f}_4$ in run-up order. We consider the sequences below $\mathfrak{b}_3 \prec \mathfrak{b}_4 \prec \mathfrak{b}_1 \prec \mathfrak{b}_2$ and $\mathfrak{f}_1 \prec \mathfrak{f}_2 \prec \mathfrak{f}_3 \prec \mathfrak{f}_4$. Thus, the builder $\mathfrak{b}_3$ can choose the flooring material $\mathfrak{f}_4$= ceramic flooring , the builder $\mathfrak{b}_2$ can choose the flooring material $\mathfrak{f}_3$= marble, the builder $\mathfrak{b}_1$ can choose the flooring material $\mathfrak{f}_3$ = flag stones and the builder $\mathfrak{b}_4$ can choose the flooring material $\mathfrak{f}_1$= line concrete.

Now we solve the same problem by using second PFNTS-MADM method as given in Algorithm 2.

Step 1 and Step 2 are same as in Algorithm 1.

Step-3: Construct the in-discernibility relation for the correlation between the symptoms is given as $\mathfrak{R} = \{\{\zeta_1, \zeta_2, \zeta_4\}, \{\zeta_3, \zeta_5\}\}$.

Step-4: Construct the picture fuzzy nano topological spaces for each builder and each flooring material with respect to the features as follows:

Picture fuzzy nano topologies for builders are $\mathfrak{C}^*_{\tau_1\eta}$

1. $\mathfrak{C}^*_{\tau_1}(\mathfrak{b}_1) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.9, 0.0, 0.1\rangle, \langle 0.5, 0.2, 0.1\rangle, \langle 0.4, 0.5, 0.1\rangle, \langle 0.3, 0.1, 0.2\rangle, \langle 0.3, 0.4, 0.5\rangle, \langle 0.0, 0.7, 0.0\rangle\}$
2. $\mathfrak{C}^*_{\tau_1}(\mathfrak{b}_2) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.6, 0.1, 0.2\rangle, \langle 0.3, 0.2, 0.3\rangle, \langle 0.3, 0.7, 0.0\rangle, \langle 0.6, 0.1, 0.1\rangle, \langle 0.6, 0.2, 0.3\rangle, \langle 0.0, 0.9, 0.0\rangle\}$
3. $\mathfrak{C}^*_{\tau_1}(\mathfrak{b}_3) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.6, 0.2, 0.1\rangle, \langle 0.2, 0.4, 0.5\rangle, \langle 0.4, 0.6, 0.0\rangle, \langle 0.6, 0.2, 0.1\rangle, \langle 0.5, 0.2, 0.1\rangle, \langle 0.1, 0.9, 0.0\rangle\}$
4. $\mathfrak{C}^*_{\tau_1}(\mathfrak{b}_4) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.5, 0.1, 0.2\rangle, \langle 0.3, 0.3, 0.3\rangle, \langle 0.2, 0.8, 0.0\rangle, \langle 0.7, 0.0, 0.1\rangle, \langle 0.2, 0.6, 0.2\rangle, \langle 0.5, 0.4, 0.0\rangle\}$

Picture fuzzy nano topologies for flooring material are $\mathfrak{C}^*_{\tau_2\zeta}$

1. $\mathfrak{C}^*_{\tau_2}(\mathfrak{f}_1) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.4, 0.1, 0.2\rangle, \langle 0.3, 0.4, 0.3\rangle, \langle 0.1, 0.7, 0.0\rangle, \langle 0.6, 0.1, 0.2\rangle, \langle 0.5, 0.2, 0.3\rangle, \langle 0.1, 0.9, 0.0\rangle\}$
2. $\mathfrak{C}^*_{\tau_2}(\mathfrak{f}_2) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.7, 0.1, 0.1\rangle, \langle 0.3, 0.3, 0.2\rangle, \langle 0.4, 0.6, 0.0\rangle, \langle 0.6, 0.1, 0.2\rangle, \langle 0.2, 0.4, 0.3\rangle, \langle 0.4, 0.6, 0.0\rangle\}$
3. $\mathfrak{C}^*_{\tau_2}(\mathfrak{f}_3) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.9, 0.0, 0.0\rangle, \langle 0.6, 0.2, 0.2\rangle, \langle 0.3, 0.7, 0.0\rangle, \langle 0.4, 0.2, 0.3\rangle, \langle 0.3, 0.2, 0.3\rangle, \langle 0.1, 0.9, 0.0\rangle\}$
4. $\mathfrak{C}^*_{\tau_2}(\mathfrak{f}_4) = \{1_\mathfrak{p}, 0_\mathfrak{p}, \langle 0.7, 0.1, 0.1\rangle, \langle 0.5, 0.3, 0.1\rangle, \langle 0.2, 0.8, 0.0\rangle, \langle 0.7, 0.0, 0.2\rangle, \langle 0.5, 0.2, 0.2\rangle, \langle 0.2, 0.8, 0.0\rangle\}$

Step-5: Computation of picture fuzzy score functions for the builders and flooring materials are done as in step-5 of the algorithm are as follows:

Score values for the builders are

$$\mathfrak{S}_{cr}(\mathfrak{b}_1) = 0.740625, \mathfrak{S}_{cr}(\mathfrak{b}_2) = 0.7375, \mathfrak{S}_{cr}(\mathfrak{b}_3) = 0.734375, \mathfrak{S}_{cr}(\mathfrak{b}_4) = 0.74375$$

Score values for the flooring materials are

$$\mathfrak{S}_{cr}(\mathfrak{f}_1) = 0.675, \mathfrak{S}_{cr}(\mathfrak{f}_2) = 0.771875, \mathfrak{S}_{cr}(\mathfrak{f}_3) = 0.76875, \mathfrak{S}_{cr}(\mathfrak{f}_4) = 0.80625$$

As the score functions are not equal, the accuracy function does not need to be calculated.

Step-6: Arrange the picture fuzzy score values for the alternatives $\mathfrak{b}_1, \mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_4$ and the attributes $\mathfrak{f}_1, \mathfrak{f}_2, \mathfrak{f}_3, \mathfrak{f}_4$ in run-up order. We consider the sequences below $\mathfrak{b}_3 \prec \mathfrak{b}_2 \prec \mathfrak{b}_1 \prec \mathfrak{b}_4$ and $\mathfrak{f}_1 \prec \mathfrak{f}_3 \prec \mathfrak{f}_2 \prec \mathfrak{f}_4$. Thus, the builder $\mathfrak{b}_3$ can choose the flooring material $\mathfrak{f}_4$= ceramic flooring, the builder $\mathfrak{b}_2$ can choose the flooring material $\mathfrak{f}_2$= flag stones, the builder $\mathfrak{b}_1$ can choose the flooring material $\mathfrak{f}_3$ = marble and the builder $\mathfrak{b}_4$ can choose the flooring material $\mathfrak{f}_1$= line concrete.

We propose two algorithms for MADM of the real world problems. The first two steps and last two steps of Algorithm 1 and Algorithm 2 are the same. In Step 3 of Algorithm 1 we find PFTS while in step 3 and step 4 of Algorithm 2 we find in-discernibility relations and PFNTSs. Both algorithms give approximately same ranks; this difference does not mean incomplete information. This is only because both algorithms have different formulae. The constructed algorithms are valid and practical. Finally both algorithms gives the same final decision.

The comparison Table 3 shows the difference between novel picture fuzzy nano topological space with existing work.

Table 3. The difference between novel picture fuzzy nano topological space with existing work

Sets	Uncertainty	Truth Value of an Element	False Value of an Element	Abstinence of an Element	Roughness & Boundary of a Set
Zafer ref-Int. J. Fuzzy Syst. RFS	✓	✓	-	-	✓
Atanassov ref-Fuzzy sets and systems IFT	✓	✓	✓	-	-
Wei ref-Iranian Journal of Fuzzy Systems PFS	✓	✓	✓	✓	-
Proposed Algorithm PFT	✓	✓	✓	✓	-
Proposed Algorithm PFNT	✓	✓	✓	✓	✓

6. Conclusions and Future Work

The application of the rough picture fuzzy set gained attention among researchers. However, the boundary of the region was not studied further by Nguyen [12]. In this paper, we introduced boundary of a region on picture fuzzy set along with upper and lower approximation. It is our opinion that picture fuzzy information can be best dealt with by unclear, vague, indeterminate, contradictory, and incomplete periodic / redundant information work. This paper aimed at bringing out the picture fuzzy nano topology which is more versatile and adaptable to real-time issues than rest of the types of general fluffy sets. Definitions of nano topology in picture fuzzy sets were identified, followed by the closure and interior operations. A new form of MADM technique in the picture fuzzy set has been introduced and applied to a building material selection process. To show the advantages and applicability, a comparison was made between the proposed method and the existing methods. The results are critical for enriching the picture fuzzy set awareness provided for decision making applications. Future research plans are to use the MADM technique for more practical applications and advance the practical interval valued complex picture fuzzy nano topological logic method for prediction of forecasting problems.

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Abbreviations

MADM	multiple attribute decision making
RFS	Rough Fuzzy Set
NS	neutrosophic set
IFS	intuitionistic fuzzy sets
NT	nano topology
PFS	picture fuzzy set
PFNT	picture fuzzy nano Topological spaces
PFNCS	picture fuzzy nano closed set
PFN	picture fuzzy nano
PFNOS	picture fuzzy nano open set
PFNTS	picture fuzzy nano topological space

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