This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2925344, IEEE Access

IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.

Digital Object Identifier 10.1109/ACCESS.2017.Doi Number

Multiple Attribute Group Decision Making Based on 2-Tuple Linguistic Neutrosophic Dombi Power Heronian Mean Operators

Peide Liu[1], Qaisar Khan[2], Tahir Mahmood[2], Florentin Smarandache[3] and Ying Li[1,*]

[1,*] School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan Shandong 250014, China. Tel.: +86 53182222188.

[2,] Department of Mathematics and Statistics International Islamic University Islamabad Pakistan

[3]Department of Mathematics and Sciences University of New Mexico, 705 Gurly Ave, Gallup NM-87301, USA

*Corresponding author: Liying@mail.sdufe.edu.cn

This paper is supported by the National Natural Science Foundation of China (Nos. 71771140 and 71471172), the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045).

ABSTRACT As an expansion of 2-tuple linguistic intuitionistic fuzzy set, the newly developed 2-tuple linguistic neutrosophic set (2-TLNS) is more satisfactory to define decision maker's assessment information in decision making problems. 2-TLN aggregation operators are of great significance in multiple attribute group decision making (MAGDM) problems with a 2-tuple linguistic environment. Therefore, in this article our main contribution is to develop novel 2-TLN power Heronian aggregation (2-TLNPHM) operators. Firstly, we develop new operational laws established on Dombi T-norm (DTN) and Dombi T-conorm (DTCN). Secondly, Taking full advantages of the power average (PA) operator and Heronian mean (HM) operator, we develop some new novel power Heronian mean operator and discuss its related properties and special cases. The main advantages of developed aggregation operators are that not only remove the effect of awkward data which may be too high or too low, but also have a good capacity to model the extensive correlation between attributes, making them more worthy for successfully solving more and more complicated MAGDM problems. Thus, we develop a new algorithm to handle MAGDM based on the developed aggregation operators. Lastly, we apply the proposed method and algorithm to risk assessment for construction of engineering projects to show the efficiency of the developed method and algorithm. The dominant novelties of this contribution are triplex. Firstly, new operational laws are proposed for 2-TLNNs. Secondly, novel 2-TLNPHM operators are developed. Thirdly, a new approach for 2-tuple linguistic neutrosophic MAGDM is developed.

INDEX TERMS 2-TLNS, Dombi T-norm, Dombi T-conorm, PA operator, Heronian mean, MAGDM.

I. INTRODUCTION

In actual life, multiple attribute group decision making (MAGDM) problems are the vital part of decision theory in which we select the optimal one from the group of finite alternatives based on the overall information. Conventionally, it has been accepted that the information concerning acquiring the alternatives is taken in the form of real number. But in our daily life, it is hard for a decision maker to give his evaluations regarding the object in crisp values due to vagueness and insufficient information. Rather, it has been enhance acceptable that these evaluations are given by fuzzy set (FS) or its extended form. Intuitionistic fuzzy set (IFS) [1] is the vigorous augmentation of FS [2] to deal with vagueness by including an identical falsity-membership into the analysis. A lot of studies by different researchers were conducted on IFS in different fields. IFSs have good capability to explain and articulate decision maker's (DMs) fuzzy decision information in MAGDM problems. However, IFS still have shortcomings and there exist relatively a few situations in which it is inappropriate to employ IFS to articulate DMs preference information. The key motive is that the hesitancy/indeterminacy degree is dependent of membership degree and non-membership degree in IFSs, for example when a DM utilizes an IFN (0.6, 0.2) to represent his/her assessment on a certain attribute. Then, the indeterminacy/hesitancy degree of the DM is 1 − 0.6 − 0.2 = 0.2. In simple words, once the truth-membership and falsity-membership degrees are determined, the degree of indeterminacy is determined automatically. Some other generalizations FS are proposed by some scholars such as Pythagorean fuzzy sets [3], hesitant Pythagorean fuzzy sets [4]. However, these are rather different from real MAGDM problems. In real MAGDM, the indeterminacy/hesitancy degrees should not be determined automatically and should be provided by DMs. For example, if a DM thinks the membership degree is 0.6, the membership degree is 0.4, and the degree that he/she is not sure about the result is 0.2, then the DMs evaluation value can be denoted as (0.6, 0.4, 0.2), which cannot be represented by IFSs. In order to deal with this case, Smarandache [5, 6] initially developed the concept of neutrosophic set (NS), which has the capacity of dealing inconsistent and indeterminate information. In the NS, its degree of membership $\overline{\overline{TR}}_A(a)$, degree of indeterminacy $\overline{\overline{IN}}_A(a)$ and degree of falsity $\overline{\overline{FL}}_A(a)$ are expressed independently, which lie real standard or non-standard subsets of $\left]0^-,1^+\right[$, that is $\overline{\overline{TR}}_A(a):U\to\left]0^-,1^+\right[$, $\overline{\overline{IN}}_A(a):U\to\left]0^-,1^+\right[$ and $\overline{\overline{FL}}_A(a):U\to\left]0^-,1^+\right[$, such that $0^-\le\overline{\overline{TR}}_A(a)+\overline{\overline{IN}}_A(a)+\overline{\overline{FL}}_A(a)\le 3^+$. Thus, the use of nonstandard interval $\left]0^-,1^+\right[$ may verdict some difficulty in real applications. To utilize NS easily in real application Wang et al. [7] proposed the concept of single valued neutrosophic set (SVNS) by changing the non-standard unit interval into the standard unit interval $[0,1]$. Further, Wang et al. [8] proposed the concept of interval neutrosophic set (INS). Ye [9] developed simplified neutrosophic set (SNS), which consist of both concepts of SVNS and INS. Some researcher developed improved operational laws for these sets [10,11].

In recent time, information aggregation operators [12-15] have enticed comprehensive recognitions of researchers and have become a vital part of MAAGDM. Generally, for aggregating a group of data, it is mandatory to assess the functions and the operations of aggregation operators. For the functions, the conventional aggregation operator developed Xu, Xu and Yager [16, 17] only can aggregate a group of real values into a single real value. In the past few years, some expanded aggregation operators have been developed by different researchers. For example, Sun et al. [18] developed some Choquet integral operator for INS. Liu and Tang, Peng et al. [19, 20] extended the power average (PA) operator developed by Yager [21] to interval neutrosophic and multi-valued neutrosophic environment, which has the capacity of removing the bad impact of awkward data. Wu et al [22] developed cross entropy and prioritized aggregation operators for SNNs, which take the priorities of criterion by priority weights. Besides, some aggregation operators can consider interrelationship among aggregated arguments. That is Bonferroni mean (BM) operator developed by Bonferroni [23], Heronian mean (HM) operator developed by Sykora [24].

All the above aggregation operators are capable to deal with information available in the form of real numbers. However, in various actual situations, mostly for various actual MAGDM problems, the assessment information associated with every alternatives are normally unpredictable or vague, due to the increasing complexity such as lack of time, lack of knowledge and various other limitations. Therefore, it is often hard for DMs to represent the assessment information about alternatives in the form of numeric values. Hence, to deal with such type of situations, Zadeh [25] initially proposed the concept of linguistic variable. It has also been generalized to various linguistic environments such as 2-tuple linguistic representation model [26-30], intuitionistic 2-tuple linguistic model [31] and so on [32, 33]. These developed concepts have also the same limitations to that of FS and IFS have. To overcome these limitations, Wang et al. [34] developed the concept of 2-tuple linguistic neutrosophic set (2-TLNS) based on the SVNS and 2-tuple linguistic information model , which is the generalization of several concepts such as 2-tuple linguistic set, 2-tuple linguistic fuzzy set and 2-tuple linguistic intuitionistic fuzzy set [35]. They described some operational laws for 2-tuple linguistic neutrosophic number (2-TLNN) , proposed some aggregation operators and apply these aggregation operators to solve MADM problems. Wang et al. [36, 37] further developed MAGDM method based TODIM and Muirhead mean operators to deal with 2-tuple linguistic environment. Wu et al. [38] proposed some 2-tuple linguistic neutrosophic Hamy mean (2-TLNHM) operators. Wu et al. [39] proposed the idea of SVN 2-tuple linguistic set (SVN2TLS), SVN 2 tuple linguistic number (SVN2TN), basic operational laws based on Hamacher triangular norm and conorm. Then based on these operational laws propose some aggregation operators and apply these aggregation operator to deal with MAGDM problem under SVN2TL information.

The Dombi t-norm (DTN) and Dombi t-conorm (DTCN) proposed by Dombi [40] have general parameter, which makes the information aggregation process more flexible. In the past few years, some researchers proposed Dombi operational laws for various sets and based on these Dombi operational laws they developed different aggregation operators [41-56].

Due to the increasing complexity in real decision making problems day by day, we have to look at the following questions, when selecting the best alternative. (1) In various situations, the assessment values of the attributes presented by the DMs may be too high or too low, have a negative effect on the final ranking results. The PA operator is a useful aggregation operator that authorizes the assessed values to equally supported and improved. Therefore, we may utilize the PA operator to vanish such bad effect by choosing different weights constructed by the support measure. (2) In various practical decision making problems the assessment values of attribute are dependent. Therefore, the interrelationship among the values of the attributes should be scrutinized. The HM operator can gain this function. However, HM operator has some advantages over BM. From the existing literature, we can notice that there is a need to combine PA operator with HM operator to deal with 2-TLN environment and achieved the above advantages.

Therefore, the main aim of this article is to propose some Dombi operational laws for 2-TLNNs, combine PA operator with HM operator, and extend the idea to 2-TLN environment, and develop some new aggregation operators such as 2-TLN power HM (2-TLNHM) operator, its weighted form, 2-LN power geometric HM (2-TLNHM) operator, its weighted form and discussed some special cases of the developed aggregation operator and apply them to MAGDM to achieve the two requirements discussed above.

To do so, the rest of the article is organized as follows.

In section 1, some basic definitions about SVNS, 2-TLNS, PA operator, HM operator and related properties are discussed. In section 3, we developed some operational laws for 2-TLNNs. In section 4, based on these operational laws we developed some 2-tuple linguistic Dombi power Heronian mean operators, related properties and special cases are discussed. In section 5, MAGDM method is developed based on these newly developed aggregation operators and a numerical example is given to show the effectiveness of the proposed MAGDM approach. In section 6, comparison of the developed approach and some existing approaches are given. At the end Conclusion, future work and references are given.

II. Preliminaries

In this part, we gave some basic definitions and results about 2-TLNSs, PA operator and HM operator.

A. 2-TLNS and their operations

Definition 1[7]. Let Θ be a space of points (objects), with a common component in Θ denoted by η. A SVNS $\widetilde{SN}$ in Θ is expressed by,

$$\widetilde{SN}=\left\{\left\langle \eta,\xi_{\widetilde{SN}}(\eta),\psi_{\widetilde{SN}}(\eta),\zeta_{\widetilde{SN}}(\eta)\right\rangle \mid \eta\in\Theta\right\} \tag{1}$$

Where $\xi_{\widetilde{SN}}(\eta),\psi_{\widetilde{SN}}(\eta)$ and $\zeta_{\widetilde{SN}}(\eta)$ respectively denote the TMD, IMD and FMD of the element $\eta\in\Theta$ to the set $\widetilde{SN}$. For each point $\eta\in\Theta$, we have $\xi_{\widetilde{SN}}(\eta),\psi_{\widetilde{SN}}(\eta),\zeta_{\widetilde{SN}}(\eta)\in[0,1]$, and $0\le\xi_{\widetilde{SN}}(\eta)+\psi_{\widetilde{SN}}(\eta)+\zeta_{\widetilde{SN}}(\eta)\le 3.$

Definition 2 [34]. Suppose that $\Gamma=\{\Gamma_1,\Gamma_2,....,\Gamma_p\}$ is a 2-TLSs with $p+1$ cardinality. That is the order of 2-TLSs is odd. If $\Gamma=\left\langle (s_t,\Xi),(s_i,\Psi),(s_f,\Upsilon)\right\rangle$ is described for $(s_t,\Xi),(s_i,\Psi),(s_f,\Upsilon)\in\Gamma$ and $\Xi,\Psi,\Upsilon\in[o,p]$, where $(s_t,\Xi),(s_i,\Psi)$ and (s_f,Υ) respectively, represents the truth-membership degree, indeterminacy-membership degree and falsity-membership degree by 2-TLNSs, then the 2-TLNSs is described as follows:

$$\Gamma_g=\left\{\left\langle\left(s_{t_g},\Xi_g\right),\left(s_{i_g},\Psi_g\right),\left(s_{f_g},\Upsilon_g\right)\right\rangle\right\} \tag{2}$$

where, $0\le\Delta^{-1}\left(s_{t_g},\Xi\right)\le p, 0\le\Delta^{-1}\left(s_{i_g},\Psi\right)\le p, 0\le\Delta^{-1}\left(s_{f_g},\Upsilon\right)\le p$ such that $0\le\Delta^{-1}\left(s_{t_g},\Xi\right)+\Delta^{-1}\left(s_{i_g},\Psi\right)+\Delta^{-1}\left(s_{f_g},\Upsilon\right)\le 3p.$

Definition 3 [34]. Let $\Gamma=\left\langle (s_t,\Xi),(s_i,\Psi),(s_f,\Upsilon)\right\rangle$ be a 2-TLNN. Then, the score and accuracy functions are described as follows:

$$\overline{\overline{SR}}(\Gamma)=\Delta\left\{\frac{2p+\Delta^{-1}(s_t,\Xi)-\Delta^{-1}(s_i,\Psi)-\Delta^{-1}(s_f,\Upsilon)}{3p}\right\},\overline{\overline{SR}}(\Gamma)\in[0,1]; \tag{3}$$

$$\overline{\overline{AC}}(\Gamma)=\Delta\left\{\Delta^{-1}(s_t,\Xi)-\Delta^{-1}(s_f,\Upsilon)\right\},\overline{\overline{AC}}(\Gamma)\in[-p,p]. \tag{4}$$

Definition 4 [34]. Let $\Gamma_1=\left\langle (s_{t_1},\Xi_1),(s_{i_1},\Psi_1),(s_{f_1},\Upsilon_1)\right\rangle$ and $\Gamma_2=\left\langle (s_{t_2},\Xi_2),(s_{i_2},\Psi_2),(s_{f_2},\Upsilon_2)\right\rangle$ be any two arbitrary 2-TLNNs. Then, the comparison rules are described as follows:

(1) If $\overline{\overline{SR}}(\Gamma_1)>\overline{\overline{SR}}(\Gamma_2)$, then $\Gamma_1>\Gamma_2$;

(2) If $\overline{\overline{SR}}(\Gamma_1)=\overline{\overline{SR}}(\Gamma_2)$, then

i. If $\overline{\overline{AC}}(\Gamma_1)>\overline{\overline{AC}}(\Gamma_2)$, then $\Gamma_1>\Gamma_2$;

ii. If $\overline{\overline{AC}}(\Gamma_1)=\overline{\overline{AC}}(\Gamma_2)$, then $\Gamma_1=\Gamma_2$.

Definition 5 [36]. Let $\Gamma_1=\left\langle (s_{t_1},\Xi_1),(s_{i_1},\Psi_1),(s_{f_1},\Upsilon_1)\right\rangle$ and $\Gamma_2=\left\langle (s_{t_2},\Xi_2),(s_{i_2},\Psi_2),(s_{f_2},\Upsilon_2)\right\rangle$ be any two arbitrary 2-TLNNs. Then, the normalized Hamming distance is described as follows:

$$\overline{\overline{D}}_H(\Gamma_1,\Gamma_2)=\frac{1}{3p}\left\{\begin{array}{l}\left|\Delta^{-1}\left(s_{t_1},\Xi_1\right)-\Delta^{-1}\left(s_{t_2},\Xi_2\right)\right|+\left|\Delta^{-1}\left(s_{i_1},\Psi_1\right)-\Delta^{-1}\left(s_{i_2},\Psi_2\right)\right|+\\ \left|\Delta^{-1}\left(s_{f_1},\Upsilon_1\right)-\Delta^{-1}\left(s_{f_2},\Upsilon_2\right)\right|\end{array}\right\} \tag{5}$$

B. The PA operator

Yager [21] was the first one who presented the concept of the PA which is one of the important aggregation operators. The PA operator diminishes some negative effects of unnecessarily high or unnecessarily low arguments given by experts. The conventional PA operator can only deal with crisp numbers, and is defined as follows.

Definition 6 [21]. Let $b_i(i=1,2,...,m)$ be a group of non-negative crisp numbers, the PA is a function defined by

$$PA(b_1,b_2,....,b_m)=\frac{\sum_{i=1}^{m}\left(1+T(b_i)\right)b_i}{\sum_{i=1}^{m}\left(1+T(b_i)\right)} \tag{6}$$

Where $T(b_i)=\sum_{\substack{j=1\\ j\ne i}}^{m}Sup(b_i,b_j)$ and $Sup(b,c)$ is the support degree for b from c, which satisfies some axioms. 1) $Sup(b,c)\in[0,1]$; 2) $Sup(b,c)=Sup(c,b)$; 3) $Sup(b,c)\ge Sup(d,e)$, if $|b-c|<|d-e|$.

C. HM operator

HM [24] is also an important tool, which can represent the interrelationships of the input values, and it is defined as follows:

Definition 7 [24]. Let $I=[0,1], x,y\geq 0, H^{x,y}:I^m\to I$, if $H^{x,y}$ satisfies;

$$H^{x,y}\left(b_1,b_2,...,b_m\right)=\left(\frac{2}{m^2+m}\sum_{i=1}^{m}\sum_{j=i}^{m}b_i^x b_j^y\right)^{\frac{1}{x+y}} \tag{7}$$

Then the mapping $H^{x,y}$ is said to be HM operator with parameters. The HM satisfies the properties of idem potency, boundedness and monotonicity.

III. Dombi operational laws for 2-TLNNs

A. Dombi TN and TCN

Dombi operations consist of the Dombi sum and Dombi product.

Definition 8 [40]. Let α and β be any two real number. Then, the DTN and DTCN among α and β are explain as follows:

$$T_D(\alpha,\beta)=\frac{1}{1+\left\{\left(\frac{1-\alpha}{\alpha}\right)^{\Im}+\left(\frac{1-\beta}{\beta}\right)^{\Im}\right\}^{\frac{1}{\Im}}}; \tag{8}$$

$$T_D^{*}(\alpha,\beta)=1-\frac{1}{1+\left\{\left(\frac{\alpha}{1-\alpha}\right)^{\Im}+\left(\frac{\beta}{1-\beta}\right)^{\Im}\right\}^{\frac{1}{\Im}}}. \tag{9}$$

Where $\Im\geq 1$, and $(\alpha,\beta)\in[0,1]\times[0,1]$.

According to the DTN and DTCN, we develop few operational rules for 2-TLNNs.

Definition 9. Let $\Gamma_1=\left\langle\left(s_{t_1},\Xi_1\right),\left(s_{i_1},\Psi_1\right),\left(s_{f_1},\Upsilon_1\right)\right\rangle$ and $\Gamma_2=\left\langle\left(s_{t_2},\Xi_2\right),\left(s_{i_2},\Psi_2\right),\left(s_{f_2},\Upsilon_2\right)\right\rangle$ be an arbitrary 2-TLNNs and $\Im>0$, for simplicity, we assume that $\frac{\Delta^{-1}\left(s_{t_g},\Xi_g\right)}{h}=\bar{\bar{t}}_g, \frac{\Delta^{-1}\left(s_{i_g},\Psi_g\right)}{h}=\bar{\bar{i}}_g, \frac{\Delta^{-1}\left(s_{f_g},\Upsilon_g\right)}{h}=\bar{\bar{f}}_g$ for $g=1,2$. Then, the operational laws can be described as follows:

$$(1)\Gamma_1\otimes\Gamma_2=\left\langle\begin{array}{l}\Delta\left(h\left(\frac{1}{1+\left(\left(\frac{1-\bar{\bar{t}}_1}{\bar{\bar{t}}_1}\right)^{\Im}+\left(\frac{1-\bar{\bar{t}}_2}{\bar{\bar{t}}_2}\right)^{\Im}\right)^{1/\Im}}\right)\right),\\ \Delta\left(h\left(1-\frac{1}{1+\left(\left(\frac{\bar{\bar{i}}_1}{1-\bar{\bar{i}}_1}\right)^{\Im}+\left(\frac{\bar{\bar{i}}_2}{1-\bar{\bar{i}}_2}\right)^{\Im}\right)^{1/\Im}}\right)\right),\Delta\left(h\left(1-\frac{1}{1+\left(\left(\frac{\bar{\bar{f}}_1}{1-\bar{\bar{f}}_1}\right)^{\Im}+\left(\frac{\bar{\bar{f}}_2}{1-\bar{\bar{f}}_2}\right)^{\Im}\right)^{1/\Im}}\right)\right)\end{array}\right\rangle \tag{10}$$

$$(2)\Gamma_1\otimes\Gamma_2=\left\langle\begin{array}{l}\Delta\left(h\left(1-\frac{1}{1+\left(\left(\frac{\bar{\bar{t}}_1}{1-\bar{\bar{t}}_1}\right)^{\Im}+\left(\frac{\bar{\bar{t}}_2}{1-\bar{\bar{t}}_2}\right)^{\Im}\right)^{1/\Im}}\right)\right),\\ \Delta\left(h\left(\frac{1}{1+\left(\left(\frac{1-\bar{\bar{i}}_1}{\bar{\bar{i}}_1}\right)^{\Im}+\left(\frac{1-\bar{\bar{i}}_2}{\bar{\bar{i}}_2}\right)^{\Im}\right)^{1/\Im}}\right)\right),\Delta\left(h\left(\frac{1}{1+\left(\left(\frac{1-\bar{\bar{f}}_1}{\bar{\bar{f}}_1}\right)^{\Im}+\left(\frac{1-\bar{\bar{f}}_2}{\bar{\bar{f}}_2}\right)^{\Im}\right)^{1/\Im}}\right)\right)\end{array}\right\rangle; \tag{11}$$

$$(3)\,\xi\Gamma_1=\left\langle\begin{array}{l}\Delta\left(h\left(1-\frac{1}{1+\left(\xi\left(\frac{\bar{\bar{t}}_1}{1-\bar{\bar{t}}_1}\right)^{\Im}\right)^{1/\Im}}\right)\right),\Delta\left(h\left(\frac{1}{1+\left(\xi\left(\frac{1-\bar{\bar{i}}_1}{\bar{\bar{i}}_1}\right)^{\Im}\right)^{1/\Im}}\right)\right),\\ \Delta\left(h\left(\frac{1}{1+\left(\xi\left(\frac{1-\bar{\bar{f}}_1}{\bar{\bar{f}}_1}\right)^{\Im}\right)^{1/\Im}}\right)\right)\end{array}\right\rangle,\xi>0; \tag{12}$$

$$(4)\,\Gamma_1^{\xi}=\left\langle\begin{array}{l}\Delta\left(h\left(\frac{1}{1+\left(\xi\left(\frac{1-\bar{\bar{t}}_1}{\bar{\bar{t}}_1}\right)^{\Im}\right)^{1/\Im}}\right)\right),\Delta\left(h\left(1-\frac{1}{1+\left(\xi\left(\frac{\bar{\bar{i}}_1}{1-\bar{\bar{i}}_1}\right)^{\Im}\right)^{1/\Im}}\right)\right),\\ \Delta\left(h\left(1-\frac{1}{1+\left(\xi\left(\frac{\bar{\bar{f}}_1}{1-\bar{\bar{f}}_1}\right)^{\Im}\right)^{1/\Im}}\right)\right)\end{array}\right\rangle,\xi>0. \tag{13}$$

IV. The 2-tuple linguistic neutrosophic Dombi Heronian aggregation operators

In this part, based on the Dombi operational laws for 2-TLNNs, we combine PA operator and HM operator to propose 2-TLNDPHM operator, 2-TLNDWPHM operator, 2-TLNDPGHM operator, 2-TLNDWPGHM operator and discuss some related properties.

A. The 2-LNDPHM and 2-LNDWPHM operators

Definition 10. Let $\Gamma_g\,(g=1,2,...,p)$ be a group of 2-TLNNs, $x,y\geq 0$. Then, the 2-TLNNDPHM operator is described as follows:

$$2-TLNDPHM^{x,y}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)$$

$$=\left(\frac{2}{p^2+p}\sum_{g=1}^{p}\sum_{q=g}^{p}\left(\frac{p\left(1+T\left(\Gamma_g\right)\right)}{\sum_{r=1}^{p}\left(1+T\left(\Gamma_r\right)\right)}\Gamma_g\right)^{x}\otimes_D\left(\frac{p\left(1+T\left(\Gamma_q\right)\right)}{\sum_{r=1}^{p}\left(1+T\left(\Gamma_r\right)\right)}\Gamma_q\right)^{y}\right)^{\frac{1}{x+y}}. \tag{14}$$

Where $T(\Gamma_g)=\sum_{q=1,g\neq q}^{p} Sup(\Gamma_g,\Gamma_q)$, $Sup(\Gamma_g,\Gamma_q)=1-\overline{\overline{D}}(\Gamma_g,\Gamma_q)$ is the support degree for Γ_g from Γ_q, which satisfy the following conditions: (1) $Sup(\Gamma_g,\Gamma_q)\in[0,1]$; (2) $Sup(\Gamma_g,\Gamma_q)=Supp(\Gamma_g,\Gamma_q)$; (3) $Sup(\Gamma_g,\Gamma_q)\geq Sup(\Gamma_r,\Gamma_s)$, if $\overline{\overline{D}}(\Gamma_g,\Gamma_q)<\overline{\overline{D}}(\Gamma_r,\Gamma_s)$, in which $\overline{\overline{D}}(\Gamma_g,\Gamma_q)$ is the distance measure between 2-TLNNs Γ_g and Γ_q defined in Definition (5).

In order, to represent Equation (14) in a simple form, we assume that

$$\aleph_g=\frac{\left(1+T(\Gamma_g)\right)}{\sum_{r=1}^{p}\left(1+T(\Gamma_r)\right)} \tag{15}$$

Therefore, Equation (14) takes the form

$$2-TLNDPHM^{x,y}(\Gamma_1,\Gamma_2,...,\Gamma_p)$$
$$=\left(\frac{2}{p^2+p}\sum_{g=1}^{p}\sum_{q=g}^{p}\left(p\aleph_g\Gamma_g\right)^x\otimes_D\left(\aleph_q\Gamma_q\right)^y\right)^{\frac{1}{x+y}}. \tag{16}$$

Theorem 1. Let $x,y\geq 0,$ and x,y do not take the value 0 at the same time, $\Gamma_g(g=1,2,...,p)$ be a group of 2-TLNNs and let $\frac{\Delta^{-1}(s_{t_g},\Xi_g)}{h}=\overline{\overline{t}}_g,\frac{\Delta^{-1}(s_{i_g},\Psi_g)}{h}=\overline{\overline{i}}_g,\frac{\Delta^{-1}(s_{f_g},\Upsilon_g)}{h}=\overline{\overline{f}}_g$. Then, the aggregated value utilizing Equation (14), is still a 2-TLNN, and

$$2-TLNPHM(\Gamma_1,\Gamma_2,...,\Gamma_p)=$$
$$\left\langle\begin{array}{l}\Delta\left(h\left(1\middle/\left(1+\left(\frac{p^2+p}{2(x+y)}\times 1\middle/\left(\sum_{g=1,q=q}^{p}1\middle/\left(\frac{x}{p\aleph_g\left(\frac{\overline{\overline{t}}_g}{1-\overline{\overline{t}}_g}\right)^3}+\frac{y}{p\aleph_q\left(\frac{\overline{\overline{t}}_q}{1-\overline{\overline{t}}_q}\right)^3}\right)\right)\right)\right)^{\frac{1}{3}}\right)\right),\\ \Delta\left(h\left(1-1\middle/\left(1+\left(\frac{p^2+p}{2(x+y)}\times 1\middle/\left(\sum_{g=1,q=g}^{p}1\middle/\left(\frac{x}{p\aleph_g\left(\frac{1-\overline{\overline{i}}_g}{\overline{\overline{i}}_g}\right)^3}+\frac{y}{p\aleph_q\left(\frac{1-\overline{\overline{i}}_q}{\overline{\overline{i}}_q}\right)^3}\right)\right)\right)\right)^{\frac{1}{3}}\right)\right),\\ \Delta\left(h\left(1-1\middle/\left(1+\left(\frac{p^2+p}{2(x+y)}\times 1\middle/\left(\sum_{g=1,q=g}^{p}1\middle/\left(\frac{x}{p\aleph_g\left(\frac{1-\overline{\overline{f}}_g}{\overline{\overline{f}}_g}\right)^3}+\frac{y}{p\aleph_q\left(\frac{1-\overline{\overline{f}}_q}{\overline{\overline{f}}_q}\right)^3}\right)\right)\right)\right)^{\frac{1}{3}}\right)\right)\end{array}\right\rangle. \tag{17}$$

Proof. According to operational laws, we have

$$p\aleph_g\Gamma_g=\left\langle\Delta\left(h\left(1-1\middle/\left(1+\left(p\aleph_g\left(\frac{\overline{\overline{t}}_g}{1-\overline{\overline{t}}_g}\right)^3\right)^{\frac{1}{3}}\right)\right)\right),\right.$$
$$\left.\Delta\left(h\left(1\middle/\left(1+\left(p\aleph_g\left(\frac{1-\overline{\overline{i}}_g}{\overline{\overline{i}}_g}\right)^3\right)^{\frac{1}{3}}\right)\right)\right),\Delta\left(h\left(1\middle/\left(1+\left(p\aleph_g\left(\frac{1-\overline{\overline{f}}_g}{\overline{\overline{f}}_g}\right)^3\right)^{\frac{1}{3}}\right)\right)\right)\right\rangle,$$

and

$$p\aleph_q\Gamma_q=\left\langle\Delta\left(h\left(1-1\middle/\left(1+\left(p\aleph_q\left(\frac{\overline{\overline{t}}_q}{1-\overline{\overline{t}}_q}\right)^3\right)^{\frac{1}{3}}\right)\right)\right),\right.$$
$$\left.\Delta\left(h\left(1\middle/\left(1+\left(p\aleph_q\left(\frac{1-\overline{\overline{i}}_q}{\overline{\overline{i}}_q}\right)^3\right)^{\frac{1}{3}}\right)\right)\right),\Delta\left(h\left(1\middle/\left(1+\left(p\aleph_q\left(\frac{1-\overline{\overline{f}}_q}{\overline{\overline{f}}_q}\right)^3\right)^{\frac{1}{3}}\right)\right)\right)\right\rangle.$$

Let $\overline{\overline{a}}_g=\frac{\overline{\overline{t}}_g}{1-\overline{\overline{t}}_g},\overline{\overline{a}}_q=\frac{\overline{\overline{t}}_q}{1-\overline{\overline{t}}_q},\overline{\overline{b}}_g=\frac{1-\overline{\overline{i}}_g}{\overline{\overline{i}}_g},\overline{\overline{b}}_q=\frac{1-\overline{\overline{i}}_q}{\overline{\overline{i}}_q},\overline{\overline{c}}_g=\frac{1-\overline{\overline{f}}_g}{\overline{\overline{f}}_g},\overline{\overline{c}}_q=\frac{1-\overline{\overline{f}}_q}{\overline{\overline{f}}_q}.$

Then, we can obtain

$$p\aleph_g\Gamma_g=\left\langle\Delta\left(h\left(1-1\middle/1+\left(p\aleph_g\right)^{\frac{1}{3}}\overline{\overline{a}}_g\right)\right),\right.$$
$$\left.\Delta\left(h\left(1\middle/1+\left(p\aleph_g\right)^{\frac{1}{3}}\overline{\overline{b}}_g\right)\right),\Delta\left(h\left(1\middle/1+\left(p\aleph_g\right)^{\frac{1}{3}}\overline{\overline{c}}_g\right)\right)\right\rangle,$$
$$p\aleph_q\Gamma_q=\left\langle\Delta\left(h\left(1-1\middle/1+\left(p\aleph_q\right)^{\frac{1}{3}}\overline{\overline{a}}_q\right)\right),\right.$$
$$\left.\Delta\left(h\left(1\middle/1+\left(p\aleph_q\right)^{\frac{1}{3}}\overline{\overline{b}}_q\right)\right),\Delta\left(h\left(1\middle/1+\left(p\aleph_q\right)^{\frac{1}{3}}\overline{\overline{c}}_q\right)\right)\right\rangle,$$

and

$$\left(p\aleph_g\Gamma_g\right)^x=\left\langle\Delta\left(h\left(1\middle/1+x^{\frac{1}{3}}\middle/\left(p\aleph_g\right)^{\frac{1}{3}}\overline{\overline{a}}_g\right)\right),\right.$$
$$\left.\Delta\left(h\left(1-1\middle/1+x^{\frac{1}{3}}\middle/\left(p\aleph_g\right)^{\frac{1}{3}}\overline{\overline{b}}_g\right)\right),\Delta\left(h\left(1-1\middle/1+x^{\frac{1}{3}}\middle/\left(p\aleph_g\right)^{\frac{1}{3}}\overline{\overline{c}}_g\right)\right)\right\rangle,$$
$$\left(p\aleph_q\Gamma_q\right)^y=\left\langle\Delta\left(h\left(1\middle/1+y^{\frac{1}{3}}\middle/\left(p\aleph_q\right)^{\frac{1}{3}}\overline{\overline{a}}_q\right)\right),\right.$$
$$\left.\Delta\left(h\left(1-1\middle/1+y^{\frac{1}{3}}\middle/\left(p\aleph_q\right)^{\frac{1}{3}}\overline{\overline{b}}_q\right)\right),\Delta\left(h\left(1-1\middle/1+y^{\frac{1}{3}}\middle/\left(p\aleph_q\right)^{\frac{1}{3}}\overline{\overline{c}}_q\right)\right)\right\rangle.$$

Furthermore, we can have

$$\left(p\aleph_g\Gamma_g\right)^x\otimes_D\left(p\aleph_q\Gamma_q\right)^y=\left\langle\Delta\left(h\left(1\middle/\left(1+\left(x\middle/p\aleph_g\overline{\overline{a}}_g^{3}+y\middle/p\aleph_q\overline{\overline{a}}_q^{3}\right)^{\frac{1}{3}}\right)\right)\right),\right.$$
$$\left.\Delta\left(h\left(1-1\middle/\left(1+\left(x\middle/p\aleph_g\overline{\overline{b}}_g^{3}+y\middle/p\aleph_q\overline{\overline{b}}_q^{3}\right)^{\frac{1}{3}}\right)\right)\right),\Delta\left(h\left(1-1\middle/\left(1+\left(x\middle/p\aleph_g\overline{\overline{c}}_g^{3}+y\middle/p\aleph_q\overline{\overline{c}}_q^{3}\right)^{\frac{1}{3}}\right)\right)\right)\right\rangle,$$

and

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2925344, IEEE Access

$$\sum_{\substack{g=1,\\q=g}}^{p}\left(p\aleph_g\Gamma_g\right)^{x}\otimes_D\left(p\aleph_q\Gamma_q\right)^{y}$$

$$=\left\langle \Delta\left(h\left(1-1\Big/\left(1+\left(\sum_{\substack{g=1,\\q=g}}^{p}\left(\left(\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\Im}}\right)^{\frac{1}{\Im}}}\right)\Big/\left(1-\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\Im}}\right)^{\frac{1}{\Im}}}\right)\right)^{\Im}\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1\Big/\left(1+\left(\sum_{\substack{g=1,\\q=g}}^{p}\left(\left(1-1+\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\Im}}\right)^{\frac{1}{\Im}}}\right)\Big/\left(1-\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\Im}}\right)^{\frac{1}{\Im}}}\right)\right)^{\Im}\right)^{\frac{1}{\Im}}\right)\right)\right),$$

$$\left.\Delta\left(h\left(1\Big/\left(1+\left(\sum_{\substack{g=1,\\q=g}}^{p}\left(\left(1-1+\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\Im}}\right)^{\frac{1}{\Im}}}\right)\Big/\left(1-\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\Im}}\right)^{\frac{1}{\Im}}}\right)\right)^{\Im}\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle,$$

$$=\left\langle \Delta\left(h\left(1-1\Big/\left(1+\left(\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1\Big/\left(1+\left(\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),$$

$$\left.\Delta\left(h\left(1\Big/\left(1+\left(\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle.$$

So, we can have

$$\frac{2}{p^2+p}\sum_{\substack{g=1,\\q=g}}^{p}\left(p\aleph_g\Gamma_g\right)^{x}\otimes_D\left(p\aleph_q\Gamma_q\right)^{y}$$

$$=\left\langle \Delta\left(h\left(1-1\Big/\left(1+\left(\frac{2}{p^2+p}\left(\left(1-1\Big/\left(1+\left(\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\Big/\left(1-1+1\Big/1+\left(\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)^{\Im}\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1\Big/\left(1+\left(\frac{2}{p^2+p}\left(\left(1-1\Big/\left(1+\left(\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\Big/\left(1\Big/1+\left(\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)^{\Im}\right)^{\frac{1}{\Im}}\right)\right)\right),$$

$$\left.\Delta\left(h\left(1\Big/\left(1+\left(\frac{2}{p^2+p}\left(\left(1-1\Big/\left(1+\left(\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\Big/\left(1\Big/1+\left(\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)^{\Im}\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle,$$

$$=\left\langle \Delta\left(h\left(1-1\Big/\left(1+\left(\frac{2}{p^2+p}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1\Big/\left(1+\left(\frac{2}{p^2+p}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),$$

$$\left.\Delta\left(h\left(1\Big/\left(1+\left(\frac{2}{p^2+p}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle.$$

Then

$$\left(\frac{2}{p^2+p}\sum_{\substack{g=1,\\q=g}}^{p}\left(p\aleph_g\Gamma_g\right)^{x}\otimes_D\left(p\aleph_q\Gamma_q\right)^{y}\right)^{\frac{1}{x+y}}$$

$$=\left\langle \Delta\left(h\left(1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\quad (18)$$

$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\Im}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle.$$

Now put $\bar{\bar{a}}_g=\frac{\bar{\bar{t}}_g}{1-\bar{\bar{t}}_g},\bar{\bar{a}}_q=\frac{\bar{\bar{t}}_q}{1-\bar{\bar{t}}_q},\bar{\bar{b}}_g=\frac{1-\bar{\bar{i}}_g}{\bar{\bar{i}}_g},\bar{\bar{b}}_q=\frac{1-\bar{\bar{i}}_q}{\bar{\bar{i}}_q},\bar{\bar{c}}_g=\frac{1-\bar{\bar{f}}_g}{\bar{\bar{f}}_g},\bar{\bar{c}}_q=\frac{1-\bar{\bar{f}}_q}{\bar{\bar{f}}_q}$ in Equation (18), we can have

$$=\left\langle \Delta\left(h\left(1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{\bar{\bar{t}}_g}{1-\bar{\bar{t}}_g}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{\bar{\bar{t}}_q}{1-\bar{\bar{t}}_q}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{1-\bar{\bar{i}}_g}{\bar{\bar{i}}_g}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{1-\bar{\bar{i}}_q}{\bar{\bar{i}}_q}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),$$

$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{1-\bar{\bar{f}}_g}{\bar{\bar{f}}_g}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{1-\bar{\bar{f}}_g}{\bar{\bar{f}}_g}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle.$$

This completes the proof of Theorem (1).

Theorem 2 (Idempotency). Let $\Gamma_g(g=1,2,...,p)$ be a group of 2-TLNNs, if all $\Gamma_g(g=1,2,...,p)$ are same, that is

IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

$\Gamma_g=\Gamma=\left\langle (s_t,\Xi),(s_i,\Psi),(s_f,\Upsilon)\right\rangle (g=1,2,...,p)$. Assume that $\frac{\Delta^{-1}\left(s_{t_g},\Xi_g\right)}{h}=\bar{\bar{t}}_g,\frac{\Delta^{-1}\left(s_{i_g},\Psi_g\right)}{h}=\bar{\bar{i}}_g,\frac{\Delta^{-1}\left(s_{f_g},\Upsilon_g\right)}{h}=\bar{\bar{f}}_g$, then

$$2-TLNPHM\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)=\Gamma. \tag{19}$$

Proof. Since all $\Gamma_g=\Gamma=\left\langle (s_t,\Xi),(s_i,\Psi),(s_f,\Upsilon)\right\rangle (g=1,2,...,p)$, so we can have $Sup\left(\Gamma_g,\Gamma_q\right)=1$, for all $g,q=1,2,...,p$, so $\aleph_g=\frac{1}{p}$, for all $g=1,2,...,p$. Then

$$2-TLNPHM^{x,y}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)=2-TLNPHM^{x,y}\left(\Gamma,\Gamma,...,\Gamma\right)$$

$$=\left\langle \Delta\left(h\left(1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\frac{1}{p}\left(\frac{\bar{\bar{t}}}{1-\bar{\bar{t}}}\right)^{\Im}}+\frac{y}{p\frac{1}{p}\left(\frac{\bar{\bar{t}}}{1-\bar{\bar{t}}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\frac{1}{p}\left(\frac{1-\bar{\bar{i}}}{\bar{\bar{i}}}\right)^{\Im}}+\frac{y}{p\frac{1}{p}\left(\frac{1-\bar{\bar{i}}}{\bar{\bar{i}}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),$$

$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\frac{1}{p}\left(\frac{1-\bar{\bar{f}}}{\bar{\bar{f}}}\right)^{\Im}}+\frac{y}{p\frac{1}{p}\left(\frac{1-\bar{\bar{f}}}{\bar{\bar{f}}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle,$$

$$=\left\langle \Delta\left(h\left(1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x+y}{\left(\frac{\bar{\bar{t}}}{1-\bar{\bar{t}}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x+y}{\left(\frac{1-\bar{\bar{i}}}{\bar{\bar{i}}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),$$

$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x+y}{\left(\frac{1-\bar{\bar{f}}}{\bar{\bar{f}}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle,$$

$$=\left\langle \Delta\left(h\left(1\Big/\left(1+\left(\frac{1}{(x+y)}\times 1\Big/\left(\frac{x+y}{\left(\frac{\bar{\bar{t}}}{1-\bar{\bar{t}}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{1}{(x+y)}\times 1\Big/\left(\frac{x+y}{\left(\frac{1-\bar{\bar{i}}}{\bar{\bar{i}}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right),$$

$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{1}{(x+y)}\times 1\Big/\left(\frac{x+y}{\left(\frac{1-\bar{\bar{f}}}{\bar{\bar{f}}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle,$$

$$=\left\langle (s_t,\Xi),(s_i,\Psi),(s_f,\Upsilon)\right\rangle=\Gamma$$

Theorem 3 (Boundedness). Let $\Gamma_g(g=1,2,...,p)$ be a group of 2-TLNNs. If $\bar{\bar{m}}^{-}=\left\langle \min_g\left(s_{t_g},\Xi_g\right),\max_g\left(s_{i_g},\Psi_g\right),\max_g\left(s_{f_g},\Upsilon_g\right)\right\rangle$ and $\bar{\bar{m}}^{+}=\left\langle \max_g\left(s_{t_g},\Xi_g\right),\min_g\left(s_{i_g},\Psi_g\right),\min_g\left(s_{f_g},\Upsilon_g\right)\right\rangle$, then

$$\bar{\bar{m}}^{-}\le 2-TLNPHM^{x,y}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)\le \bar{\bar{m}}^{+} \tag{20}$$

Proof. To prove this let us assume that,

$$\frac{\Delta^{-1}\left(s_{t_g},\Xi_g\right)}{h}=\bar{\bar{t}}_g,\frac{\Delta^{-1}\left(s_{i_g},\Psi_g\right)}{h}=\bar{\bar{i}}_g,\frac{\Delta^{-1}\left(s_{f_g},\Upsilon_g\right)}{h}=\bar{\bar{f}}_g,$$

$$\frac{\Delta^{-1}\left(s_{t_g^-},\Xi_g^-\right)}{h}=\bar{\bar{t}}_g^{-},\frac{\Delta^{-1}\left(s_{i_g^-},\Psi_g^-\right)}{h}=\bar{\bar{i}}_g^{-},\frac{\Delta^{-1}\left(s_{f_g^-},\Upsilon_g^-\right)}{h}=\bar{\bar{f}}_g^{-},$$

$$\frac{\Delta^{-1}\left(s_{t_g^+},\Xi_g^+\right)}{h}=\bar{\bar{t}}_g^{+},\frac{\Delta^{-1}\left(s_{i_g^+},\Psi_g^+\right)}{h}=\bar{\bar{i}}_g^{+},\frac{\Delta^{-1}\left(s_{f_g^+},\Upsilon_g^+\right)}{h}=\bar{\bar{f}}_g^{+}.$$

Since $\bar{\bar{m}}^{+}=\left\langle \max_g\left(s_{t_g},\Xi_g\right),\min_g\left(s_{i_g},\Psi_g\right),\min_g\left(s_{f_g},\Upsilon_g\right)\right\rangle$, $\bar{\bar{m}}^{-}=\left\langle \min_g\left(s_{t_g},\Xi_g\right),\max_g\left(s_{i_g},\Psi_g\right),\max_g\left(s_{f_g},\Upsilon_g\right)\right\rangle$. Then, there are $\bar{\bar{t}}^{-}\le \bar{\bar{t}}\left(\Gamma_g\right)\le \bar{\bar{t}}^{+}, \bar{\bar{i}}^{-}\le \bar{\bar{i}}\left(\Gamma_g\right)\le \bar{\bar{i}}^{+}, \bar{\bar{f}}^{-}\le \bar{\bar{f}}\left(\Gamma_g\right)\le \bar{\bar{f}}^{+}$ for all $g=1,2,....,p$. So, we have

$$\overline{\overline{t}}(\Gamma_g)=\Delta\left(h\left(1\Big/\left(1+\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{\overline{\overline{t}}_g}{1-\overline{\overline{t}}_g}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{\overline{\overline{t}}_q}{1-\overline{\overline{t}}_q}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)$$

$$\geq\Delta\left(h\left(1\Big/\left(1+\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{\overline{\overline{t}}_g^{-}}{1-\overline{\overline{t}}_g^{-}}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{\overline{\overline{t}}_q^{-}}{1-\overline{\overline{t}}_q^{-}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)=\overline{\overline{t}}^{-},$$

$$\overline{\overline{i}}(\Gamma_g)=\Delta\left(h\left(1-1\Big/\left(1+\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{1-\overline{\overline{i}}_g}{\overline{\overline{i}}_g}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{1-\overline{\overline{i}}_q}{\overline{\overline{i}}_q}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)$$

$$\leq\Delta\left(h\left(1-1\Big/\left(1+\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{1-\overline{\overline{i}}_g^{+}}{\overline{\overline{i}}_g^{+}}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{1-\overline{\overline{i}}_q^{+}}{\overline{\overline{i}}_q^{+}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)=\overline{\overline{i}}^{+},$$

$$\overline{\overline{f}}(\Gamma_g)=\Delta\left(h\left(1-1\Big/\left(1+\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{1-\overline{\overline{f}}_g}{\overline{\overline{f}}_g}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{1-\overline{\overline{f}}_q}{\overline{\overline{f}}_q}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)$$

$$\leq\Delta\left(h\left(1-1\Big/\left(1+\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{1-\overline{\overline{f}}_g^{+}}{\overline{\overline{f}}_g^{+}}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{1-\overline{\overline{f}}_q^{+}}{\overline{\overline{f}}_q^{+}}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)=\overline{\overline{f}}^{+}.$$

Then, there is the following comparison:

(1) For the expected value:

$$\overline{\overline{SR}}(\Gamma)=\Delta\left\{\frac{2h+\Delta^{-1}(s_t,\Xi)-\Delta^{-1}(s_i,\Psi)-\Delta^{-1}(s_f,\Upsilon)}{3h}\right\}$$

$$\geq\Delta\left\{\frac{2h+\Delta^{-1}(s_{t^-},\Xi^-)-\Delta^{-1}(s_{i^+},\Psi^+)-\Delta^{-1}(s_{f^+},\Upsilon^+)}{3h}\right\}$$

$$=\overline{\overline{SR}}\left(\left\langle(s_{t^-},\Xi)-\Delta^{-1}(s_{i^-},\Psi)-\Delta^{-1}(s_{f^-},\Upsilon)\right\rangle\right).$$

If $\overline{\overline{SR}}(\Gamma)>\overline{\overline{SR}}\left(\left\langle(s_{t^-},\Xi)-\Delta^{-1}(s_{i^-},\Psi)-\Delta^{-1}(s_{f^-},\Upsilon)\right\rangle\right)$,

then

$\left\langle(s_{t^-},\Xi)-\Delta^{-1}(s_{i^-},\Psi)-\Delta^{-1}(s_{f^-},\Upsilon)\right\rangle<2-TLNPHM^{x,y}(\Gamma_1,\Gamma_2,\ldots,\Gamma_p)$,

Else $\overline{\overline{SR}}(\Gamma)=\overline{\overline{SR}}\left(\left\langle(s_{t^-},\Xi)-\Delta^{-1}(s_{i^-},\Psi)-\Delta^{-1}(s_{f^-},\Upsilon)\right\rangle\right)$,

then we have the score function

(2)

$$\overline{\overline{AC}}(\Gamma)=\Delta\left\{\Delta^{-1}(s_t,\Xi)-\Delta^{-1}(s_f,\Upsilon)\right\}\geq\Delta\left\{\Delta^{-1}(s_{t^-},\Xi^-)-\Delta^{-1}(s_{f^+},\Upsilon^+)\right\}$$

$$=\overline{\overline{AC}}\left(\left\langle(s_{t^-},\Xi)-\Delta^{-1}(s_{i^-},\Psi)-\Delta^{-1}(s_{f^-},\Upsilon)\right\rangle\right).$$

If $\overline{\overline{AC}}(\Gamma)>\overline{\overline{AC}}\left(\left\langle(s_{t^-},\Xi)-\Delta^{-1}(s_{i^-},\Psi)-\Delta^{-1}(s_{f^-},\Upsilon)\right\rangle\right)$,

then

$\left\langle(s_{t^-},\Xi)-\Delta^{-1}(s_{i^-},\Psi)-\Delta^{-1}(s_{f^-},\Upsilon)\right\rangle<2-TLNPHM^{x,y}(\Gamma_1,\Gamma_2,\ldots,\Gamma_p)$,

Else

$\overline{\overline{AC}}(\Gamma)=\overline{\overline{AC}}\left(\left\langle(s_{t^-},\Xi)-\Delta^{-1}(s_{i^-},\Psi)-\Delta^{-1}(s_{f^-},\Upsilon)\right\rangle\right)$,

$=2-TLNPHM^{x,y}(\Gamma_1,\Gamma_2,\ldots,\Gamma_p)$

So, we have

$\overline{\overline{m}}^{-}\leq 2-TLNPHM^{x,y}(\Gamma_1,\Gamma_2,\ldots,\Gamma_p)$.

In a similar way, we can show that $2-TLNPHM^{x,y}(\Gamma_1,\Gamma_2,\ldots,\Gamma_p)\leq\overline{\overline{m}}^{+}$.

Hence we have

$\overline{\overline{m}}^{-}\leq 2-TLNPHM^{x,y}(\Gamma_1,\Gamma_2,\ldots,\Gamma_p)\leq\overline{\overline{m}}^{+}$.

In the following, we shall discuss some special cases with respect to the parameter parameters x *and* y.

(1) When $y\to 0,\Im>0$, we can have

$$2-TLNDPHM^{x,0}(\Gamma_1,\Gamma_2,\ldots,\Gamma_p)$$

$$=\left(\frac{2}{p^2+p}\sum_{g=1}^{p}\sum_{q=g}^{p}\left(\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_g\right)^{x}\otimes\left(\frac{p(1+T(\Gamma_q))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_q\right)^{y}\right)^{\frac{1}{x+y}}$$

$$=\left(\frac{2}{p^2+p}\sum_{g=1}^{p}\left((p+1-g)\left(\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_g\right)^{x}\right)\right)^{\frac{1}{x}}.$$

That is, the 2-TLDPHM operator degenerates into the 2-tuple linguistic neutrosophic descending Dombi power average operator.

(2) When $x\to 0,\Im>0$, we can have

$$2-TLNDPHM^{o,y}(\Gamma_1,\Gamma_2,\ldots,\Gamma_p)$$

$$=\left(\frac{2}{p^2+p}\sum_{g=1}^{p}\sum_{q=g}^{p}\left(\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_g\right)^{x}\otimes\left(\frac{p(1+T(\Gamma_q))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_q\right)^{y}\right)^{\frac{1}{x+y}}$$

$$=\left(\frac{2}{p^2+p}\sum_{g=1}^{p}\left((g)\left(\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_g\right)^{y}\right)\right)^{\frac{1}{y}}.$$

That is, the 2-TLDPHM operator degenerates into the 2-tuple linguistic neutrosophic ascending Dombi power average operator.

IEEE Access
Multidisciplinary ⁝ Rapid Review ⁝ Open Access Journal

(3) When $y \to 0, \Im > 0,$ and $Sup(\Gamma_g, \Gamma_q) = \beta(\beta \in [0,1])$ for all $g \neq q,$ then, we can have

$$2-TLNDPHM^{x,0}(\Gamma_1, \Gamma_2, ..., \Gamma_p)$$

$$= \left(\frac{2}{p^2+p}\sum_{g=1}^{p}\sum_{q=g}^{p}\left(\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_g\right)^x \otimes \left(\frac{p(1+T(\Gamma_q))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_q\right)^y\right)^{\frac{1}{x+y}}$$

$$= \left(\frac{2}{p^2+p}\sum_{g=1}^{p}\left((p+1-g)(\Gamma_g)^x\right)\right)^{\frac{1}{x}}.$$

That is, the 2-TLDPHM operator degenerates into the 2-tuple linguistic neutrosophic linear descending Dombi weighted average operator. Certainly, the weight vector of Γ_g^x is $(p, p-1, ..., 1)$.

(4) When $x \to 0, \Im > 0,$ and $Sup(\Gamma_g, \Gamma_q) = \beta(\beta \in [0,1])$ for all $g \neq q,$ then, we can have

$$2-TLNDPHM^{0,y}(\Gamma_1, \Gamma_2, ..., \Gamma_p)$$

$$= \left(\frac{2}{p^2+p}\sum_{g=1}^{p}\sum_{q=g}^{p}\left(\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_g\right)^x \otimes \left(\frac{p(1+T(\Gamma_q))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}\Gamma_q\right)^y\right)^{\frac{1}{x+y}}$$

$$= \left(\frac{2}{p^2+p}\sum_{g=1}^{p}\left((g)(\Gamma_g)^y\right)\right)^{\frac{1}{y}}.$$

That is, the 2-TLDPHM operator degenerates into the 2-tuple linguistic neutrosophic linear descending Dombi weighted average operator.

(5) When $x = y = 1, \Im > 0,$ and $Sup(\Gamma_g, \Gamma_q) = \beta(\beta \in [0,1])$ for all $g \neq q,$ then, we can have

$$2-TLNDPHM^{0,y}(\Gamma_1, \Gamma_2, ..., \Gamma_p) = \left(\frac{2}{p^2+p}\sum_{g=1}^{p}\sum_{q=g}^{p}\left((\Gamma_g)\otimes(\Gamma_q)\right)\right)^{\frac{1}{2}}.$$

That is, the 2-TLDPHM operator degenerates into the 2-tuple linguistic neutrosophic linear Dombi Heronian mean operator.

In the above developed 2-TLNDPHM operator, only power weight vector and the correlation between input arguments are taken under consideration and are not to consider the weight vector of the input arguments. Therefore, to remove this deficiency, we will propose it weighted form, that is 2-TPLNDWPHM operator.

Definition 11. Let $\Gamma_g\,(g = 1, 2, ..., p)$ be a group of 2-TLNNs, $x, y \geq 0, \overline{\overline{W}} = \left(\overline{\overline{w}}_1, \overline{\overline{w}}_2, ..., \overline{\overline{w}}_p\right)^T$ be the weight vector such that $\overline{\overline{w}}_g \in [0,1]$ and $\sum_{g=1}^{p}\overline{\overline{w}}_g = 1.$ Then, the 2-TLNNDWPHM operator is described as follows:

$$2-TLNDWPHM^{x,y}(\Gamma_1, \Gamma_2, ..., \Gamma_p)$$

$$= \left(\frac{2}{p^2+p}\sum_{g=1}^{p}\sum_{q=g}^{p}\left(\frac{p\overline{\overline{w}}_g(1+T(\Gamma_g))}{\sum_{r=1}^{p}\overline{\overline{w}}_t(1+T(\Gamma_r))}\Gamma_g\right)^x \otimes \left(\frac{p\overline{\overline{w}}_q(1+T(\Gamma_q))}{\sum_{r=1}^{p}\overline{\overline{w}}_t(1+T(\Gamma_r))}\Gamma_q\right)^y\right)^{\frac{1}{x+y}}. \tag{21}$$

Where $T(\Gamma_g) = \sum_{q=1, g\neq q}^{p} Sup(\Gamma_g, \Gamma_q), Sup(\Gamma_g, \Gamma_q) = 1 - \overline{\overline{D}}(\Gamma_g, \Gamma_q)$ is the support degree for Γ_g from Γ_q, which satisfy the following conditions: (1) $Sup(\Gamma_g, \Gamma_q) \in [0,1]$; (2) $Sup(\Gamma_g, \Gamma_q) = Supp(\Gamma_g, \Gamma_q)$; (3) $Sup(\Gamma_g, \Gamma_q) \geq Sup(\Gamma_r, \Gamma_s)$, if $\overline{\overline{D}}(\Gamma_g, \Gamma_q) < \overline{\overline{D}}(\Gamma_r, \Gamma_s)$, in which $\overline{\overline{D}}(\Gamma_g, \Gamma_q)$ is the distance measure between 2-TLNNs Γ_g and Γ_q defined in Definition (5).

In order, to represent Equation (21) in a simple form, we assume that

$$\Theta_g = \frac{\overline{\overline{w}}_g(1+T(\Gamma_g))}{\sum_{r=1}^{p}\overline{\overline{w}}_r(1+T(\Gamma_r))} \tag{22}$$

Therefore, Equation (21) takes the form

$$2-TLNDWPHM^{x,y}(\Gamma_1, \Gamma_2, ..., \Gamma_p)$$

$$= \left(\frac{2}{p^2+p}\sum_{g=1}^{p}\sum_{q=g}^{p}\left(p\Theta_g\Gamma_g\right)^x \otimes \left(\Theta_q\Gamma_q\right)^y\right)^{\frac{1}{x+y}}. \tag{23}$$

Theorem 4. Let $x, y \geq 0,$ and x, y do not take the value 0 at the same time, $\Gamma_g\,(g = 1, 2, ..., p)$ be a group of 2-TLNNs and let $\frac{\Delta^{-1}\left(s_{t_g}, \Xi_g\right)}{h} = \overline{\overline{t}}_g, \frac{\Delta^{-1}\left(s_{i_g}, \Psi_g\right)}{h} = \overline{\overline{i}}_g, \frac{\Delta^{-1}\left(s_{f_g}, \Upsilon_g\right)}{h} = \overline{\overline{f}}_g$. Then, the aggregated value utilizing Equation (21), is still a 2-TLNN, and

$$2-TLNDWPHM(\Gamma_1, \Gamma_2, ..., \Gamma_p)$$

$$= \left\langle \Delta\left(h\left(1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\times 1\Big/\left(\sum_{g=1, q=q}^{p} 1\Big/\left(\frac{x}{p\Theta_g\left(\frac{\overline{\overline{t}}_g}{1-\overline{\overline{t}}_g}\right)^{\Im}}+\frac{y}{p\Theta_q\left(\frac{\overline{\overline{t}}_q}{1-\overline{\overline{t}}_q}\right)^{\Im}}\right)\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\right.$$

$$\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\times 1\Big/\left(\sum_{g=1, q=g}^{p} 1\Big/\left(\frac{x}{p\Theta_g\left(\frac{1-\overline{\overline{i}}_g}{\overline{\overline{i}}_g}\right)^{\Im}}+\frac{y}{p\Theta_q\left(\frac{1-\overline{\overline{i}}_q}{\overline{\overline{i}}_q}\right)^{\Im}}\right)\right)\right)^{\frac{1}{\Im}}\right)\right)\right),$$

$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\times 1\Big/\left(\sum_{g=1, q=g}^{p} 1\Big/\left(\frac{x}{p\Theta_g\left(\frac{1-\overline{\overline{f}}_g}{\overline{\overline{f}}_g}\right)^{\Im}}+\frac{y}{p\Theta_q\left(\frac{1-\overline{\overline{f}}_q}{\overline{\overline{f}}_q}\right)^{\Im}}\right)\right)\right)^{\frac{1}{\Im}}\right)\right)\right)\right\rangle. \tag{24}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2925344, IEEE Access

Proof. Same is Theorem 1.

It is worthy to note that the 2-TLNDWPHM operator has only the property of boundedness and does not have the properties of idempotency and monotonicity.

B. The 2-TLNDPGHM Operator and 2-TLNDWPGHM operator

Definition 12. Let $\Gamma_g\,(g=1,2,\ldots,p)$ be a group of 2-TLNNs, $x,y\geq 0.$ Then, the 2-TLNNDPGHM operator is described as follows:

$$2-TLNDPGHM^{x,y}\left(\Gamma_1,\Gamma_2,\ldots,\Gamma_p\right)=\frac{1}{x+y}\left(\prod_{g=1}^{p}\prod_{q=g}^{p}\left(x\left(\Gamma_g\right)^{\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\oplus y\left(\Gamma_q\right)^{\frac{p(1+T(\Gamma_q))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\right)\right)^{\frac{2}{p^2+p}}. \tag{25}$$

Where $T\left(\Gamma_g\right)=\sum_{q=1,g\neq q}^{p}Sup\left(\Gamma_g,\Gamma_q\right)$,$Sup\left(\Gamma_g,\Gamma_q\right)=1-\overline{\overline{D}}\left(\Gamma_g,\Gamma_q\right)$ is the support degree for Γ_g from Γ_q, which satisfy the following conditions: (1) $Sup\left(\Gamma_g,\Gamma_q\right)\in[0,1]$; (2) $Sup\left(\Gamma_g,\Gamma_q\right)=Supp\left(\Gamma_g,\Gamma_q\right)$; (3) $Sup\left(\Gamma_g,\Gamma_q\right)\geq Sup\left(\Gamma_r,\Gamma_s\right)$, if $\overline{\overline{D}}\left(\Gamma_g,\Gamma_q\right)<\overline{\overline{D}}\left(\Gamma_r,\Gamma_s\right)$, in which $\overline{\overline{D}}\left(\Gamma_g,\Gamma_q\right)$ is the distance measure between 2-TLNNs Γ_g and Γ_q defined in Definition (5).

In order, to represent Equation (25) in a simple form, we assume that

$$\aleph_g=\frac{\left(1+T\left(\Gamma_g\right)\right)}{\sum_{r=1}^{p}\left(1+T\left(\Gamma_r\right)\right)} \tag{26}$$

Therefore, Equation (25) takes the form

$$2-TLNDPGHM^{x,y}\left(\Gamma_1,\Gamma_2,\ldots,\Gamma_p\right)=\frac{1}{x+y}\left(\prod_{g=1}^{p}\prod_{q=g}^{p}\left(x\left(\Gamma_g\right)^{p\aleph_g}\oplus y\left(\Gamma_q\right)^{p\aleph_q}\right)\right)^{\frac{2}{p^2+p}}. \tag{27}$$

Theorem 5. Let $x,y\geq 0,$ and x,y do not take the value 0 at the same time, $\Gamma_g(g=1,2,\ldots,p)$ be a group of 2-TLNNs and let $\frac{\Delta^{-1}\left(s_{t_g},\Xi_g\right)}{h}=\overline{\overline{t}}_g,\frac{\Delta^{-1}\left(s_{i_g},\Psi_g\right)}{h}=\overline{\overline{i}}_g,\frac{\Delta^{-1}\left(s_{f_g},\Upsilon_g\right)}{h}=\overline{\overline{f}}_g$. Then, the aggregated value utilizing Equation (25), is still a 2-TLNN, and

$$2-TLNDWPGHM\left(\Gamma_1,\Gamma_2,\ldots,\Gamma_p\right)$$
$$=\left\langle\Delta\left(h\left(1-1\middle/\left(1+\frac{p^2+p}{2(x+y)}\times 1\middle/\left(\sum_{\substack{g=1\\q=g}}^{p}1\middle/\left(\frac{x}{p\aleph_g\left(\frac{1-\overline{\overline{t}}_g}{\overline{\overline{t}}_g}\right)^{\backepsilon}}+\frac{y}{p\aleph_q\left(\frac{1-\overline{\overline{t}}_q}{\overline{\overline{t}}_q}\right)^{\backepsilon}}\right)\right)\right)^{\frac{1}{\backepsilon}}\right)\right)\right),\right.$$
$$\Delta\left(h\left(1\middle/\left(1+\frac{p^2+p}{2(x+y)}\times 1\middle/\left(\sum_{\substack{g=1,\\q=q}}^{p}1\middle/\left(\frac{x}{p\aleph_g\left(\frac{\overline{\overline{i}}_g}{1-\overline{\overline{i}}_g}\right)^{\backepsilon}}+\frac{y}{p\aleph_q\left(\frac{\overline{\overline{i}}_q}{1-\overline{\overline{i}}_q}\right)^{\backepsilon}}\right)\right)\right)^{\frac{1}{\backepsilon}}\right)\right),$$
$$\left.\Delta\left(h\left(1\middle/\left(1+\frac{p^2+p}{2(x+y)}\times 1\middle/\left(\sum_{\substack{g=1,\\q=q}}^{p}1\middle/\left(\frac{x}{p\aleph_g\left(\frac{\overline{\overline{f}}_g}{1-\overline{\overline{f}}_g}\right)^{\backepsilon}}+\frac{y}{p\aleph_q\left(\frac{\overline{\overline{f}}_q}{1-\overline{\overline{f}}_q}\right)^{\backepsilon}}\right)\right)\right)^{\frac{1}{\backepsilon}}\right)\right)\right\rangle. \tag{28}$$

Proof. According to operational laws, we have

$$\Gamma_g^{p\aleph_g}=\left\langle\Delta\left(h\left(1\middle/1+\left(p\aleph_g\left(\frac{1-\overline{\overline{t}}_g}{\overline{\overline{t}}_g}\right)^{\backepsilon}\right)^{\frac{1}{\backepsilon}}\right)\right),\right.$$
$$\left.\Delta\left(h\left(1-1\middle/1+\left(p\aleph_g\left(\frac{\overline{\overline{i}}_g}{1-\overline{\overline{i}}_g}\right)^{\backepsilon}\right)^{\frac{1}{\backepsilon}}\right)\right),\Delta\left(h\left(1-1\middle/1+\left(p\aleph_g\left(\frac{\overline{\overline{f}}_g}{1-\overline{\overline{f}}_g}\right)^{\backepsilon}\right)^{\frac{1}{\backepsilon}}\right)\right)\right\rangle,$$

and

$$\Gamma_q^{p\aleph_q}=\left\langle\Delta\left(h\left(1\middle/1+\left(p\aleph_q\left(\frac{1-\overline{\overline{t}}_q}{\overline{\overline{t}}_q}\right)^{\backepsilon}\right)^{\frac{1}{\backepsilon}}\right)\right),\right.$$
$$\left.\Delta\left(h\left(1-1\middle/1+\left(p\aleph_q\left(\frac{\overline{\overline{i}}_q}{1-\overline{\overline{i}}_q}\right)^{\backepsilon}\right)^{\frac{1}{\backepsilon}}\right)\right),\Delta\left(h\left(1-1\middle/1+\left(p\aleph_q\left(\frac{\overline{\overline{f}}_q}{1-\overline{\overline{f}}_q}\right)^{\backepsilon}\right)^{\frac{1}{\backepsilon}}\right)\right)\right\rangle,$$

Let

$$\overline{\overline{a}}_g=\frac{1-\overline{\overline{t}}_g}{\overline{\overline{t}}_g},\overline{\overline{a}}_q=\frac{1-\overline{\overline{t}}_q}{\overline{\overline{t}}_q},\overline{\overline{b}}_g=\frac{\overline{\overline{i}}_g}{1-\overline{\overline{i}}_g},\overline{\overline{b}}_q=\frac{\overline{\overline{i}}_q}{1-\overline{\overline{i}}_q},\overline{\overline{c}}_g=\frac{\overline{\overline{f}}_g}{1-\overline{\overline{f}}_g},\overline{\overline{c}}_q=\frac{\overline{\overline{f}}_q}{1-\overline{\overline{f}}_q}.$$

Then, we can obtain

$$\Gamma_g^{p\aleph_g}=\left\langle\Delta\left(h\left(1\middle/1+\left(p\aleph_g\right)^{\frac{1}{\backepsilon}}\overline{\overline{a}}_g\right)\right),\right.$$
$$\left.\Delta\left(h\left(1-1\middle/1+\left(p\aleph_g\right)^{\frac{1}{\backepsilon}}\overline{\overline{b}}_g\right)\right),\Delta\left(h\left(1-1\middle/1+\left(p\aleph_g\right)^{\frac{1}{\backepsilon}}\overline{\overline{c}}_g\right)\right)\right\rangle,$$

$$\Gamma_q^{p\aleph_q}=\left\langle\Delta\left(h\left(1\middle/1+\left(p\aleph_q\right)^{\frac{1}{\backepsilon}}\overline{\overline{a}}_q\right)\right),\right.$$
$$\left.\Delta\left(h\left(1-1\middle/1+\left(p\aleph_q\right)^{\frac{1}{\backepsilon}}\overline{\overline{b}}_q\right)\right),\Delta\left(h\left(1-1\middle/1+\left(p\aleph_q\right)^{\frac{1}{\backepsilon}}\overline{\overline{c}}_q\right)\right)\right\rangle,$$

and

$$x\Gamma_g^{p\aleph_g}=\left\langle\Delta\left(h\left(1-1\middle/1+x^{\frac{1}{\backepsilon}}\middle/\left(p\aleph_g\right)^{\frac{1}{\backepsilon}}\overline{\overline{a}}_g\right)\right),\right.$$
$$\left.\Delta\left(h\left(1\middle/1+x^{\frac{1}{\backepsilon}}\middle/\left(p\aleph_g\right)^{\frac{1}{\backepsilon}}\overline{\overline{b}}_g\right)\right),\Delta\left(h\left(1\middle/1+x^{\frac{1}{\backepsilon}}\middle/\left(p\aleph_g\right)^{\frac{1}{\backepsilon}}\overline{\overline{c}}_g\right)\right)\right\rangle,$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2925344, IEEE Access

$$y\Gamma_q^{p\aleph_q} = \left\langle \Delta\left(h\left(1-1\Big/1+y^{\frac{1}{\mathfrak{I}}}\Big/\left(p\aleph_q\right)^{\frac{1}{\mathfrak{I}}}\bar{\bar{a}}_q\right)\right),\right.$$
$$\left.\Delta\left(h\left(1\Big/1+y^{\frac{1}{\mathfrak{I}}}\Big/\left(p\aleph_q\right)^{\frac{1}{\mathfrak{I}}}\bar{\bar{b}}_q\right)\right), \Delta\left(h\left(1\Big/1+y^{\frac{1}{\mathfrak{I}}}\Big/\left(p\aleph_q\right)^{\frac{1}{\mathfrak{I}}}\bar{\bar{c}}_q\right)\right)\right\rangle,$$

Furthermore, we can have

$$x\Gamma_g^{p\aleph_g} \oplus_D x\Gamma_q^{p\aleph_q} = \left\langle \Delta\left(h\left(1-1\Big/1+\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\,\mathfrak{I}}}\right)^{\frac{1}{\mathfrak{I}}}\right)\right),\right.$$
$$\Delta\left(h\left(1\Big/1+\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\,\mathfrak{I}}}\right)^{\frac{1}{\mathfrak{I}}}\right)\right),$$
$$\left.\Delta\left(h\left(1\Big/1+\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\,\mathfrak{I}}}\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right\rangle,$$

and

$$\prod_{\substack{g=1,\\ q=g}}^{p} x\Gamma_g^{p\aleph_g} \oplus_D y\Gamma_q^{p\aleph_q}$$

$$= \left\langle \Delta\left(h\left(1\Big/\left(1+\left(\sum_{\substack{g=1,\\ q=g}}^{p}\left(\left(1-1+\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\,\mathfrak{I}}}\right)^{\frac{1}{\mathfrak{I}}}}\right)\Big/\left(1-\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\,\mathfrak{I}}}\right)^{\frac{1}{\mathfrak{I}}}}\right)\right)^{\mathfrak{I}}\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right),\right.$$
$$\Delta\left(h\left(1-1\Big/\left(1+\left(\sum_{\substack{g=1,\\ q=g}}^{p}\left(\left(\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\,\mathfrak{I}}}\right)^{\frac{1}{\mathfrak{I}}}}\right)\Big/\left(1-\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\,\mathfrak{I}}}\right)^{\frac{1}{\mathfrak{I}}}}\right)\right)^{\mathfrak{I}}\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right),$$
$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\sum_{\substack{g=1,\\ q=g}}^{p}\left(\left(\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\,\mathfrak{I}}}\right)^{\frac{1}{\mathfrak{I}}}}\right)\Big/\left(1-\frac{1}{1+\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\,\mathfrak{I}}}\right)^{\frac{1}{\mathfrak{I}}}}\right)\right)^{\mathfrak{I}}\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right)\right\rangle,$$

$$= \left\langle \Delta\left(h\left(1\Big/\left(1+\left(\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right),\right.$$
$$\Delta\left(h\left(1-1\Big/\left(1+\left(\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right),$$
$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right)\right\rangle.$$

So, we can have

$$\left(\prod_{\substack{g=1,\\ q=g}}^{p} x\Gamma_g^{p\aleph_g} \oplus_D y\Gamma_q^{p\aleph_q}\right)^{\frac{2}{p^2+p}}$$

$$= \left\langle \Delta\left(h\left(1\Big/\left(1+\left(\frac{2}{p^2+p}\left(\left(1-1\Big/\left(1+\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\Big/\left(1\Big/1+\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)^{\mathfrak{I}}\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right),\right.$$
$$\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{2}{p^2+p}\left(\left(1-1\Big/\left(1+\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\Big/\left(1-1+1\Big/1+\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)^{\mathfrak{I}}\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right),$$
$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{2}{p^2+p}\left(\left(1-1\Big/\left(1+\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\Big/\left(1-1+1\Big/1+\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)^{\mathfrak{I}}\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right)\right\rangle,$$

$$= \left\langle \Delta\left(h\left(1\Big/\left(1+\left(\frac{2}{p^2+p}\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right),\right.$$
$$\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{2}{p^2+p}\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right),$$
$$\left.\Delta\left(h\left(1-1\Big/\left(1+\left(\frac{2}{p^2+p}\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right)\right\rangle.$$

Then

$$\frac{1}{x+y}\left(\prod_{\substack{g=1,\\ q=g}}^{p} x\Gamma_g^{p\aleph_g} \oplus_D y\Gamma_q^{p\aleph_q}\right)^{\frac{2}{p^2+p}}$$

$$= \left\langle \Delta\left(h\left(1-1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{a}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{a}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right),\right.$$
$$\Delta\left(h\left(1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{b}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{b}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right), \quad (29)$$
$$\left.\Delta\left(h\left(1\Big/\left(1+\left(\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\ q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\bar{\bar{c}}_g^{\,\mathfrak{I}}}+\frac{y}{p\aleph_q\bar{\bar{c}}_q^{\,\mathfrak{I}}}\right)\right)^{\frac{1}{\mathfrak{I}}}\right)\right)\right)\right\rangle.$$

Now put

$$\bar{\bar{a}}_g = \frac{1-\bar{\bar{t}}_g}{\bar{\bar{t}}_g}, \bar{\bar{a}}_q = \frac{1-\bar{\bar{t}}_q}{\bar{\bar{t}}_q}, \bar{\bar{b}}_g = \frac{\bar{\bar{i}}_g}{1-\bar{\bar{i}}_g}, \bar{\bar{b}}_q = \frac{\bar{\bar{i}}_q}{1-\bar{\bar{i}}_q}, \bar{\bar{c}}_g = \frac{\bar{\bar{f}}_g}{1-\bar{\bar{f}}_g}, \bar{\bar{c}}_q = \frac{\bar{\bar{f}}_q}{1-\bar{\bar{f}}_q}$$

in Equation (29), we can have

$$=\left\langle \Delta\left(h\left(1-1\Big/\left(1+\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{1-\bar{\bar{t}}_g}{\bar{\bar{t}}_g}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{1-\bar{\bar{t}}_q}{\bar{\bar{t}}_q}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right),\right.$$

$$\Delta\left(h\left(1\Big/\left(1+\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{\bar{\bar{i}}_g}{1-\bar{\bar{i}}_g}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{\bar{\bar{i}}_q}{1-\bar{\bar{i}}_q}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right),$$

$$\left.\Delta\left(h\left(1\Big/\left(1+\frac{p^2+p}{2(x+y)}\sum_{\substack{g=1,\\q=g}}^{p}1\Big/\left(\frac{x}{p\aleph_g\left(\frac{\bar{\bar{f}}_g}{1-\bar{\bar{f}}_g}\right)^{\Im}}+\frac{y}{p\aleph_q\left(\frac{\bar{\bar{f}}_q}{1-\bar{\bar{f}}_q}\right)^{\Im}}\right)\right)^{\frac{1}{\Im}}\right)\right)\right\rangle.$$

This completes the proof of Theorem.

Theorem 6 (Idempotency). Let $\Gamma_g(g=1,2,...,p)$ be a group of 2-TLNNs, if all $\Gamma_g(g=1,2,...,p)$ are same, that is $\Gamma_g=\Gamma=\langle(s_t,\Xi),(s_i,\Psi),(s_f,\Upsilon)\rangle(g=1,2,...,p)$. Assume that $\frac{\Delta^{-1}\left(s_{t_g},\Xi_g\right)}{h}=\bar{\bar{t}}_g,\frac{\Delta^{-1}\left(s_{i_g},\Psi_g\right)}{h}=\bar{\bar{i}}_g,\frac{\Delta^{-1}\left(s_{f_g},\Upsilon_g\right)}{h}=\bar{\bar{f}}_g$, then

$$2-TLNPGHM\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)=\Gamma. \tag{30}$$

Theorem 7 (Boundedness). Let $\Gamma_g(g=1,2,...,p)$ be a group of 2-TLNNs. If $\bar{\bar{m}}^{-}=\left\langle\min_g\left(s_{t_g},\Xi_g\right),\max_g\left(s_{i_g},\Psi_g\right),\max_g\left(s_{f_g},\Upsilon_g\right)\right\rangle$ and $\bar{\bar{m}}^{+}=\left\langle\max_g\left(s_{t_g},\Xi_g\right),\min_g\left(s_{i_g},\Psi_g\right),\min_g\left(s_{f_g},\Upsilon_g\right)\right\rangle$, then

$$\bar{\bar{m}}^{-}\le 2-TLNPHM^{x,y}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)\le\bar{\bar{m}}^{+}. \tag{31}$$

By specifying different values to the parameters x and y, some particular cases of the 2-TLNDPGHM operator are described below:

(1) If $y\to 0,\Im>0$, then we can have

$$2-TLNDPGHM^{x,0}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)$$
$$=\frac{1}{x+y}\left(\prod_{g=1}^{p}\prod_{q=g}^{p}\left(x\left(\Gamma_g\right)^{\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\oplus y\left(\Gamma_q\right)^{\frac{p(1+T(\Gamma_q))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\right)\right)^{\frac{2}{p^2+p}}$$
$$=\frac{1}{x}\left(\prod_{g=1}^{p}\left(x\left(\Gamma_g\right)^{\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\right)^{(p+1-g)}\right)^{\frac{2}{p^2+p}}.$$

That is, the 2-TLNDPGHM operator degenerates into the 2-tuple linguistic neutrosophic Dombi descending power geometric average operator.

(2) If $x\to 0,\Im>0$, then we can have

$$2-TLNDPGHM^{0,y}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)$$
$$=\frac{1}{x+y}\left(\prod_{g=1}^{p}\prod_{q=g}^{p}\left(x\left(\Gamma_g\right)^{\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\oplus y\left(\Gamma_q\right)^{\frac{p(1+T(\Gamma_q))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\right)\right)^{\frac{2}{p^2+p}}$$
$$=\frac{1}{y}\left(\prod_{g=1}^{p}\left(y\left(\Gamma_g\right)^{\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\right)^{(g)}\right)^{\frac{2}{p^2+p}}.$$

That is, the 2-TLNDPGHM operator degenerates into the 2-tuple linguistic neutrosophic Dombi descending power geometric average operator.

(3) If $y\to 0,\Im>0,$ and $Sup\left(\Gamma_g,\Gamma_q\right)=\beta\left(\beta\in[0,1]\right)$ for all $g\neq q.$ Then, we can have

$$2-TLNDPGHM^{x,0}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)$$
$$=\frac{1}{x+y}\left(\prod_{g=1}^{p}\prod_{q=g}^{p}\left(x\left(\Gamma_g\right)^{\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\oplus y\left(\Gamma_q\right)^{\frac{p(1+T(\Gamma_q))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\right)\right)^{\frac{2}{p^2+p}}$$
$$=\frac{1}{x}\left(\prod_{g=1}^{p}\left(x\Gamma_g\right)^{(p+1-g)}\right)^{\frac{2}{p^2+p}}.$$

That is, the 2-TLNDPGHM operator degenerates into 2-tuple linguistic neutrosophic Dombi descending geometric average operator.

(4) If $x\to 0,\Im>0,$ and $Sup\left(\Gamma_g,\Gamma_q\right)=\beta\left(\beta\in[0,1]\right)$ for all $g\neq q.$ Then, we can have

$$2-TLNDPGHM^{x,0}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)$$
$$=\frac{1}{x+y}\left(\prod_{g=1}^{p}\prod_{q=g}^{p}\left(x\left(\Gamma_g\right)^{\frac{p(1+T(\Gamma_g))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\oplus y\left(\Gamma_q\right)^{\frac{p(1+T(\Gamma_q))}{\sum_{r=1}^{p}(1+T(\Gamma_r))}}\right)\right)^{\frac{2}{p^2+p}}$$
$$=\frac{1}{y}\left(\prod_{g=1}^{p}\left(y\Gamma_g\right)^{(g)}\right)^{\frac{2}{p^2+p}}.$$

That is, the 2-TLNDPGHM operator degenerates into 2-tuple linguistic neutrosophic Dombi ascending geometric average operator.

Similar to 2-TLNDPHM operator, the 2-TLNDPGHM operator have only power weight vector and the correlation between input arguments are taken under consideration and are not to consider the weight vector of the input arguments. Therefore, to remove this deficiency, we will propose its weighted form, that is 2-TPLNDWPGHM operator.

Definition 13. Let $\Gamma_g\,(g=1,2,...,p)$ be a group of 2-TLNNs, $x,y\geq 0.$ Then, the 2-TLNNDWPGHM operator is described as follows:

$$2-TLNDWPGHM^{x,y}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)$$

$$=\frac{1}{x+y}\left(\prod_{g=1}^{p}\prod_{q=g}^{p}\left(x\left(\Gamma_g\right)^{\frac{p\overline{\overline{w}}_g\left(1+T\left(\Gamma_g\right)\right)}{\sum_{r=1}^{p}\overline{\overline{w}}_r\left(1+T\left(\Gamma_r\right)\right)}}\oplus y\left(\Gamma_q\right)^{\frac{p\overline{\overline{w}}_g\left(1+T\left(\Gamma_q\right)\right)}{\sum_{r=1}^{p}\overline{\overline{w}}_r\left(1+T\left(\Gamma_r\right)\right)}}\right)\right)^{\frac{2}{p^2+p}}. \tag{32}$$

Where $T\left(\Gamma_g\right)=\sum_{q=1,g\neq q}^{p}Sup\left(\Gamma_g,\Gamma_q\right), Sup\left(\Gamma_g,\Gamma_q\right)=1-\overline{\overline{D}}\left(\Gamma_g,\Gamma_q\right)$ is the support degree for Γ_g from Γ_q, which satisfy the following conditions: (1) $Sup\left(\Gamma_g,\Gamma_q\right)\in[0,1]$; (2) $Sup\left(\Gamma_g,\Gamma_q\right)=Supp\left(\Gamma_g,\Gamma_q\right)$; (3) $Sup\left(\Gamma_g,\Gamma_q\right)\geq Sup\left(\Gamma_r,\Gamma_s\right)$, if $\overline{\overline{D}}\left(\Gamma_g,\Gamma_q\right)<\overline{\overline{D}}\left(\Gamma_r,\Gamma_s\right)$, in which $\overline{\overline{D}}\left(\Gamma_g,\Gamma_q\right)$ is the distance measure between 2-TLNNs Γ_g and Γ_q defined in Definition (5).

In order, to represent Equation (32) in a simple form, we assume that

$$\Theta_g=\frac{\overline{\overline{w}}_g\left(1+T\left(\Gamma_g\right)\right)}{\sum_{r=1}^{p}\overline{\overline{w}}_r\left(1+T\left(\Gamma_r\right)\right)} \tag{33}$$

Therefore, Equation (32) takes the form

$$2-TLNDWPGHM^{x,y}\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)$$

$$=\frac{1}{x+y}\left(\prod_{g=1}^{p}\prod_{q=g}^{p}\left(x\left(\Gamma_g\right)^{p\Theta_g}\oplus y\left(\Gamma_q\right)^{p\Theta_q}\right)\right)^{\frac{2}{p^2+p}}. \tag{34}$$

Theorem 8. Let $x,y\geq 0,$ and x,y do not take the value 0 at the same time, $\Gamma_g(g=1,2,...,p)$ be a group of 2-TLNNs and let $\frac{\Delta^{-1}\left(s_{t_g},\Xi_g\right)}{h}=\overline{\overline{t}}_g,\frac{\Delta^{-1}\left(s_{i_g},\Psi_g\right)}{h}=\overline{\overline{i}}_g,\frac{\Delta^{-1}\left(s_{f_g},\Upsilon_g\right)}{h}=\overline{\overline{f}}_g$. Then, the aggregated value utilizing Equation (32), is still a 2-TLNN, and

$$2-TLNDWPGHM\left(\Gamma_1,\Gamma_2,...,\Gamma_p\right)=$$

$$\left\langle\begin{array}{l}\Delta\left(h\left(1-1\middle/\left(1+\left(\frac{p^2+p}{2(x+y)}\times 1\middle/\left(\sum_{g=1,q=g}^{p}1\middle/\left(\frac{x}{p\Theta_g\left(\frac{1-\overline{\overline{t}}_g}{\overline{\overline{t}}_g}\right)^{\Im}}+\frac{y}{p\Theta_q\left(\frac{1-\overline{\overline{t}}_q}{\overline{\overline{t}}_q}\right)^{\Im}}\right)\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\\ \Delta\left(h\left(1\middle/\left(1+\left(\frac{p^2+p}{2(x+y)}\times 1\middle/\left(\sum_{g=1,q=q}^{p}1\middle/\left(\frac{x}{p\Theta_g\left(\frac{\overline{\overline{i}}_g}{1-\overline{\overline{i}}_g}\right)^{\Im}}+\frac{y}{p\Theta_q\left(\frac{\overline{\overline{i}}_q}{1-\overline{\overline{i}}_q}\right)^{\Im}}\right)\right)\right)^{\frac{1}{\Im}}\right)\right)\right),\\ \Delta\left(h\left(1\middle/\left(1+\left(\frac{p^2+p}{2(x+y)}\times 1\middle/\left(\sum_{g=1,q=q}^{p}1\middle/\left(\frac{x}{p\Theta_g\left(\frac{\overline{\overline{f}}_g}{1-\overline{\overline{f}}_g}\right)^{\Im}}+\frac{y}{p\Theta_q\left(\frac{\overline{\overline{f}}_q}{1-\overline{\overline{f}}_q}\right)^{\Im}}\right)\right)\right)^{\frac{1}{\Im}}\right)\right)\right)\end{array}\right\rangle. \tag{35}$$

Similar to 2-TLNDWPHM, the 2-TLNDWPGHM operator has only the property of boundedness and does not have the properties of idempotency and monotonicity.

V. An application of 2-TLNDWPHM and 2-TLNDWPGHM operator to group decision making

In this section, we pertains the afore-presented Dombi power Heronian aggregation operators to establish constructive approach for MAGDM under 2-TLNN environments. Let $\overline{\overline{AT}}=\left\{\overline{\overline{AT}}_1,\overline{\overline{AT}}_2,...,\overline{\overline{AT}}_m\right\}$ be the set of discrete alternatives, the set of attributes is expressed by $\overline{\overline{CT}}=\left\{\overline{\overline{CT}}_1,\overline{\overline{CT}}_2,...,\overline{\overline{CT}}_n\right\}$, the weight vector of the attributes is represented by $\overline{\overline{W}}=\left(\overline{\overline{w}}_1,\overline{\overline{w}}_2,...,\overline{\overline{w}}_n\right)^T$ such that $\overline{\overline{w}}_e\in[0,1],\sum_{e=1}^{n}\overline{\overline{w}}_e=1,$ and $\overline{\overline{DE}}=\left(\overline{\overline{de}}_1,\overline{\overline{de}}_2,...,\overline{\overline{de}}_a\right)$ denote the set of a decision makers, with weight vector expressed by $\varpi=\left(\varpi_1,\varpi_2,...,\varpi_a\right)^T$ such that $\varpi_b\in[0,1],\sum_{b=1}^{a}\varpi_b=1.$ Assume that $\overline{\overline{DT}}^b=\left(\Gamma_{ce}^b\right)_{m\times n}$ is the decision matrix, where $\Gamma_{ce}^b=\left\langle\left(s_{t_{ce}^b},\Xi_{ce}^b\right),\left(s_{i_{ce}^b},\Psi_{ce}^b\right),\left(s_{f_{ce}^b},\Upsilon_{ce}^b\right)\right\rangle$ takes the form of 2-TLNN, given by decision maker $\overline{\overline{de}}_b$ for alternative $\overline{\overline{AT}}_c$ with respect to the attribute $\overline{\overline{CT}}_e$.

Then, depending on real decision situations where the weight vector of both attributes and decision makers are completely known in advance. Therefore, in the following we present a MAGDM approach based on the developed 2-TLNDWPHM and 2-TLNDWPGHM operators. To do so, just follow the step below:

Step 1. Calculate the support degrees by the following formula:

$$Sup\left(\Gamma_{ce}^b,\Gamma_{ce}^l\right)=1-\overline{\overline{D}}_H\left(\Gamma_{ce}^b,\Gamma_{ce}^l\right),(b,l=1,2,...,a;c=1,2,...,m;e=1,2,...,n). \tag{36}$$

Which satisfy the axioms for support functions, $\overline{\overline{D}}_H\left(\Gamma_{ce}^b,\Gamma_{ce}^l\right)$ is the distance measure given in Definition (5).

Step 2. Determine the support degree $T\left(\Gamma_{ce}^b\right)$ that IFN Γ_{ce}^b receives from other 2-TLNNs $\Gamma_{ce}^l\,(l=1,2,...,a;l\neq b)$, where

$$T\left(\Gamma_{ce}^b\right)=\sum_{l=1,l\neq b}^{a}\sup\left(\Gamma_{ce}^b,\Gamma_{ce}^l\right). \tag{37}$$

Step 3. Utilize weights $\varpi_b\,(b=1,2,....,a)$ for decision makers $\overline{\overline{de}}_b$ to determine weights $\aleph_{ce}^b$ associated with the 2-TLNN Γ_{ce}^b,

$$\aleph_{ce}^b=\frac{\varpi_b\left(1+T\left(\Gamma_{ce}^b\right)\right)}{\sum_{b=1}^{a}\varpi_b\left(1+T\left(\Gamma_{ce}^b\right)\right)},(b=1,2,...,a). \tag{38}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2925344, IEEE Access

Where $\aleph_{ce}^{b} \geq 0$ and $\sum_{b=1}^{a} \aleph_{ce}^{b} = 1.$

Step 4. Aggregate all the individual decision matrices $\overline{\overline{DT}}^{b} = \left(\Gamma_{ce}^{b}\right)_{m \times n} (b = 1, 2, ..., a)$ into group decision matrix $\widetilde{DT} = \left(\Gamma_{ce}\right)_{m \times n}$ by utilizing 2-TLNDWPHM or 2-TLNDWPGHM operators, where

$$\Gamma_{ce}^{b} = 2-TLNDWPHM\left(\Gamma_{ce}^{1}, \Gamma_{ce}^{2}, ..., \Gamma_{ce}^{a}\right) \tag{39}$$

Or

$$\Gamma_{ce}^{b} = 2-TLNDWPGHM\left(\Gamma_{ce}^{1}, \Gamma_{ce}^{2}, ..., \Gamma_{ce}^{a}\right) \tag{40}$$

Step 5. Determine support degrees $Sup\left(\Gamma_{ce}, \Gamma_{cx}\right)$ by the following formula;

$$\begin{aligned} &Sup\left(\Gamma_{ce}, \Gamma_{cx}\right) = 1 - \overline{\overline{D}}_{H}\left(\Gamma_{ce}, \Gamma_{cx}\right); \\ &(c = 1, 2, ..., m, e = 1, 2, ..., n, e \neq x) \end{aligned} \tag{41}$$

where $\overline{\overline{D}}_{H}\left(\Gamma_{ce}, \Gamma_{cx}\right)$ is distance measure given in Definition(5).

Step 6. Determine the support degree $T\left(\Gamma_{ce}\right)$ that 2-TLNNs Γ_{ce} collects from other 2-TLNNs $\Gamma_{cx} (x = 1, 2, ..., n; e \neq x)$, where

$$T\left(\Gamma_{ce}\right) = \sum_{x=1, x \neq e}^{n} \overline{\overline{w}}_{x} Sup\left(\Gamma_{ce}, \Gamma_{cx}\right). \tag{42}$$

Step 7. Determine weighting vector $\Phi_{ce} (c = 1, 2, ..., m, e = 1, 2, ..., n)$ associated with Γ_{ce},

$$\Phi_{ce} = \frac{\overline{\overline{w}}_{e}\left(1 + T\left(\Gamma_{ce}\right)\right)}{\sum_{e=1}^{n} \overline{\overline{w}}_{e}\left(1 + T\left(\Gamma_{ce}\right)\right)}. \tag{43}$$

Step 8. Utilize 2-TLNDWPHM or 2-TLNNDWPGHM operators to aggregate all assessment values $\Gamma_{ce} (c = 1, 2, ..., m, e = 1, 2, ..., n)$ into overall assessment value $\Gamma_{c} (c = 1, 2, ..., m)$ corresponding to the alternatives $\overline{\overline{AL}}_{c} (c = 1, 2, ..., m)$:

$$\Gamma_{c} = 2-TLNDWPHM\left(\Gamma_{c1}, \Gamma_{c2}, ..., \Gamma_{cn}\right) \tag{44}$$

Or

$$\Gamma_{c} = 2-TLNDWPGHM\left(\Gamma_{c1}, \Gamma_{c2}, ..., \Gamma_{cn}\right) \tag{45}$$

Step 9. Determine the scores $\overline{\overline{SC}}\left(\overline{\overline{if}}_{d}\right)$ for the overall IFN of the alternatives $\overline{\overline{AL}}_{d} (d = 1, 2, ..., g)$ by utilizing Definition (3).

Step 10. Rank all alternatives $\overline{\overline{AL}}_{d} (d = 1, 2, ..., g)$ and select the optimal one (s) with the ranking order $\Gamma_{d} (d = 1, 2, ..., g)$.

A. Numerical Examples and Comparative analysis

The following example is adapted from [38], to show the validity and practicality of the developed aggregation operators.

Example 1. Let us assume that there are five potential construction engineering projects (alternatives) $\overline{\overline{AL}}_{b} (b = 1, 2, ..., 5)$ to be assess. These five potential alternatives are assessed by decision makers with respect to the following four attributes (1) the construction work environment denoted by $\overline{\overline{CT}}_{1}$;
(2) the construction site safety protection measure denoted by $\overline{\overline{CT}}_{2}$; (3) The safety management ability of the engineering projects management denoted by $\overline{\overline{CT}}_{3}$ and (4) the safety production responsibility system denoted by $\overline{\overline{CT}}_{4}$, with weight vector $(0.5, 0.3, 0.1, 0.1)^{T}$ and expert weight vector is $(0.2, 0.5, 0.3)^{T}$. The experts provide information in the form of 2-TLNNs, which are listed in Tables 1-3.

Table.1 The 2-TLN decision matrix $\overline{\overline{DT}}^{1}$

	$\overline{\overline{CT}}_{1}$	$\overline{\overline{CT}}_{2}$	$\overline{\overline{CT}}_{3}$	$\overline{\overline{CT}}_{4}$
$\overline{\overline{AL}}_{1}$	$\left\langle (s_4,0),(s_3,0),(s_2,0) \right\rangle$	$\left\langle (s_5,0),(s_3,0),(s_1,0) \right\rangle$	$\left\langle (s_4,0),(s_1,0),(s_2,0) \right\rangle$	$\left\langle (s_2,0),(s_3,0),(s_2,0) \right\rangle$
$\overline{\overline{AL}}_{2}$	$\left\langle (s_3,0),(s_2,0),(s_4,0) \right\rangle$	$\left\langle (s_4,0),(s_2,0),(s_2,0) \right\rangle$	$\left\langle (s_3,0),(s_2,0),(s_2,0) \right\rangle$	$\left\langle (s_4,0),(s_3,0),(s_3,0) \right\rangle$
$\overline{\overline{AL}}_{3}$	$\left\langle (s_5,0),(s_4,0),(s_3,0) \right\rangle$	$\left\langle (s_4,0),(s_4,0),(s_3,0) \right\rangle$	$\left\langle (s_2,0),(s_1,0),(s_2,0) \right\rangle$	$\left\langle (s_4,0),(s_2,0),(s_2,0) \right\rangle$
$\overline{\overline{AL}}_{4}$	$\left\langle (s_2,0),(s_1,0),(s_2,0) \right\rangle$	$\left\langle (s_5,0),(s_1,0),(s_2,0) \right\rangle$	$\left\langle (s_4,0),(s_3,0),(s_5,0) \right\rangle$	$\left\langle (s_3,0),(s_1,0),(s_1,0) \right\rangle$
$\overline{\overline{AL}}_{5}$	$\left\langle (s_4,0),(s_3,0),(s_1,0) \right\rangle$	$\left\langle (s_5,0),(s_2,0),(s_2,0) \right\rangle$	$\left\langle (s_3,0),(s_2,0),(s_1,0) \right\rangle$	$\left\langle (s_3,0),(s_2,0),(s_2,0) \right\rangle$

Table.2 The 2-TLN decision matrix $\overline{\overline{DT}}^{2}$

	$\overline{\overline{CT}}_{1}$	$\overline{\overline{CT}}_{2}$	$\overline{\overline{CT}}_{3}$	$\overline{\overline{CT}}_{4}$
$\overline{\overline{AL}}_{1}$	$\left\langle (s_3,0),(s_2,0),(s_3,0) \right\rangle$	$\left\langle (s_3,0),(s_3,0),(s_2,0) \right\rangle$	$\left\langle (s_3,0),(s_1,0),(s_2,0) \right\rangle$	$\left\langle (s_4,0),(s_1,0),(s_3,0) \right\rangle$
$\overline{\overline{AL}}_{2}$	$\left\langle (s_2,0),(s_3,0),(s_4,0) \right\rangle$	$\left\langle (s_3,0),(s_3,0),(s_3,0) \right\rangle$	$\left\langle (s_3,0),(s_4,0),(s_3,0) \right\rangle$	$\left\langle (s_2,0),(s_4,0),(s_4,0) \right\rangle$
$\overline{\overline{AL}}_{3}$	$\left\langle (s_2,0),(s_3,0),(s_3,0) \right\rangle$	$\left\langle (s_3,0),(s_2,0),(s_2,0) \right\rangle$	$\left\langle (s_2,0),(s_3,0),(s_1,0) \right\rangle$	$\left\langle (s_3,0),(s_2,0),(s_4,0) \right\rangle$
$\overline{\overline{AL}}_{4}$	$\left\langle (s_3,0),(s_2,0),(s_2,0) \right\rangle$	$\left\langle (s_2,0),(s_2,0),(s_3,0) \right\rangle$	$\left\langle (s_3,0),(s_4,0),(s_2,0) \right\rangle$	$\left\langle (s_3,0),(s_1,0),(s_2,0) \right\rangle$
$\overline{\overline{AL}}_{5}$	$\left\langle (s_3,0),(s_2,0),(s_1,0) \right\rangle$	$\left\langle (s_3,0),(s_4,0),(s_3,0) \right\rangle$	$\left\langle (s_4,0),(s_1,0),(s_1,0) \right\rangle$	$\left\langle (s_2,0),(s_3,0),(s_2,0) \right\rangle$

Table.3 The 2-TLN decision matrix $\overline{\overline{DT}}^{3}$

	$\overline{\overline{CT}}_{1}$	$\overline{\overline{CT}}_{2}$	$\overline{\overline{CT}}_{3}$	$\overline{\overline{CT}}_{4}$

$\overline{\overline{AL}}_1$	$\left\langle (s_3,0),(s_3,0),(s_1,0) \right\rangle$	$\left\langle (s_4,0),(s_2,0),(s_1,0) \right\rangle$	$\left\langle (s_4,0),(s_4,0),(s_3,0) \right\rangle$	$\left\langle (s_4,0),(s_1,0),(s_3,0) \right\rangle$
$\overline{\overline{AL}}_2$	$\left\langle (s_2,0),(s_3,0),(s_3,0) \right\rangle$	$\left\langle (s_4,0),(s_4,0),(s_4,0) \right\rangle$	$\left\langle (s_2,0),(s_4,0),(s_3,0) \right\rangle$	$\left\langle (s_2,0),(s_3,0),(s_4,0) \right\rangle$
$\overline{\overline{AL}}_3$	$\left\langle (s_2,0),(s_1,0),(s_2,0) \right\rangle$	$\left\langle (s_3,0),(s_2,0),(s_2,0) \right\rangle$	$\left\langle (s_3,0),(s_4,0),(s_5,0) \right\rangle$	$\left\langle (s_2,0),(s_4,0),(s_4,0) \right\rangle$
$\overline{\overline{AL}}_4$	$\left\langle (s_3,0),(s_1,0),(s_2,0) \right\rangle$	$\left\langle (s_2,0),(s_3,0),(s_2,0) \right\rangle$	$\left\langle (s_3,0),(s_4,0),(s_5,0) \right\rangle$	$\left\langle (s_5,0),(s_3,0),(s_2,0) \right\rangle$
$\overline{\overline{AL}}_5$	$\left\langle (s_3,0),(s_3,0),(s_2,0) \right\rangle$	$\left\langle (s_3,0),(s_2,0),(s_2,0) \right\rangle$	$\left\langle (s_3,0),(s_2,0),(s_3,0) \right\rangle$	$\left\langle (s_5,0),(s_3,0),(s_4,0) \right\rangle$

Step 1. Calculate the support degrees by utilizing formula (36). For simplicity we shall denote $Sup\left(\Gamma_{ce}^{b},\Gamma_{ce}^{l}\right)=S_{ce}^{bl},(b,l=1,2,3;c=1,...,5;e=1,...,4)$.

$S_{11}^{12}=S_{11}^{21}=0.8333,S_{11}^{13}=S_{11}^{31}=0.8889,S_{11}^{23}=S_{11}^{23}=0.8333,$
$S_{12}^{12}=S_{12}^{21}=0.8333,S_{12}^{13}=S_{12}^{31}=0.8889,S_{12}^{23}=S_{12}^{32}=0.8333;$
$S_{13}^{12}=S_{13}^{21}=0.9444,S_{13}^{13}=S_{13}^{31}=0.7778,S_{13}^{23}=S_{13}^{32}=0.7222,$
$S_{14}^{12}=S_{14}^{21}=0.7222,S_{14}^{13}=S_{14}^{31}=0.7222,S_{14}^{23}=S_{14}^{32}=1.000;$
$S_{21}^{12}=S_{21}^{21}=0.8889,S_{21}^{13}=S_{21}^{31}=0.8333,S_{21}^{23}=S_{21}^{32}=0.9444,$
$S_{22}^{12}=S_{22}^{21}=0.8333,S_{22}^{13}=S_{22}^{31}=0.7778,S_{22}^{23}=S_{22}^{32}=0.8333;$
$S_{23}^{12}=S_{23}^{21}=0.8333,S_{23}^{13}=S_{23}^{31}=0.7778,S_{23}^{23}=S_{23}^{32}=0.9444,$
$S_{24}^{12}=S_{24}^{21}=0.7778,S_{24}^{13}=S_{24}^{31}=0.8333,S_{24}^{23}=S_{24}^{32}=0.9444;$
$S_{31}^{12}=S_{31}^{21}=0.777778,S_{31}^{13}=S_{31}^{31}=0.6111,S_{31}^{23}=S_{31}^{23}=0.8333,$
$S_{32}^{12}=S_{32}^{21}=0.7778,S_{32}^{13}=S_{32}^{31}=0.7778,S_{32}^{23}=S_{32}^{32}=1.000;$
$S_{33}^{12}=S_{33}^{21}=0.833333,S_{33}^{13}=S_{33}^{31}=0.6111,S_{33}^{23}=S_{33}^{32}=0.6667,$
$S_{34}^{12}=S_{34}^{21}=0.8333,S_{34}^{13}=S_{34}^{31}=0.6667,S_{34}^{23}=S_{34}^{32}=0.8333;$
$S_{41}^{12}=S_{41}^{21}=0.8889,S_{41}^{13}=S_{41}^{31}=0.9444,S_{41}^{23}=S_{41}^{32}=0.9444,$
$S_{42}^{12}=S_{42}^{21}=0.722222,S_{42}^{13}=S_{42}^{31}=0.7222,S_{42}^{23}=S_{42}^{32}=0.8889;$
$S_{43}^{12}=S_{43}^{21}=0.7222,S_{43}^{13}=S_{43}^{31}=0.8889,S_{43}^{23}=S_{43}^{32}=0.8333,$
$S_{44}^{12}=S_{44}^{21}=0.9444,S_{44}^{13}=S_{44}^{31}=0.7222,S_{44}^{23}=S_{44}^{32}=0.7778;$
$S_{51}^{12}=S_{51}^{21}=0.8889,S_{51}^{13}=S_{51}^{31}=0.8889,S_{51}^{23}=S_{51}^{32}=0.8889,$
$S_{52}^{12}=S_{52}^{21}=0.722222,S_{52}^{13}=S_{52}^{31}=0.8889,S_{52}^{23}=S_{52}^{32}=0.8333;$
$S_{53}^{12}=S_{53}^{21}=0.8889,S_{53}^{13}=S_{53}^{31}=0.8889,S_{53}^{23}=S_{53}^{32}=0.7778,$
$S_{54}^{12}=S_{54}^{21}=0.8889,S_{54}^{13}=S_{54}^{31}=0.7222,S_{54}^{23}=S_{54}^{32}=0.7222;$

Step 2. Determine the support degree $T\left(\Gamma_{ce}^{b}\right)$ by utilizing formula (37). For simplicity, we shall denote $T\left(\Gamma_{ce}^{b}\right)$ by $T_{ce}^{b}\left(b=1,2,3;c=1,...,5;e=1,...,4\right)$.

$T_{11}^{1}=1.7222,T_{11}^{2}=1.6667,T_{11}^{3}=1.7222,T_{12}^{1}=1.7222,T_{12}^{2}=1.6667,T_{12}^{3}=1.7222;$
$T_{13}^{1}=1.7222,T_{13}^{2}=1.6667,T_{13}^{3}=1.5000,T_{14}^{1}=1.4444,T_{14}^{2}=1.7222,T_{14}^{3}=1.7222;$
$T_{21}^{1}=1.7222,T_{21}^{2}=1.8333,T_{21}^{3}=1.7778,T_{22}^{1}=1.6111,T_{22}^{2}=1.6667,T_{22}^{3}=1.6111;$
$T_{23}^{1}=1.6111,T_{23}^{2}=1.7778,T_{23}^{3}=1.7222,T_{24}^{1}=1.6111,T_{24}^{2}=1.7222,T_{24}^{3}=1.7222;$
$T_{31}^{1}=1.3889,T_{31}^{2}=1.6111,T_{31}^{3}=1.4444,T_{32}^{1}=1.5556,T_{32}^{2}=1.7778,T_{32}^{3}=1.7778;$
$T_{33}^{1}=1.444444,T_{33}^{2}=1.5000,T_{33}^{3}=1.2778,T_{34}^{1}=1.5000,T_{34}^{2}=1.6667,T_{34}^{3}=1.6667;$
$T_{41}^{1}=1.8333,T_{41}^{2}=1.8333,T_{41}^{3}=1.8889,T_{42}^{1}=1.4444,T_{42}^{2}=1.6111,T_{42}^{3}=1.6111;$
$T_{43}^{1}=1.6111,T_{43}^{2}=1.5556,T_{43}^{3}=1.7222,T_{44}^{1}=1.6667,T_{44}^{2}=1.7222,T_{44}^{3}=1.7222;$
$T_{51}^{1}=1.7778,T_{51}^{2}=1.7778,T_{51}^{3}=1.7778,T_{52}^{1}=1.6111,T_{52}^{2}=1.5556,T_{52}^{3}=1.7222;$
$T_{53}^{1}=1.7778,T_{53}^{2}=1.6667,T_{53}^{3}=1.6667,T_{54}^{1}=1.6111,T_{54}^{2}=1.6111,T_{54}^{3}=1.6111;$

Step 3. Utilize weights $\varpi_b\left(b=1,2,....,a\right)$ for decision makers $\overline{\overline{de}}_b$ to determine weights $\aleph_{ce}^{b}$ utilizing formula (38), we have

$\aleph_{11}^{1}=0.2021,\aleph_{11}^{2}=0.4949,\aleph_{11}^{3}=0.3031,\aleph_{12}^{1}=0.2021,$
$\aleph_{12}^{2}=0.4949,\aleph_{12}^{3}=0.3031;\aleph_{13}^{1}=0.2072,\aleph_{13}^{2}=0.5074,$
$\aleph_{13}^{3}=0.2854,\aleph_{14}^{1}=0.1833,\aleph_{14}^{2}=0.5104,\aleph_{14}^{3}=0.3063;$
$\aleph_{21}^{1}=0.1948,\aleph_{21}^{2}=0.5070,\aleph_{21}^{3}=0.2982,\aleph_{22}^{1}=0.1979,$
$\aleph_{22}^{2}=0.5053,\aleph_{22}^{3}=0.2968;\aleph_{23}^{1}=0.1915,\aleph_{23}^{2}=0.5092,$
$\aleph_{23}^{3}=0.2994,\aleph_{24}^{1}=0.1934,\aleph_{24}^{2}=0.5041,\aleph_{24}^{3}=0.3025;$
$\aleph_{31}^{1}=0.1898,\aleph_{31}^{2}=0.5188,\aleph_{31}^{3}=0.2914,\aleph_{32}^{1}=0.1870,$
$\aleph_{32}^{2}=0.5081,\aleph_{32}^{3}=0.3049;\aleph_{33}^{1}=0.2018,\aleph_{33}^{2}=0.5161,$
$\aleph_{33}^{3}=0.2821,\aleph_{34}^{1}=0.1899,\aleph_{34}^{2}=0.5063,\aleph_{34}^{3}=0.3038;$
$\aleph_{41}^{1}=0.1988,\aleph_{41}^{2}=0.4971,\aleph_{41}^{3}=0.3041,\aleph_{42}^{1}=0.1897,$
$\aleph_{42}^{2}=0.5065,\aleph_{42}^{3}=0.3039;\aleph_{43}^{1}=0.1996,\aleph_{43}^{2}=0.4883,$
$\aleph_{43}^{3}=0.3121,\aleph_{44}^{1}=0.1967,\aleph_{44}^{2}=0.5020,\aleph_{44}^{3}=0.3012;$
$\aleph_{51}^{1}=0.2000,\aleph_{51}^{2}=0.5000,\aleph_{51}^{3}=0.3000,\aleph_{52}^{1}=0.1996,$
$\aleph_{52}^{2}=0.4883,\aleph_{52}^{3}=0.3121;\aleph_{53}^{1}=0.2066,\aleph_{53}^{2}=0.4959,$
$\aleph_{53}^{3}=0.2975,\aleph_{54}^{1}=0.2000,\aleph_{54}^{2}=0.5000,\aleph_{54}^{3}=0.3000.$

Step 4. Aggregate all the individual decision matrices $\overline{\overline{DT}}^{b}=\left(\Gamma_{ce}^{b}\right)_{m\times n}(b=1,2,3;c=1,...,5;e=1,2....,4)$ into group decision matrix $\widetilde{DT}=\left(\Gamma_{ce}\right)_{m\times n}$ by utilizing formula (39) or (40), we have (assume $x=y=1,\Im=2$)

Table 4. Overall decision matrix utilizing 2-TLNDWPHM operator

	$\overline{\overline{CT}}_1$	$\overline{\overline{CT}}_2$
$\overline{\overline{AL}}_1$	$\left\langle (s_3,0.3121),(s_3,-0.4416),(s_2,-0.2546) \right\rangle$	$\left\langle (s_4,0.1659),(s_3,-0.3613),(s_1,0.2902) \right\rangle$
$\overline{\overline{AL}}_2$	$\left\langle (s_2,0.2801),(s_3,-0.3045),(s_4,-0.3296) \right\rangle$	$\left\langle (s_4,-0.3597),(s_3,-0.0893),(s_3,-0.0893) \right\rangle$
$\overline{\overline{AL}}_3$	$\left\langle (s_4,-0.2651),(s_2,-0.0778),(s_3,-0.3501) \right\rangle$	$\left\langle (s_3,0.2982),(s_2,0.3239),(s_2,0.2406) \right\rangle$
$\overline{\overline{AL}}_4$	$\left\langle (s_2,-0.2647),(s_1,0.3901),(s_2,0.0337) \right\rangle$	$\left\langle (s_4,-0.2654),(s_2,-0.2069),(s_2,0.3631) \right\rangle$
$\overline{\overline{AL}}_5$	$\left\langle (s_3,0.3096),(s_3,-0.4428),(s_1,0.2318) \right\rangle$	$\left\langle (s_4,-0.0301),(s_3,-0.4330),(s_2,0.3512) \right\rangle$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2925344, IEEE Access

Table 4. Overall decision matrix utilizing 2-TLNDWPHM operator

	$\overline{\overline{CT}}_3$	$\overline{\overline{CT}}_4$
$\overline{\overline{AL}}_1$	$\langle(s_4,-0.3581),(s_1,0.3112),(s_2,0.2905)\rangle$	$\langle(s_4,-0.3215),(s_1,0.2170),(s_3,-0.2948)\rangle$
$\overline{\overline{AL}}_2$	$\langle(s_3,-0.3265),(s_2,0.2579),(s_3,-0.3015)\rangle$	$\langle(s_3,-0.2540),(s_3,0.3221),(s_4,-0.3122)\rangle$
$\overline{\overline{AL}}_3$	$\langle(s_2,0.3217),(s_2,0.1066),(s_2,-0.4061)\rangle$	$\langle(s_3,0.0781),(s_2,0.4413),(s_3,0.2604)\rangle$
$\overline{\overline{AL}}_4$	$\langle(s_3,0.3115),(s_4,-0.2868),(s_3,-0.0090)\rangle$	$\langle(s_4,0.1354),(s_1,0.3029),(s_2,-0.3720)\rangle$
$\overline{\overline{AL}}_5$	$\langle(s_3,0.4424),(s_1,0.4666),(s_3,0.2970)\rangle$	$\langle(s_4,0.0490),(s_3,-0.3096),(s_2,0.4329)\rangle$

Table 5. Overall decision matrix utilizing 2-TLNDWPGHM operator

	$\overline{\overline{CT}}_1$	$\overline{\overline{CT}}_2$
$\overline{\overline{AL}}_1$	$\langle(s_3,0.2654),(s_3,-0.3807),(s_2,0.2226)\rangle$	$\langle(s_3,0.3613),(s_3,-0.3211),(s_1,0.4161)\rangle$
$\overline{\overline{AL}}_2$	$\langle(s_2,0.2434),(s_3,-0.2657),(s_4,-0.2991)\rangle$	$\langle(s_4,-0.4152),(s_3,0.2068),(s_3,0.2068)\rangle$
$\overline{\overline{AL}}_3$	$\langle(s_2,0.3580),(s_3,-0.0182),(s_3,-0.3261)\rangle$	$\langle(s_3,0.2628),(s_3,-0.2657),(s_2,0.2741)\rangle$
$\overline{\overline{AL}}_4$	$\langle(s_3,-0.2587),(s_1,0.4168),(s_2,-0.0334)\rangle$	$\langle(s_2,0.3539),(s_2,0.1742),(s_2,0.4146)\rangle$
$\overline{\overline{AL}}_5$	$\langle(s_3,0.2659),(s_3,-0.3839),(s_1,0.3244)\rangle$	$\langle(s_3,0.3685),(s_3,0.1333),(s_2,0.4107)\rangle$

Table 5. Overall decision matrix utilizing 2-TLNDWPHM operator

	$\overline{\overline{CT}}_3$	$\overline{\overline{CT}}_4$
$\overline{\overline{AL}}_1$	$\langle(s_4,-0.4167),(s_3,-0.3578),(s_2,0.3254)\rangle$	$\langle(s_3,0.2725),(s_1,0.6568),(s_3,-0.2619)\rangle$
$\overline{\overline{AL}}_2$	$\langle(s_3,-0.3557),(s_4,-0.3280),(s_3,-0.2649)\rangle$	$\langle(s_2,-0.3282),(s_3,0.4432),(s_4,-0.2421)\rangle$
$\overline{\overline{AL}}_3$	$\langle(s_2,0.2928),(s_3,0.1344),(s_4,-0.0620)\rangle$	$\langle(s_3,-0.2141),(s_3,0.0877),(s_4,-0.3258)\rangle$
$\overline{\overline{AL}}_4$	$\langle(s_3,0.2627),(s_4,-0.2405),(s_5,-0.4560)\rangle$	$\langle(s_3,0.4863),(s_2,0.1564),(s_2,-0.2633)\rangle$
$\overline{\overline{AL}}_5$	$\langle(s_3,0.3799),(s_2,-0.3948),(s_2,-0.1612)\rangle$	$\langle(s_3,-0.2028),(s_3,-0.2659),(s_3,0.0921)\rangle$

Step 5. Calculate the support degrees of Table4, by utilizing formula (41). For simplicity we shall denote $Sup(\Gamma_{ce},\Gamma_{ce}) = S_c^{ce}, (c=1,...,5; e=1,...,4)$.

$S_1^{12}=S_1^{21}=0.9228, S_1^{13}=S_1^{31}=0.8821, S_1^{14}=S_1^{41}=0.8518,$
$S_1^{23}=S_1^{32}=0.8416, S_1^{24}=S_1^{42}=0.8153, S_1^{34}=S_1^{43}=0.9697;$
$S_2^{12}=S_2^{21}=0.8703, S_2^{13}=S_2^{31}=0.8929, S_2^{14}=S_2^{41}=0.9383,$
$S_2^{23}=S_2^{32}=0.9152, S_2^{24}=S_2^{42}=0.8843, S_2^{34}=S_2^{43}=0.9374;$
$S_3^{12}=S_3^{21}=0.9307, S_3^{13}=S_3^{31}=0.8526, S_3^{14}=S_3^{41}=0.9008,$
$S_3^{23}=S_3^{32}=0.8977, S_3^{24}=S_3^{42}=0.9246, S_3^{34}=S_3^{43}=0.8468;$
$S_4^{12}=S_4^{21}=0.9038, S_4^{13}=S_4^{31}=0.7857, S_4^{14}=S_4^{41}=0.8948,$
$S_4^{23}=S_4^{32}=0.8349, S_4^{24}=S_4^{42}=0.9097, S_4^{34}=S_4^{43}=0.7446;$
$S_5^{12}=S_5^{21}=0.9006, S_5^{13}=S_5^{31}=0.9284, S_5^{14}=S_5^{41}=0.8848,$
$S_5^{23}=S_5^{32}=0.8510, S_5^{24}=S_5^{42}=0.9842, S_5^{34}=S_5^{43}=0.8352;$

or

Calculate the support degrees of Table 5, by utilizing formula (41). For simplicity we shall denote

$S_1^{12}=S_1^{21}=0.9261, S_1^{13}=S_1^{31}=0.9754, S_1^{14}=S_1^{41}=0.9175,$
$S_1^{23}=S_1^{32}=0.9393, S_1^{24}=S_1^{42}=0.8444, S_1^{34}=S_1^{43}=0.9051;$
$S_2^{12}=S_2^{21}=0.8718, S_2^{13}=S_2^{31}=0.8720, S_2^{14}=S_2^{41}=0.9527,$
$S_2^{23}=S_2^{32}=0.8957, S_2^{24}=S_2^{42}=0.8864, S_2^{34}=S_2^{43}=0.9129;$
$S_3^{12}=S_3^{21}=0.9138, S_3^{13}=S_3^{31}=0.9177, S_3^{14}=S_3^{41}=0.9168,$
$S_3^{23}=S_3^{32}=0.8314, S_3^{24}=S_3^{42}=0.8858, S_3^{34}=S_3^{43}=0.9456;$
$S_4^{12}=S_4^{21}=0.9115, S_4^{13}=S_4^{31}=0.6977, S_4^{14}=S_4^{41}=0.9221,$
$S_4^{23}=S_4^{32}=0.7431, S_4^{24}=S_4^{42}=0.8811, S_4^{34}=S_4^{43}=0.7252;$
$S_5^{12}=S_5^{21}=0.9052, S_5^{13}=S_5^{31}=0.9089, S_5^{14}=S_5^{41}=0.8794,$
$S_5^{23}=S_5^{32}=0.8827, S_5^{24}=S_5^{42}=0.9185, S_5^{34}=S_5^{43}=0.8455;$

Step 6. Determine the support degree $T(\Gamma_{ce})$ by utilizing formula (42)

$T_{11}=2.6567, T_{12}=2.5797, T_{13}=2.6934, T_{14}=2.6368, T_{21}=2.7015,$
$T_{22}=2.6698, T_{23}=2.7456, T_{24}=2.7601; T_{31}=2.6840, T_{32}=2.7530,$
$T_{33}=2.5971, T_{34}=2.6721, T_{41}=2.5844, T_{42}=2.6484, T_{43}=2.3653,$
$T_{44}=2.5491; T_{51}=2.7138, T_{52}=2.7358, T_{53}=2.6146, T_{54}=2.7042.$

Or

Determine the support degree $T(\Gamma_{ce})$ by utilizing formula (42)

$T_{11}=2.8189, T_{12}=2.7097, T_{13}=2.8197, T_{14}=2.6669, T_{21}=2.6965,$
$T_{22}=2.6539, T_{23}=2.6806, T_{24}=2.7521; T_{31}=2.7482, T_{32}=2.6311,$
$T_{33}=2.6947, T_{34}=2.7482, T_{41}=2.5313, T_{42}=2.5357, T_{43}=2.1660,$
$T_{44}=2.5284; T_{51}=2.6936, T_{52}=2.7064, T_{53}=2.6372, T_{54}=2.6434.$

Step 7. Determine weighting vector Φ_{ce} by utilizing formula (43),

$\Phi_{11}=0.5029, \Phi_{12}=0.2954, \Phi_{13}=0.1016, \Phi_{14}=0.1000, \Phi_{21}=0.4999,$
$\Phi_{22}=0.2974, \Phi_{23}=0.1012, \Phi_{24}=0.1016; \Phi_{31}=0.4985, \Phi_{32}=0.3047,$
$\Phi_{33}=0.0974, \Phi_{34}=0.0994, \Phi_{41}=0.5009, \Phi_{42}=0.3059, \Phi_{43}=0.0941,$
$\Phi_{44}=0.0992; \Phi_{51}=0.5006, \Phi_{52}=0.3021, \Phi_{53}=0.0974, \Phi_{54}=0.0999.$

Or

Determine weighting vector Φ_{ce} by utilizing formula (43),

$\Phi_{11}=0.5063, \Phi_{12}=0.2951, \Phi_{13}=0.1013, \Phi_{14}=0.0972, \Phi_{21}=0.5012,$
$\Phi_{22}=0.2973, \Phi_{23}=0.0998, \Phi_{24}=0.1017; \Phi_{31}=0.5055, \Phi_{32}=0.2938,$
$\Phi_{33}=0.0996, \Phi_{34}=0.1011, \Phi_{41}=0.1430, \Phi_{42}=0.3034, \Phi_{43}=0.0906,$
$\Phi_{44}=0.1009; \Phi_{51}=0.5009, \Phi_{52}=0.3016, \Phi_{53}=0.0987, \Phi_{54}=0.0988.$

Step 8. Utilize 2-TLNDWPHM or 2-TLNNDWPGHM operators given in formula (44) or formula (45) to aggregate all assessment values (assume $x=y=1, \Im=2$)

$\overline{\overline{AL}}_1 = \langle(s_4,-0.3978),(s_2,0.0053),(s_2,-0.1452)\rangle;$

$\overline{\overline{AL}}_2 = \langle(s_3,-0.2250),(s_3,0.0835),(s_3,0.3512)\rangle;$

$\overline{\overline{AL}}_3 = \langle (s_3, 0.1907), (s_2, 0.2747), (s_2, 0.4501) \rangle$;
$\overline{\overline{AL}}_4 = \langle (s_3, 0.3506), (s_2, -0.2546), (s_2, 0.2681) \rangle$;
$\overline{\overline{AL}}_5 = \langle (s_4, -0.4224), (s_2, 0.4077), (s_2, -0.2722) \rangle$.

or

$\overline{\overline{AL}}_1 = \langle (s_4, -0.4206), (s_2, 0.3839), (s_2, 0.0258) \rangle$;
$\overline{\overline{AL}}_2 = \langle (s_3, -0.2382), (s_3, 0.0972), (s_3, 0.3152) \rangle$;
$\overline{\overline{AL}}_3 = \langle (s_3, -0.2307), (s_3, -0.1970), (s_3, -0.0111) \rangle$;
$\overline{\overline{AL}}_4 = \langle (s_3, 0.2394), (s_2, 0.0406), (s_3, -0.3743) \rangle$;
$\overline{\overline{AL}}_5 = \langle (s_3, 0.3430), (s_3, -0.4474), (s_2, -0.0295) \rangle$.

Step 9. Calculate the score values utilizing Definition (3), we have

$\overline{\overline{SR}}(\overline{\overline{AL}}_1) = 0.6523, \overline{\overline{SR}}(\overline{\overline{AL}}_2) = 0.4634, \overline{\overline{SR}}(\overline{\overline{AL}}_3) = 0.5814,$
$\overline{\overline{SR}}(\overline{\overline{AL}}_4) = 0.6298, \overline{\overline{SR}}(\overline{\overline{AL}}_5) = 0.6357.$

Calculate the score values utilizing Definition (3), we have

$\overline{\overline{SR}}(\overline{\overline{AL}}_1) = 0.6205, \overline{\overline{SR}}(\overline{\overline{AL}}_2) = 0.4639, \overline{\overline{SR}}(\overline{\overline{AL}}_3) = 0.4987,$
$\overline{\overline{SR}}(\overline{\overline{AL}}_4) = 0.5874, \overline{\overline{SR}}(\overline{\overline{AL}}_5) = 0.6011.$

Step 10. Rank all the alternatives and select the best one according to their score values.

$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_3.$

or

$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_3.$

$\overline{\overline{AL}}_1$ is the best one while the worst one is $\overline{\overline{AL}}_3$.

VI. Discussion

In the following, we will further analyze the effect of the parameters x, y and $\Im$ on the final ranking result of Example 1. Then we can adopt the different values of x and y in step 4 and step 8, while the value $\Im$ is fix. The results are given in Table 6 and Table 7. Moreover, the effect of general parameter $\Im$, is shown in Table 8 and Table 9, while the parameters x, y are fix.

From Table 6 and Table 7, we can notice that the ranking orders are different for different values of the parameters x, y. However, the best alternative $\overline{\overline{AL}}_1$ or $\overline{\overline{AL}}_5$. From Table 6 and Table 7, we can also notice that, when the values of the parameter x or y increases, the score values increases utilizing 2-TLNDWPHM operator, while the score values decreases utilizing2-TLNDWPGHM operator. Generally, for computational simplicity one may select $x = y = 1, \ or \ x = y = \frac{1}{2}$ according to the actual need of decision making problems.

Table 6. Effect of parameter x and y on ranking result utilizing 2-TLNDWPHM operator

Parameter values	Score values	Ranking orders
$x=1, y=2, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6537, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4574, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5764, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6273, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6325.$	$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$
$x=3, y=5, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6530, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4585, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5772, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6275, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6328.$	$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$
$x=2, y=7, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6570, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4560, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5757, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6284, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6332.$	$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$
$x=6, y=19, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6563, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4561, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5756, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6280, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6329.$	$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$
$x=14, y=30, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6540, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4571, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5761, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6273, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6324.$	$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$
$x=2, y=100, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6754, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4631, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5926, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6476, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6526.$	$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$
$x=50, y=2, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6897, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.5290, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.6451, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6745, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6976.$	$\overline{\overline{AL}}_5 > \overline{\overline{AL}}_1 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$
$x=35, y=6, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6685, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4998, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.6142, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6526, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6669.$	$\overline{\overline{AL}}_5 > \overline{\overline{AL}}_1 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$
$x=80, y=4, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6869, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.5256, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.6409, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6713, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6937.$	$\overline{\overline{AL}}_5 > \overline{\overline{AL}}_1 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$

Table 7. Effect of parameter x and y on decision result utilizing 2-TLNDWPGHM operator

Parameter values	Score values	Ranking orders
$x=1, y=2, \Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6252, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4663, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5037, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5810, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5998.$	$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$

IEEE Access Multidisciplinary ⁝ Rapid Review ⁝ Open Access Journal

$x=3, y=5,$ $\Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6245, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4662, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5029, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5832, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6006.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=2, y=7,$ $\Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6251, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4643, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5042, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5718, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5951.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=6, y=19,$ $\Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6254, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4649, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5043, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5737, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5961.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=14, y=30,$ $\Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6253, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4662, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.5039, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5800, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5994.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=2, y=100,$ $\Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6082, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4388, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.4828, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5177, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5575.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=50, y=2,$ $\Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.5583, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4136, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.4440, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5533, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5620.$	$\overline{\overline{AL}}_5>\overline{\overline{AL}}_1>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=35, y=6,$ $\Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.5868, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4367, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.4683, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5771, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5835.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=80, y=4,$ $\Im=2$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.5618, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4163, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.4470, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5569, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5650.$	$\overline{\overline{AL}}_5>\overline{\overline{AL}}_1>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$

Table 8. Effect of parameter $\Im$ on decision result 2-TLNDWPHM operator

Parameter values	Score values	Ranking orders
$x=1, y=2,$ $\Im=3$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.6855, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4851, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.6316, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.6667, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.6713.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=1, y=2,$ $\Im=5$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.7326, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.5291, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.7017, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.7181, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.7226.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=1, y=2,$ $\Im=9$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.7795, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.5840, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.7632, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.7668, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.7734.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=1, y=2,$ $\Im=15$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.8025, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.6176, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.7934, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.7954, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.7991.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=1, y=2,$ $\Im=30$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.8184, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.6425, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.8142, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.8152, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.8169.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=1, y=2,$ $\Im=100$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.8290, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.6595, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.8278, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.8281, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.8285.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=1, y=2,$ $\Im=200$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.8312, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.6631, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.8306, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.8307, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.8310.$	$\overline{\overline{AL}}_5>\overline{\overline{AL}}_1>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$

Table 9. Effect of parameter $\Im$ on decision result 2-TLNDWPGHM operator

Parameter values	Score values	Ranking orders
$x=1, y=2,$ $\Im=3$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.5940, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4411, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.4610, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.5084, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5644.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2.$
$x=1, y=2,$ $\Im=5$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.5531, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.4105, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.4008, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.4233, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.5123.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_2>\overline{\overline{AL}}_3.$
$x=1, y=2,$ $\Im=9$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.5146, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.3794, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.3443, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.3570, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.4478.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_2>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3.$
$x=1, y=2,$ $\Im=15$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.4875, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.3609, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.3163, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.3240, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.4019.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_2>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3.$
$x=1, y=2,$ $\Im=30$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.4656, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.3470, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.2964, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.3003, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.3670.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_2>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3.$
$x=1, y=2,$ $\Im=100$	$\overline{\overline{SR}}(\overline{\overline{AL}}_1)=0.4507, \overline{\overline{SR}}(\overline{\overline{AL}}_2)=0.3374, \overline{\overline{SR}}(\overline{\overline{AL}}_3)=0.2833, \overline{\overline{SR}}(\overline{\overline{AL}}_4)=0.2844, \overline{\overline{SR}}(\overline{\overline{AL}}_5)=0.3433.$	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_2>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3.$

$x=1, y=2$, $\Im=200$	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.4476, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.3354$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.2805, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.2811$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.3383$.	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_2>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3$.

From Table 8 and Table 9, we can notice that the ranking orders are different for different values of the parameters $\Im$. However, the best alternative $\overline{\overline{AL}}_1$ or $\overline{\overline{AL}}_5$. From Table 8 and Table 9, we can also notice that, when the values of the parameter $\Im$ increases, the score values increases utilizing 2-TLNDWPHM operator, while the score values decreases utilizing2-TLNDWPGHM operator. So, one may select the parameter value according to the actual need of decision making problem.

A. Compare with existing methods

In order to confirm the efficacy of the developed approach and describe its advantages, we can compare our developed method with some existing methods.

B. Validity of the developed method

In order to confirm the validity of the developed approach, we can utilize some existing methods to solve the same example. Since the developed approach is based on the combination of PA, HM operators and Dombi operations. So, we can utilize the methods in which the interrelationships between two input arguments are considered. Therefore, the reference methods of comparison are 2-TLNNWBM, 2-TLNNWGBM operators and 2-TLNHM, 2-TLNDHM operators. The score values and ranking orders of the above example by solving these two methods and the developed method as given in Table 10. From Table 10, we can notice that the ranking order obtained by the existing methods is the same as that obtained from the proposed approach. This shows the developed approach is valid.

Table 10. The score values and ranking orders obtained from different methods

Approach	Score values	Ranking order
2-TLNNWBM [34] ($p=q=1$)	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.6298, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.4648$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.5642, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.6145$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.6243$.	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2$.
2-TLNNWGBM [34] ($p=q=1$)	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.6259, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.4606$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.5622, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.6080$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.6198$.	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2$.
2-TLNWHM[42] ($k=2$)	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.9013, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.8395$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.8751, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.8895$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.8962$.	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2$.
2-TLNWDHM[42] ($k=2$)	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.2062, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.1327$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.1718, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.1946$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.2005$.	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2$.
Proposed 2-TLNDWPHM operator	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.6523, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.4634$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.5814, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.6298$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.6357$.	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2$.
Proposed 2-TLNDW PGHM	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.6205, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.4639$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.4987, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.5874$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.6011$.	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2$.

From Table 10, we can see that the ranking order obtained from the proposed method based on developed aggregation operator and the methods developed Wang et al. [34], Wu et al. [38] are same. This shows the validity of the proposed method. Yet, it cannot manifest the advantages of the developed method due to same ranking results.

Further, in the following we will show the advantages of the developed method.

C. The advantages of the developed method

(1) The developed method is based on the 2-TLNDWPHM operator and the method presented by Wei [34] is based on 2-TLNNWBM operator. Both the methods have the characteristics of considering interrelationship among two input arguments and the only difference between them is that the developed aggregation operators also remove the effect of awkward data which may be too low or too high. In order to show this advantage, we give the following example.

Example 2. We can only change some data in the Example 1. We slightly change the value of alternative $\overline{\overline{AL}}_1$ with respect to the attribute $\overline{\overline{CT}}_4$. That is the value $\left\langle (s_4,0),(s_1,0),(s_3,0)\right\rangle$ is changed to $\left\langle (s_3,0),(s_2,0),(s_4,0)\right\rangle$ and the score values and ranking order are given in Table 11.

Table 11. The score values and ranking orders obtained from different methods

Approach	Score values	Ranking order
2-LNNWBM [34]	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.6215, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.4648$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.5642, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.6145$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.6243$.	$\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2$.
2-TLNNWGBM [34]	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.6178, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.4606$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.5622, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.6080$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.6198$.	$\overline{\overline{AL}}_5>\overline{\overline{AL}}_1>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2$.
Proposed Method 2-TLNDWPHM operator	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.6392, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.4629$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.5814, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.6298$, $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.6357$.	$\overline{\overline{AL}}_1>\overline{\overline{AL}}_5>\overline{\overline{AL}}_4>\overline{\overline{AL}}_3>\overline{\overline{AL}}_2$.

Proposed Method 2-TLNDWPGHM	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.6056, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.4639,$ $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.4987, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.5874,$ $\overline{\overline{SR}}\left(\overline{\overline{AL}}_5\right)=0.6011.$	$\overline{\overline{AL}}_1 > \overline{\overline{AL}}_5 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2.$

From Table 11, we can notice that when we slightly change the value of the alternative $\overline{\overline{AL}}_1$ with respect to the attribute $\overline{\overline{CT}}_4$ in Table 2, then the ranking order obtained from the proposed method remain the same, while that acquired from the method developed by Wang et al.[34] is totally different. The best alternative remains the same in the proposed approach while utilizing the Wang et el. [34] approach based on 2-TLNNWBM and 2-TLNNWGBM, the best alternative is $\overline{\overline{AL}}_5$. The main reason behind these different ranking orders is that, the aggregation operators developed by Wang et al. [34] just only consider the interrelationship among input arguments and does not have the capacity of removing the bad impact of awkward data on final ranking result. While, the proposed approach is based on the proposed aggregation operators have the property of removing the effect of awkward data and consider the interrelationship among input arguments. The proposed aggregation operators are based on Dombi operational laws which have a general parameter, that makes the decision process more flexible. So the developed aggregation operator in this article is more general and practical to be used in solving MAGDM problems.

(2) Compare with the approach based on Hamy mean operator

To compare the developed approach with that of Hamy mean operator proposed by Wu et al. [38], we take another Example adapted from [12]. The Hamy mean operator proposed by Wu et al. [38] can also consider the interrelationship among input arguments.

Example 3. Let there is an investment company who wants to invest some money in the available four companies as a group of alternatives $\overline{\overline{AL}}_b\,(b=1,2,...,4)$. These four companies are respectively, a car company denoted by $\overline{\overline{AL}}_1$, a food company denoted by $\overline{\overline{AL}}_2$, , a computer company denoted by $\overline{\overline{AL}}_3$ and an arm company denoted by $\overline{\overline{AL}}_4$. These four potential alternatives are assessed by decision makers with respect to the following three attributes (1) the risk denoted by $\overline{\overline{CT}}_1$; (2) the growth denoted by $\overline{\overline{CT}}_2$; and (3) The environmental impact denoted by $\overline{\overline{CT}}_3$ with weight vector $(0.4,0.2,0.4)^T$. The assessment information is provided in the form of 2-TLNNs and is given in Table 12.

Table.12. The 2-TLN decision matrix

	$\overline{\overline{CT}}_1$	$\overline{\overline{CT}}_2$	$\overline{\overline{CT}}_3$
$\overline{\overline{AL}}_1$	$\langle (s_2,0),(s_4,0),(s_4,0) \rangle$	$\langle (s_4,0),(s_1,0),(s_3,0) \rangle$	$\langle (s_2,0),(s_4,0),(s_4,0) \rangle$
$\overline{\overline{AL}}_2$	$\langle (s_4,0),(s_3,0),(s_3,0) \rangle$	$\langle (s_2,0),(s_3,0),(s_2,0) \rangle$	$\langle (s_4,0),(s_1,0),(s_2,0) \rangle$
$\overline{\overline{AL}}_3$	$\langle (s_5,0),(s_1,0),(s_3,0) \rangle$	$\langle (s_3,0),(s_2,0),(s_2,0) \rangle$	$\langle (s_2,0),(s_4,0),(s_2,0) \rangle$
$\overline{\overline{AL}}_4$	$\langle (s_3,0),(s_5,0),(s_1,0) \rangle$	$\langle (s_3,0),(s_1,0),(s_2,0) \rangle$	$\langle (s_3,0),(s_1,0),(s_2,0) \rangle$

The score values and ranking results obtained by the proposed aggregation operators and the 2-TLNWHM operator, 2-TLNWDHM operator are given in Table 13.

From Table 13, one can notice that the ranking order obtained from the developed aggregation operators and that of obtained by 2-TLNWHM operator, and 2-TLNWDHM operator are totally different. From the proposed aggregation operator the best alternative is $\overline{\overline{AL}}_3$, while the worst one is $\overline{\overline{AL}}_1$, and from the 2-TLNWHM operator or 2-TLNWDHM operator proposed in Wu et al. [38], the best alternative is $\overline{\overline{AL}}_4$, while the worst one remain the same. The main reason behind different ranking order is that the both the aggregation operators can consider the interrelationship between input arguments, but the developed aggregation operator have two more characteristics. It can remove the effect of awkward data and proposed aggregation operators are based on Dombi operational laws, which have a general parameter that makes the information aggregation process more flexible. Therefore the developed aggregation operators are more flexible and general to be used in solving MAGDM problems.

Table 13. The score values and ranking orders obtained from different methods

Approach	Score values	Ranking order
2-LNNWHM [38]	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.7337, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.7917,$ $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.8367, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.8406.$	$\overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2 > \overline{\overline{AL}}_1.$
2-TLNNWDHM [38]	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.1691, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.2082,$ $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.2695, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.2868.$	$\overline{\overline{AL}}_4 > \overline{\overline{AL}}_3 > \overline{\overline{AL}}_2 > \overline{\overline{AL}}_1.$
Proposed Method 2-TLNDWPHM operator	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.5228, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.5246,$ $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.7052, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.6902.$	$\overline{\overline{AL}}_3 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_2 > \overline{\overline{AL}}_1.$
Proposed Method 2-TLNDWPGHM	$\overline{\overline{SR}}\left(\overline{\overline{AL}}_1\right)=0.3678, \overline{\overline{SR}}\left(\overline{\overline{AL}}_2\right)=0.4755,$ $\overline{\overline{SR}}\left(\overline{\overline{AL}}_3\right)=0.4998, \overline{\overline{SR}}\left(\overline{\overline{AL}}_4\right)=0.4921.$	$\overline{\overline{AL}}_3 > \overline{\overline{AL}}_4 > \overline{\overline{AL}}_2 > \overline{\overline{AL}}_1.$

VII CONCLUSION

IEEE Access
Multidisciplinary ¦ Rapid Review ¦ Open Access Journal

In this article firstly, we proposed some new operational laws for 2-TLNNs based on Dombi T-norm and Dombi T-conorm. Secondly, we proposed some new aggregation operators on these operational laws such as 2-tuple linguistic neutrosophic Dombi power Heronian mean operator, 2-tuple linguistic neutrosophic Dombi weighted power Heronian mean operator, 2-tuple linguistic neutrosophic Dombi power geometric Heronian mean operator and 2-tuple linguistic neutrosophic Dombi weighted power geometric Heronian mean operator. We also discussed it properties and few special cases with respect to parameters. Furthermore, we developed an algorithm for solving MAGDM problems under 2-tuple linguistic neutrosophic environment. We also show the advantages of the developed MAGDM approaches by comparing with some existing MAGDM approaches. The main advantages of the developed aggregation operators are The developed aggregation operators are based on Dombi operational laws, which consists of general parameter, that makes the information aggregation process more flexible. The developed aggregation operators have two characteristics at a time, firstly, it can vanish the effect of awkward data by taking the advantage of PA operator, Secondly, it can consider the interrelationship among the input arguments by taking the advantages of HM operator. For these reasons the developed MAGDM method based on these developed aggregation operator is more general and reasonable.

In future research, we will extend power Heronian mean operators to some new extension such as 2-tuple linguistic cubic neutrosophic, 2-tuple linguistic Double valued neutrosophic and so on. At the same time, we also research on some applications in energy and supply chain management.

REFERENCES

[1] K. TAtanassov. "Intuitionistic fuzzy sets." *Fuzzy sets and Systems*, vol.20, no.1 pp. 87-96, 1986.

[2] L. A. Zadeh “Fuzzy sets”, *Inform Control.*; vol. 8, no.3, pp. 338–356, 1965.

[3] Yager, R. R. (2013, June). Pythagorean fuzzy subsets. In *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)* (pp. 57-61). IEEE 2013.

[4] Q. Wu, W. Lin, L. Zhou, Y. Chen & H. Chen, Enhancing multiple attribute group decision making flexibility based on information fusion technique and hesitant Pythagorean fuzzy sets. *Computers & Industrial Engineering*, *127*, 954-970, 2019.

[5] F. Smarandache. “Neutrosophy: Neutrosophic Probability, Set, and Logic” American Research Press: Rehoboth, DE, USA, 1998.

[6] F. Smarandache. “Neutrosophic set-a generalization of the intuitionistic fuzzy set”, *International Journal of Pure and Applied Mathematics*, vol. 24,no.3, 287, 2005.

[7] H. Wang, F. Smarandache, Y. Zhang, & R. Sunderraman, “Single valued neutrosophic sets”, Proceedings of 10th 476 International Conference on Fuzzy Theory and Technology, Salt Lake City, 477 Utah.

[8] H. Wang, P. Madiraju, Y. Zhang & R. Sunderraman. “Interval neutrosophic sets and logic Theory and applications in computing”, Hexis, Phoenix.

[9] J. Ye. “A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets”. *Journal of Intelligent & Fuzzy Systems*, vol.26, no.5,pp. 2459-2466, 2014.

[10] H. Y. Zhang, J. Q., Wang & X. H. Chen. “Interval neutrosophic sets and their application in multi-criteria decision making problems.” *The Scientific World Journal*, 2014.

[11] J. J. Peng, J. Q., Wang, J. Wang, ,H. Y. Zhang, & X. H. Chen, “Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems”. *International journal of systems science*, vol.47, no.10, pp-2342-2358, 2016.

[12] P. Liu, T. Mahmood, and Q. Khan. "Group decision making based on power Heronian aggregation operators under linguistic neutrosophic environment." *International Journal of Fuzzy Systems*, vol.20.no.3: pp-970-985,2018.

[13] P. Liu, and Y. Wang. "Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean." *Neural Computing and Applications* vol. 25.no.7-8 ,pp-2001-2010, 2014.

[14] J. Q. Wang, Y. Yang, and L. Li. "Multi-criteria decision-making method based on single-valued neutrosophic linguistic Maclaurin symmetric mean operators." *Neural Computing and Applications*, vol.30, no.5, pp-1529-1547, 2018.

[15] P. Liu and L. Shi. "Some neutrosophic uncertain linguistic number Heronian mean operators and their application to multi-attribute group decision making." *Neural Computing and Applications* vol.28,no.5, pp-1079-1093, 2017.

[16] Z. Xu, and R. R. Yager. "Some geometric aggregation operators based on intuitionistic fuzzy sets." *International journal of general systems*, vol.35.vol.4, pp-417-433, 2006.

[17] Z. Xu,. "Intuitionistic fuzzy aggregation operators." *IEEE Transactions on fuzzy systems*, vol.15,no.6, pp-1179-1187, 2007.

[18] H. X. Sun, H. X., Yang, J. Z. Wu & Y. Ouyang, “Interval neutrosophic numbers Choquet integral operator for multi-criteria decision making.” *Journal of Intelligent & Fuzzy Systems*, vol.28, no.6, pp-2443-2455, 2015.

[19] P. Liu, & G. Tang, “Some power generalized aggregation operators based on the interval neutrosophic sets and their application to decision making.” *Journal of Intelligent & Fuzzy Systems*, vol.30, no.5, pp-2517-2528, 2016.

[20] J. J. Peng, , J.Q. Wang, X. H., Wu, J., Wang, & X. H. Chen, “Multi-valued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems”. *International Journal of Computational Intelligence Systems*, vol.8. no.2, pp-345-363, 2015.

[21] R. R. Yager, “The power average operator.” *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, vol.31,no.6, pp-724-731, 2001.

[22] X, H. Wu, J. Q., Wang, J. J., Peng, & X. H. Chen,. “Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems”. *International Journal of Fuzzy Systems*, vol.18,no.6, pp-1104-1116, 2016.

[23] C. Bonferroni, .” Sulle medie multiple di potenze”. *Bollettino dell’Unione Matematica Italiana*, vol.5, no.(3-4), pp-267-270, 1950.

[24] S. Sykora “Mathematical means and averages: generalized Heronian means,” Sykora S. Stan’s Library.2009.

[25] L. A. Zadeh, ‘‘The concept of a linguistic variable and its application to approximate reasoning—I,’’ Inf. Sci., vol. 8, no. 3, pp. 199–249, 1975.

[26] F. Herrera and L. Martinez, ‘‘A 2-tuple fuzzy linguistic representation model for computing with words,’’ IEEE Trans. Fuzzy Syst., vol. 8, no. 6, pp. 746–752, Dec. 2000.

[27] F. Herrera and L. Martinez, ‘‘An approach for combining linguistic and numerical information based on the 2-tuple fuzzy linguistic representation model in decision-making,’’ Int. J. Uncertainty Fuzziness Knowl.-Based Syst., vol. 8, no. 5, pp. 539–562, 2000.

[28] Q. Wu, F. Wang, L. Zhou & H. Chen. Method of multiple attribute group decision making based on 2-dimension interval type-2 fuzzy aggregation operators with multi-granularity linguistic information. *International Journal of Fuzzy Systems*, *19*(6), 1880-1903, 2017.

[29] Q. Wu P. Wu, Y. Zhou, L. Zhou, H. Chen & X. Ma. Some 2-tuple linguistic generalized power aggregation operators and their applications to multiple attribute group decision making. *Journal of Intelligent & Fuzzy Systems*, *29*(1), 423-436. 2015.

[30] P. Xiao, Q. Wu, H. Li, L. Zhou, Z. Tao & J. Liu, Novel Hesitant Fuzzy Linguistic Multi-Attribute Group Decision Making Method Based on Improved Supplementary Regulation and Operational Laws. *IEEE Access*. 2019.

[31]P. Liu and S. Chen, ‘‘Multiattribute group decision making based on intuitionistic 2-tuple linguistic information,’’ Inf. Sci., vols. 430–431, pp. 599–619, Mar. 2018.

[32] H. Liao, L. Jiang, Z. Xu, J. Xu, and F. Herrera, ‘‘A linear programming method for multiple criteria decision making with probabilistic linguistic information,’’ Inf. Sci., vols. 415–416, pp. 341–355, Nov. 2017.

[33] P. Liu and Y. Wang, ‘‘Multiple attribute group decision making methods based on intuitionistic linguistic power generalized aggregation operators,’’ Appl. Soft Comput., vol. 17, pp. 90–104, Apr. 2014.

[34] J. Wang, G. Wei, and W. Yu. “Models for Green Supplier Selection with Some 2-Tuple Linguistic Neutrosophic Number Bonferroni Mean Operators.”*Symmetry*, vol.10,no.5, 131,2018.

[35] Y. Zhang, H. Ma, B. Liu, & J. Liu,” Group decision making with 2-tuple intuitionistic fuzzy linguistic preference relations.” *Soft Computing*, vol.16, no.8, pp-1439-1446, 2012.

[36] J. Wang, , G. Wei, and M. Lu. “TODIM method for multiple attribute group decision making under 2-tuple linguistic neutrosophic environment.” *Symmetry* vol.10.no.10 ,486, 2018.

[37] W. Jie, H. Gao, and G. Wei. “Some 2-tuple linguistic neutrosophic number Muirhead mean operators and their applications to multiple attribute decision making.” *Journal of Experimental & Theoretical Artificial Intelligence* pp-1-31, 2018.

[38] S. Wu, J., Wang, G., Wei, G., & Y. Wei,. “Research on construction engineering project risk assessment with some 2-tuple linguistic neutrosophic hamy mean operators”, Sustainability vol.10 no.5, 1536,2018.

[39] Q. Wu, P. Wu, L. Zhou, H. Chen & X. Guan, Some new Hamacher aggregation operators under single-valued neutrosophic 2-tuple linguistic environment and their applications to multi-attribute group decision making. *Computers & Industrial Engineering*, *116*, 144-162, 2018.

[40] J. Dombi, “A general class of fuzzy operators, the De-Morgan class of fuzzy operators and fuzziness measures induced by fuzzy operators.” *Fuzzy sets and systems*, vol.8,no. 2, pp-149-163,1982.

[41] G. Wei, and Y. Wei. “Some single-valued neutrosophic Dombi prioritized weighted aggregation operators in multiple attribute decision making.” *Journal of Intelligent & Fuzzy Systems*Preprint: pp-1-13,2018.

[42] L. Wu, G., Wei, H., Gao, & Y.,Wei, “Some interval-valued intoitionistic fuzzy 22 ombi hamy mean operators and their application for evaluating the elderly tourism service quality in tourism destination”. *Mathematics*, vol. 6, no.12, 294, 2018.

[43] Q. Khan, P. Liu, T. Mahmood, F. Smarandache, & K. Ullah, “Some Interval Neutrosophic Dombi Power Bonferroni Mean Operators and Their Application in Multi–Attribute Decision–Making.” *Symmetry*, vol. 10. No.10, 459, 2018.

[44] P. Liu, J. Liu, and S. Chen. “Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making.” *Journal of the Operational Research Society* vol.69.no1,pp- 1-24,2018.

[45] S. Lilian, and J. Ye. “Dombi aggregation operators of neutrosophic cubic sets for multiple attribute decision-making.” *Algorithms* vol.11.no. 3, 29,2018.

[46] L. Xueping, and J. Ye. "Dombi aggregation operators of linguistic cubic variables for multiple attribute decision making." *Information* vol.9,no.8 : 188, 2018.